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Firm Productivity Dispersion and the Matching Role of UI Policy

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ABSTRACT

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This paper studies optimal UI policy from the perspective of worker assignment to heterogenous jobs in an environment of random matching. Workers react to UI policy through job acceptance decisions; firms react to UI policy through wage posting. There is endogenous assortative matching as a result of the fact that UI policy induces a time profile for reservation wages, shifting the labor force towards the more productive firms. The relation between productivity dispersion and UI policy is mediated by the wage posting policies of firms that take both productivity and policy into account. Optimal UI policy is shown to crucially depend on the properties of the firm productivity distribution, such as its variance and skewness.

JEL Classification: E24, J64, J65

Keywords: productivity, heterogeneity, UI policy, endogenous assortative matching, search

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FIRM PRODUCTIVITY DISPERSION
AND THE MATCHING ROLE OF UI POLICY[1]

1 Introduction

While there is growing evidence on the importance of productivity dispersion across firms,[2] there is little modelling of labor market policy in the presence of such dispersion. In particular, UI policy, which is a key labor market policy tool, has, generally, not been linked with such productivity differences. This paper shows how UI policy can enhance the matching of workers to jobs in the face of firm productivity dispersion, leading to output gains. It proposes a mechanism whereby a declining UI time profile produces positive assortative matching.

The motivation for UI benefits is usually discussed in the context of the provision of a tool for consumption smoothing over periods of employment and unemployment. Most of the literature has been concerned with trade-offs involving consumption smoothing and moral hazard. While it has been recognized that even when agents are risk-neutral UI can be used to induce them to search more intensively, thereby improving the quality of matches, not much attention has been devoted to an assortative matching role for UI policy. Moreover, the recognition of the importance of job productivity dispersion and its influence on such policy has been absent, with few exceptions. The current paper links these issues, providing a modelling framework for UI policy as a function of the firm productivity distribution.

The intuition for the main result of the paper may be summarized as follows. Consider a frictional environment where the assignment of unemployed workers to jobs with heterogenous productivity is uncoordinated and governed by a random matching process. All firms pay the

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[2] As found, for example, by Abowd, Kramarz and Margolis (1999), Davis, Haltiwanger and Schuh (1996), and Hatiwanger, Lane and Speltzer (2000). See the discussion in Section 3.4 below.

reservation wage. At the beginning of their unemployment spell agents are relatively more choosy and tend to proceed with search unless faced with a sufficiently attractive offer. Later on, the reservation wage drops and agents are willing to take less attractive job offers. Heterogenous firms respond by introducing endogenous market segmentation. In equilibrium, more productive firms offer higher wages, anticipating a tighter 'sub-market', namely lower vacancy risk. Less productive firms offer lower wages and face higher vacancy risk as only long-term unemployed workers will accept the offer. A declining UI profile results in voluntary unemployment by the short term unemployed but features improved matching, as it shifts the labor force towards the more productive firms. Random matching thus turns assortative.

The model is a general equilibrium framework with homogenous workers, heterogenous firms, endogenous wage posting, and random search and matching. We assume that agents are risk neutral and abstract from the traditional trade-offs. Assortative matching emerges endogenously. The mechanism described above implies that heterogenous firms respond to the reservation wage time-profile induced by UI policy. Wage setting takes this reservation profile into account so as to attract workers, while UI policy takes wage setting into account to produce output gains. Optimal UI policy is shown to crucially depend on the nature of technological dispersion, in particular on the variance and skewness of the productivity distribution.

The paper makes two main contributions: first, it shows how productivity dispersion should affect UI policy, providing a link between two key labor market issues that were unrelated thus far – productivity dispersion and aggregate policy. This link is mediated by the wage posting policies of firms that take both productivity and policy into account. Second, it demonstrates the assortative or assignment role of optimal UI policy, against the background of random firm-worker matching. It does so within a framework that is shown to be empirically relevant.

The paper is organized as follows. Section 2 presents the model. Section 3 discusses optimal UI policy in the face of firm productivity dispersion. Section 4 demonstrates the results with an illustrative numerical solution and discusses the empirical relevance of key elements of the model. Section 5 presents the relation of the paper with the relevant literature. Section 6 concludes. In what follows we shall use the terms assortative matching, worker assignment, and worker sorting interchangeably.

2 The model

In what follows we describe the general set up (2.1) and then look at two alternative time paths for UI policy: constant (2.2) and declining (2.3).

2.1 The General Set-Up

There is a continuum of workers whose measure is given by $L > 0$, and a continuum of firms whose measure is given by $M >> L$. Firms differ in the technology they possess. Each firm can post a vacancy, incurring no costs, w.l.o.g. Once the vacancy is filled, the job produces x units of the single perishable consumption good, which price in normalized to unity. Production terminates with an exogenous Poisson parameter, $s > 0$. Otherwise the vacancy produces nothing.

The technological parameter, x, is assumed to be distributed according to the cumulative distribution function:

$$G(\cdot) \sim [\underline{x}, \overline{x}]$$

with strictly positive densities.

Workers are ex-ante identical in all respects and are assumed to be risk neutral. Let us note that this homogeneity assumption renders the model more tractable but is not crucial for the analysis. If workers were heterogenous, what would be needed is that their degree of heterogeneity be lower than the extent of technological dispersion. Firms are assumed to be expected-profit maximizers. To close the general equilibrium model we assume that workers possess an equal stake in each of the firms.

In every period (time is assumed to be discrete) firms post wage rates. Then, unemployed workers are randomly assigned to vacancies. Without loss of generality we assume that each active firm posts a single vacancy. Due to lack of coordination there are cases where multiple applicants arrive at a job vacancy and others where either no one is assigned or where a single applicant arrives. Upon receiving a job offer, the worker decides whether to accept or reject the offer. Thus we allow for voluntary unemployment. If more than one applicant accepts the offer, the assignment choice is taken randomly. We assume that search intensity is fixed and normalized to one application per period. We further assume that all unassigned individuals are eligible for UI, and that UI benefits are financed by neutral lump-sum taxes.

We now turn to analyze the optimal behavior of the agents in steady-state equilibrium under different UI time paths.

2.2 A Constant UI Profile

Consider first a UI scheme with a constant time profile. The sole purpose of this discussion is to serve as a benchmark for the analysis which follows. Note that throughout we assume that agents search only when unemployed.

Following Diamond's (1971) seminal contribution, by the homogeneity of workers, and since on-the-job-search is ruled out and firms are committed to the wages posted prior to the arrival of the job applicants, the wage distribution collapses to a singleton. Denote the equilibrium wage rate, which coincides with the uniform reservation wage, by w. It therefore follows that:

$$w = a + h \tag{1}$$

where a and $h \leq \underline{x}$ denote the constant UI benefit and the imputed value of leisure, respectively.

The typical worker's optimal acceptance rule is trivial, namely accept the first offer received. Turning next to firms, it is easy to observe that firms, provided that they decide to operate, will choose to offer the uniform wage rate, w. This emerges from the fact that workers are assumed not to search on the job. Thus, applicants accept any job offer above the reservation instantaneously, and firms choose to offer precisely the reservation wage. Cutting this wage even slightly will increase vacancy risk to infinity, making profits drop to zero. Offering above it does not bring any gain, as it does not affect the vacancy risk because of the reservation strategy.

Firms differ in their productivity and correspondingly need to take a strategic decision, whether to enter the market or remain idle. Given w, the equilibrium wage rate, all firms possessing a technology $x \geq w$ will participate, for all expect the arrival of applicants with strictly positive probability.

Denote by U and V, the measure of unemployed workers and unfilled vacancies in steady state equilibrium, respectively. Let F denote the steady state measure of active firms, which is also the number of jobs, filled or vacant, in the economy. In equilibrium the following conditions hold true, in addition to the wage determination condition given by equation (1) above:

$$L - U = F - V \tag{2}$$

$$M\left(1 - G(w)\right) = F \tag{3}$$

$$V\left(1 - e^{-\frac{U}{F}}\right) = s\left(F - V\right) \tag{4}$$

The interpretation of the equations above is straightforward. Equation (2) defines the equilibrium condition according to which the measure of matched workers (on the left-hand side) is equal to the measure of filled vacancies (on the right-hand side). Equation (3) defines the entry condition, given that the prevailing wage rate is w. Equation (4) states the familiar worker flow condition (the Beveridge curve). We assume urn-ball random matching so the distribution of the number of applicants that arrive to any posted vacancy is Poisson, where the expected number of applicants is given by the $\frac{U}{F}$ ratio.[3] On the left-hand side we have, therefore, the flow into the pool

[3] From the worker's point of view, the frictions are that the worker may approach a job already assigned and that there exists a possibility of multiple applicants at the same open vacancy.

of filled vacancies (successful matches), given by the probability of having at least one applicant times the measure of unfilled vacancies. The right-hand side gives the flow out of the pool of filled vacancies.[4]

With equations (1)-(4) we can solve for the equilibrium recursively, for any level of the constant level of UI benefit a. First we substitute for w (from (1)) into (3) and obtain an explicit solution for F. We then obtain a system of two equations ((2) and (4)) solved for two unknowns, U and V.[5]

By relaxing the fixed arrival rate paradigm of conventional wage-posting search equilibrium models, we obtain the inherent trade-off between sorting and employment using an extremely simple framework. To see this, let welfare be measured by per-capita utility, that is per-capita consumption plus the value imputed to per-capita leisure. By lowering a, the constant UI benefit, we lower the wage rate, w, thereby reducing unemployment. On the other hand, the fall in w, brings in less productive firms (the lower tail of the productivity distribution) that previously failed to break even.

