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ABSTRACT

Efficiency in a Matching Model with Heterogeneous Agents: Too Many Good or Bad Jobs?*

This paper analyses the efficiency of the equilibrium allocation in a matching model with two types of workers and jobs. The basic assumption is that high-skill workers can perform both skilled and unskilled jobs, while low-skill workers can only perform unskilled jobs. Our first result shows that the equilibrium with ex-post bargaining is never efficient. Second, under Hosios' (1990) condition we show that high-skill workers are under-valued in equilibrium, while the opposite holds for low-skill workers. Firms therefore tend to create too few unskilled jobs, resulting in a suboptimally high unemployment rate for low-skill workers. We show that these results generalize to environments with more types of agents and different production technologies. Finally, in an extension we derive a tax scheme that restores efficiency and we show how workers' bargaining strength affects unemployment and skill-mismatch.

JEL Classification: C78, D61, J64

Keywords: matching, ex post bargaining, heterogeneity, efficiency

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1 Introduction

This paper analyses the efficiency of the resource allocation in a matching model with two types of workers and jobs. Workers' skills are exogenous and we assume that high-skill workers can do both jobs, skilled and unskilled, while low-skill workers can only perform unskilled jobs. This setup, first introduced by Albrecht and Vroman (2002), is useful to study phenomena like skill-biased technological change or low-wage competition. However, so far little is known about the efficiency properties of these models.

Our objective in this paper is to characterize the entire set of possible steady state equilibria with random search and ex post bargaining. As in Albrecht and Vroman (henceforth AV) we therefore need to distinguish between two types of equilibria. A *cross-skill matching* equilibrium in which high-skill workers accept both types of jobs and an *ex post segmentation* equilibrium in which these workers only match with skilled jobs. To achieve this goal we compare the equilibrium with the efficient allocation chosen by a social planner. This planner controls the mass of the two types of vacancies and the optimal matching rule for high-skill workers and her objective is to maximize the value of net-output.

Our first result shows that the equilibrium with ex post bargaining is never efficient. Second, under Hosios' (1990) condition we show that high-skill workers are undervalued in equilibrium while the opposite holds for low-skill workers. In a cross-skill matching equilibrium this results in an insufficient number of unskilled jobs as firms need to pay low-skill workers more than their shadow value. Furthermore, due to the undervaluation of high-skill workers, there may be cases in which these workers accept unskilled jobs while the efficient allocation features ex post segmentation.

The result that the labour market tends to create too few unskilled jobs achieves our first goal. Namely, to show that the presence of high-skill workers tends to harm low-skill workers by raising their unemployment rate above the socially optimal level. Furthermore, our results suggest that this distortion becomes even stronger when we move from cross-skill matching to ex-post segmentation. In the latter case high-skill workers no longer occupy unskilled jobs, but this effect is more than offset by a discrete fall in the supply of these jobs. Finally, it should be noted that our model can also generate the opposite result, *i.e.* over-creation of unskilled jobs. The latter occurs when workers' surplus share is sufficiently smaller than the Hosios' value. In this parameter region, firms create too many jobs and a suboptimally large share of these jobs is unskilled. In contrast, for sufficiently high values of workers' bargaining power, the fraction of unskilled jobs is above the efficient value, but because the overall number of vacancies per job seeker is too low, the unemployment rate of low-skill workers is still suboptimally high.

To understand the intuition behind our results, it is convenient to make a comparison with the results of Hosios (1990). For an environment with ex ante homogeneous agents he demonstrated that the equilibrium is efficient if workers' bargaining power is equal to the elasticity of the matching function with respect to the mass of unemployed workers. In that case, the search externalities are perfectly internalized in the wage and unemployed workers' expected income coincides with their shadow value.

In contrast, in our economy an additional unemployed worker congests the market for workers with different skill levels. The absolute value of this externality is the same for both types of workers. However, in the case of a high-skill worker it corresponds to a lower share of her expected future productivity than in the case of a low-skill worker. When Hosios' condition is satisfied, high-skill workers are therefore under-valued in equilibrium while the opposite holds for low-skill workers.

