Incentives in Competitive Search Equilibrium and Wage Rigidity*

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Abstract

In this paper we derive competitive search equilibrium when workers have private information regarding effort and/or "type". Wage contracts are used to enhance efficiency. We then investigate the effects of economy-wide shocks on the unemployment- and vacancy rates.

In the standard competitive search equilibrium, the planner trades off a high wage (or the rents associated with employment) to employed workers and a high exit rate from unemployment. Asymmetric information brings in an additional effect: Worker rents ease the constraints imposed by the workers' private information and thereby enhance efficiency. We derive a modified Hosios rule determining the allocation of resources. When the information problems are more severe, fewer resources are used to create vacancies.

Shocks to the economy may change the productivity-enhancing value of worker rents, and this influences the responsiveness of the wage- and unemployment rate. We find that asymmetric information reduces the responsiveness of the unemployment rate to changes in

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matching technology. However, it increases the responsiveness of the unemployment rate to changes in the deterministic part of the production function. The responsiveness of the unemployment rate to changes in the information structure may be large even if the changes in expected productivity are small.

1 Introduction

In this paper we derive the competitive search equilibrium when workers have private information regarding effort and /or a match-specific "type". We then investigate how private information influences the responsiveness of wages and unemployment to aggregate shocks.

In any search market, the resource constraint implies that there exists a trade-off between high wages and a high exit rate from unemployment. In competitive search equilibrium, a market maker optimally balances this trade-off. The resulting wage, or equivalently, employment rent (expected difference in future income when employed and unemployed) ensures that the agents on both sides of the market have the correct incentives to enter the market and search for a trading partner.

In a pure contractual setting with asymmetric information, rents play a different role. In a standard principal-agent model, where the agent has private information about his type, this information makes him better off, he receives information rents. The stronger incentives the principal gives the agent to exert effort, the higher will this information rent typically be. The principal thus faces a trade-off between rent extraction and effort provision, and chooses the wage contract so as to optimally balance these two considerations.

Our model combines the principal agent model and the competitive search model. When the market maker trades off employment rents and a high exit rate from unemployment, he or she will take into account the fact that employment rent eases the constraints imposed by workers' private information and thereby enhances efficiency. We derive a modified Hosios rule determining the constrained efficient allocation of resources. When the value of relaxing the private information constraints are large, employment rents are large while few resources are used to create new jobs.

Recent studies by Shimer (2004, 2005a,) have demonstrated a series of empirical regularities of the business cycle that seem to fit poorly with the standard matching model of the labor market. He finds that the fluctuations in the unemployment rate predicted by the model as a response to the observed productivity shocks are much smaller than the fluctuations in the unemployment rate that are actually observed. The reason is that wages are too flexible, and thereby absorb too much of the shock. He also finds that a low job creation rate underlies the high unemployment rate during a the recession, not a high job destruction rate. Similar findings are reported in Hall (2005).

A seemingly robust result for our model is that a negative productivity shock, which reduces the productivity of all matches with the same amount, tightens the constraints imposed by worker private information. It follows that employment rents become more important relative to creating new jobs, and as a result the wages become less responsive and unemployment more responsive such shocks than in the standard search model. We interpret such a shock as an increase in input prices (oil prices). If, in addition, worker effort is more crucial after a negative shock (for instance because effort and other inputs with higher prices are substitutes), this will further exacerbate the responsiveness of the unemployment rate. The same may be true if a negative shock is associated with (or caused by) more private information to workers.

On the other hand, private information may actually stabilize the unemployment rate for other kinds of macroeconomic shocks. After a negative shock to the matching technology, private information will dampen the responsiveness of the unemployment rate.

Two papers that are closely related to ours are Shimer (2004) and Shimer and Wright (2004). Shimer (2004) suggests that private information may increase the volatility of the unemployment rate. Shimer and Wright (2004) construct a model where firms (not workers) have private information about productivity and workers have private information about effort. They study how private information may distort trade, and show that this increases the unemployment rate. They do not analyze how the allocation of rents between workers and firms may influence these distortions.

Another related model is developed in Faig and Jerez (2004). They analyze trade in the retail market when buyers have private information about their willingness to pay for a product. They find that in competitive search equilibrium there are too many buyers relative to their full information benchmark. However, they do not study the effects of shocks. From a technical point of view, their model is similar to the model sketched in our "Example 2" below .

In Hall (2004 a and b) it is shown that the volatility of the unemployment rate will increase dramatically if wages are sticky. As a rationale for wage stickiness, Hall refers to social norms.[1] Wage stickiness implies that a larger share of the match surplus is allocated to the workers in a recession than in a boom. Our model gives a an alternative micro-foundation as to why this may be the case. Furthermore, in our model the countercyclicality of of the surplus allocated to the worker is an optimal response by the market maker in the presence of private information among workers.

Another related paper is Kennan (2004), who also studies the effect of information rents on unemployment fluctuations. In his model, workers and firms bargain over wages once they meet. Firms have private information in booms but not in recessions, and thus earns information rents in booms. This increases the profits in booms, and thus also unemployment volatility. Nagypal (2004) and Kraus and Lubik (2004) allow for on-the-job search in a matching model, and show how this may amplify the effects of productivity shocks on the unemployment rate. Menzio (2004) shows that if firms have private information, it may be in their interest to keep wages fixed if hit by high-frequency shocks. Again, this increases volatility.