More formally, given the welfare measure defined above, the maximization problem solved by the social planner is given by:

$$\max_a \{(L-U)E[x \mid x > w] + Uh\} \tag{5}$$

s.t. equations (1)-(4), where, $E[\cdot]$ denotes the expectation operator. Using equation (3), the maximization may be re-formulated as a function of F :

$$\max_F \{(F-V)\left(E[x \mid x > G^{-1}(1-\frac{F}{M})] - h\right) + Lh\} \tag{6}$$

subject to equations (2)-(4). Note that G^{-1} is well defined since by assumption densities are strictly

[4] Observe that by the law of large numbers the product on the left-hand side of equation (4) gives the actual measure of successful matches, rather than the expected measure.

[5] Note that the solution is unique, as equation (2) is upward sloping in U-V space, whereas equation (4) is downward sloping, so that there exists a single crossing of the two curves.

positive. Equations (5) and (6) indicate that we can solve for the optimum by setting the number of posted vacancies F optimally, and implement it via setting the wage w through the determination of unemployment benefits a.

By fully differentiating equation (2) and (4) with respect to F, it follows that $\frac{dV}{dF} < 1$, thus the term $F - V$ is rising with respect to F. At the same time the second term $(E[x \mid x > G^{-1}(1 - \frac{F}{M})])$ decreases, since firms possessing technologies of lower productivity enter the market, thereby reducing expected productivity. The optimal F balances these two opposing effects, namely employment versus enhanced matching. Note that without UI (i.e., $a = 0$) wages are determined by the imputed value of leisure (h) and are, therefore, independent of the productivity distribution.

2.3 A Declining UI Time Path

We now allow for a declining UI profile. We confine attention to a two-tiered regime, in which agents are eligible for a short period of regular UI benefits, followed by an indefinite period of reduced compensation, which we refer to as income support. Let the level of income support be denoted by a, and let z denote the UI benefit ($z > a$). We assume that z is paid during the first two periods of unemployment and that all agents who exhaust their eligibility for UI benefits are henceforth indefinitely eligible for income support. Later, we relax the two period assumption. UI eligibility is assumed to be independent of work history, for simplicity.

First, consider the intuition. A declining profile implies a non-degenerate wage distribution, while, as we saw, with a constant profile, there exists a unique wage rate in equilibrium. This key feature derives from the wage-posting setting and the fact the agents search only when unemployed. While agents are ex-ante identical in all respects, the declining profile implies that short-term unemployed agents, faced with a non-degenerate distribution of wage offers, will have a higher reservation wage rate than long-term unemployed agents who have already exhausted their eligibility for UI benefits. The declining UI profile yields ex-post heterogeneity among ex-ante identical agents. For a sufficiently dispersed set of technologies, a two-wage equilibrium exists. Voluntary unemployment by short-term unemployed agents induces firms possessing more productive technologies to offer higher wages, thereby reducing their vacancy risk. Assuming two levels of UI benefits and two periods when the first level is in place, we can confine attention to a two-wage equilibrium.

We turn now to a formal presentation of the model and the optimal policy.

Unemployed Workers Value Function. We start with the value functions for a typical agent in the economy. There are four states to consider: employment, two states of short-term unemployment (for each period of UI eligibility), and long term unemployment (income support recipients). We start with the three unemployment states, denoting by $H_1, H_2,\ H$, the continuation value functions for short-term unemployed agents (during the first and second period of the unemployment spell, respectively) and income support recipients. In steady-state equilibrium the following asset-value conditions hold:

$$H_1 = z + h + \beta[\overline{m}\max(\overline{J}, H_2) + \underline{m}\max(\underline{J}, H_2) + (1 - \overline{m} - \underline{m})H_2] \qquad (7)$$

$$H_2 = z + h + \beta[\overline{m}\max(\overline{J}, H) + \underline{m}\max(\underline{J}, H) + (1 - \overline{m} - \underline{m})H] \qquad (8)$$

$$H = a + h + \beta[\overline{m}\max(\overline{J}, H) + \underline{m}\max(\underline{J}, H) + (1 - \overline{m} - \underline{m})H] \qquad (9)$$

where $\overline{J}$, $\underline{J}$ denote the high and low wage jobs continuation values, respectively, $\overline{m}$, $\underline{m}$ denote re-employment chances in firms offering high and low wage rates, respectively, and $\beta \in (0, 1)$ denotes the discount factor. Short-term unemployed get UI benefits z and long-term unemployed get income support a in addition to the value of leisure h. In the following period they move either to a job or to the next stage of unemployment.

Employment Value Functions. The steady state value functions for the two types of jobs (those with high wage rate and low wage rate, respectively) are given by $\overline{J}$ and $\underline{J}$:

$$\overline{J} = \overline{w} + \beta[(1-s)\overline{J} + sH_1] \tag{10}$$

$$\underline{J} = \underline{w} + \beta[(1-s)\underline{J} + sH_1] \tag{11}$$

Workers get the relevant wage in each job and face the exogenous separation probability s.

Wages. By virtue of wage posting and the assumption that individuals search only while unemployed, in equilibrium firms offer such wages so as to satisfy the following reservation asset values:

$$\overline{J} = H_2 \tag{12}$$

$$\underline{J} = H \tag{13}$$

It is easy to verify that the reservation wage property is satisfied, by observing that $\overline{J} = H_2 > \underline{J} = H$. Thus, short-term unemployed agents will accept only high wage offers during their first period of UI eligibility. All other unemployed agents will accept any offer.

Manipulating equations (7)-(13) yields the following two conditions which determine the equilibrium wage offers as a function of the policy parameters (z and a), the re-employment probabilities ($\overline{m}, \underline{m}$), the discount factor ($\beta$) and the separation rate (s):[6]

$$z - a = \frac{\overline{w} - \underline{w}}{1 - \beta(1-s)} \tag{14}$$

$$\overline{w} = (z-a)[(1-\beta) + \beta\overline{m} - \beta^2 s(1-\overline{m})] + h + a \tag{15}$$

Matching. We now explicitly formulate the re-employment probabilities. Using a Taylor's expansion, it is straightforward to verify that the urn-ball matching process (assuming a single

[6] See appendix A for the full derivation.

job application per individual per period) implies a Poisson distribution of applicants, yielding the following conditions (for an explicit derivation see appendix B):

$$\overline{m} = \frac{\overline{V}}{U}(1 - e^{-\frac{U}{F}}) \tag{16}$$

$$\underline{m} = \frac{\underline{V}}{U - U_1}(1 - e^{-\frac{U-U_1}{F}}) \tag{17}$$

where $\overline{V}$ and $\underline{V}$ denote the measures of unfilled vacancies posted by firms offering the high wage rate and low wage rate, respectively; F denotes the measure of posted jobs, U denotes the measure of aggregate unemployment and U_1 denotes the measure of short-term unemployed agents during the first period of UI eligibility.

Steady State Flow Equations. The standard steady state flow conditions are given by:

$$U_2 = (1 - \overline{m})U_1 \tag{18}$$

$$(U - U_1 - U_2)(\overline{m} + \underline{m}) = (1 - \overline{m} - \underline{m})U_2 \tag{19}$$

$$\overline{V}\left(1 - e^{-\frac{U}{F}}\right) = s\left(\overline{F} - \overline{V}\right) \tag{20}$$

$$\underline{V}\left(1 - e^{-\frac{U-U_1}{F}}\right) = s\left(F - \overline{F} - \underline{V}\right) \tag{21}$$

where , U_j, $j = 1, 2$ denotes the measure of unemployed agents during the jth period of UI eligibility and $\overline{F}$ denotes the measure of vacancies posted by firms offering the high wage rate in equilibrium.

Steady State Equilibrium. To complete the characterization of steady state equilibrium we introduce the following conditions and interpret them:

$$L - U = F - \overline{V} - \underline{V} \tag{22}$$

$$F = M\left(1 - G(\underline{w})\right) \tag{23}$$

$$\left(1 - e^{-\frac{U}{F}}\right)\left(\widehat{x} - \overline{w}\right) = \left(1 - e^{-\frac{U-U_1}{F}}\right)\left(\widehat{x} - \underline{w}\right) \tag{24}$$

$$\overline{F} = M\left(1 - G(\widehat{x})\right) \tag{25}$$

Equation (22) is the condition according to which the measure of filled vacancies is equal to the measure of employed agents. Equation (23) is a consistency condition, which requires that the total measure of active firms (hence posted vacancies) should be equal to the fraction of the firms possessing a technology above the lower bound wage rate times the measure of potential firms. Equation (24) determines the wage distribution by defining a cutoff technology, $\widehat{x}$, above which all firms maximize expected profits by offering the high wage rate, and below which all firms maximize expected profits by offering the low wage rate. Equation (25) is a consistency condition similar to (23) with regard to the firms offering the high wage rate in equilibrium.

Optimality Problem. The optimality problem may be written as follows:

$$\begin{aligned}&\max_{a,z}\{(F - \overline{F} - \underline{V})E[x \mid \underline{w} \leq x \leq \widehat{x}]\\&+(\overline{F} - \overline{V})E[x \mid x > \widehat{x}]\\&+Uh\}\end{aligned} \tag{26}$$

subject to the above 12 equations [equations (14) – (25)]. Equilibrium is defined by this system of equations, where a and z are solved according to (26).

