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## A Collective Labor Supply Model with Detailed Time Use and Intrahousehold Expenditure Information

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# ABSTRACT

## Married with Children: A Collective Labor Supply Model with Detailed Time Use and Intrahousehold Expenditure Information[*]

We propose a collective labor supply model with household production that generalizes an original model of Blundell, Chiappori and Meghir (2005). In our model, adults' individual preferences do not only depend on own leisure and individual private consumption of market goods. They also depend on the consumption of domestic goods, which are produced by combining goods bought at the market with individuals' time. We apply our model to new and unique data on Dutch couples with children. The data contains detailed information about the spouses' time use and the intrahousehold allocation of all expenditures. Our application uses a novel estimation strategy that builds upon the familiar two-stage allocation representation of the collective model. We obtain interesting (and plausible) empirical results. Spouses' preferences depend on the consumption of domestically produced goods (including children's welfare). Next, Pareto weights depend on variables like the individual wages and the share in the household's nonlabor income. Finally, and perhaps surprisingly, we do not find evidence that mothers care more for their children than fathers.

JEL Classification:    D11, D12, D13

Keywords:    collective model, labor supply, time use, public goods, household production

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# 1   Introduction

Parents care for their children. This impacts on their decisions regarding time use and consumption. How can we take this into account when analyzing household behavior? Recently, Blundell, Chiappori and Meghir (2005; henceforth BCM) presented a collective labor supply model that accounts for caring parents, and that has a number of attractive theoretical and conceptual features. We provide a first empirical application of BCM's theoretical ideas. In doing so, we also present a number of extensions of BCM's original analysis to enhance economic realism and to facilitate empirical implementation. In this introductory section, we motivate BCM's collective model to account for caring parents in the analysis of household behavior, and we articulate the main contributions of the current study.

It is by now well established that the unitary model, which assumes that households behave as single decision makers, is not adequate to describe the observed behavior of households consisting of multiple individuals. A most popular alternative to the unitary model is the collective model, which was originally suggested by Chiappori (1988, 1992) and Apps and Rees (1988). This collective model explicitly recognizes that the household consists of multiple individuals who have their own rational preferences. These individuals are assumed to take Pareto efficient decisions that result from an intrahousehold bargaining process.

Chiappori (1988, 1992) originally proposed a collective labor supply model that starts from the rather standard assumption that individuals divide their time between leisure and market work. Interestingly, this model effectively does provide a better fit of household labor supply data than the unitary model.[1] However, as Becker (1965) already noticed, the underlying assumption is too restrictive: in usual settings, not all non-market work can be considered as pure leisure, since time is also spent on household production. Importantly, Apps and Rees (1996) point out that a model which starts from the simple dichotomization of time into leisure and market work may result in misleading welfare recommendations.[2]

In response to this rightful critique, Chiappori (1997) has proposed a collective labor supply model that does include household production.[3] However, the model is again rather specific. It assumes a setting with a single market good and a single domestically produced good, which are both privately consumed. In addition, the sole inputs for the domestic good are the spouses' time allocated to its production. As such, the model does not allow for public consumption in a direct way, and it abstracts from market goods that act as inputs in the household production process.

BCM presented an alternative model to account for household production in a

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[1]See, for example, the empirical applications of Fortin and Lacroix (1997) and Cherchye and Vermeulen (2008).

[2]Donni (2008) derived the conditions that need to be satisfied for welfare analyses still to be valid in the absence of information about the time allocation between market work and household work.

[3]See Aronsson, Daunfeldt and Wikström, (2001), Couprie (2007), and Rapoport, Sofer and Solaz (2010) for empirical applications of the model.

collective labor supply setting.[4] Their model has a number of attractive features. Firstly, it assumes that the parents' individual preferences depend not only on own private consumption and leisure, but also on the children's welfare. Children's welfare then acts as a public good, which is characterized as a Beckerian (1965) domestic good that is produced by means of expenditures on children (like clothing and toys) and parental time invested in children. The model is fully identifiable in the sense that parents' individual preferences and the decision process, as summarized by the Pareto weights, can be recovered from observed couples' behavior alone.

From a conceptual point of view, an important motivation for BCM's collective model is that it is particularly well-suited for intrahousehold welfare analysis. For example, as argued by BCM, the model provides a natural framework to analyze issues related to the targeting view. This view takes as a starting point that the effectiveness of a specific benefit or tax also depends on the particular household member to whom it has been targeted. This last point is most notably illustrated by the rejection of the income pooling hypothesis on numerous occasions.[5] In addition, the abovementioned identifiability result enables a sound analysis of statements such as "mothers care more for children than fathers".[6]

The theoretical and conceptual attractiveness of BCM's model begs for an empirical application. However, such an application does not yet exist. A likely explanation is that the model implies severe data requirements. For example, it requires detailed knowledge about how the household's total expenditures are broken down in different expenditure categories (including expenditures on children). Furthermore, the model needs specific information on time use (including parental time invested in children).

The current paper fills this gap in the literature, and brings BCM's theoretical ideas to observational data. More specifically, we provide three contributions. Firstly, we present a first application to a (novel and unique) data set that contains all necessary information to implement BCM's model. The data is drawn from the new LISS (Longitudinal Internet Studies for the Social sciences) panel that is gathered by CentERdata; this panel is representative for the Dutch population. The LISS panel is a regular social survey, to which a questionnaire was added about the intrahousehold allocation of expenditures and detailed time use.[7]

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[4] See Blundell, Chiappori and Meghir (2004) for additional discussion of the model.

[5] See, for example, Thomas (1990), Browning, Bourguignon, Chiappori and Lechene (1994), Lundberg, Pollak and Wales (1997) and Duflo (2003).

[6] Lewbel and Pendakur (2010) propose an alternative collective model that focuses on children. This model differs from our model in two important respects. Firstly, Lewbel and Pendakur only focus on the allocation of expenditures and do not consider the allocation of time to market work, leisure and parental time invested in children. Secondly, for identification purposes, they need to make the assumption that individual resource shares do not vary with total household expenditures. In addition, they need one of two restrictions on the individual preferences (either involving that preferences for a particular good are similar across individuals or that individual preferences for a particular good are similar across household types).

[7] Our data set is comparable to the one of Bonke and Browning (2009) in the sense that the intrahousehold allocation of all expenditures is known. However, our data set is richer because it

Secondly, we generalize BCM's original model to a setting with more than one domestic good. This generalization seems warranted for the very same reason as why a simple dichotomization between market work and leisure may obtain distorted welfare analyses: parents do not only allocate their time to market work, parental time and leisure, but also to household activities such as cleaning or gardening. We will argue that BCM's identifiability result extends to this generalized model.

Our final contribution is that we propose and apply a novel estimation strategy for our collective model. The strategy is directly based upon insights obtained through the well-known two-stage allocation representation of the collective model (see, Chiappori, 1988, 1992), and starts from a dual characterization of the model. We will argue that this approach considerably facilitates the derivation of a flexible functional form for the observables. In turn, this greatly benefits the empirical implementation of the theoretical model under study.

As we will discuss, the estimated model obtains intuitive results in terms of the factors that impact on the parental preferences and the intrahousehold Pareto weights. In addition, we show the usefulness of the model for empirical analysis of intrahousehold welfare issues. For example, we will assess the impact of male and female wage changes on the intrahousehold consumption of private goods and leisure, and on the production of public goods (including children's welfare); this complies with the targeting view mentioned above. Next, we will empirically evaluate whether or not mothers care more for their children than fathers.

The remainder of this paper unfolds as follows. Section 2 presents our theoretical model, which generalizes BCM's original model. Section 3 contains our empirical application of this model. It presents our data set, proposes our estimation strategy and discusses our empirical results. Section 4 concludes.

## 2   Theory

### 2.1   The model

In this section, we introduce our collective labor supply model with home production, which extends BCM's original model. Throughout, we will focus on a household with two adults ($i = 1, 2$). The household further contains children who do not have any bargaining power on their own. The adult members spend their time on leisure (denoted by $l^i$), market work (denoted by $m^i$) and household work. We make a distinction between two types of household work: parental time invested in children (denoted by $h_k^i$) and other household work like cleaning or gardening (denoted by

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also contains detailed information about the time use of the household members. In this respect, another comparable survey is described in Browning and Gørtz (2006). This last survey also includes detailed time use information. However, it focuses on a more limited set of consumption categories.

$h_p^i$).[8] For each adult member $i$, the time budget constraint thus equals:

$$l^i + m^i + h_k^i + h_p^i = 1, \qquad (1)$$

where we normalize the time endowment to one.

A unit of market work by member $i$ is associated with a wage $w^i$. The household has a nonlabor income $y$. The income of the household is allocated to a Hicksian composite good with a price that is normalized to one. We assume that the Hicksian composite good is used for the private consumption of the adult members ($c^1$ and $c^2$), expenditures on children ($k$) and expenditures on other public goods ($p$). This results in the following household budget constraint:

$$c^1 + c^2 + k + p = y + w^1 m^1 + w^2 m^2. \qquad (2)$$

In contrast to what is usually the case, we observe the complete intrahousehold consumption allocation in our data set (see Section 3). In other words, we do not only observe how much of the household's resources go to children and other public goods, but also how the household's private consumption $c^1 + c^2$ is allocated to both adult members.