Our model is also related to a large literature on efficiency wage models (see for instance Weiss 1980 and Shapiro and Stiglitz 1984), and in particular to studies of how efficiency wages influence unemployment volatility. Strand (1992) finds that efficiency wages reduce unemployment volatility. He assumes that firms, after a negative aggregate shock, may be tempted to fire workers. As a result, firms are reluctant to hire more workers during a boom as this will increase the wage necessary to deter shirking. Thus, if the productivity differences are relatively small, employment does not change over the cycle. Dantine and Donaldson (1990) argue that efficiency wages may exacerbate the effect of productivity shocks on the unemployment rate if the shocks are short-lived compared with the time it takes to fire shirking workers. Ramey and Watson (1997) analyze how contractual fragility caused by a lacking ability of firms to commit to a wage contract may increase the volatility of the unemployment rate. Rocheteau (2001) introduces shirking

[1] In Hall 2005 it is shown that wage stickiness may be the result of alternative specifications of the bargaining procedure or self-selection among workers.

in a search model, and show that the non-shriking constraint forms a lower bound on wages paid to workers.

In this paper we first analyze the relationship between employment rents and unemployment within a reduced-form specification of the link between rents and efficiency. We also give two examples by using standard models of efficiency wages. Then we set up a specified model of optimal wage contracts taken from Moen and Rosen (2004), and derive the relationship between employment rents and productivity from first principles. This allows us to study in more detail the relationship between private information and unemployment.

2 A general model

The model consists of two parts. The first part is the matching framework, which is the standard Diamond-Mortensen-Pissarides framework (Diamond 1982, Mortensen 1986, Pissarides 1985). The second part links employment rents and worker productivity.

All agents are risk neutral and discount the future with the same discount factor r. There exists a continuum of ex ante identical workers with measure normalized to one. Workers leave the market at an exogenous rate s. New workers enter the market as unemployed at the same rate. The unemployment rate is denoted by u.

There exists a continuum of firms in the economy. A firm is either matched with a worker and producing or unmatched an searching for a worker. The flow cost associated with search is denoted by c. The number (measure) of searching firms is denoted by v.

The number of contacts in the economy is determined by a concave, constant return to scale matching function $x(u, v)$. Let p denote contact rate for workers and q the contact rate of firms. Since the matching function is constant return to scale it follows that we can write $q = q(p)$, with $q'(p) < 0$.

The surplus generated by a contact may be stochastic, and only contacts that generates a positive surplus leads to a match (i.e., an employment relationship). Let $\widetilde{H}$ denote the probability that a contact leads to employment ($\widetilde{H}$ will be endogenized later on). The expected discounted income (utility) of an unemployed worker is given by

$$(r+s)U = z + p\widetilde{H}(W-U)$$

where z is income (utility) flow when unemployed, $\widetilde{H}$, and W the expected discounted value of employment. The expected *rent* associated with a contact can thus be written as $R \equiv \widetilde{H}(W-U)$.

Let y_i be the net output of a given worker-firm pair. The expected income for this match is given by $y_i/(r+s)$. The expected income generated by a contact is written as Y. If $y^e = y$ denotes the expected productivity of a contact given that the contact leads to a match, it follows that we can write $Y = \widetilde{H}y^e/(r+s)$.

Let V and J denote the expected value of a firm with a vacancy and a firm that gets in contact with a worker, respectively. It follows that we can write

$$rV = -c + qJ$$

By definition we have that

$$J \equiv Y - R - \widetilde{H}U \tag{1}$$

Finally, let S denote the expected surplus of a contact, defined as

$$S = J + R = Y - \widetilde{H}U$$

A key feature in our paper is that Y may depend positively on $R, Y \equiv Y(R)$. There may be several reasons for this relationship:

- Asymmetric information and moral hazard. As will be clear below, incentivizing workers in the presence of asymmetric information and moral hazard may require that workers receive rents.

- More conventional efficiency wage arguments. Workers may have to receive rents when employed in order to exert unobservable effort (Shapiro and Stiglitz 1984). Alternatively, workers may have private information about non-pecuniary aspects of a given job (similar to Weiss 1980).

As Y depends on R, so does the contact surplus S, we write $S = S(R,U)$. Note that

$$S(R, U) = E \max[Y(R) - U, 0] \tag{2}$$

In what follows we assume that $S_R > 0$ for $R < R^*$ and $S_R(R) = 0$ for $R \geq R^*$.[2] We write $S^*(U) = S^*(R^*, U)$, and refer to S^* as the first best production level. In any search model the search frictions imply that $R > 0$. Thus, rents only affect output if R^* is greater than the rents that prevail in search equilibrium.

Finally, note that if $\widetilde{H}$ is independent of U on an interval (for instance because all workers are hired and $\widetilde{H} = 1$), then $S_{RU} = 0$. Thus, the marginal value of worker rent in this case is independent of U. [3]

2.1 First best competitive search equilibrium

Our equilibrium solution concept is the competitive search equilibrium. In competitive search equilibrium, the expected utility of unemployed workers are maximized given the resource constraint of the economy (essentially the free entry condition of firms). With asymmetric information, additional constraints must be added, namely the incentive compatibility constraint and the individual rationality constraint of the worker. These constraints will be specified later on. Now we just assume that any such constraints bind also for the market maker.