It can be shown that the above formulation, which focuses on choosing the levels of UI and income support, can be mapped into the following equivalent structure, focusing on the allocation of production:

$$\max_{\underline{\underline{x}},\widehat{x}} \left\{ M \int_{\underline{\underline{x}}}^{\overline{x}} xdG(x) - \underline{V} \frac{\int_{\underline{\underline{x}}}^{\widehat{x}} xdG(x)}{G(\widehat{x}) - G(\underline{\underline{x}})} - \overline{V} \frac{\int_{\widehat{x}}^{\overline{x}} xdG(x)}{1 - G(\widehat{x})} + Uh \right\} \tag{27}$$

subject to equations (20), (21),(22), (25) and:

$$U_1 = s\left(L - U\right). \tag{28}$$

$$F = M\left(1 - G(\underline{\underline{x}})\right) \tag{29}$$

In other words, the policymaker chooses $\underline{\underline{x}}$, the threshold productivity level above which firms enter the labor market, and $\widehat{x}$, the productivity level above which firms offer the high wage rate in equilibrium. The optimal allocation can be implemented by appropriate wage rates, $\underline{w}$ and $\overline{w}$ (employing equations (23) and (24)) through the choice of the two UI policy instruments a and z (employing equations (14) and (15)).[7] Note that the firm productivity distribution $G(x)$ enters into the objective function (27) and into the constraints (25) and (29).

3 Optimal UI Policy and the Productivity Distribution

The question we would like to address is a normative one, namely under what circumstances is a declining UI profile optimal. Moreover, we want to link this policy with the properties of the firm productivity distribution. Intuitively it seems that the answer should relate to the extent to which the set of technologies is dispersed. As the above problem has no closed form solution, it needs to be addressed by numerical methods. We opt for the simplest possible set-up, i.e., a

[7] The flat UI profile is obtained as a special case of the allocation problem in (27) when $\underline{\underline{x}} = \widehat{x}$. In such a case, a would be set equal to z, so as to induce $\underline{w} = \overline{w}$.

discrete distribution of technologies, which comprises two elements in its support. We find that it is a rich enough formulation to demonstrate the linkages between the moments of the productivity distribution and optimal UI policy. We characterize the types of worker sorting which emerge under the different UI paths analyzed above (3.1). We then study the relationship between the welfare levels induced by the different paths and the properties of the productivity distribution (3.2). Finally, we examine the case where policy determines the duration of the first tier of the UI path (3.3).

3.1 Firm Technologies, Worker Sorting, and Optimal UI Paths

Suppose there are two technologies, denoted by $\overline{x}$ and $\underline{x}$, where $\overline{x} > \underline{x} > h$. Denote by $0 < p < 1$ the fraction of firms (which measure is given by $M >> 0$) possessing $\overline{x}$. We henceforth restrict attention to pure-strategy equilibria.

There are three equilibrium configurations to consider. A benchmark case is the one in which the UI profile is constant and the benefit is set sufficiently low, so that all firms are active in equilibrium. We refer to this configuration as maximum employment or no-sorting, interchangeably. A second case, is the one in which the UI regime is constant, but the benefit is set high enough so as to crowd out the low-productivity firms. We refer to this configuration as high-sorting. In the third configuration both technologies are active, but due to a declining UI profile, voluntary unemployment by short-term unemployed agents yields partial-sorting.

Denote by $N^{HS}, N^{PS}$and N^{NS}, the steady-sate measures of employed workers in the high-sorting, partial-sorting and no-sorting configurations, respectively (where $N = L - U$).

We had already observed that $N^{NS} > N^{HS}$(see the characterization of equilibrium in the constant profile UI regime). Close inspection of (20)-(22) yields that $N^{NS} > N^{PS}$. This condition derives directly from the existence of voluntary unemployment in the partial-sorting configuration.[8]

[8]By aggregating (20) and (21) and comparing the expression to (4), it can be seen that for a given measure of active firms and for any level of unemployment, the market clears with a higher measure of aggregate unfilled

However, one cannot relate N^{HS} and N^{PS} without making further assumptions. This ambiguity derives from the trade-off between involuntary unemployment in the high sorting case (when a lower measure of firms participates) and voluntary unemployment in the partial sorting case (due to workers' ex-post heterogeneity emerging from the declining UI profile).

Formulating the welfare measures for each one of the three configurations (denoted W, and maintaining our definition of welfare from the previous section), we obtain the following:

$$W^{NS} = N^{NS}[p\overline{x} + (1-p)\underline{x} - h] + Lh \tag{30}$$

$$W^{PS} = N^{PS}[q\overline{x} + (1-q)\underline{x} - h] + Lh \tag{31}$$

$$W^{HS} = N^{HS}[\overline{x} - h] + Lh \tag{32}$$

where $q = \frac{(pM - \overline{V})}{M - \overline{V} - \underline{V}}$ and it is easy to show, using (20) and (21), that $1 > q > p$.

Equation (30) is the "benchmark" case: a fraction p of workers go to the high technology firms and $1-p$ to the low technology ones and N^{NS} is determined solely through random matching. At the other extreme there is (32) with a constant UI profile: here only the high technology is in operation. The two equations (30) and (32) express the trade-off between employment and sorting as $N^{NS} > N^{HS}$ and $[\overline{x} - h] > [p\overline{x} + (1-p)\underline{x} - h]$. The intermediate case is that of a declining UI profile – equation (31). This policy balances the two considerations, employment and productivity. In order to increase the degree of sorting in the market, namely to shift workers away from low-productivity firms to high-productivity ones, the policymaker sacrifices a rise in unemployment. The optimal policy depends on the properties of the productivity distribution, as illustrated below.

It is easy to verify the existence of policy instruments that implement the social optimum. For the no sorting case set $\underline{x} \geq a$ and for the high sorting case set $\overline{x} \geq a \geq \underline{x}$. For the partial

vacancies relative to the constant UI regime.

sorting configuration, see appendix B. Note that if taxation were distortionary,[9] then when $N^{PS} > N^{HS}$ the case for a declining UI profile is reinforced. This is so because expenditures on UI benefits are lower under the PS regime due to lower unemployment.

3.2 UI Policy and the Variance and Skewness of the Productivity Distribution

We now study the relationship between the optimal UI time path, welfare, and the productivity distribution. To do so we fix the average productivity in the economy and denote it by μ, where $\mu = p\overline{x} + (1-p)\underline{x} > h$. Consider a mean-preserving spread $\triangle \geq 0$ whereby $\overline{x} = \mu + \triangle$ and $\underline{x} = \mu - \frac{p\triangle}{1-p}$, with $\underline{x} \geq h$. Then the moments of this productivity distribution are given by:[10]

mean	μ
standard deviation σ	$\sqrt{\frac{p}{1-p}}\triangle$
skewness	$(1-2p)\sqrt{(1-p)p}$

Reformulation of equations (30) -(32) yields the following:

$$W^{NS} = N^{NS}[\mu - h] + Lh \tag{33}$$

$$W^{PS} = N^{PS}[\mu - h] + N^{PS}\frac{q-p}{1-p} \triangle + Lh \tag{34}$$

$$W^{HS} = N^{HS}[\mu - h] + N^{HS} \triangle + Lh \tag{35}$$

We turn now to characterize the general properties of optimal policy (3.2.1) and then present a numerical simulation that provides further illustration of this policy (3.2.2).

[9]Recall that we have assumed throughout that benefits are financed by lump-sum taxation.

[10]Note that when $p = 0.5$ then $\sigma = \triangle$ and the skewness is zero; when $0.5 < p < 1$, the skewness is negative, and when $0 < p < 0.5$ the skewness is positive.

3.2.1 General Properties

The following proposition establishes some general properties of optimal UI policy. It refers to the following illustrative figure, where we depict each of the three configurations in welfare-productivity spread space ($W - \triangle$).[11] Note that social optimum is given by the welfare frontier (the upper envelope of the figure).

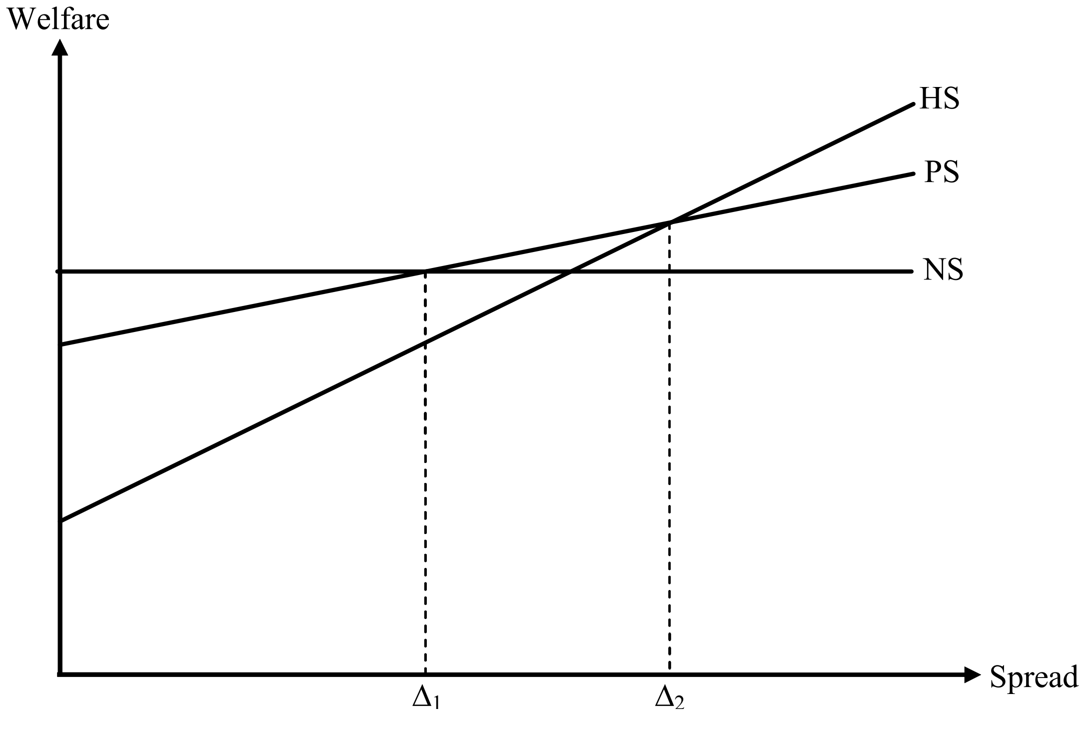

Figure 1: Optimality and Productivity Variance (spread)