The allocation of the household's income (as well as the size of this income) depends on the individual preferences of the adult members and their respective bargaining positions inside the household. The preferences of member $i$ are represented by the utility function

$$u^i = u^i \left( c^i, l^i, u^k \left( k, h_k^1, h_k^2 \right), u^p \left( p, h_p^1, h_p^2 \right) \right). \qquad (3)$$

We will assume that the function $u^i \left( c^i, l^i, u^k \left( k, h_k^1, h_k^2 \right), u^p \left( p, h_p^1, h_p^2 \right) \right)$ is twice continuously differentiable, strictly increasing and strongly concave.[9] The subutility functions $u^k \left( k, h_k^1, h_k^2 \right)$ and $u^p \left( p, h_p^1, h_p^2 \right)$ are assumed to have the same properties and, in addition, to be homothetic (i.e., the household production process is characterized by constant returns to scale).[10] These subutilities represent two domestic goods of which the output is unobserved. More specifically, the domestic good $u^k$ stands for the children's utility that acts as a public good in the adult members' preferences. Similarly, the domestic good $u^p$ can be interpreted as the joy of a clean and cosy house (though other interpretations are of course also possible). Contrary to $u^k$ and $u^p$, individual consumption $c^i$ and leisure $l^i$ are assumed to be private goods (which do not entail any external effects).[11]

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[8] All our results are easily generalized to apply to any number of household work types and public goods. For simplicity, we restrict to two types of household work and public goods in what follows. The same setting will be considered in our empirical application in Section 3.

[9] To save on notation, we will use expressions like $f = f(x)$ in what follows, where $f$ refers to a function and $f(x)$ to the value the function takes when its argument equals $x$.

[10] The assumption of homotheticity (or constant returns to scale) follows Pollak and Wachter (1975) and will prove useful for our empirical specification (see Section 2.3).

[11] As is clear from above, the production process focused on is a household technology, which is not necessarily related to individual technologies. See Pollak (2007) for a discussion on the connection between individual and household production technologies.

To conclude this introduction of the individual preferences, one important remark is in order. It is clear from (3) that we assume that the children's utility $u^k$ is produced by means of parental time invested in children and expenditures on children. Still, one can argue that $u^k$ also depends on, say, expenditures on housing. This poses the problem that housing affects the adult household members' utility both through $u^k$ and $u^p$. This problem is very reminiscent to Gorman's (1978) and Blundell and Robin's (2000) concept of latent separability.[12] Latent separability generalizes weak separability in the sense that goods are allowed to enter more than one subutility function. However, in the current setting the data forces us to focus on a Hicksian composite commodity with a price that is normalized to one, so that we cannot make use of latent separability. Still, we do see the exploration of this latent separability idea as an interesting avenue for follow-up research; e.g., future waves of the data set used in the present study may generate the required price variation.

The adult members' bargaining positions depend on their wages $w^1$ and $w^2$, the household's nonlabor income $y$ and (at least) two so-called distribution factors $z_1$ and $z_2$. The latter are defined as variables that affect the bargaining position of the adult members without affecting their preferences nor the household budget constraint (after controlling for total income). See Bourguignon, Browning and Chiappori (2009) for a more detailed discussion.

Following Chiappori (1988, 1992) and BCM, we assume that the adult members choose Pareto efficient intrahousehold allocations. Therefore, observed allocations are assumed to result from the following optimization programme:

$$\max_{l^1,l^2,h_k^1,h_k^2,h_p^1,h_p^2,c^1,c^2,k,p} \lambda\left(w^1,w^2,y,z_1,z_2\right) u^1\left(c^1,l^1,u^k\left(k,h_k^1,h_k^2\right),u^p\left(p,h_p^1,h_p^2\right)\right) (4)$$
$$+ \left(1 - \lambda\left(w^1,w^2,y,z_1,z_2\right)\right) u^2\left(c^2,l^2,u^k\left(k,h_k^1,h_k^2\right),u^p\left(p,h_p^1,h_p^2\right)\right)$$

subject to

$$c^1 + c^2 + k + p = y + w^1 m^1 + w^2 m^2$$
$$l^i + m^i + h_k^i + h_p^i = 1 \ (i=1,2).$$

The Pareto weight $\lambda\left(w^1,w^2,y,z_1,z_2\right)$ represents the relative bargaining power of member 1 and, as indicated earlier, depends on the adult members' wages, the household's nonlabor income and the distribution factors. In what follows, we assume that the Pareto weight is continuously differentiable in its arguments. The household's optimal choices are observable functions of the same variables $(i = 1, 2)$:

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[12] In this respect, see also Pollak and Wachter's (1975) discussion on the issue of joint production in the household (i.e. the same inputs are used for multiple domestic outputs).

$$
\begin{aligned}
l^i &= l^i \left( w^1, w^2, y, z_1, z_2 \right) \\
c^i &= c^i \left( w^1, w^2, y, z_1, z_2 \right) \\
h^i_k &= h^i_k \left( w^1, w^2, y, z_1, z_2 \right) \\
h^i_p &= h^i_p \left( w^1, w^2, y, z_1, z_2 \right) \\
k &= k \left( w^1, w^2, y, z_1, z_2 \right) \\
p &= p \left( w^1, w^2, y, z_1, z_2 \right).
\end{aligned}
\tag{5}
$$

At this point, it is worth stressing that the above system of equations is assumed to be completely observed. A natural question now is whether this system of equations allows us to recover the underlying structural model that consists of both adult members' individual preferences and the decision process inside households (as summarized by the Pareto weight).

The identifiability of our model is guaranteed by Corollary 10 of Chiappori and Ekeland (2009). This corollary focuses on a general collective model with both private and public consumption. For this general case, Chiappori and Ekeland have shown that if each adult member is associated with at least one exclusive good (i.e., a good that is exclusively consumed by only one member), then the members' indirect collective utility functions (i.e., indirect utility functions that take into account how resources are allocated inside the household for given exogenous variables) can be identified from observable household demand (5) alone. In our case, each adult member $i$ is associated with the exclusively consumed leisure $l^i$, while the nonexclusive goods $u^k$ and $u^p$ are publicly consumed. Moreover, as we will discuss in Section 3, in our data set $c^i$ is an assignable good (i.e., a privately consumed good of which the consumption is observable). Chiappori and Ekeland's Corollary 10 further shows that for any cardinalization of the indirect utilities, the Pareto weights are also identified. This result thus implies complete identifiability of our collective model.

We will give more intuition for this identifiability result in the next two subsections. To this end, we will use that we can also represent the solution to the optimization programme (4) as stemming from a two-stage allocation process. In the first stage, the household members agree on the household public goods (via choosing the expenditures for children, the other public expenditures and the time devoted to household production) and an intrahousehold allocation of the residual nonlabor income. In the second stage, each member maximizes her or his utility by choosing own leisure and own private consumption conditional on the level of both domestic goods and the budget constraint defined in the first stage. The analysis of the two-stage allocation process will also prove instrumental for our empirical application: we will use it to design the empirical model that we will bring to the data.

## 2.2 Second stage of the allocation process

The identifiability of our model is best explained by first focusing on the second stage. The next subsection deals with the first stage of the allocation process.

It is clear that the outputs $u^k$ and $u^p$ of the household production process are not observable. Still, the fact that we observe the inputs (i.e. $(k, h_k^1, h_k^2)$ and $(p, h_p^1, h_p^2)$) as functions of $(w^1, w^2, y, z_1, z_2)$ allows us to recover $u^k(k, h_k^1, h_k^2)$ and $u^p(p, h_p^1, h_p^2)$ up to a monotonically increasing transformation. This happens through the assumption of cost minimization in the production of the public goods, which implies $(j = k, p)$:

$$\frac{\frac{\partial u^j\left(j, h_j^1, h_j^2\right)}{\partial h_j^1}}{\frac{\partial u^j\left(j, h_j^1, h_j^2\right)}{\partial j}} \equiv \phi_j^1\left(j, h_j^1, h_j^2\right) = w^1 \text{ and } \frac{\frac{\partial u^j\left(j, h_j^1, h_j^2\right)}{\partial h_j^2}}{\frac{\partial u^j\left(j, h_j^1, h_j^2\right)}{\partial j}} \equiv \phi_j^2\left(j, h_j^1, h_j^2\right) = w^2. \qquad (6)$$

Making use of Frobenius' theorem (see, e.g., Afriat, 1977), these systems of partial differential equations can be integrated to respectively $u^k(k, h_k^1, h_k^2)$ and $u^p(p, h_p^1, h_p^2)$ (up to an increasing transformation) if the following symmetry conditions are satisfied $(j = k, p)$:

$$\frac{\partial \phi_j^1\left(j, h_j^1, h_j^2\right)}{\partial h_j^2} + \frac{\partial \phi_j^1\left(j, h_j^1, h_j^2\right)}{\partial j}\phi_j^2 = \frac{\partial \phi_j^2\left(j, h_j^1, h_j^2\right)}{\partial h_j^1} + \frac{\partial \phi_j^2\left(j, h_j^1, h_j^2\right)}{\partial j}\phi_j^1. \qquad (7)$$

In what follows, we will assume that particular cardinalizations for $u^k(k, h_k^1, h_k^2)$ and $u^p(p, h_p^1, h_p^2)$ have been identified in this way. In other words, we assume that $u^k(k, h_k^1, h_k^2)$ and $u^p(p, h_p^1, h_p^2)$ are known functions of respectively $(k, h_k^1, h_k^2)$ and $(p, h_p^1, h_p^2)$. The latter variables are themselves known functions of $(w^1, w^2, y, z_1, z_2)$, which is inherited by $u^k(k, h_k^1, h_k^2)$ and $u^p(p, h_p^1, h_p^2)$. This obtains the functions $u^k(w^1, w^2, y, z_1, z_2)$ and $u^p(w^1, w^2, y, z_1, z_2)$.