As in Mortensen and Wright (2002), the equilibrium can be interpreted as follows: A market maker determines the wage contract in his market. Free entry of market makers then ensures that the only market makers that survive in the market are the one that maximizes the utility of unemployed workers given the free entry condition on firms.

As a benchmark, suppose there are no information problems, and that $S = S^*$. The competitive search equilibrium p^c, R^c, U^c solves

$$\begin{aligned} \max_R (r+s)U^c &= z + p^c R^c \\ \frac{c}{q(p^c)} &= S^*(U^c) - R^c \end{aligned}$$

[2] Note that R^* may depend on U.

[3] We have that $S(R, U) = \widetilde{H}[EY(R) - U]$, where the expectation is taken conditional on Y being greater than U. If $\widetilde{H}$ is constant it follows that $S_R = \widetilde{H}EY'(R)$, independently of U.

Our first lemma, below, states that if the information rent R^* is less than the "search rent" R^c determined in the full-information equilibrium, the market maker can (and will) implement first best even in the presence of private information. Thus, in this case asymmetric information plays no role.

Lemma 1 *Suppose the search rent R^c exceeds the information rent R^*. Then the market maker can implement first best.*

Proof: Omitted

If R^* exceeds R^c, first best can still be obtained if the market maker can use cross subsidization, by collecting an entry fee from workers and a subsidy for vacancies.

Proposition 2 *(Irrelevance of private information) Suppose the market maker can collect an entry fee from the workers, and subsidize vacancies that enter their market. Then the first best competitive search equilibrium is always feasible.*

Proof: We know that efficiency can be obtained if $R^c > R^*$. Suppose therefore that $R^c < R^*$. Then first best can be obtained as follows. When the worker and the firm meets, the worker receives an expected rent R^* so that first best production is ensured. To obtain the optimal vacancy rate, the market maker gives the vacancies a subsidy $D = q(p^c)(R^* - R^c)$ when entering the search market, so that the expected value of entering is $q(Y^* - R^* - U^c) + D = q(Y^* - R^* - U^c) + q(R^* - R^c) = q(Y^* - R^c - U^c)$. It follows that the correct number of firms enter the market. The unemployed workers are charged a fee fee $T = p^c(R^* - R^c)$ when entering. Since $qv = x = pu$, this scheme balances the budget. QED

Cross subsidization between workers and firms breaks the link between the workers' rent when employed and the firms' incentives to enter the market. Thus, the market maker can solve for the optimal trade-off between wages and job finding rate without influencing worker productivity once hired.

A sign-on fee paid by the worker to the firm may play the same role as an entry fee. If the worker has private information the sign-on fee must be agreed upon before the private information is revealed to the worker.

2.2 Constrained competitive search equilibrium

In what follows we do not allow for cross-subsidization between workers and firms. As the market maker has to obey the individual rationality constraint and the incentive compatibility constraint of workers, the market maker faces a relationship $S(R;U)$ between productivity and worker rents. The competitive search equilibrium then solves

$$\max_R (r+s)U = z + pR \tag{3}$$

$$\text{S.T.} \quad \frac{c}{q(p)} = S(R,U) - R \tag{4}$$

For any given R, there exists a corresponding value of p and U, hence we can write $p = p(R)$ and $U = U(R)$. By definition, $U'(R) = 0$ in optimum. From equation (3) it follows that

$$el_R p = -1 \tag{5}$$

where $el_R p$ denotes the elasticity of p with respect to R. Let $\alpha \equiv S_R(R)$. From equation (4) it follows that

$$el_R[\frac{c}{q(p(R))}] = -(1-\alpha)\frac{R}{J} \tag{6}$$

Substituting in for $el_R p = -1$ gives

$$\begin{aligned} el_R[\frac{c}{q(p(R))}] &= -el_q q(p) el_R p(R) = el_p q(p) \\ &= -\frac{\eta}{1-\eta} \end{aligned}$$

where $\eta = |el_\theta \widetilde{q}(\theta)|$, the absolute value of the elasticity of q with respect to $\theta = v/u$. [4] The equilibrium in the search market is thus given by

[4] To see that that this is true, let $p = \widetilde{p}(\theta)$. Then

$$\begin{aligned} el_p q(p) &= el_p \widetilde{q}(\widetilde{p}^{-1}(p)) \\ &= \frac{el_\theta \widetilde{q}(\theta)}{el_\theta \widetilde{p}(\theta)} \end{aligned}$$

Since $el_\theta \widetilde{q}(\theta) = -\eta$ and $el_\theta \widetilde{p}(\theta) = el_\theta[\theta \widetilde{q}(\theta)]$, it follows that $el_p q(p) = -\frac{\eta}{1-\eta}$.

$$\frac{\eta}{1-\eta} = (1-\alpha)\frac{R}{J} \tag{7}$$

When $\alpha = 0$, the equation is identical to the Hosios condition for efficiency in search models (Hosios 1990). We will refer to this equation as the modified Hosios condition.