Proposition 1 *Holding fixed the mean (μ) and the skewness (p), there exist thresholds* $\triangle_1$ *and* $\triangle_2$

[11] The mean and the skewness in the figure are fixed. Due to the fact that the skewness depends on the parameter p only, shifts in the (mean preserving) spread parameter $\triangle$ measure shifts in the standard deviation of the productivity distribution.

where $0 < \triangle_1 \leq \triangle_2 \leq \frac{1-p}{p}[\mu - h]$*, such that: for all* $0 < \triangle \leq \triangle_1$ *social welfare is maximized by the no sorting configuration, for all* $\triangle_1 \leq \triangle \leq \triangle_2$*social welfare is maximized by the partial sorting configuration, and for all* $\triangle_2 \leq \triangle \leq \frac{1-p}{p}[\mu - h]$ *social welfare is maximized by the high sorting configuration.*

Proof. First observe that by virtue of linearity with respect to $\triangle$ each one of the schedules (33)-(34), each configuration will appear at most once on the welfare frontier (single crossing property). Next, note that the finiteness of set of configurations ensures non-emptiness.

Note further that for $\triangle = 0$, the no-sorting configuration is welfare maximizing, since $\mu > h$ and $N^{NS} > \max[N^{PS}, N^{HS}]$. Then, by continuity, for sufficiently small $\triangle$, the no-sorting configuration is socially desirable.

Let NS denote the set of all $\triangle$, for which the no-sorting configuration is welfare maximizing. The set NS is non-empty (as just shown) and bounded from above (by construction of the spread $\triangle$). Thus, it has a least upper bound. We denote it by $\triangle_1$.

Next, note that if the high-sorting configuration is welfare maximizing for some $\triangle'$, then it remains the maximizing configuration for all $\triangle' \geq \triangle$. To see that, suppose, by way of contradiction, that the opposite holds true. Since the schedule of the no-sorting configuration is flat, whereas the high-sorting configuration is rising with respect to $\triangle$, the only case we need to examine is the possibility where partial-sorting attains a higher level of welfare than high sorting for some $\triangle$, $\triangle \geq \triangle'$ This necessarily implies that the slope of the partial sorting schedule with respect to $\triangle$ is steeper than the corresponding slope of the high sorting schedule. Formally:

$$N^{PS}\frac{q-p}{1-p} > N^{HS}$$

which implies

$$N^{PS} > N^{HS}$$

Thus, we obtain a contradiction, since it follows that for all $\triangle$, partial sorting is preferred to high sorting.

Let HS denote the set of all $\triangle$ for which the high-sorting configuration is welfare maximizing. The set is bounded from below (by construction of the spread $\triangle$). If it is non-empty, it has a highest lower bound. Let $\triangle_2$ denote the highest lower bound (if it exists) and set $\triangle_2 = \frac{1-p}{p(\mu-h)}$, otherwise. This completes the proof. ■

While the discrete two-technology case is stylized, it provides us with some clear insights regarding the forces at play. When the set of technologies is almost degenerate, i.e. the spread $\triangle$ converges to zero, there is little to gain from introducing voluntary unemployment and shifting the pool of workers away from low-productivity firms towards high-productivity ones. In this case UI is redundant. When, however, technologies are sufficiently dispersed, the partial sorting configuration is preferred to the no-sorting one. In graphical terms, the partial sorting solution dominates in the interval $\triangle_1 - \triangle_2$. This interval is well-defined only when there is a substantial difference between the intercepts of the HS and PS schedules (on the vertical axis). This implies a sufficient degree of right-skewness of the productivity distribution. When dispersion is very large, it is desirable to increase unemployment by eliminating employment at low-productivity firms altogether and obtaining high (full) sorting. For intermediate values of the technological spread, social welfare is maximized by the partial sorting configuration, hence a declining UI profile is optimal.[12] In the two other cases, a constant profile (either no sorting or high-sorting) suffices.

3.2.2 Numerical Solution

We turn now to a numerical solution of the model. In Figure 2 we plot four different curves in skewness (skew) – s.d. (σ) space.[13]

[12] This discrete example extends to continuous cases with sufficient skewness. In cases that are not skewed (or not sufficiently skewed), such as the uniform distribution, the constant profile dominates.

[13] The figure is based on the numerical solution using the following parametric assumptions: $M = 100, L = 70, s = 0.01, h = 0$ and $\mu = 10$.

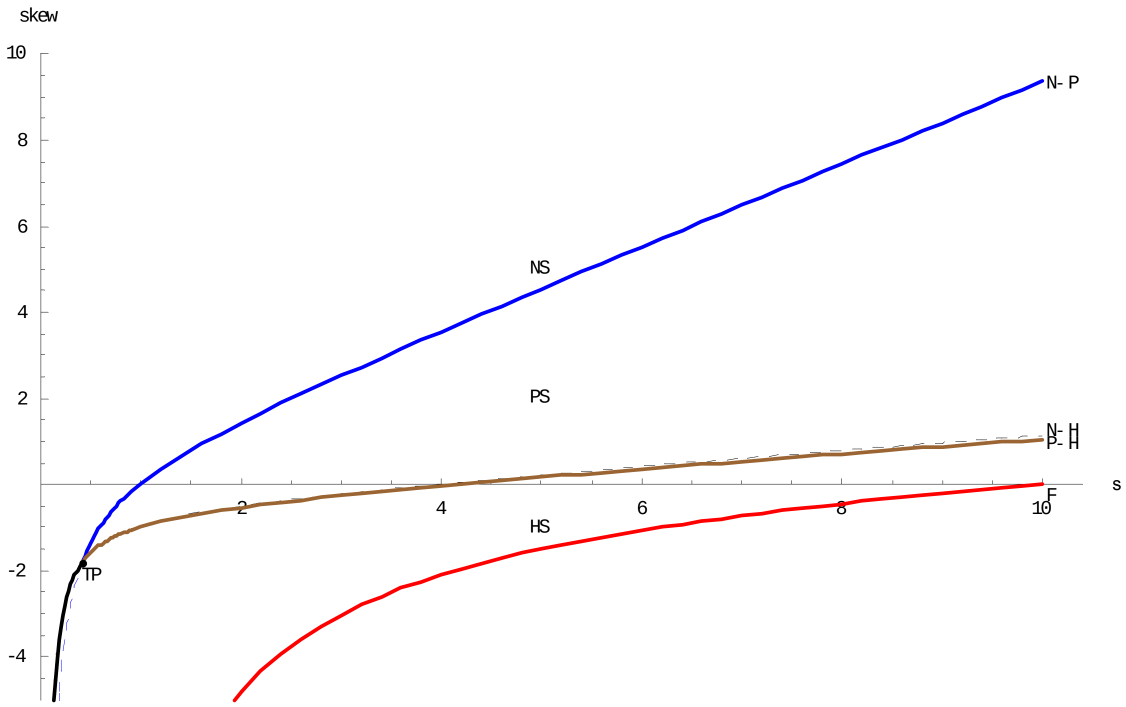

Figure 2: Skewness and Variance Relationships

The four curves depict the following:

The curve labeled F represents a feasibility constraint defined by the non-negativity of the lower-bound technology, $\underline{x}$. The set of feasible points lies above the curve.

Along the N-H curve the welfare attained by a no-sorting equilibrium configuration is equal to the welfare attained by the high-sorting configuration, where the lower-bound technology is crowded out of the market. Note that as the welfare for the no-sorting configuration depends on the mean productivity of the distribution of technologies, and is completely insensitive to the other two moments, the curve N-H is essentially an indifference curve for the high-sorting configuration along which the welfare attained by the high-sorting configuration is constant. The positive slope of the curve is due to the fact that the gains from sorting increase with respect to the standard deviation of the distribution of technologies and decrease with respect to its skewness. A larger value of skewness implies that a higher weight is shifted to the lower bound technology which is crowded out in the high-sorting configuration, which in turn implies a higher unemployment rate. A higher dispersion (captured by a higher value of the standard deviation) implies that the upper bound technology is farther away from the mean productivity, which implies larger gains from sorting. As we move either downwards (decreasing skewness) or rightwards (increasing the s.d.) the welfare obtained by the high-sorting configuration rises. Thus, for any point which lies below the N-H curve the high-sorting configuration dominates the no-sorting one and vice-versa.

The N-P curve represents points for which the welfare attained by the partial-sorting configuration is equal to the welfare attained by the no-sorting one. The positive slope of the curve derives from the same reasons given for the N-H curve. Similarly, any point lying above the curve implies that the no-sorting configuration dominates the partial-sorting one and vice-versa.