To exploit the idea of the conditional sharing rule, the output associated with the domestic goods is fixed to arbitrary levels: $u^k(w^1, w^2, y, z_1, z_2) = \overline{u}^k$ and $u^p(w^1, w^2, y, z_1, z_2) = \overline{u}^p$. Assuming that the matrix $\begin{bmatrix} \frac{\partial u^k(.)}{\partial z_1} & \frac{\partial u^k(.)}{\partial z_2} \\ \frac{\partial u^p(.)}{\partial z_1} & \frac{\partial u^p(.)}{\partial z_2} \end{bmatrix}$ is nonsingular in an appropriately defined subset of the domain of $u^k(w^1, w^2, y, z_1, z_2)$ and $u^p(w^1, w^2, y, z_1, z_2)$, we can make use of the implicit function theorem to express the distribution factors as functions of the observable exogenous variables and the levels $\overline{u}^k$ and $\overline{u}^p$: $z_1 = z_1\left(w^1, w^2, y, \overline{u}^k, \overline{u}^p\right)$ and $z_2 = z_2\left(w^1, w^2, y, \overline{u}^k, \overline{u}^p\right)$. The role of the two distribution factors becomes immediately clear: they serve to keep the output of the domestic goods constant while allowing variation in the individual wages and the nonlabor income.

We define the adult members' conditional shares as follows $(i = 1, 2)$:

$$\rho^i\left(w^1, w^2, y, z_1, z_2\right) = w^i l^i\left(w^1, w^2, y, z_1, z_2\right) + c^i\left(w^1, w^2, y, z_1, z_2\right) - w^i. \qquad (8)$$

The shares $\rho^1\left(w^1, w^2, y, z_1, z_2\right)$ and $\rho^2\left(w^1, w^2, y, z_1, z_2\right)$ define the conditional sharing rule by distributing among the adult members the residual nonlabor income that is left after purchasing the inputs in the household production process. We thus get:

$$
\begin{aligned}
\rho^1&\left(w^1, w^2, y, z_1, z_2\right) + \rho^2\left(w^1, w^2, y, z_1, z_2\right) = \\
&y - k\left(w^1, w^2, y, z_1, z_2\right) - w^1 h_k^1\left(w^1, w^2, y, z_1, z_2\right) - w^2 h_k^2\left(w^1, w^2, y, z_1, z_2\right) \\
&- p\left(w^1, w^2, y, z_1, z_2\right) - w^1 h_p^1\left(w^1, w^2, y, z_1, z_2\right) - w^2 h_p^2\left(w^1, w^2, y, z_1, z_2\right).
\end{aligned} \tag{9}
$$

Let us introduce the following notation: $\rho\left(w^1, w^2, y, z_1, z_2\right) = \rho^1\left(w^1, w^2, y, z_1, z_2\right)$ and $\rho^2\left(w^1, w^2, y, z_1, z_2\right) = y - k - w^1 h_k^1 - w^2 h_k^2 - p - w^1 h_p^1 - w^2 h_p^2 - \rho$. Given the above, $l^1\left(w^1, w^2, y, z_1, z_2\right)$, $l^2\left(w^1, w^2, y, z_1, z_2\right)$ and $\rho\left(w^1, w^2, y, z_1, z_2\right)$ are functions of $\left(w^1, w^2, y, z_1\left(w^1, w^2, y, \overline{u}^k, \overline{u}^p\right), z_2\left(w^1, w^2, y, \overline{u}^k, \overline{u}^p\right)\right)$. However, because $\overline{u}^k$ and $\overline{u}^p$ are fixed, $l^1\left(w^1, w^2, y, z_1, z_2\right)$, $l^2\left(w^1, w^2, y, z_1, z_2\right)$ and $\rho\left(w^1, w^2, y, z_1, z_2\right)$ solely depend on $\left(w^1, w^2, y\right)$. Given cost minimization in the household production process and the abovementioned properties of the subutility functions $u^k\left(k, h_k^1, h_k^2\right)$ and $u^p\left(p, h_p^1, h_p^2\right)$, there will be unique values for the inputs in the household production process that generate the outputs $\overline{u}^k$ and $\overline{u}^p$. We denote these optimal input values by $\left(\overline{k}, \overline{h}_k^1, \overline{h}_k^2, \overline{p}, \overline{h}_p^1, \overline{h}_p^2\right)$. Using $\overline{y} = y - \overline{k} - w^1 \overline{h}_k^1 - w^2 \overline{h}_k^2 - \overline{p} - w^1 \overline{h}_p^1 - w^2 \overline{h}_p^2$, we can then define the following individual maximization programmes for the second stage of the allocation problem:

$$
\max_{l^1, c^1} u^1\left(l^1, c^1, \overline{u}^k, \overline{u}^p\right) \tag{10}
$$

subject to

$$
w^1 l^1 + c^1 = w^1 + \rho,
$$

and

$$
\max_{l^2, c^2} u^2\left(l^2, c^2, \overline{u}^k, \overline{u}^p\right) \tag{11}
$$

subject to

$$
w^2 l^2 + c^2 = w^2 + \overline{y} - \rho.
$$

Chiappori (1988, 1992) proved that the observability of both members' individual labor supply functions allows us to recover the sharing rule up to a constant and the individual preferences up to a translation. A similar result applies to the above setting with household production, provided that two distribution factors are available. The only difference between Chiappori's original setting and BCM's extension with household production is that the unidentified constant generally depends on $\overline{u}^k$ and $\overline{u}^p$.

Importantly, we do not have such an unidentified constant in our case, which implies that the sharing rule and individual preferences are completely identified. The reason is that we observe $c^1$ and $c^2$ in our data set, which obtains two boundary conditions in the individual integrability problems. Although Chiappori (1992) demonstrated that the unidentified constant is welfare irrelevant in the sense that it

does not affect indirect utilities, the above result is useful. For example, it allows us to uniquely characterize the shares that are going to the adult members. Such information is very valuable for policy makers; e.g., recall our discussion in the introduction on targeting benefits or taxes to particular household members. More importantly, as we will show below, observing the complete intrahousehold allocation of time and resources considerably facilitates the identification of the first stage of the allocation process.

Summarizing, the optimal allocation in the second stage entails completely identified conditional individual collective indirect utilities (up to an increasing transformation that generally depends on $\overline{u}^k$ and $\overline{u}^p$):

$$v^1 \left( w^1, \rho, \overline{u}^k, \overline{u}^p \right) \tag{12}$$

and

$$v^2 \left( w^2, \overline{y} - \rho, \overline{u}^k, \overline{u}^p \right).$$

## 2.3   First stage of the allocation process

Let us now look at the first stage of the allocation process. This stage focuses on the optimal allocation of the household's resources to the household public goods (via choosing the expenditures for children, the other public expenditures and the time devoted to household production) and to the adult members' shares of the residual nonlabor income. When we interpret $u^k \left( k, h_k^1, h_k^2 \right)$ and $u^p \left( p, h_p^1, h_p^2 \right)$ as standard direct utility functions, we can define the following cost or expenditure functions ($j = k, p$):

$$e^j \left( u^j, w^1, w^2 \right) = \min_{j, w^1, w^2} \left[ j + w^1 h_j^1 + w^2 h_j^2; u^j \left( j, h_j^1, h_j^2 \right) = u^j \right] = x^j. \tag{13}$$

These cost functions give the minimal expenditures $x^j$ on the inputs $\left( j, h_j^1, h_j^2 \right)$ needed to produce a quantity $u^j$ of the domestic good $j$. Since $u^k \left( k, h_k^1, h_k^2 \right)$ and $u^p \left( p, h_p^1, h_p^2 \right)$ are assumed to be homothetic, the above cost functions will be of the form:

$$e^j(u^j, w^1, w^2) = g^j \left( w^1, w^2 \right) u^j,$$

where $g^j$ is a linearly homogeneous price index.

We are now in a position to formulate the maximization programme associated with the first stage of the allocation process. This is achieved by substituting particular cardinalizations for the adult members' indirect utility functions in (12). The optimal choice of $\left( \rho^1, \rho^2, u^k, u^p \right)$ is a solution to the following maximization programme:

$$\max_{\rho^1, \rho^2, u^k, u^p} \lambda \left( w^1, w^2, y, z_1, z_2 \right) v^1 \left( w^1, \rho^1, u^k, u^p \right) \tag{14}$$
$$+ \left( 1 - \lambda \left( w^1, w^2, y, z_1, z_2 \right) \right) v^2 \left( w^2, \rho^2, u^k, u^p \right)$$

subject to

$$\rho^1 + \rho^2 + g^k\left(w^1, w^2\right)u^k + g^p\left(w^1, w^2\right)u^p = y.$$

This first stage maximization programme corresponds to the one of Chiappori and Ekeland's (2009) general collective model with both private and public consumption. The privately consumed goods are adult member $i$'s private consumption $c^i$ and leisure $l^i$, while the nonexclusive goods $u^k$ and $u^p$ are publicly consumed. Our model thus directly fits into Chiappori and Ekeland's (2009) Corollary 10, which implies the identifiability of our model. More specifically, we cannot only recover how the conditional shares $\rho^i$ are allocated to $c^i$ and $l^i$, but also how the individuals allocate the household's resources over $\rho^i$, $u^k$ and $u^p$ through Pareto efficient intrahousehold bargaining. In this respect, it is worth remarking that Chiappori and Ekeland obtain recovery up to an unidentified constant that is associated with $\rho^i$. Like before, we can identify this constant in our case because the complete intrahousehold allocation of time and resources is observed in our data.

To recover the Pareto weights, we can make use of the first-order conditions for (14):

$$\lambda\frac{\partial v^1\left(w^1, \rho^1, u^k, u^p\right)}{\partial\rho^1} = (1-\lambda)\frac{\partial v^2\left(w^2, \rho^2, u^k, u^p\right)}{\partial\rho^2}$$

$$\Longleftrightarrow \quad \lambda = \frac{\partial v^2\left(w^2, \rho^2, u^k, u^p\right)/\partial\rho^2}{\partial v^1\left(w^1, \rho^1, u^k, u^p\right)/\partial\rho^1 + \partial v^2\left(w^2, \rho^2, u^k, u^p\right)/\partial\rho^2}.$$

This demonstrates that the Pareto weights correspond to the relative marginal utilities of income for both adult members. As a result, we know the Pareto weights in addition to the adults' preferences over the public domestic goods and private consumption. This is obtained by only making use of observable allocations at the household level (see (5)). We thus conclude complete identifiability of our structural collective model.