Proposition 3 *The constrained competitive search equilibrium obeys the modified Hosios condition (7).*

The modified Hosios condition states that as the marginal value of worker rents increases, the share of the match surplus that is allocated to the worker increases. Thus, fewer resources will be devoted to maintaining vacancies.

In order to simplify the exposition we will assume that the matching function is Cobb Douglas so that η is constant. We thus assume that $x(u,v) = Au^{\beta}v^{1-\beta}$, in which case $\eta = \beta$. Let $\nu = \frac{\beta}{1-\beta}$. It then follows that the modified Hosios condition can be written as

$$(1-\alpha)\frac{R}{J} = \nu \tag{8}$$

2.3 Comparative statics

As mentioned in the introduction, an important issue is whether private information may influence the responsiveness of wages to economy-wide shocks. We do this by analyzing how a *change* in parameter values (for instance productivity) will change the unemployment rate. We assume that $S_{RR} < 0$. Thus, the smaller is the worker rent, the larger is the marginal value of this rent. In addition we assume that $\widetilde{H}$ is constant so that $S_{RU} = 0$.

We say that private information stabilize the unemployment rate whenever a negative shock (in, say, productivity) leads to a reduction in α and thus to a larger fraction of the match surplus being devoted to job creation. In the opposite case, the incentive contracts destabilize the unemployment rate.

In general, a shock may influence the relationship between Y and R, and this will be analyzed in detail in later sections. However, some shocks will typically not influence this relationship:

- Changes in the value of leisure (or unemployment benefit)
- Shocks to the matching function
- Contract-independent changes in productivity levels, e.g., input prices.

We first investigate the effect of an increase in the value of leisure. (Formal proofs are given after the proposition below.) This will increase the unemployment rate in competitive search equilibrium. At the same time, an increase in z increases U, the prospects for unemployed workers. This will reduce the match surplus S (see equation 2), and for a given sharing rule α this will reduce R. Since $\alpha = S_R$ and $S_{RR} < 0$ it follows that α increases. Thus, a smaller share of the surplus is allocated to job creation, and this increases the unemployment rate further. It follows that for shocks to the value of leisure, private information tends to destabilize the unemployment rate.

Let us then turn to shocks to the matching process. In competitive search equilibrium, a decrease in A or an increase in c increase the unemployment rate. A negative shock will decrease U, and thereby increase the match surplus S for a given α. Thus, from equation (8) and the assumption that $S_{RR} < 0$, α falls. For shocks to the matching technology, private information tends to stabilize the unemployment rate.

Finally, consider a contract-independent changes in productivity levels. Suppose the productivity of all contacts falls with δ. This will increase the unemployment rate in competitive search equilibrium. We want to show that for a given α, this reduces the match surplus

Lemma 4 *For a given α, a fall in productivity (an increase in δ) as described above reduces the match surplus S.*

Proof: From the envelope theorem, it follows that

$$|\frac{\partial U}{\partial \delta}| = \frac{p}{r+s+p}|\frac{\partial Y}{\partial \delta}| < |\frac{\partial Y}{\partial \delta}|$$

It follows that the lemma can only be untrue if the partial derivative $\frac{\partial Y}{\partial \delta}$ is less in absolute value than the total derivative (including the effect of changes in R). But this is only the case if R falls when productivity decreases, in which case the lemma by definition holds.

It follows from (8) that α increases. Thus, a lower share of the surplus is allocated to creating jobs after a negative productivity shock.

Proposition 5 *Consider a shock to the economy. Then the following holds*

a) Incentive contracts destabilize the unemployment rate after shocks to the value of leisure

b) Incentive contracts stabilize the unemployment rate after shocks to the matching technology and to the cost of search

c) Incentive contracts destabilize the unemployment rate after contract-independent changes in productivity shocks (the same for all worker "types")

Formal proof: We give a formal proof of a). The proof of b) and c) are similar and therefore not included. Since U is maximized in equilibrium, it follows that U is increasing in z. Suppose α falls. Then, by definition, R increases, since $\alpha = S_R$ and $S_{RR} \leq 0$. From (1) it follows that J falls. But then (8) cannot be satisfied, and we have derived a contradiction.

In addition, the probability $\widetilde{H}$ that a contact leads to a match may vary over the cycle. If productivity drop tends to decrease the number of contacts that lead to a match, the direct effect will be that the unemployment rate increases even further. However, as S_{RU} in this case is different from 0, this may also influence α.

Before we turn to our main model of private information with observable output, we will give two examples related to the efficiency wage theory.

2.3.1 Example 1. The shirking model

In the shirking model (Shapiro and Stiglitz 1984), workers are identical, but both worker effort and output is private information to the worker. Effort is either 0 or 1, and output is y if the worker exerts effort and zero otherwise.[5] The effort cost of is ψ. Let g denote the probability rate that a shirking worker is detected, in which case he is fired. The non-shirking condition is then given by

$$\psi \leq gR$$

That is, the cost of effort should be less than the probability rate of being detected when shirking times the cost of loosing the job. Let $R^{ns} = \psi/g$ denote the lowest rent that prevents the worker from shirking. Define the constrained competitive search equilibrium as the allocation that maximizes

[5] Note that S_R is not defined at $R = R^{ns}$. Thus, we cannot set $\alpha = S_R$ in this case.