Along the P-H curve lie points for which the partial-sorting and the high-sorting configurations attain the same level of utility. The positive slope derives from the fact that the gain from sorting (shifting from partial-sorting to high-sorting) is increasing with respect to the standard deviation and is decreasing with respect to the skewness. It follows that for any point above the P-H curve, the partial-sorting configuration dominates the high-sorting configuration and vice versa.

To summarize, for any point which lies above both the N-P and the N-H curves, the no-sorting configuration is socially desirable; for any point which lies below both the N-H and the P-H curves, the high-sorting configuration is the socially desirable one; while for any other point (within the feasible range of parameters), the partial-sorting configuration prevails.

To provide further illustration of the results of this simulation, we provide an illustrative figure, Figure 3, which depicts on a magnified scale the points in the neighborhood of the point T.P (in Figure 2) where all three curves (N-P, N-H, and P-H) intersect. Note that by construction, at the point T.P. all the three different configuration yield the same level of utility.

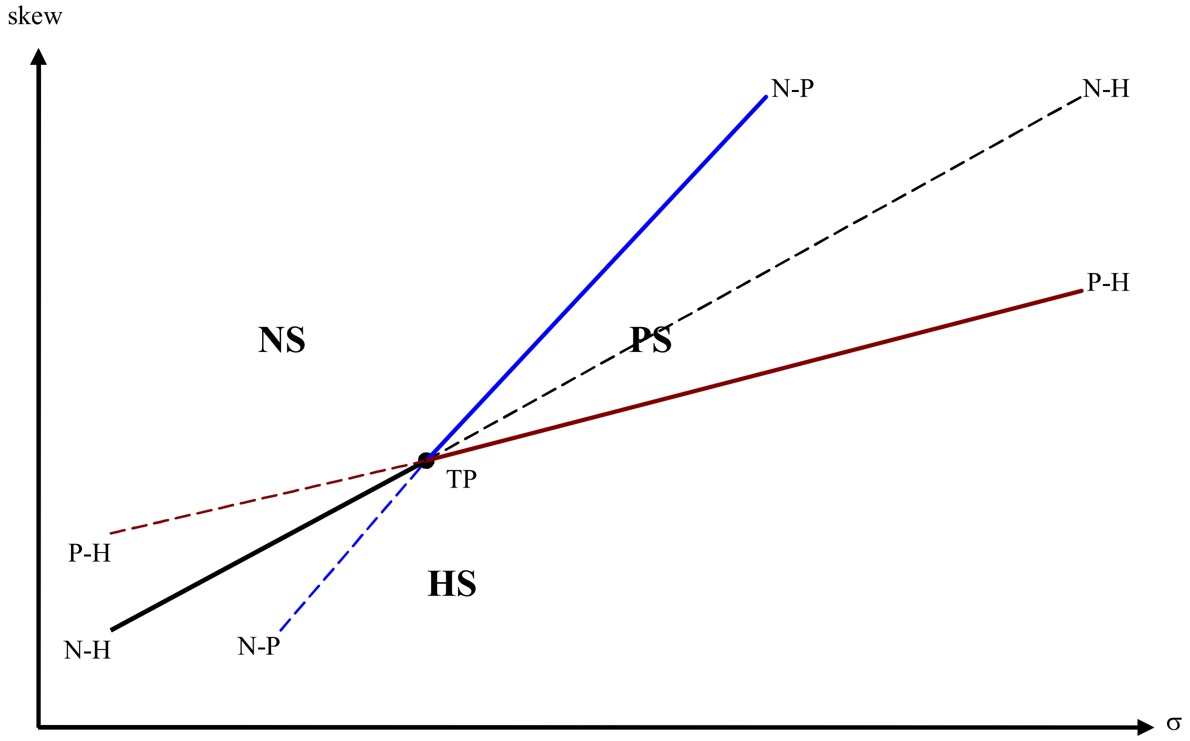

Figure 3: Optimal UI Regions

The solid parts of the curves define a fork-shape borderline which divide up the space into three regions, according to the dominating equilibrium configuration. For any point which lies above the upper envelope of the N-P and the N-H curves, the no-sorting configuration is socially desirable (as it dominates the two other configurations). By the same reasoning for any point which lies below the lower envelope of the P-H and the N-H curves, the high-sorting configuration is socially desirable. In the remaining region, the partial-sorting (namely a declining UI path) is the socially desirable allocation.

A number of conclusions emerge from inspection of Figures 2 and 3:

(i) For a wide range of parameters, the partial sorting equilibrium configuration constitutes the social optimum, implying a declining UI time path.

(ii) For this partial sorting to prevail, skewness has to be sufficiently large, exceeding the skewness associated with the T.P. intersection point. As already demonstrated in Figure 1, for a given skewness of such magnitude, partial sorting dominates the other two configurations for intermediate levels of dispersion (standard deviation).

(iii) Similarly, for intermediate levels of skewness, partial sorting dominates when dispersion is large enough, exceeding the standard deviation associated with the T.P. intersection point.

To see the rationale behind these results, note that when skewness is sufficiently small, the costs associated with crowding out the low-tech firms are relatively small. In such a case, the high-sorting configuration prevails, as it attains enhanced matching in exchange for a moderate loss in employment. Similarly, when skewness is large enough, the costs associated with foregoing the low-tech firms are significant, rendering the no-sorting configuration the socially desirable equilibrium, as the gains from sorting (partial or high) are outweighed by the increase in unemployment. In the intermediate range, partial sorting attains the right balance between employment and matching considerations. As productivity dispersion rises, the ranges in which the high-sorting and the partial sorting configurations prevail expand, at the expense of the no-sorting configuration. This is due to the increase in the gains from sorting.

Summarizing the insights of this sub-section, we have demonstrated that the socially desirable degree of sorting, attained by a particular time path for UI, entails the balancing of the gains from enhanced matching with the loss of output associated with higher unemployment. The higher the standard deviation of the productivity distribution, the larger are the gains from sorting. The higher the skewness, the larger are the costs associated with increased unemployment. Thus, as we move in a south-east direction (either towards higher standard deviation and/or towards lower skewness) the optimal degree of sorting rises going from no sorting to partial sorting to high sorting.

3.3 Duration Policy

In the previous sub-sections we have confined attention to two kinds of UI schemes. We compared a flat profile, where a constant benefit is paid indefinitely, to a two-tiered scheme, where individuals are eligible for high benefits during a limited period, and, thereafter, are eligible for reduced benefits indefinitely. In what follows we generalize our duration analysis. Denoting the duration of the first tier of the UI scheme by $\bar{t}$, we now allow for $0 \leq \bar{t} \leq \infty$, while we have thus far considered only the two schemes $\bar{t} = 0$ (flat profile) and $\bar{t} = 2$ (declining profile). We retain the structure of the two-tiered UI regime, i.e., regular UI and income support, and allow the number of periods in the first tier to be optimally determined by policy. We first characterize optimal policy, analyzing the forces that affect optimal duration (3.3.1). We then re-examine the relationship between optimal duration and the properties of the productivity distribution, in particular its standard deviation and skewness (3.3.2).

3.3.1 Optimal Duration

In order to maintain the simple property of a two wage equilibrium, but gain more flexibility with respect to duration policy, we confine attention to the case of myopic agents. Thus we assume that the discount factor is given by $\beta = 0$. This is an extreme assumption and is made to focus the analysis on the essential duration implications. The insights gained carry over to the more general case ($1 \geq \beta > 0$). Myopia implies that along each of the two tiers, the reservation wages, and hence the equilibrium wages, will be fixed. Regardless of the length of the first tier of the UI regime, these will be given respectively by:

$$\underline{w} = a + h$$

$$\overline{w} = z + h$$

This result can be verified by substituting into the wage determination equations (14) and (15).

Under this set-up the social planner has one additional degree of freedom in choosing optimal UI policy, namely, setting the duration of the first tier of the regime, $\bar{t}$. Let U_t denote the measure of unemployed agents during period t of UI (first-tier) eligibility. During the first $\bar{t}-1$ periods of UI eligibility agents will only accept high-wage offers. Maintaining our notation from previous sections, it follows that:

$$U_t = U_{t-1}(1-\overline{m}) \quad 2 \leq t \leq \bar{t}$$

Denote by $\overline{U}$ the aggregate measure of unemployed agents whose reservation wage is high (given by $\overline{w}$). By construction:

$$\overline{U} = \sum_{t=1}^{\bar{t}-1} U_t = U_1 \frac{(1-(1-\overline{m})^{\bar{t}-1})}{\overline{m}} \tag{36}$$

Reformulating the UI optimization problem (see equation (27)) yields:

$$\max_{\underline{\underline{x}},\widehat{x},\overline{U}} \left\{ M \int_{\underline{\underline{x}}}^{\overline{x}} x dG(x) - \underline{V} \frac{\int_{\underline{\underline{x}}}^{\widehat{x}} x dG(x)}{G(\widehat{x}) - G(\underline{\underline{x}})} - \overline{V} \frac{\int_{\widehat{x}}^{\overline{x}} x dG(x)}{1 - G(\widehat{x})} + Uh \right\} \tag{37}$$

subject to the same constraints as above (or re-formulated wherever relevant), reproduced here:

$$\overline{V}(1 - e^{-\frac{U}{F}}) = s(\overline{F} - \overline{V}) \tag{38}$$

$$\underline{V}\left[1 - e^{-\frac{U-\overline{U}}{F}}\right] = s\left[F - \overline{F} - \underline{V}\right] \tag{39}$$

$$L - U = F - \overline{V} - \underline{V} \tag{40}$$

$$F = M[1 - G(\underline{\underline{x}})]] \tag{41}$$

$$\overline{F} = M[1 - G(\widehat{x})]] \tag{42}$$

$$\overline{U} \geq U_1 \tag{43}$$

$$\overline{U} \leq \frac{U_1}{\overline{m}} \tag{44}$$

$$U_1 = s[L - U] \tag{45}$$

$$\overline{m} = \frac{\overline{V}}{U}[1 - e^{-\frac{U}{F}}] \tag{46}$$

With (43) and (44) denoting constraints derived from (36) for the two limiting cases of $\overline{t} = 2$ and $\overline{t} \to \infty$.