One final remark is in order. The identification result in Chiappori and Ekeland's (2009) Corollary 10 does not depend on the presence of any distribution factors. Still, in our model, and following BCM, we assume that we have two distribution factors available. The reason is that these two factors allow the two-stage allocation representation of our structural model discussed above. This will prove useful for explicitly building our empirical model (in Section 3.2).

# 3 Empirical application

## 3.1 Data

We will apply the above collective model to a sample of households drawn from the new LISS panel (Longitudinal Internet Studies for the Social sciences) that is gathered by CentERdata. The basic panel consists of 5000 households (comprising

8000 individuals) and is representative for the Dutch population.[13] The first wave of the LISS panel was gathered in 2008. The LISS Core Study is a longitudinal study, which is repeated yearly and is designed to follow changes in the life course and living conditions of the panel members. It is comparable in content to standard social surveys. In addition to the LISS Core Study, researchers can collect their own data via online questionnaires to the panel members. We made use of this possibility to gather the data necessary to conduct our analysis. This obtains a unique data set that combines detailed assignable expenditures and time use information with a battery of data gathered through a regular social survey.

We added a module on time use and consumption to the LISS panel. This questionnaire was given to all household members who were at least sixteen years old. The time use data were collected by means of survey questions about the time spent on a set of time use categories during the past seven days. As indicated by Browning and Gørtz (2006), such questions can be informative and have the advantage that they avoid infrequency problems associated with diary-based surveys. Moreover, they are less time consuming than surveys based on detailed diaries. LISS-respondents had to fill out their time use on thirteen exhaustive categories during the past seven days.[14] For each category, a number of activities was given as an example along with other useful information.[15]

The consumption module is also based on survey questions on normal individual and household (nondurable) expenditures. The set up of the questionnaire is partly based on the recommendations by Browning, Crossley and Weber (2003), who conclude that, although survey measures are more noisy than diary measures, they do contain a useful signal on individual consumption. A first set of questions refers to expenditures on twelve categories of goods and services that can be argued to be publicly consumed by the household. Examples of such expenditures are (imputed) rent, expenditures on utilities or formal day care for young children. Although expenditures on food at home are intrinsically private in nature, expenditures on food and drinks used at home and outside home with (other members of) the household appeared in the public expenditures categories. Still, we added a follow-up question where all respondents had to indicate how much of these expenditures they personally consumed. The next set of questions in the consumption module refers to the private expenditures of the respondents, which were gathered in nine categories. Examples of such personal expenditures were food and drinks consumed outside home (but not with other members of the household), clothing, expenditures on leisure activities or personal care. The questionnaire on private expenditures of children less than sixteen

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[13]Households without any internet access are provided with a basic computer (a 'SimPC') that enables them to connect to the internet.

[14]Since we wanted to maximize the response rate, the questionnaires were online in early September and early December. It is expected that the time use for regular working weeks is fairly well captured. However, the normal yearly time use on summer holidays will be underestimated.

[15]See the Appendix for detailed lists of the time use categories, public expenditure categories and private expenditure categories.

years old were responded by one of the adult members.

The set of households used for this study was subject to the following sample selection rules. We focused attention on couples with children, where both adult members participate to the labor market. After deleting the households with important missing information (mostly, incomplete information of one of the spouses), we obtained a sample of 212 couples with children. This sample is relatively small. Future research could focus on including non-participation to the labor market in our theoretical model, which would yield a considerable number of extra observations; see also are discussion in the concluding section.

As mentioned before, the time use and consumption module is complemented with information from the LISS Core Study. More specifically, information on the household composition, the ages of the household members, their wage rates and the individual and household nonlabor incomes are added. Importantly, the wage rates were not derived by dividing an individual's labor income by the number of hours worked according to the time use module, but rather through division of the labor income by the hours worked according to the LISS Core Study. This will help us to avoid division bias in our empirical application. As indicated before, we assume two distribution factors for our collective labor supply model. The distribution factors in this study will be the husband's share in the spouses' individual nonlabor incomes and their age difference.

To conclude this data subsection, we report some summary statistics on the core variables used in this study. The adult members' private expenditures ($c^1$ and $c^2$) are equal to the sum of all the individual expenditures on the nine categories of goods and services in the private expenditures questionnaire and the own share in the expenditures on food and drinks (which is obtained by the follow-up question on own food consumption in the public expenditures questionnaire). The expenditures on children ($k$) are equal to the sum of the expenditures on child care (obtained through the public expenditures questionnaire) and the sum of all the children's private expenditures (obtained through the private expenditures questionnaire filled out either by children who are at least sixteen years old or by one the adult members for children less than sixteen years old). The expenditures on the other public good ($p$) equal the sum of the expenditures on the twelve categories in the public expenditures questionnaire minus the expenditures on food and drinks and expenses on child care. The adult members' time spent on market labor ($m^1$ and $m^2$) is equal to the sum of the time spent on paid work and commuting. Parental time invested in children ($h_k^1$ and $h_k^2$) includes all time spent on activities with children (such as dressing, playing, going to the doctor, etc.). Finally, time spent on other public goods ($h_p^1$ and $h_p^2$) equals the sum of all time spent on home work (cleaning, gardening, cooking, etc.) and administrative tasks related to the own household.

As is clear from Table 1, wives have, on average, slightly less private expenditures (302 euro per month) than their husbands (311 euro per month). Expenditures on children are, on average, equal to about 479 euro per month. Most of the household's

Table 1: Summary statistics LISS couples with children

| | Husband | | Wife | | Household | |
|---|---|---|---|---|---|---|
| | Mean | Std. dev. | Mean | Std. dev. | Mean | Std. dev. |
| Expenditures | | | | | | |
| Private expenditures (EUR per month) | 311.08 | 217.10 | 302.64 | 182.41 | | |
| Expenditures on children (EUR per month) | | | | | 479.15 | 493.21 |
| Other public expenditures (EUR per month) | | | | | 1827.77 | 875.32 |
| Time use | | | | | | |
| Market labor (hours per week; incl. commuting) | 47.71 | 11.60 | 28.54 | 11.32 | | |
| Child care (hours per week) | 9.04 | 8.52 | 15.43 | 13.79 | | |
| Other home work (hours per week) | 11.66 | 8.22 | 20.65 | 11.75 | | |
| Socio-economic variables | | | | | | |
| Age | 42.89 | 7.43 | 40.61 | 7.37 | | |
| Wage rate (EUR per hour) | 10.71 | 3.76 | 9.83 | 5.19 | | |
| Household nonlabor income (EUR per month) | | | | | 130.00 | 330.11 |
| Husband's share in individual nonlabor income | | | | | 0.59 | 0.21 |
| Age difference | | | | | 2.27 | 3.14 |
| Number of children | | | | | 2.02 | 0.75 |

expenditures are spent on other public goods: on average, households spend about 1828 euro per month on these goods. Next, when looking at the adult members' time use, we find that husbands spend substantially more hours on market work than their wives: on average, husbands work about 47.7 hours per week for pay (including commuting), while wives supply about 28.5 hours per week. A different picture emerges when focusing on child care and other home work. Husbands spend on average about 9 hours per week to their children, while they are engaged in other home work for about 11.7 hours per week. The figures for women are almost double: average time devoted to child care and other home work equals respectively about 15.4 and 20.7 hours per week. If we add both market work and home work together, it appears that husbands work a bit more than their wives (68.4 versus 64.6 hours per week). This result does not differ that much from results obtained in earlier studies based on different data (see, e.g., Burda, Hamermesh and Weil, 2008).

## 3.2 Parametric specification

To introduce the parametric specification that we use for our theoretical model, we shall refer to the model's two-stage allocation representation discussed before. The reason is that it turns out to be impossible to derive a flexible closed form specification for the observables on the basis of a direct utility representation of the adult members' preferences, if we want to preserve the assumption that leisure and individual consumption are not separable from the unobserved outputs of the household production

14

process. Essentially, the two-stage allocation representation allows us to make use of individual indirect utility functions, which effectively facilitates the derivation of a rather flexible functional reduced form for the observables.

We assume that the second stage's adult members' preferences over leisure and own consumption, conditional on the amount of domestic goods produced, can be represented by the following indirect utility functions ($i = 1, 2$):

$$v^i \left(w^i, \rho^i, \overline{u}^k, \overline{u}^p\right) = \frac{\ln \left(w^i + \rho^i\right) - \ln a^i \left(w^i; \overline{u}^k, \overline{u}^p\right)}{\left(w^i\right)^{\beta^i}}, \tag{15}$$

where $\ln a^i \left(w^i; \overline{u}^k, \overline{u}^p\right) = \left(\alpha_1^i + \alpha_2^i \ln \overline{u}^k + \alpha_3^i \ln \overline{u}^p\right) \ln w^i$. These indirect utility functions belong to the PIGLOG class. In fact, using that the Hicksian composite commodity has a normalized price equal to one, they exactly coincide with the indirect utility function underlying Deaton and Muellbauer's (1980) Almost Ideal Demand System. Applying Roy's rule to these indirect utility functions results in the following (conditional) Marshallian demand for leisure and own consumption:

$$
\begin{aligned}
l^i &= \left[\left(\alpha_1^i + \alpha_2^i \ln \overline{u}^k + \alpha_3^i \ln \overline{u}^p\right) + \beta^i \ln \left(\frac{w^i + \rho^i}{a^i \left(w^i; \overline{u}^k, \overline{u}^p\right)}\right)\right] \frac{\left(w^i + \rho^i\right)}{w^i} \\
c^i &= \left[\left(1 - \alpha_1^i - \alpha_2^i \ln \overline{u}^k - \alpha_3^i \ln \overline{u}^p\right) - \beta^i \ln \left(\frac{w^i + \rho^i}{a^i \left(w^i; \overline{u}^k, \overline{u}^p\right)}\right)\right] \left(w^i + \rho^i\right).
\end{aligned}
\tag{16}
$$

Let us now focus on the first stage allocation of the household's nonlabor income $y$ to $\left(\rho^1, \rho^2, u^k, u^p\right)$ (see (14)). We first need to specify the household production technologies that transform expenditures on public goods and the spouses' time spent on home production into the domestic goods $u^k$ and $u^p$. We assume that these technologies are of the Cobb-Douglas form ($j = k, p$):

$$u^j \left(j, h_j^1, h_j^2\right) = \gamma_j^1 \ln h_j^1 + \gamma_j^2 \ln h_j^2 + \gamma_j^3 \ln j, \tag{17}$$

where $\sum_i \gamma_j^i = 1$.