U given the non-shirking constraint. it follows that $R = \min[R^c, R^{ns}]$. Suppose we are in a region where the non-shirking constraint binds. A fall in y then has no impact on R. Since the contact surplus S decreases, this requires that α increases, a larger fraction of the match surplus is given out as employment rents. Thus, shirking destabilize the economy. [6]

2.3.2 Example 2. Non-pecuniary aspects of employment

Suppose workers obtain non-pecuniary gains from the employment relationship, and that these gains are private information to the workers and thus cannot be contracted upon. In all other respects the workers have symmetric information. The model is similar to the model of Faigh and Jerez (2004).

To be more specific, suppose the utility flow of a match for a worker who is paid a wage w is equal to $w + \tau$, where τ can take a high value τ^h or a low value τ^l. We assume that τ is I.I.D. over all worker-firm pairs. Worker productivity is the same for both types of workers, and equal to y. Efficient matching requires that a contact leads to a match whenever $S(\tau) \geq 0$. Workers, by contrast, only accept jobs for which $R(\tau) \geq 0$. Suppose that initially, $R(\tau^l) \geq 0$ in the unconstrained equilibrium. Thus, both types of workers accept the job and there are no information problems. In this case, $\alpha = 0$.

Consider a fall in y. For a given value of α, this leads to a fall in R.. Thus, after the shock we may have that $R(\tau^l) < 0 < S(\tau^l)$ if the same surplus-sharing rule is applied. Thus, in order to motivate workers to stay after a low realization of τ, the market maker may increase the share of the surplus that is allocated to the workers so that workers accept all job offers. This will increase α and thus destabilize the unemployment rate.

3 Optimal incentive contracts

We now turn to our main model of worker private information. The contracting framework is taken from Laffont and Tirole's (1993) model of op-

[6] Rocheteau (2001) incorporates the shirking condition into a standard search model where workers and firms bargain over the wage. However, he does not analyze the effects of economywide shocks.

timal regulation, and the application to a labor market setting is borrowed from Moen and Rosen (2004). In Laffont and Tirole's model , the principal trades of incentive provision to and rent extraction from the agent. The market maker in our model in addition takes into account that the division of rents between workers and firms influence the unemployment rate.

The productivity of a worker in a firm is given by a function $y = f(\varepsilon, e)$. The variable ε reflects a match-specific productivity term, and is I.I.D. over all worker-firm matches.[7] In the general exposition we assume that ε is continuously distributed on some interval $[\underline{\varepsilon}, \overline{\varepsilon}]$ with cumulative distribution function H. The variable e denotes worker effort, also unobservable to the firm.

To simplify the derivations we assume that $y = \overline{y} + \varepsilon + \gamma e$. Thus, there are no cross effects between effort and worker type. Adding a cross term εe will complicate the expressions but will not bring new insights. A worker's flow utility is given by $\omega = w - \psi(e)$, where w denotes wages and $\psi(e)$ cost of effort.

A wage contract $w(y)$ specifies a relationship between a worker's wage and observed output y. We assume that firms are able to commit to wage contracts. As shown in Baron and Besanko (1984), the optimal dynamic contract repeats the optimal static contract provided that the firm can commit not to renegotiate.[8] We therefore solve for the optimal static contract.

When a worker and a firm meets, the worker first learns ε. Then the worker determines whether to accept the contract or not. If the match is not accepted, the worker starts searching again, while the vacancy dissolves. It is important that the worker learns ε before the contract is signed; this implies that the information rents to workers from knowing ε cannot be extracted by a sign-on fee.

First best requires that $e = e^* = \arg\max \gamma e - \psi(e)$ and that the cut-off productivity ε^* solves $\overline{y} + \varepsilon^* + \gamma e - \psi(e) = U^c$. In order to exert effort for all values of ε, all workers that are hired must be residual claimants of their effort. The lowest expected employment rent consistent with first best is thus given by

$$R^* = \int_{\varepsilon^*}^{\overline{\varepsilon}} \varepsilon dF$$

[7] Note that ε also may reflect ideosyncraticies of the job in question.

[8] For an instructive proof see Fudenberg and Tirole (1991), p. 299 ff.

In what follows we assume that $R^c < R^*$. Thus, in the absence of cross subsidies the first-best completive search equilibrium is infeasible. We will first characterize some situations where this typically will be the case.

3.1 More on the requirement that $R^c < R^*$

The first thing to note is that if not all workers are hired in the first best equilibrium, then $R^c < R^*$:

Lemma 6 *Suppose $\overline{y} + \underline{\varepsilon} + \gamma e^* - \psi(e^*) < U^c$. Then the first best competitive search equilibrium is infeasible.*

Proof: The proof is done by contradiction. Suppose the first best competitive search equilibrium did exist. In this equilibrium, let ε^* denote the (optimal) cut-off productivity, given by the equation $\overline{y}+\varepsilon^*+\gamma e^*-\psi(e^*) = U^c$. The marginal worker must be paid a wage equal to his productivity. Furthermore, as $w'(y) = 1$ for all other workers, first best implies zero profit to the firm. Thus, no vacancies enter the market and no workers are employed. This is inconsistent with equilibrium.