Any change in duration policy, namely in $\overline{t}$, translates into a change in $\overline{U}$ (by virtue of (36)). To examine the implication of a change in duration policy, suppose that $\overline{U}$ is increased, fixing $\underline{\underline{x}}$ and $\widehat{x}$. Fully differentiating equations (38)-(40) yields:

$$0 < \frac{dU}{d\overline{U}} < 1 \tag{47}$$

It follows that the matching probability for a firm posting a vacancy offering the high wage rate (given by the term in brackets on the left-hand-side of (38)) rises, whereas the corresponding matching probability for a low wage vacancy (given by the term in brackets on the left-hand-side of (39)) declines. Thus the rise in $\overline{U}$ implies a higher aggregate level of unemployment (U) accompanied by enhanced sorting, that is a shift from low-wage vacancies towards high-wage vacancies, which in equilibrium results in a shift from low-productivity firms towards high-productivity ones.

Balancing the two opposing forces will determine the optimum. To illustrate the point consider the following figure derived for a numerical solution of the model based on the two-point discrete example:[14]

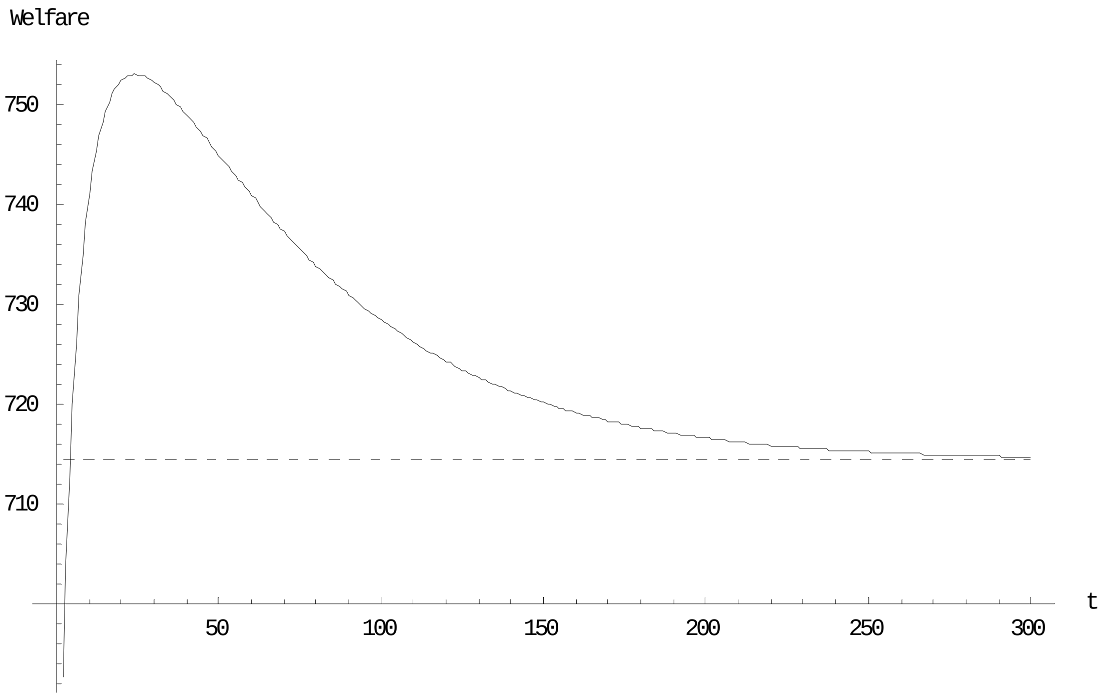

Figure 4: Optimal Duration

The figure illustrates that the optimal duration is given by $\bar{t}^* = 24$.[15] Note that for sufficiently short duration ($\bar{t} < 24$) the enhanced sorting effect dominates, whereas for sufficiently long

[14]There is no particular importance to the numerical values here, but rather to the shape. The following parameter values are used: $\overline{x} = 15, \underline{x} = 5, p = 0.5, M = 100, L = 70, s = 0.01, h = 0$.

[15]Note that when $\bar{t}^*$ is set then the control variable $\overline{U}$ is set according to (36). Note, also, that the other controls $\underline{\underline{x}}, \widehat{x}$ in (37) are set as a function of the properties of the productivity distribution, which is a two type discrete one.

duration ($\bar{t} > 24$) the increased unemployment effect dominates. The optimum at $\bar{t}^*$ is obtained at the point of balance.

3.3.2 Optimal Duration, Worker Sorting, and the Firms Productivity Distribution

We re-do the analysis of section 3.2.2, setting the duration at one period longer than in the benchmark model (i.e., using $\bar{t} = 3$ as opposed to $\bar{t} = 2$ in the previous sub-sections). We allow for an additional equilibrium configuration (to be denoted P_2S), which we shall refer to as enhanced partial sorting. Note that it is different from the partial sorting configuration, PS, discussed above, where duration was set at $\bar{t} = 2$. Figure 5 depicts seven different curves in the skewness - standard deviation space. Three new curves (labeled $N - P_2, P - P_2, P_2 - H$) are added to the four curves that appear in Figure 2 above. We maintain the notation used above. Thus, for instance, along the $P - P_2$ curve the partial sorting and the enhanced partial sorting attain the same level of welfare. The figure is based on a numerical solution, using the same parametric assumptions used to derive Figure 2.

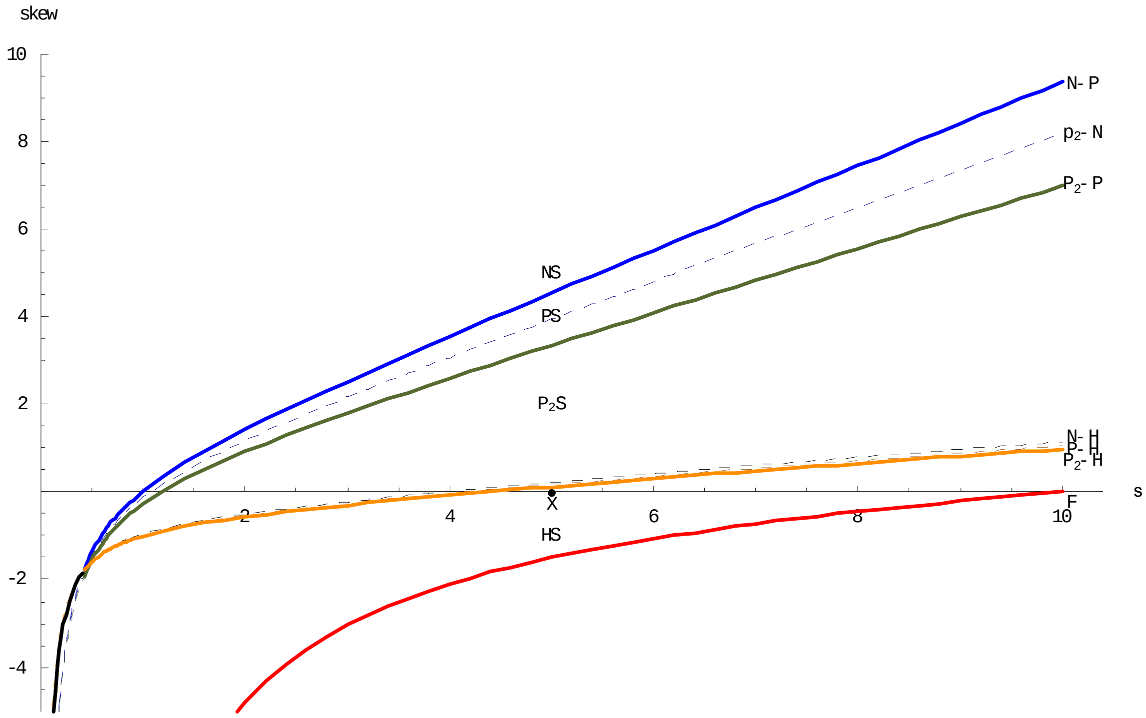

Figure 5: Skewness and Variance Relationships II

We provide again an illustrative figure on a magnified scale (Figure 6). The solid parts divide the space into four disjoint regions. For any point which lies in the region labeled NS, the no-sorting configuration attains the highest level of welfare and is therefore socially desirable. Similarly, for points that lie in the regions labeled, respectively, PS, P_2S and HS, the partial sorting, enhanced partial sorting, and high-sorting configurations prevail, correspondingly.