The above argument seems to suggest that there may exist a pair of surplus shares, one for each type of worker, that decentralizes the efficient allocation. However, this is not the case. In our model Hosios' condition is necessary to ensure that firms create the right number of jobs per job seeker. When we introduce different bargaining strengths to correct the distortion of the job distribution — the over-creation of skilled jobs and the under-creation of unskilled jobs — we would therefore end up with an inefficient number of jobs.[1] Instead, it is easy to show that the efficient allocation can be decentralized through taxation. In the paper we analyse the case of lump sum taxes on unemployed workers, showing that the government should levy a tax on unemployed workers with a low skill level, while workers with a high skill level should receive a subsidy during unemployment. The proposed tax scheme confirms our explanation about the source of the over-valuation (under-valuation) of low-skill (high-skill) workers, but it may be difficult to implement. An alternative would be to levy a hiring tax on skilled jobs. Also in this case, the relative profits of skilled jobs go down, but the hiring tax on skilled jobs avoids the negative income effect on low-skill workers.

1.1 Related literature

The seminal contribution on efficiency with heterogeneous agents is Sattinger (1995). In his model there is a fixed supply of T types of workers and firms whose matching rates are fixed. Sattinger's main result shows that heterogeneity may give rise to multiple and inefficient equilibria. Like us, he also shows that the equilibrium payoffs never coincide with the shadow value of agents. However, while computing the shadow value of a worker, Sattinger sterilizes the effects on the other workers by increasing the job supply to offset any changes in matching probabilities. Distortion of the wage payments therefore arise because workers ignore the costs of the firms with whom they interview and not because of the congestion effects on other workers as in

[1] The details of the proof are available upon request.

our case. Our work is therefore closer to Shimer and Smith (2001a, 2001b) who use a social planner's problem to derive the efficient allocations in an environment with assortative matching and endogeneous search intensity. In Shimer and Smith (2001b) the aim is to show that the model may have efficient non-stationary allocations. Our work is closer in spirit to Shimer and Smith (2001a). In this paper the authors show that the decentralized equilibrium is never efficient without search subsidies. In particular, in the decentralized equilibrium without subsidies, the most productive agents do not search hard enough and they accept too many (low-productivity) matches. In contrast, low-productivity types search too hard and they reject too many matches. An optimal tax scheme therefore involves a search subsidy for the high-productivity agents and a search penalty for the low-productivity workers.

This last result is somewhat similar to our optimal tax scheme. However, the advantage of our simple model is that we can immediately relate our results to the well-known results of Hosios. Hence, while Shimer and Smith (2001a) obtain the inefficiency of the resource allocation via the derivation of a non-trivial tax scheme, we are able to prove that low-skill workers are over-valued in equilibrium. Moreover, our results indicate that this is a general feature of models with ex ante heterogeneous agents. One of the main contributions of this paper is therefore that we provide a clear intuition for the inefficiency of the resource allocation in economies with heterogeneous agents and ex post bargaining. Finally, unlike Shimer and Smith, we allow for free entry of firms.

The outline of the rest of the paper is as follows. Section 2 introduces the model. The next section derives the two possible equilibrium configurations. The set of efficient allocation is derived in Section 4. This section starts with a characterization of efficient cross-skill matching allocations. Next, we discuss the efficient allocations with ex post segmentation and at the end of the section we derive the optimal matching rule of high-skill workers, showing that there exists an intermediate range of parameters in which high-skill workers adopt an inefficient matching rule. Finally, in section 5 we derive the optimal taxes and we show how extreme values of the bargaining strength may lead to overcreation of unskilled jobs.

2 The Model

2.1 Main Assumptions

Consider an economy populated by a continuum of risk-neutral workers with measure normalized to one. All workers live forever in continuous time and value consumption of the unique final good. The distribution of skills is exogenous. A fraction $\mu \in (0,1)$ of the population is low-skilled (l), while the remaining fraction $1-\mu$ is high-skilled (h).