It follows that as long as the distribution of ε is non-degenerate, the first best competitive search equilibrium is infeasible, provided that the search frictions measured by the search costs c are sufficiently small:

Corollary 7 *The first best competitive search equilibrium is infeasible if the search costs c are sufficiently small (provided that the distribution of ε is not degenerate).*

Proof: In competitive search equilibrium, $p \to \infty$ as $c \to 0$. As a result, the optimal cut-off approaches $\overline{\varepsilon}$. From lemma 6 it then follows that the first best competitive search equilibrium is infeasible.

However, it may well be that $R^c < R^*$ even if all worker "types" are hired. Suppose $\varepsilon = kz$, where z is a stochastic variable and k a scalar. Then the following holds

Lemma 8 *For any given combination of parameters and any distribution H of z with finite support, there exists an interval $k \in (\underline{k}, \overline{k})$ such that for any $k\varepsilon$ the following holds: 1) first best is not feasible, and 2) the cut-off level is equal to $\underline{\varepsilon}$.*

The proof utilizes properties of the optimal contract (not yet derived) and is therefore deferred to the appendix (we do not use this lemma when deriving the optimal contract).

3.2 Optimal contracts

The optimization problem facing the market maker is given by (3)-(4). From the constraint (4) it follows that, for a given value of R and U, the planner maximizes the expected surplus S of a contact. This opens up for the highest p given R.

A mechanism is a triple $(\varepsilon, e(\varepsilon), w(\varepsilon))$ that obeys the workers' incentive compatible (IC) constraints (workers choose effort so as to maximize utility) and the individual rationality (IR) constraints (workers only accept a contract if it gives an expected utility higher than continuing search). Let $R(\varepsilon)$ denote the rent to a worker of "type" ε. The incentive compatibility constraint can then be written as

$$(r+s)R'(\varepsilon) = \psi'(e(\varepsilon))/\gamma$$

This condition ensures that an agent does not have an incentive to pretend that he is of a lower type than he really is. The individual rationality constraint requires that $R(\varepsilon^*) \geq 0$. When $R^* > R^c$, this constraint binds.

The expected match surplus of a contact can be written as

$$(r+s)S = \int_{\varepsilon^*}^{\overline{\varepsilon}} [\overline{y} + \varepsilon + \gamma e - \psi(e)] - (r+s)U]dH \tag{9}$$

which is maximized subject to the IR constraint, the IC constraint, and the expected rents given to employed workers:

$$\begin{aligned} R(\varepsilon^*) &= 0 && (10) \\ (r+s)R'(\varepsilon) &= \psi'(e(\varepsilon))/\gamma \\ \int_{\varepsilon^*}^{\overline{\varepsilon}} R(\varepsilon)dH(R) &= R && (11) \end{aligned}$$

The associated Hamiltonian is given by

$$\begin{aligned}\mathcal{H} &= [\overline{y} + \gamma\varepsilon + \gamma e - \psi(\varepsilon) - (r+s)U]h(\varepsilon) \\ &\quad +\lambda\psi'(e(\varepsilon))/\gamma \ - \alpha[\int_{\varepsilon^*}^{\overline{\varepsilon}} R(\varepsilon)dH(R) - R]\end{aligned}$$

First order conditions for $e(\varepsilon)$ can be written as

$$(\gamma - \psi'(e(\varepsilon))h(\varepsilon) = \lambda\psi''(e(\varepsilon))/\gamma \tag{12}$$

Furthermore,

$$\lambda'(\varepsilon) = \delta\mathcal{H}/\delta R(\varepsilon) = -\alpha h(\varepsilon)$$

Since $\overline{\varepsilon}$ is free it follows that $\lambda(\overline{\varepsilon}) = 0$. Thus, $\lambda = \alpha(1 - H(\varepsilon))$. Inserted, this gives

$$\gamma - \psi'(e(\varepsilon) = \alpha\frac{1 - H(\varepsilon)}{h(\varepsilon)}\psi''(e(\varepsilon))/\gamma \tag{13}$$

Let us then turn to the optimal cut-off value ε^*. The optimal cut-off value is obtained by setting $\mathcal{H} = 0$:

$$[\overline{y} + \varepsilon + \gamma e - \psi(\varepsilon) - (r+s)U]h(\varepsilon^*) - \alpha(1 - H(\varepsilon^*))\frac{\psi'(e(\varepsilon))}{\gamma} = 0 \tag{14}$$

If $\mathcal{H}(\underline{\varepsilon}) < 0$, it is optimal to hire all workers, and we set $\varepsilon^* = \underline{\varepsilon}$.

From the first order condition (13), we can observe the following:

1. No distortions at the top, $\gamma = \psi'(e(\overline{\varepsilon}))$.
2. If the constraint associated with R does not bind, then all types of agents are given full incentives.
3. For all types ε, the incentive power of the contract is a decreasing function of α.
4. As $\alpha \to \infty$, no incentives are given and the effort level is equal to zero.

Note that $\frac{dS}{dR}(r+s) = \delta\mathcal{H}/\delta R = \alpha$.