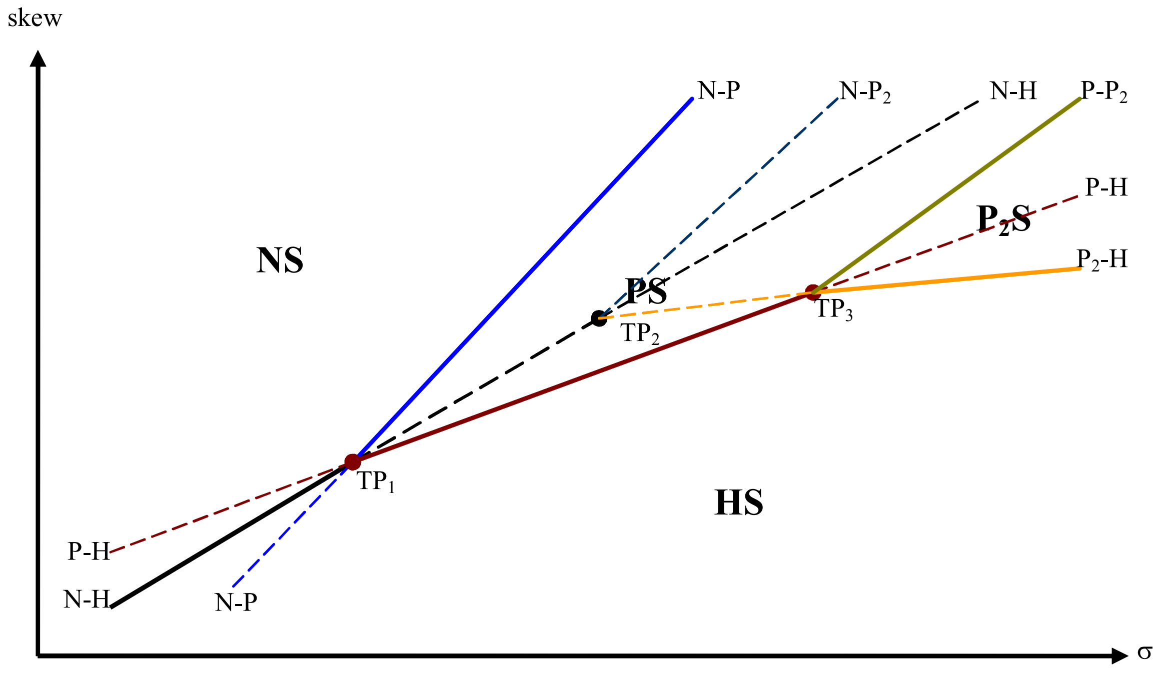

Figure 6: Optimal UI Regions II

Several insights emerge from inspection of the figure:

First, compare Figure 6 to Figure 3. It can be seen that allowing for the setting of an extended duration of the first tier of the UI scheme implies that the range in which a declining UI time path prevails (either partial sorting (PS) or enhanced partial sorting (P_2S)) expands. Notably, the added flexibility of setting the duration of the first tier implies that the enhanced partial sorting configuration crowds out both the partial sorting configuration and the high-sorting one, as the social planner can now achieve better balance between the opposing matching and employment considerations. In other words, there is some "fine tuning" of the high sorting and the partial sorting configurations.

Second, one can see that the optimal degree of sorting, which is determined by the length of the duration of the first tier, is rising with respect to the dispersion (s.d.) of the firm technological distribution, and is decreasing with respect to the skewness. Thus, as we move either rightwards (increasing s.d.) or downwards (reducing skewness), we gradually increase the optimal degree of sorting (by extending the duration). This reflects the fundamental trade-off between matching and employment considerations. Note that the high-sorting and the no-sorting configuration are essentially the limiting cases, whereby the duration is, correspondingly, set to infinity ($\bar{t} = \infty$) or zero ($\bar{t} = 0$).

While the figure demonstrates only four different durations ($\bar{t} = 0, 2, 3$ and ∞), it is straightforward to see how the analysis can be extended to any range of durations. The more flexibility we add, the more refined will be the partition of the variance-skewness space. Thus, for example, allowing for a fifth configuration, where the duration would be set at $\bar{t} = 4$, we would obtain another intersection point along the $P_2 - H$ curve to the right of TP_3, which would yield a fork-shaped region stemming from the intersection point. In this region the new configuration would prevail, crowding out the high-sorting and the enhanced partial sorting configurations. As we refine duration policy, we would obtain additional fork-shaped regions like those stemming from intersection points TP_1 and TP_3.

To sum up, optimal duration is set so as to balance between matching and employment considerations. The longer the duration, the higher the degree of sorting, and the larger is the associated unemployment. For any distribution of technologies, captured by the different points in the s.d.-skewness space, duration is set optimally. The higher the gains from sorting and/or the lower the costs entailed by the induced unemployment, namely, the higher the spread and/or the lower the skewness, the longer will optimal duration be.

Another way to demonstrate the relationship between duration policy and the properties of the productivity distribution is by relating Figure 4 and Figure 5. Note that Figure 4 illustrates welfare as a function of the duration, for a given distribution of technologies, while Figure 5 partitions the technology distribution space (skewness-σ space) into different lengths of duration, according to the dominating configuration. The numerical solution, as expressed in Figure 4, has the welfare level associated with the partial-sorting configuration converging to an asymptote of approximately 714, given by the horizontal dashed line in the figure. This is the welfare level associated with the high-sorting configuration (recall that the high-sorting configuration is essentially a limiting case of the partial-sorting configuration when $\bar{t} \to \infty$). When the duration is set to be sufficiently short, $\bar{t} < 5$, the partial sorting configuration is dominated by the high-sorting one, while for duration of $\bar{t} > 5$ the partial-sorting configuration prevails. This observation can also be demonstrated using Figure 5. To do so, note that the technological distribution used in Figure 4 is characterized by the parameters: $\mu = 10, \sigma = 5$ and $skewness = 0$. These values correspond to point X in Figure 5. Note too that this point lies below the $P_2 - H$ line, which implies that in this case, where $\bar{t} = 3$, the high-sorting configuration dominates the partial sorting one, as shown in Figure 4. As $\bar{t}$ increases, the borderline below which the HS dominates moves downwards. Figure 6 illustrates this movement, as the boundary $P_2 - H$ lies below the $P - H$ curve (to the right of the intersection point, TP_3). More generally, by extending the duration further, say, setting $\bar{t} = n$, where $n > 3$, the $P_{n-1} - H$ curve would lie below the $P_2 - H$ curve. Eventually, for n sufficiently large ($\bar{t} \geq 5$ in terms of Figure 4) the point X would lie above the $P_{n-1} - H$ curve, which would indicate that the partial sorting with duration n dominates the high sorting configuration.

3.4 Empirical Consistency

The model has examined optimal UI policy from a normative perspective, focusing on the response of policy to the productivity distribution. It has built on a number of key elements: heterogeneity in firm productivity, a positive relationship between productivity and wages, random matching, and a declining UI time profile. How relevant empirically are these elements?

The assumption of firm heterogeneity is well supported by empirical studies. In U.S. data, Davis, Haltiwanger and Schuh (1996) using LRD establishment level data, and Haltiwanger, Lane and Spletzer (2000) using matched employer-employee data of the Bureau of Census find that: (i) firms locate along a productivity/earnings/skill locus with some firms being high productivity, high wage, and high skill while others are low productivity, low wage, and low skill; (ii) firm performance and behavior, even within quite narrowly defined industries, is quite heterogeneous and this is a substantial and persistent phenomenon.

Abowd, Kramraz and Margolis (1999), using a longitudinal sample from France of over one million workers from more than five hundred thousand firms, find that firms that hire high wage workers are more productive per worker. Mortensen (2003) argues that dispersion in wages paid for observably equivalent workers is to be explained by differences in firm productivity and that more productive firms offer higher wages. He provides evidence from a matched employer-employee data set for the Danish economy.

For empirical evidence on random matching see the survey by Petrongolo and Pissarides (2001) and the structural estimates in Yashiv (2000). Note too, that the current paper is fully consistent with w including non pecuniary elements as well as wages, so workers may be compensated not only by wage payments but also by other job attributes, such as job risk, work environment, promotion chances, amenities etc. In this setting it seems reasonable to assume imperfections in worker information about vacant jobs.

A declining UI time profile is a very prevalent phenomenon. UI benefits duration is typically limited and is then replaced by social or income assistance which is lower (and often means-tested, thereby leading to lower take up rates). Thus data from OECD (2004) indicates that UI duration in OECD countries ranges between 6 and 60 months across 28 member countries (with only Belgium having unlimited duration in some cases). In particular, 13 countries have a benefits duration of 12 months and less.

4 Relation to the Literature

We briefly discuss the place of the current paper in two literatures: the search literature and the literature on optimal UI policy. Starting with the former, a recent survey by Rogerson, Shimer and

Wright (2005) characterizes three main classes of search models: random matching and bargaining (see Pissarides (2000)), directed search and wage posting (see Moen (1997)), and random matching and wage posting (two key models and many references are discussed in Section 6 of Rogerson, Shimer and Wright (2005)). The current paper belongs in the last class. The first class, that includes bargaining, does not allow for the wage posting behavior of firms, which is crucial for the effects of UI policy in the current set up. Indeed that class of models is not geared to explain wage dispersion. The second class, sometimes referred to as "competitive search theory," does share a key feature with the current approach: firms set wages optimally, knowing that the probability of filling a job rises with the wage offer. Additionally, as in Moen (2003), labor market segmentation arises due to the fact that firms cannot condition wage offers on the worker type and workers' productivities differ across matches. But the segmentation here does not take the form of sub-markets and the operation of market makers; rather, it is due to exogenous productivity dispersion and the effects of UI policy.