The above technologies are homothetic, which implies that the budget constraint associated with the first stage of the allocation process has a simple form that is linear in $\left(\rho^1, \rho^2, u^k, u^p\right)$. Given the above specifications of the individual indirect utility functions and the household production technologies, the first stage maximization programme boils down to:

$$
\begin{aligned}
\max_{\rho^1, \rho^2, u^k, u^p} &\ \lambda \left(w^1, w^2, y, z_1, z_2\right) \left(\frac{\ln \left(w^1 + \rho^1\right) - \ln a^1 \left(w^1; u^k, u^p\right)}{\left(w^1\right)^{\beta^1}}\right) \\
&+ \left(1 - \lambda \left(w^1, w^2, y, z_1, z_2\right)\right) \left(\frac{\ln \left(w^2 + \rho^2\right) - \ln a^2 \left(w^2; u^k, u^p\right)}{\left(w^2\right)^{\beta^2}}\right)
\end{aligned}
$$

subject to

$$\rho^1 + \rho^2 + g^k\left(w^1, w^2\right) u^k + g^p\left(w^1, w^2\right) u^p = y,$$

where $g^j\left(w^1, w^2\right) = \left[\left(w^1\right)^{\gamma_j^1}\left(w^2\right)^{\gamma_j^2}\right]$ $(j = k, p)$. A sufficient condition for a theoretically consistent first stage allocation is that the parameters $\alpha_2^i$ and $\alpha_3^i$ in the functions $\ln a^i\left(w^1; u^k, u^p\right)$ $(i = 1, 2)$ are negative. This condition will be imposed in the estimation process by the functions $\alpha_l^i = -\exp\left(\widetilde{\alpha}_l^i\right)$, where $\widetilde{\alpha}_l^i$ is estimated $(i = 1, 2; l = 2, 3)$.

Assuming an interior solution, the Lagrangian associated with the above maximization problem results in the following first-order conditions (where $\mu$ is the Lagrange multiplier):

$$\frac{\partial \mathcal{L}}{\partial \rho^1} = \frac{\lambda}{\left(w^1\right)^{\beta^1}} \frac{1}{\left(w^1 + \rho^1\right)} - \mu = 0$$

$$\frac{\partial \mathcal{L}}{\partial \rho^2} = \frac{1 - \lambda}{\left(w^2\right)^{\beta^2}} \frac{1}{\left(w^2 + \rho^2\right)} - \mu = 0$$

$$\frac{\partial \mathcal{L}}{\partial u^k} = -\frac{\lambda}{\left(w^1\right)^{\beta^1}} \frac{\alpha_2^1 \ln w^1}{u^k} - \frac{(1 - \lambda) \alpha_2^2 \ln w^2}{\left(w^2\right)^{\beta^2}} \frac{}{u^k} - \mu g^k\left(w^1, w^2\right) = 0$$

$$\frac{\partial \mathcal{L}}{\partial u^p} = -\frac{\lambda}{\left(w^1\right)^{\beta^1}} \frac{\alpha_3^1 \ln w^1}{u^p} - \frac{(1 - \lambda) \alpha_3^2 \ln w^2}{\left(w^2\right)^{\beta^2}} \frac{}{u^p} - \mu g^p\left(w^1, w^2\right) = 0$$

$$\frac{\partial \mathcal{L}}{\partial \mu} = y - \rho^1 - \rho^2 - g^k\left(w^1, w^2\right) u^k - g^p\left(w^1, w^2\right) u^p.$$

Rewriting obtains:

$$w^1 + \rho^1 = \frac{1}{\mu} \frac{\lambda}{\left(w^1\right)^{\beta^1}}$$

$$w^2 + \rho^2 = \frac{1}{\mu} \frac{(1 - \lambda)}{\left(w^2\right)^{\beta^2}} \tag{18}$$

$$g^k\left(w^1, w^2\right) u^k = \frac{1}{\mu}\left[-\frac{\lambda}{\left(w^1\right)^{\beta^1}} \alpha_2^1 \ln w^1 - \frac{(1 - \lambda)}{\left(w^2\right)^{\beta^2}} \alpha_2^2 \ln w^2\right]$$

$$g^p\left(w^1, w^2\right) u^p = \frac{1}{\mu}\left[-\frac{\lambda}{\left(w^1\right)^{\beta^1}} \alpha_3^1 \ln w^1 - \frac{(1 - \lambda)}{\left(w^2\right)^{\beta^2}} \alpha_3^2 \ln w^2\right].$$

Summing left- and right-hand sides of these equations, while taking into account that they add up to the household's full budget, results in:

$$\begin{aligned}
w^1 + w^2 + y &= \frac{1}{\mu}\left[\frac{\lambda}{\left(w^1\right)^{\beta^1}}\left[1 - \left(\alpha_2^1 + \alpha_3^1\right) \ln w^1\right] + \frac{(1 - \lambda)}{\left(w^2\right)^{\beta^2}}\left[1 - \left(\alpha_2^2 + \alpha_3^2\right) \ln w^2\right]\right] \\
&\equiv \frac{1}{\mu} X\left(w^1, w^2, \lambda\right),
\end{aligned}$$

which allows us to derive an expression for $\frac{1}{\mu}$. Substituting this expression in (18) gives the following closed form solutions to the first stage maximization problem:

$$\rho^1 = \frac{w^1 + w^2 + y}{X\left(w^1, w^2, \lambda\right)} \frac{\lambda}{\left(w^1\right)^{\beta^1}} - w^1$$

$$\rho^2 = \frac{w^1 + w^2 + y}{X\left(w^1, w^2, \lambda\right)} \frac{(1 - \lambda)}{\left(w^2\right)^{\beta^2}} - w^2$$

$$u^k = \frac{w^1 + w^2 + y}{X\left(w^1, w^2, \lambda\right)} \frac{1}{g^k\left(w^1, w^2\right)} \left[-\frac{\lambda}{\left(w^1\right)^{\beta^1}} \alpha_2^1 \ln w^1 - \frac{(1 - \lambda)}{\left(w^2\right)^{\beta^2}} \alpha_2^2 \ln w^2\right] \qquad (19)$$

$$u^p = \frac{w^1 + w^2 + y}{X\left(w^1, w^2, \lambda\right)} \frac{1}{g^p\left(w^1, w^2\right)} \left[-\frac{\lambda}{\left(w^1\right)^{\beta^1}} \alpha_3^1 \ln w^1 - \frac{(1 - \lambda)}{\left(w^2\right)^{\beta^2}} \alpha_3^2 \ln w^2\right].$$

Finally, following Browning, Chiappori and Lewbel (2008) we assume that the Pareto weight of the first adult member is of the form:

$$\lambda\left(w^1, w^2, y, z_1, z_2\right) = \frac{\exp\left(\Lambda_1 + \Lambda_2 \frac{w^1}{w^2} + \Lambda_3 y + \Lambda_4 z_1 + \Lambda_5 z_2\right)}{1 + \exp\left(\Lambda_1 + \Lambda_2 \frac{w^1}{w^2} + \Lambda_3 y + \Lambda_4 z_1 + \Lambda_5 z_2\right)}. \qquad (20)$$

Clearly, this Pareto weight will be between zero and one as the theory requires. By construction, this property extends to the spouse's Pareto weight.