Lemma 9 *The following holds:*

a) The shadow value α of worker rents is decreasing in R (for given U).

b) If $\varepsilon^* > \underline{\varepsilon}$, an increase in U reduces the shadow value α of worker rents. If $\varepsilon^* = \underline{\varepsilon}$, α is independent of U.

c) The cut-off level ε^* is increasing in α for a given U.

Proof: a) Suppose not. Then it follows from (13) that effort, and therefore rents, fall. However, it then follows that S falls, otherwise we are not in optimum initially. Thus we have a contradiction.

b) and c) are proven in the appendix.

Since $S_{RR} = \frac{d\alpha}{dR}$, result a) implies that $S_{RR} < 0$. At first glance, result c) may seem surprising. An increase in α tends to reduce e, which again reduces the cut-off. However, a reduction in e also reduces the value of hiring a person. Given the shadow cost α of worker rents, the value of $e(\varepsilon^*)$ is optimally set, and the envelope theorem thus applies.

Let (a, b) denote a linear contract of the form $w = a + by$. It is well known that the optimal non-linear contract can be represented by a menu $(a(\epsilon), b(\epsilon))$ of linear contracts (see, e.g., Laffont and Tirole 1993). For any b, the worker chooses the effort level such that $\psi'(e) = b\gamma$. Henceforth, we refer to b as the incentive power of the associated linear contract. Using the condition $\psi'(e) = b\gamma$ in equation (13), we obtain

$$b(\epsilon) = 1 - \alpha \frac{1 - H(\epsilon)}{h(\epsilon)} \frac{\psi''(e)}{\gamma^2}. \tag{15}$$

4 Comparative statics

Earlier we referred to three kinds of shocks that may affect the unemployment rate in our model: changes in the value of leisure (unemployment benefits), shocks related with the cost of matching (changes in A or in c), and shocks that are related to the production function.

Let us first consider a shock to $\overline{y}$. Suppose all workers are hired ($\varepsilon^* = \underline{\varepsilon}$). Then $S_{RU} = 0$. The first thing to note is that $S_{RR} < 0$, therefore proposition 5 applies. In particular, private information destabilizes the unemployment rate after a negative shock to $\overline{y}$.

Assume then that $\varepsilon^* > 0$. The cutoff is given by

$$\overline{y} + \varepsilon^* - (r+s)U = \alpha\frac{(1-H(\varepsilon^*))}{h(\varepsilon^*)}\psi'(e) - (\gamma e - \psi(e))$$

We first keep ε^* fixed. A fall in $\overline{y}$ implies that the left-hand side falls (since $\overline{y}$ falls more than $(r+s)U$), while an increase in α implies that the right-hand side increases. Thus, ε^* will increase. This will increase the unemployment rate even further. Note however, that as ε^* increases this will reduce α. Thus, we cannot say a priori whether total effect on α of a fall in $\overline{y}$ is negative.

Let us then turn to changes in the importance of effort. We first rearrange our production function to

$$y = \widetilde{y} + \varepsilon + \gamma(e - e^0)$$

where $e^0 > e^*$, the optimal value of e. Note that this is equivalent with our initial formulation with $\widetilde{y} = \overline{y} + \gamma e^0$. Suppose that a negative shock is driven by an increase in γ (the importance that the worker exert effort). If input prices drive the shock, this may be interpreted as worker effort and other inputs being substitutes. In the appendix, we show the following:

Proposition 10 *Suppose $\psi''/(\psi')^2$ is nonincreasing in e, and suppose $\varepsilon^* = \underline{\varepsilon}$. Consider a positive shift in γ. For a given cut-off level, such a shift will increase α for a given S. In addition it reduces S, thus increasing α further.*

Note that the destabilizing effects of contracts on the unemployment rate is stronger in this case, as it consists of two components. First, an increase in γ increases α for a given value of S. This is not the case for changes in $\overline{y}$. This comes in addition to the effects through a reduction in R induced by the fall in productivity.

What about the cut-off level. An increase in α will tend to increase the cut-off level. The same is true for a reduction in S (as for $\overline{y}$). Furthermore, the increased value of b is higher for the low types than for the high type. On the other hand, low-type workers exert less effort than high-productivity workers and will therefore *cet par* be less vulnerable to shocks. The net effect is therefore uncertain.

We will now consider the effects of an increase in the spread of ε. Suppose $\varepsilon = kz$, where z is a stochastic variable and k a scalar. To simplify the analysis we assume that $\varepsilon^* = \underline{\varepsilon}$. An increase in k then reduces output, because the workers will be given weaker incentives.

An increase in k increases the amount of private information the workers posses, and for a given contract this increases worker rents. Thus, for a given R the incentive power of the contract is reduced. This tends to increase α. On the other hand, an increase in k makes it more costly in terms of rents to incentivize workers, and this will tend to reduce the value of α. If the private information problems are moderate, the first effect dominates, and an increase in k increases α.

Proposition 11 *There exists a value $\widetilde{R} < R^*$ such that an increase in k increases α whenever $R \in [\widetilde{R}, R^*]$*

The proposition follows from the fact that at R^*, $\alpha = 0$, while for all $R < R^*$ workers receive positive rents. From the analysis in Moen and Rosen (2004) it follows that for a given U, α increases if and only if the average value of b is above $1/2$ initially.