The main line of research on the optimal design of UI policy has focused on issues of moral hazard and consumption smoothing (see Holmlund (1998) and Karni (1999) for surveys). This literature examines the impact of work disincentives on the design of optimal schemes (the seminal papers are by Baily (1978), Flemming (1978) and Shavell and Weiss (1979)). The main insight provided by the early models was the desirability of a declining schedule, i.e. benefits should decline over the spell of unemployment so as to mitigate the moral hazard effect. The early models have been recently extended in several directions, some of them into general equilibrium frameworks. Hopenhayn and Nicolini (1997), as a notable example, enlarge the set of instruments by allowing for a wage tax after re-employment. This model preserves the sequencing structure of Shavel and Weiss (1979) and attains enhanced consumption smoothing.

The current paper does not belong in the above strand, as it does not consider issues of risk aversion, consumption smoothing, or moral hazard. Rather it focuses on the role UI benefits can play in attaining a better match between jobs and workers, deriving optimal policy in the face of productivity dispersion. A seminal contribution in this context has been made by Diamond (1981), who discussed the role of UI in enhancing efficiency in the context of a steady state search model. In his model UI makes job-taking use more stringent standards, thereby raising the vacancy rate and improving the distribution of job offers.

There are a number of more recent contributions that have dealt with related issues and it is worthwhile delineating their relation to the current paper: in Marimon and Zilibotti (1999) UI improves matching between ex-ante heterogenous workers and ex-ante heterogenous firms under random matching. UI serves to reduce worker-job mismatch, as without UI workers would tend to accept unsuitable jobs. This paper however does not deal with optimal UI policy, as does the current one, and does not make any connection between UI policy and heterogeneous firm productivity. The model of Acemoglu and Shimer (1999) shares with the current paper the idea that UI generates an increase in output, whereby more productive firms choose to offer higher wages and more workers are assigned to those firms. However Aceomglu and Shimer (1999) have risk aversion at the heart of their analysis and UI has an insurance role. By offering UI benefits, apart from the consumption smoothing argument, the policymaker induces risk-averse workers to take on a higher degree of unemployment risk, boosting investment by firms. Their set-up is one with directed search, so externality issues do not arise. In the model here a key point is UI policy turning random matching into assortative matching against the backdrop of heterogeneity in productivity. Thus the mechanism studied is entirely different; it does not relate to risk aversion (agents are risk-neutral) but rather explores the role of UI policy in affecting firm and worker behavior to obtain enhanced matching. Fredriksson and Holmlund (2001) use a standard Pissarides (2000) framework to analyze the equilibrium effects of time-varying UI. They find that an optimal scheme – under certain conditions – has benefits decline over time. This is so because of an 'entitlement effect', according to which raising the compensation offered to the insured induces additional search effort among the uninsured, bringing them more quickly to employment, which results in future UI eligibility. In their model workers and firms are homogenous and there is Nash bargaining, hence all workers are paid the same wage. It is clear therefore that the current paper's focus on heterogenous firms and the reaction of UI policy to this productivity dispersion is absent. Finally, Albrecht and Vroman (2005) present a model of wage posting, matching, declining UI, and a two tier wage system in equilibrium, as is the case here. They show how time-varying unemployment benefits can generate wage dispersion even though firms and workers are homogenous . However, their paper does not contain two key ingredients of the current paper: firm productivity dispersion (in their model firms are identical) and a normative analysis of optimal policy.

5 Conclusions

This paper has studied optimal UI policy from the perspective of worker assignment to heterogenous jobs. Workers react to UI policy through job acceptance decisions; firms react to UI policy through wage posting, with the associated market segmentation. In a world of random matching, efficiency gains are induced by a declining UI profile that induces worker sorting. The longer the duration of the first phase, the higher the degree of induced heterogeneity and sorting. The main elements of the model – productivity dispersion, positive association of productivity and wages, random matching, and a declining UI profile – were shown to be empirically relevant. Thus, while the model is essentially normative, it is consistent with known empirical regularities.

A key lesson is that even in the absence of moral hazard arguments there is a role for a declining time profile of UI. By enhancing matching it operates to increase output and efficiency (in a constrained setting). Particularly important is the dependence of optimal policy on the properties of the productivity distribution. This kind of connection has received little attention thus far. In future work we hope to provide a mapping from empirically-relevant dispersion of firms' productivities to a larger set of policy instruments.

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6 Appendix A

Derivation of the Wage Equations

We reproduce the relevant equations for convenience:

$$H_1 = z + h + \beta[\overline{m}\max(\overline{J}, H_2) + \underline{m}\max(\underline{J}, H_2) + (1 - \overline{m} - \underline{m})H_2] \tag{48}$$

$$H_2 = z + h + \beta[\overline{m}\max(\overline{J}, H) + \underline{m}\max(\underline{J}, H) + (1 - \overline{m} - \underline{m})H] \tag{49}$$

$$H = a + h + \beta[\overline{m}\max(\overline{J}, H) + \underline{m}\max(\underline{J}, H) + (1 - \overline{m} - \underline{m})H] \tag{50}$$

$$\overline{J} = \overline{w} + \beta[(1 - s)\overline{J} + sH_1] \tag{51}$$

$$\underline{J} = \underline{w} + \beta[(1 - s)\underline{J} + sH_1] \tag{52}$$

$$\overline{J} = H_2 \tag{53}$$

$$\underline{J} = H \tag{54}$$

Subtracting (52) from (51) yields:

$$\overline{J} - \underline{J} = \frac{\overline{w} - \underline{w}}{1 - \beta(1 - s)} \tag{55}$$

Subtracting (50) from (49) yields:

$$H_2 - H = z - a \tag{56}$$

Substituting (53) and (54) into (55), and then substituting (55) into (56) yields:

$$z - a = \frac{\overline{w} - \underline{w}}{1 - \beta(1 - s)}$$

This is equation (14) in the main text.

Substituting (53) into (48) and noting that $\overline{J} = H_2 > \underline{J}$ yields:

$$H_1 = z + h + \beta H_2 \tag{57}$$

Substituting (53) into (51) yields:

$$[1 - \beta(1 - s)]H_2 = \overline{w} + \beta s H_1 \tag{58}$$

Solving (57) and (58) for H_2 and simplifying yields:

$$[1 - \beta(1 - s) - \beta^2 s]H_2 = \overline{w} + \beta s(z + h) \tag{59}$$

Substituting (53) and (54) into (49) and re-formulating yields:

$$(1 - \beta)H_2 = z + h - \beta(1 - \overline{m})[H_2 - H] \tag{60}$$

Substituting (56) into (60), then substituting from (59) into (60) for H_2 and simplifying, yields:

$$\overline{w} = (z - a)[(1 - \beta) + \beta\overline{m} - \beta^2 s(1 - \overline{m})] + h + a$$

This is equation (15) in the main text.

7 Appendix B

Derivation of the Re-Employment Probabilities

We show that:

$$\underline{m} = \frac{V}{U - U_1}(1 - e^{-\frac{U-U_1}{F}})$$

A symmetric argument proves the condition with respect to $\overline{m}$.

By definition:

$\underline{m} = Prob$[being assigned to a vacancy offering $\underline{w}$]$\times Prob$[obtaining a job conditional on being assigned].

Because of random matching across posted jobs, the probability of being assigned to a vacancy offering the lower wage rate is simply given by $\frac{V}{F}$.

The conditional probability takes into account the fact, that conditional on assignment, if another k applicants (who are willing to accept a low wage offer) arrive contemporaneously, a random draw from a pool of $k+1$ applicants determines the winner.

Since agents during the first period of UI eligibility reject any low wage offer, it follows that the conditional probability is given by:

$$\begin{aligned} Prob[\text{obtaining a job conditional on being assigned}] &= \sum_{k=0}^{\infty} \frac{\frac{e^{-\lambda}\lambda^k}{k!}}{k+1} \\ &= \sum_{k=0}^{\infty} \frac{e^{-\lambda}\lambda^k}{(k+1)!} \end{aligned}$$

where $\lambda = \frac{U-U_1}{F}$

Using a Taylor expansion it follows that:

$$Prob[\text{obtaining a job conditional on being assigned}] = \frac{1 - e^{-\lambda}}{\lambda}$$

Substitution completes the derivation.

8 Appendix C

Implementation of the Partial Sorting Equilibrium

We set $F = M$ and $\overline{F} = pM$.

We substitute (16) and (17) in (18) and (19) correspondingly, and solve the system (18)-(22) for five unknowns: $\overline{V}, \underline{V}, U_1, U_2$, and U.

To insure existence of partial-sorting equilibrium, we need to verify that our solution satisfies (14), (15) and (24). Modified to the discrete case, eq. (24) turns into two inequality conditions:

$$\left(1 - e^{-\frac{U-U_1}{F}}\right)(\underline{x} - \underline{w}) \geq \left(1 - e^{-\frac{U}{F}}\right)(\underline{x} - \overline{w}) \tag{61}$$

$$\left(1 - e^{-\frac{U}{F}}\right)(\overline{x} - \overline{w}) \geq \left(1 - e^{-\frac{U-U_1}{F}}\right)(\overline{x} - \underline{w}) \tag{62}$$

This is easy to observe for we have two instruments at our disposal – a and z. Using (14) we fix some $\varepsilon > 0$ arbitrarily small, and set $z - a$ small enough such that $\overline{w} - \underline{w} = \varepsilon$. Using (15), we adjust a, such that $\overline{w} = \underline{x} + \frac{\varepsilon}{2}$. For $\varepsilon > 0$ sufficiently small, (24) is satisfied i.e. all high-productivity firms choose to offer $\overline{w}$, whereas all low-productivity firms choose to offer $\underline{w}$.