To obtain the individuals' leisure and private consumption as functions of $(w^1, w^2, y, z_1, z_2)$, we substitute the first stage functions (19) in the second stage functions (16) $(i = 1, 2)$:

$$l^i = \left[A^i + \beta^i \left(\ln\left(\frac{w^1 + w^2 + y}{X\left(w^1, w^2, \lambda\right)} \frac{\lambda^i}{\left(w^i\right)^{\beta^i}}\right) - A^i \ln w^i\right)\right] \times \frac{\left(\frac{w^1 + w^2 + y}{X\left(w^1, w^2, \lambda\right)} \frac{\lambda^i}{\left(w^i\right)^{\beta^i}}\right)}{w^i}$$

$$c^i = \left[\left(1 - A^i\right) - \beta^i \left(\ln\left(\frac{w^1 + w^2 + y}{X\left(w^1, w^2, \lambda\right)} \frac{\lambda^i}{\left(w^i\right)^{\beta^i}}\right) - A^i \ln w^i\right)\right] \times \left(\frac{w^1 + w^2 + y}{X\left(w^1, w^2, \lambda\right)} \frac{\lambda^i}{\left(w^i\right)^{\beta^i}}\right),$$

where

$$A^i \equiv \alpha_1^i + \alpha_2^i \ln\left(\frac{w^1 + w^2 + y}{X\left(w^1, w^2, \lambda\right)} \frac{1}{g^k\left(w^1, w^2\right)} \left[-\frac{\lambda}{\left(w^1\right)^{\beta^1}} \alpha_2^1 \ln w^1 - \frac{(1 - \lambda)}{\left(w^2\right)^{\beta^2}} \alpha_2^2 \ln w^2\right]\right)$$

$$+ \alpha_3^i \ln\left(\frac{w^1 + w^2 + y}{X\left(w^1, w^2, \lambda\right)} \frac{1}{g^p\left(w^1, w^2\right)} \left[-\frac{\lambda}{\left(w^1\right)^{\beta^1}} \alpha_3^1 \ln w^1 - \frac{(1 - \lambda)}{\left(w^2\right)^{\beta^2}} \alpha_3^2 \ln w^2\right]\right)$$

and

$$
\begin{aligned}
\lambda &= \lambda\left(w^1, w^2, y, z_1, z_2\right) \\
\lambda^1 &= \lambda\left(w^1, w^2, y, z_1, z_2\right) \\
\lambda^2 &= 1 - \lambda\left(w^1, w^2, y, z_1, z_2\right).
\end{aligned}
$$

To obtain the inputs of the household production process as functions of $(w^1, w^2, y, z_1, z_2)$, we start from the cost functions $g^k\left(w^1, w^2\right) u^k$ and $g^p\left(w^1, w^2\right) u^p$ that are associated with the two domestic goods (see (13)). Applying Shephard's lemma to these cost functions, and substituting the observable expressions for $u^k$ and $u^p$ obtained via the first stage allocation (19) in the resulting Hicksian demands, obtains the following specification for the observable inputs of the household production process:

$$
\begin{aligned}
h_k^1 &= \frac{\gamma_k^1}{w^1} \frac{w^1 + w^2 + y}{X\left(w^1, w^2, \lambda\right)} \left[ -\frac{\lambda}{\left(w^1\right)^{\beta^1}} \alpha_2^1 \ln w^1 - \frac{(1-\lambda)}{\left(w^2\right)^{\beta^2}} \alpha_2^2 \ln w^2 \right] \\
h_k^2 &= \frac{\gamma_k^2}{w^2} \frac{w^1 + w^2 + y}{X\left(w^1, w^2, \lambda\right)} \left[ -\frac{\lambda}{\left(w^1\right)^{\beta^1}} \alpha_2^1 \ln w^1 - \frac{(1-\lambda)}{\left(w^2\right)^{\beta^2}} \alpha_2^2 \ln w^2 \right] \\
k &= \left(1 - \gamma_k^1 - \gamma_k^2\right) \frac{w^1 + w^2 + y}{X\left(w^1, w^2, \lambda\right)} \left[ -\frac{\lambda}{\left(w^1\right)^{\beta^1}} \alpha_2^1 \ln w^1 - \frac{(1-\lambda)}{\left(w^2\right)^{\beta^2}} \alpha_2^2 \ln w^2 \right] \\
h_p^1 &= \frac{\gamma_p^1}{w^1} \frac{w^1 + w^2 + y}{X\left(w^1, w^2, \lambda\right)} \left[ -\frac{\lambda}{\left(w^1\right)^{\beta^1}} \alpha_3^1 \ln w^1 - \frac{(1-\lambda)}{\left(w^2\right)^{\beta^2}} \alpha_3^2 \ln w^2 \right] \\
h_p^2 &= \frac{\gamma_p^2}{w^2} \frac{w^1 + w^2 + y}{X\left(w^1, w^2, \lambda\right)} \left[ -\frac{\lambda}{\left(w^1\right)^{\beta^1}} \alpha_3^1 \ln w^1 - \frac{(1-\lambda)}{\left(w^2\right)^{\beta^2}} \alpha_3^2 \ln w^2 \right] \\
p &= \left(1 - \gamma_p^1 - \gamma_p^2\right) \frac{w^1 + w^2 + y}{X\left(w^1, w^2, \lambda\right)} \left[ -\frac{\lambda}{\left(w^1\right)^{\beta^1}} \alpha_3^1 \ln w^1 - \frac{(1-\lambda)}{\left(w^2\right)^{\beta^2}} \alpha_3^2 \ln w^2 \right].
\end{aligned}
$$

The system that is brought to the data thus consists of 10 equations. More specifically, we will model $\left(l^1, c^1, l^2, c^2, h_k^1, h_k^2, k, h_p^1, h_p^2, p\right)$ as observable functions of $(w^1, w^2, y, z_1, z_2)$.

To avoid an overspecified model, we decided not to include any taste shifters in the individual preferences. Given the restrictions on the selected sample, it can be argued that this is not too much of an issue here. However, we do account for observed heterogeneity with respect to the number of children $(K)$ in the production process, by using the functions $\gamma_j^i = \frac{\exp\left(\gamma_{j1}^i + \gamma_{j2}^i K\right)}{1 + \exp\left(\gamma_{j1}^i + \gamma_{j2}^i K\right) + \exp\left(\gamma_{j1}^i + \gamma_{j2}^i K\right)}$ $(i = 1, 2; j = k, p)$, where $\gamma_{j1}^1, \gamma_{j2}^1, \gamma_{j1}^2$ and $\gamma_{j2}^2$ are estimated. These functions guarantee that the $\gamma_j^i$'s are all positive and are smaller than one, as the theory requires.

To account of unobservable heterogeneity across households, we add additive errors to the system equations. These errors are assumed to be uncorrelated across

households but are allowed to correlate across commodities within households. The system is estimated by means of the feasible generalized nonlinear least squares estimator (see Greene, 2008).

## 3.3 Results

### 3.3.1 Estimation results

The system of equations forming our structural collective model is highly nonlinear. Therefore, it comes as no surprise that we found several local minima. We selected the lowest local minimum found. Still, we also conducted a robustness check with the parameter values from the other minima found. It is comforting that these additional exercises provided a broad picture that is qualitatively similar to the one reported here.

Table 2 shows our estimation results; adult member 1 is the husband and adult member 2 is the wife in the household. Despite our relatively small dataset, most parameters turn out to be precisely estimated.[16] Leisure appears to be a luxury good for both husbands and wives since the estimates of $\beta^1$ and $\beta^2$ are positive. Leisure and individual consumption turn out to be nonseparable from the outputs of the household production process: both domestic goods have a significant impact on both spouses' leisure and consumption (see the estimates of $\widetilde{\alpha}_2^1$, $\widetilde{\alpha}_3^1$, $\widetilde{\alpha}_2^2$ and $\widetilde{\alpha}_3^2$).

Let us now focus on the household production technologies. The estimates are most easily interpreted when we take into account how the estimates of $\gamma_{jl}^i$ ($i = 1, 2$; $j = p, k$; $l = 1, 2$) feed into the estimate of $\gamma_j^i$ (which -to recall- equals $\frac{\exp\left(\gamma_{j1}^1 + \gamma_{j2}^1 K\right)}{1 + \exp\left(\gamma_{j1}^1 + \gamma_{j2}^1 K\right) + \exp\left(\gamma_{j1}^2 + \gamma_{j2}^2 K\right)}$ for $K$ the number of children). The latter coefficients can be interpreted as the cost shares of the inputs for given total production costs of the domestic goods. It turns out that a higher number of children is, all else equal, associated with a lower average cost share of male time spent on children in the production of the public good $u^k$ while it is associated with higher cost shares of female time spent on children and expenditures on children. The same story applies to the production of the public good $u^p$.

To conclude, we consider the parameters in the husband's Pareto weight. An increase in the husband's relative wage turns out to have a significantly positive impact on his Pareto weight, ceteris paribus. Importantly, because the husband and the wife have different preferences, this implies a strong rejection of the unitary model (which -to recall- models households as if they were single decision making units). Further, we find that the household's nonlabor income does not seem to

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[16]The covariance matrix associated with feasible generalized nonlinear least squares makes use of so-called pseudoregressors that involve derivatives of the regression functions. These derivatives were obtained numerically by means of the method of Goldfeldt and Quandt. Still, the approximation error can be substantial given our highly nonlinear system with parameters simultaneously appearing in many terms (see Greene, 2008).

Table 2: Structural estimation results

| | Estimate | Std. error |
|---|---|---|
| **Preference parameters** | | |
| $\alpha_{11}^1$ | 0.98* | 0.07 |
| $\widetilde{\alpha}_2^1$ $[u^k]$ | -2.32* | 0.14 |
| $\widetilde{\alpha}_3^1$ $[u^p]$ | -1.34* | 0.28 |
| $\beta^1$ | 0.20* | 0.02 |
| $\alpha_{11}^2$ | 0.94 | 0.79 |
| $\widetilde{\alpha}_2^2$ $[u^k]$ | -2.88* | 0.02 |
| $\widetilde{\alpha}_3^2$ $[u^p]$ | -2.70* | 0.02 |
| $\beta^2$ | 0.14* | 0.02 |
| **Household production parameters** | | |
| $\gamma_{k1}^1$ | 1.54* | 0.02 |
| $\gamma_{k2}^1$ [children] | -0.49* | 0.02 |
| $\gamma_{k1}^2$ | 0.27* | 0.05 |
| $\gamma_{k2}^2$ [children] | -0.02 | 0.02 |
| $\gamma_{p1}^1$ | -0.16* | 0.07 |
| $\gamma_{p2}^1$ [children] | -0.10* | 0.03 |
| $\gamma_{p1}^2$ | -1.65* | 0.11 |
| $\gamma_{p2}^2$ [children] | 0.12* | 0.01 |
| **Pareto weight parameters** | | |
| $\Lambda_1$ | -1.43* | 0.06 |
| $\Lambda_2$ $[w^1/w^2]$ | 1.25* | 0.29 |
| $\Lambda_3$ $[y]$ | 1.65 | 3.46 |
| $\Lambda_4$ [husband's share in nonlabor income] | 0.00* | 0.00 |
| $\Lambda_5$ [age husband minus age wife] | 0.16* | 0.05 |

Note: Coefficient estimates were obtained by the feasible generalized nonlinear least squares estimator. An asterisk denotes significance at the 5 per cent significance level. The expressions in brackets refer to the objects that are related to the respective parameters.

have a significant impact on the husband's Pareto weight. However, the husband's share in the household's nonlabor income does have a significantly positive (albeit economically negligible) impact on his Pareto weight. We remark that this implies a further rejection of the income pooling hypothesis (consistent with earlier results mentioned in the introduction). Finally, we observe that the age difference between the husband and his spouse positively and significantly influences his Pareto weight.