As for γ, an increase in k will have a double effect on α. First, it may increase α for a given value of S. In addition, the productivity effect lead to a reduced value of S (for a given α), which further increases α.

5 Conclusion

In this paper, we have analyzed how private information among workers may influence the responsiveness of the unemployment rate to aggregate shocks within the context of a matching model. First we show that the equilibrium allocation obeys a simple, modified Hosios rule. Then we show that private information stabilizes the unemployment rate when the economy is hit by shocks to the matching technology. On the other hand, if the economy is hit by productivity shocks that reduce the value of all matches, private information destabilizes the unemployment rate. This is also true if the economy is hit by shocks to the value of leisure.

Appendix

Proof of lemma 8

For $k = 0$, all workers are hired and first best is obtained. In this case, an increase in k does not influence U. We have to show that the market maker starts reducing the incentive power of the contract before he increases the cut-off. As $R^c > 0$, we must have that $\overline{y} + \underline{\varepsilon} + \gamma e^* - \psi'(e^*) > U^c$ at the point where $R^c = R^*$. At this point, increasing the cut-off level has a first-order effect on expected output. Reducing the incentive power of the contract slightly only gives a second-order effect on expected output. It thus follows that the market maker will reduce the incentive power of the contract before he increases the cut-off level (i.e., for a lower value of k).

Proof of b) and c) in lemma (9).

b) U only enters the contract through the cut-off equation, which can be written as

$$\overline{y} + \varepsilon^* - (r+s)U + (\gamma e(\varepsilon^*) - \psi(e)) = \alpha \frac{1 - H(\varepsilon^*)}{h(\varepsilon^*)} \frac{\psi'(e(\varepsilon))}{\gamma} \tag{16}$$

We first want to show that an increase in U leads to a fall in α if and only if it leads to an increase in ε^*. This follows from the fact that for a given contract, an increase in ε^* results in a lower value of R, as we are integrating over a shorter interval. Thus, the rent-constraint allows for more incentive-powered contracts. As a result, the shadow value of R (that is, α) falls. Second, for a given α, a reduces the gain from hiring workers (reduces the left-hand side of 16), and by a revealed preference argument it follows that ε^* increases (still for a given α). Put together, it follows α decreases and ε^* increases in U.

c) To prove c) it is convenient to rewrite (16) as

$$\overline{y} + \varepsilon^* - (r+s)U = \alpha \frac{1 - H(\varepsilon^*)}{h(\varepsilon^*)} \frac{\psi'(e(\varepsilon))}{\gamma} - (\gamma e(\varepsilon^*) - \psi(e))$$

It is sufficient to show that the right-hand side is increasing in α. Now the derivative of the right-hand side of the equation is given by

$$\frac{d()}{d\alpha} = \frac{1 - H(\varepsilon^*)}{h(\varepsilon^*)} \frac{\psi'(e(\varepsilon))}{\gamma} + \alpha \frac{1 - H(\varepsilon^*)}{h(\varepsilon^*)} \frac{\psi''(e(\varepsilon))}{\gamma} \frac{de}{d\alpha} - (\gamma - \psi'(e)) \frac{de}{d\alpha}$$

Now from (13) $(\gamma - \psi'(e)) = \alpha \frac{1-H}{h} \frac{\psi''}{\gamma}$. Hence the two last terms cancel out, and

$$\frac{d()}{d\alpha} = \frac{1 - H(\varepsilon^*)}{h(\varepsilon^*)} \frac{\psi'(e(\varepsilon))}{\gamma} > 0$$

By a revealed preference argument the proposition follows. QED

Proof of proposition 10

Worker rent for any given type ε' is given by

$$R(\varepsilon') = \int_{\varepsilon^*}^{\varepsilon'} \frac{\psi'(e(\varepsilon))}{\gamma} dH = \int_{\varepsilon^*}^{\varepsilon'} b(\varepsilon) dH$$

Suppose an increase in γ increases $b(\varepsilon)$ for all ε for a given α. Then R must increase. Thus, for a given value of R, α increases. Due to the envelope theorem, small changes in R does not influence U. Thus, in keeping R constant, it follows that p and hence J are constant as well. In order to satisfy (8) both R and α must increase relative to their initial value.

It is thus sufficient to show that $b(\varepsilon)$ is increasing for all ε for a given α. First note that $e(\varepsilon)$ must be increasing, otherwise (13) cannot be satisfied. Denote by b^{old} and b^{new} the value of b before and after the increase in γ. Suppose $b^{old} > b^{new}$. Substituting in $\gamma = b^{new}/\psi'(e)$ into (13) gives

$$\begin{aligned} b^{new} &= 1 - \alpha \frac{1-H}{h} \frac{b^{new}\psi''(e^{new})}{\psi'(e^{new})^2} \\ &> 1 - \alpha \frac{1-H}{h} \frac{b^{old}\psi''(e^{new})}{\psi'(e)^2} \\ &> 1 - \alpha \frac{1-H}{h} \frac{b^{old}\psi''(e^{old})}{\psi'(e^{old})^2} \\ &= b^{old} \end{aligned}$$

which is a contradiction.

In addition, an increase in γ reduces output, and as for a reduction in $\overline{y}$ this will reduce R for a given α. This will further increase α.

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