### 3.3.2 Changing wages: effects on the intrahousehold allocation of resources

Given the complexity of our model, the magnitudes of the estimated parameters are not always easy to grasp. Therefore, in what follows we provide some graphical illustrations of the impact of male and female wage changes on the dependent variables in our model. Generally, this impact depends on the complex interaction between individual preferences, intrahousehold bargaining and the household's production technologies. Our following discussion will illustrate the usefulness of the collective model for assessing the effects of wage changes on the intrahousehold consumption of private goods and leisure, and on the production of public goods (including children's welfare). As discussed in the introduction, such an analysis can be instrumental for evaluating targeting considerations related to (in casu earned income) benefits or taxes.

Figure 1 focuses on the impact of a change in the husband's wage on the dependent variables. The wage change runs from the first decile to the tenth decile in the male wage distribution, while the other explanatory variables are fixed at their means (including the female's wage). The upper left panel of the figure focuses on the leisure and the private consumption of both husbands and wives. The upper right panel shows the time spent on market labor. The two panels at the bottom show the time and expenditures spent on children and the other domestic good (respectively in the left and the right panel).

As is clear from the figure, the husband's time spent on market work increases when his wage increases. Such a result would also be observed in a standard labor supply model when the substitution effect dominates the income effect. This increase is accompanied by not only a decrease in his leisure but also a decrease in the time spent on children and other household work. At the same time, the husband's own private consumption rises dramatically. In addition, also the household's expenditures on both public goods increase. This clearly illustrates the trade-off between, on the one hand, own consumption and, on the other hand, leisure and the utility derived from the public goods ($u^k$ and $u^p$).

A different picture emerges when we look at the impact on the female dependent variables of changing the male wage. It turns out that female leisure decreases as well, while we observe an increase in time spent on house work and market labor (at least when the initial male wage is sufficiently high). Note that this compensates the male's
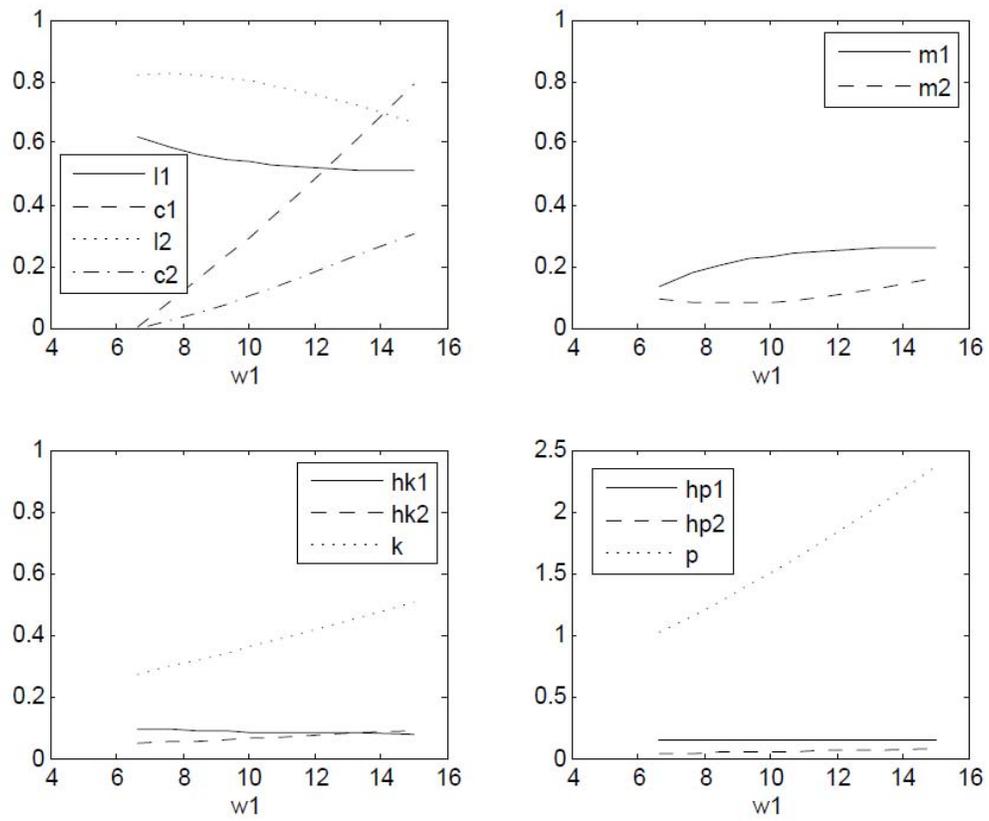
Figure 1: Impact of change in male wage on dependent variables

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decrease in time spent on children and the other domestic good. Interestingly, also the wife's private consumption increases when the husband's wage increases, but at a slower pace than the husband's private consumption. Of course, this should not be too surprising. Since both spouses supply more market labor hours, the couple's income increases. Part of this income is spent on the public goods, while the remainder is allocated to both spouses' private consumption. The latter happens more in favor of the husband, given his increased Pareto weight following his wage rise.

Figure 2 illustrates the impact of a change in the wife's wage on the dependent variables (while, again, keeping the other explanatory variables fixed at their means). The pattern in this figure is somewhat different from the one in Figure 1. While an increase in the male wage decreases the husband's leisure, we now find that the wife's leisure initially increases and then slightly decreases when her wage increases. A reverse picture applies to market labor: when the initial wage is low, female time spent on market labor decreases when wage increases, while it increases again when her wage gets above the average wage.

Next, we obtain a similar picture as before for household production. Specifically, time spent on children and the other domestic good decreases when female wage increases. This decrease is compensated by an increase in the time spent on both public goods by the husband. At the same time male leisure decreases while male time spent on market work initially increases. This last effect clearly demonstrates the negative impact on the husband's Pareto weight of an increase in the wife's wage.

As for consumption, it is clear from Figure 2 that a rise in the wife's wage also implies an increase in both spouses' private consumption, though the pattern clearly differs. In addition, there is an increase in the expenditures on both public goods. This again illustrates the trade-off between, on the one hand, private consumption and leisure and, on the other hand, the utility derived from the public goods.

### 3.3.3 Mothers versus fathers: who cares more for their children (and other public consumption)?

So far, we have focused on the impact of male and female wage changes on the key variables of the model. A closely related exercise analyzes the extent to which differences between the husbands' and wives' preferences influence the 'amounts' of public goods ($u^k$ and $u^p$) that are produced inside the household. Such an analysis allows us to shed light on a statement like "mothers care more for children than fathers", which is often made with respect to the widely observed phenomenon that changes of the male and female nonlabor incomes have a differential impact on the health of children and expenditures on children (see, for example, Thomas, 1990, and Lundberg, Pollak and Wales, 1997). To tackle this question, we focus on couples with characteristics equal to their average in the population. To facilitate a ceteris paribus comparison, the average of the spouses' wages is the average across both sexes (which equals about ten euro). Specifically, the full (respectively dotted) lines in Figure 3
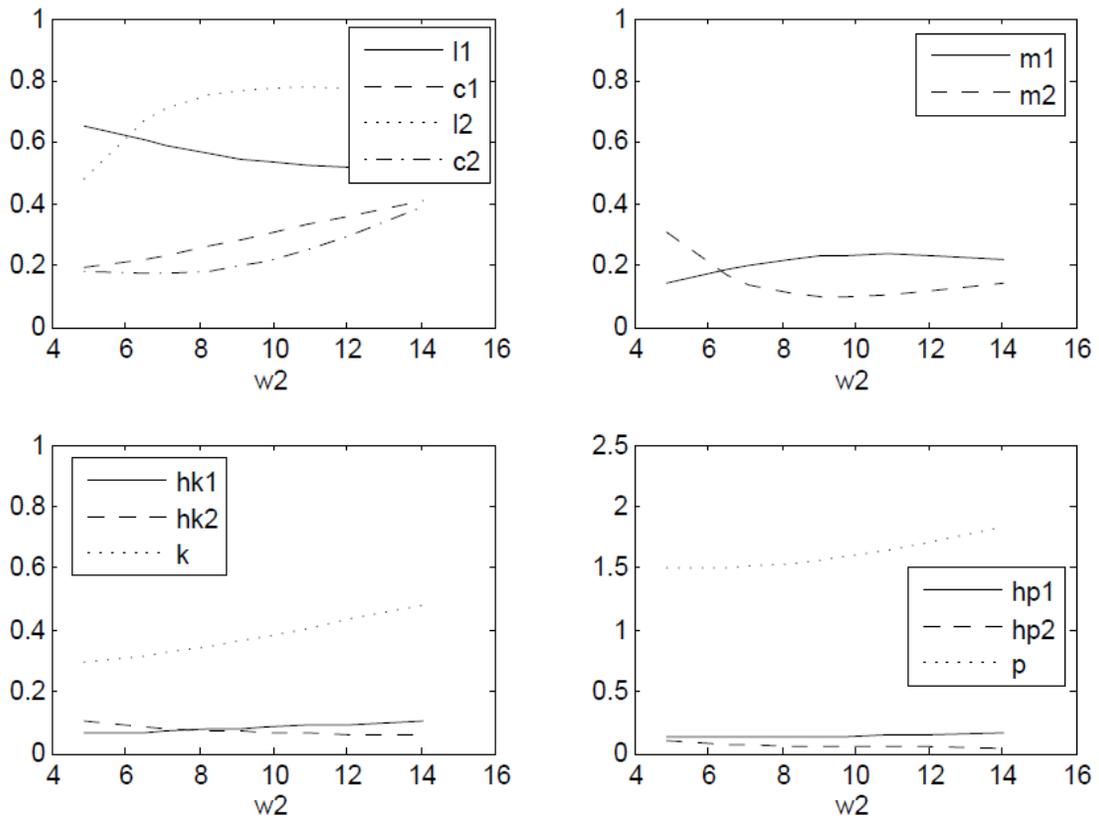
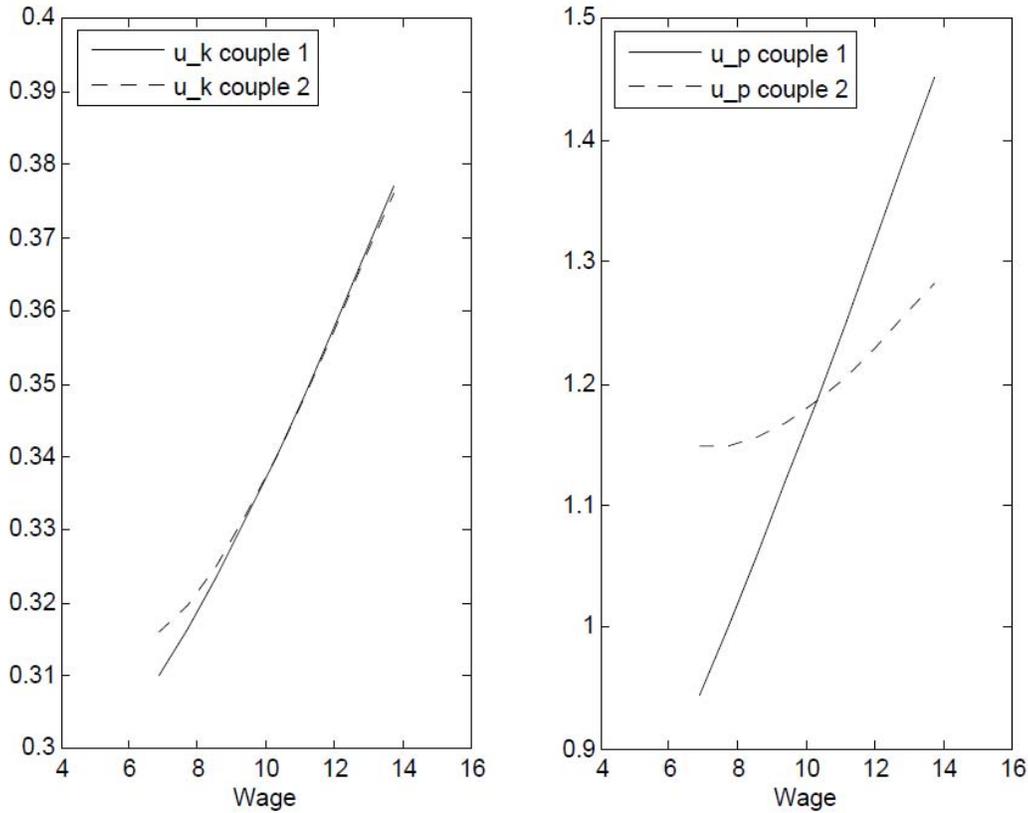Figure 2: Impact of change in female wage on dependent variables

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Figure 3: Impact of change in male and female wages on household goods

correspond to a couple where the wife's (respectively husband's) wage is fixed to this average wage. The left panel then shows the impact of an increase in respectively the husband's and the female's wage on the children's utility ($u^k$). The right panel shows the impact of wage changes on the production of the other domestic good ($u^p$).

The left panel of Figure 3 all but suggests that mothers effectively care more for children than fathers. To see this, compare two couples. The first couple is situated on the full line (which -to recall- corresponds to an average wage for the female that equals ten euro). Assume for this couple that the husband has an hourly wage of, say, seven euro (which is below the average wage). The second couple is situated on the dotted line (with an average wage for the male that equals ten euro), and we now assume that the female has an hourly wage of seven euro. The figure illustrates that the children's utility is actually higher in the second couple. This provides evidence against the above statement, since in the first couple the wife has a higher bargaining power than in the second couple. This higher bargaining power is reflected in a higher Pareto weight for the wife, which implies that the household's choices are more in line with her own preferences. A similar conclusion holds if we compare a couple on the full line where the husband has a higher than average wage (say, thirteen

euro) with a couple on the dotted line where the wife has a higher than average wage (again thirteen euro): the utility of children is higher for the first couple, although the difference is now much less pronounced than before. Again, this implies that a higher bargaining power for the female is associated with a (slightly) lower utility for the children, which provides all but evidence in favor of the above statement. Finally, if we consider the right panel of Figure 3, which pertains to the other domestic good $(u^p)$, we observe basically the same pattern as for the children's utility. In fact, the effects are even more pronounced in this case.

# 4   Conclusion

We proposed a collective labor supply model with household production that generalizes an original model of BCM. Adults' individual preferences do not only depend on own leisure and the individual private consumption of market goods, but also on the consumption of domestic goods. This last category of goods is the output of combining goods bought at the market with individuals' time. For example, one of our domestic goods is the children's welfare, which is produced by expenditures on children and parental time invested in the children. In our model, we also allow for other domestic goods, which extends the original analysis of BCM.

   We applied our model to new and unique data with detailed information about the individuals' time use and the intrahousehold allocation of all expenditures. The application uses a novel estimation strategy that builds upon the familiar two-stage allocation representation of the collective model. This estimation strategy allows for a flexible functional specification of observables, which is interesting from an empirical point of view.

   Our empirical results for a sample of Dutch couples with children reveal some interesting patterns. Firstly, the spouses' preferences turn out to significantly depend on the consumption of domestically produced goods (including children's welfare). Next, the spouses' Pareto weights vary significantly with the individual wages and the share in the household's nonlabor income. Finally, we do not find evidence that mothers care more for their children than fathers. By contrast, when focusing on a couple with average characteristics, our results even suggest an opposite pattern for the sample under study. This is at odds with earlier results that were mainly obtained for data from developing countries (see, e.g., Thomas, 1990).

   One issue that we left for future work is that of non-participation in employment; see Donni (2003) and Blundell, Chiappori, Magnac and Meghir (2007) for collective models with non-participation in a setting without household production. Such an extension with non-participation would not only be useful from a theoretical perspective. It would also greatly benefit practical applications, in that it can imply a lot of additional degrees of freedom (e.g., women with young children who do not supply any market labor). A closely related issue is that children should probably be treated as endogenous in the household model (as suggested by Apps and Rees, 2001). We

have ignored this in the current study, but we do consider it as an important topic for further research.

# Appendix

## A. Time use categories

(1) paid work (excluding time spent on commuting); (2) commuting (for work or school); (3) home work (cleaning, doing the dishes, cooking, shopping, gardening, do it yourself, etc. but no tasks related to caring for children or other persons); (4) personal care (washing, dressing, eating, going to the hairdresser, going to the doctor, etc.); (5) activities with children (washing, dressing, playing, reading, going to the doctor, etc.); (6) helping parents (administrative tasks, washing, dressing, going to the doctor, etc.); (7) helping other family members (administrative tasks, washing, dressing, going to the doctor, etc.); (8) helping other persons who are not family members (administrative tasks, washing, dressing, going to the doctor, etc.); (9) leisure activities (watching TV, reading, sports, hobbies, visiting friends or family, travelling, going out, etc.); (10) schooling (day or evening education, vocational training, language training, etc.); (11) administrative tasks related to own household; (12) sleeping and relaxing (sleeping, thinking, meditating, etc.); (13) other activities not mentioned above.

## B. Public expenditure categories

The public good categories in the data are the following: (1) expenditures on mortgages (rent and payment); (2) rent without expenditures on electricity and heating; (3) utilities (heating, electricity, water, telephone, internet, etc. but without insurances); (4) transportation costs (public transport, gasoline, etc., but no insurances and purchase of transportation means); (5) insurances (house, car, health, etc.); (6) child care (kindergarten, after school care, guest parent, home work supervision, etc.); (7) alimony and financial support to children who do not live at home (any longer); (8) expenditures to service debt (but no mortgages); (9) trips and holidays with (part of) the family (airplane tickets, hotel, restaurant, etc.); (10) expenditures related to cleaning the house or gardening; (11) food and drinks used at home; and (12) other public expenditures not mentioned above.

## C. Private expenditure categories

The private good categories in the data are the following: (1) food and drinks outside home (restaurant, pub, company restaurant, etc. but no expenditures consumed with (part of) the family; (2) cigarettes and other tobacco products; (3) clothing (clothing, shoes, jewelry, etc.); (4) personal care and services (hair care, body care, manicure,

hair dresser's, etc. but no medical expenditures); (5) medical expenditures not covered by an insurance (medicines, doctor, dentist, hospital, maternity grant, glasses, hearing device, etc.); (6) leisure activities (film, theater, hobbies, sports, photography, books, CD's, DVD's, expenditures related to traveling without the family, etc.); (7) schooling (courses, tuition fees, etc.); (8) gifts (to family members, friends, charity, etc.); and (9) other private expenditures not mentioned above.

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