There are also two types of jobs: Unskilled jobs (n) can be filled by any worker, while skilled jobs (s) require a high-skill worker. Furthermore, we assume that high-skill workers are more productive in skilled jobs, while all workers are equally productive in unskilled jobs. Formally, let $y(i,j)$ define the flow output of a job $j(=n,s)$ that is filled by a worker of type $i(=l,h)$. Our assumptions on the production technology can then be summarized as follows:

$$y(h,s) = y(s) > y(h,n) = y(l,n) = y(n) > y(l,s) = 0.$$

For convenience, we assume that firms can open at most one job. The choice of the type of job is irreversible, and the mass of each type of job is determined by a free-entry condition.

Finally, job destruction is exogenous and follows a Poisson process with arrival rate δ that is common to both types of jobs. Whenever a job is destroyed, the worker becomes unemployed while the job becomes vacant.

2.2 Matching

Unemployed workers and vacant jobs are matched together in pairs through an imperfect matching technology. The total number of random meetings between a worker and firm is determined by the standard (Cobb-Douglas) matching function

$$M(v, u) = u^{\alpha} v^{1-\alpha},$$

where v is the mass of vacancies and u is the mass of unemployed workers.

In our environment only part of the meetings will be consummated. Formally, let $\theta = v/u$ denote the labour market tightness and let ϕ denote the fraction of unskilled vacancies. The matching rate of unemployed workers is then equal to $\theta^{1-\alpha}$. However, since low-skill workers are not qualified for skilled jobs, they exit unemployment at rate $\phi\theta^{1-\alpha}$. Similarly, let η denote the fraction of low-skill workers among the unemployed. The matching rate of vacant jobs can then be expressed as $\theta^{-\alpha}$, while skilled jobs meet an appropriately qualified high-skill worker at rate $(1 - \eta)\theta^{-\alpha}$.

The assumption of random matching follows AV. In this manner we capture the idea that, for given overall labour market conditions, low-skill workers are better off the greater the fraction of unskilled jobs and vice versa for high-skill workers and skilled jobs.

2.3 Wages and Asset Values

When a match is formed, the firm-worker pair divides the surplus of the match according to the (asymmetric) Nash bargaining solution. The worker's share of the surplus is exogenous and denoted by $\beta \in (0,1)$.

Formally, let $U(i)$ be the value of an unemployed worker of type i, and let $V(j)$ denote the value of a vacant job of type j. Similarly, let $W(i,j)$ denote the value of employment for a worker of type i on a job of type j, and let $J(i,j)$ denote the value of a type j job filled by a worker of type i. The surplus of a match between a worker of type i and a job of type j is then given by $S(i,j) = W(i,j) + J(i,j) - V(j) - U(i)$, while the associated wage $w(i,j)$ solves the Nash bargaining solution:

$$(1-\beta)\left[W(i,j) - U(i)\right] = \beta\left[J(i,j) - V(j)\right]. \tag{1}$$

Finally, we say that workers' bargaining strength satisfies Hosios' condition when β is equal to α, where α corresponds to the elasticity of $M(.,.)$ with respect to u.

We now proceed with the derivation of the asset value equations for workers. The common discount rate of workers and firms is denoted by r. Furthermore, we assume that unemployed workers earn a flow income $b < y(n)$ from home production. Accordingly,

$$rU(h) = b + \theta^{1-\alpha}\left\{\phi \max\left[W(h,n) - U(h), 0\right] + (1-\phi)\left[W(h,s) - U(h)\right]\right\} \tag{2}$$

$$rW(h,s) = w(h,s) - \delta\left[W(h,s) - U(h)\right] \tag{3}$$

$$rW(h,n) = w(h,n) - \delta\left[W(h,n) - U(h)\right] \tag{4}$$

$$rU(l) = b + \theta^{1-\alpha}\phi\left[W(l,n) - U(l)\right] \tag{5}$$

$$rW(l,n) = w(l,n) - \delta\left[W(l,n) - U(l)\right] \tag{6}$$

Equation (2) shows that high-skill workers accept unskilled jobs when $W(h,n) - U(h) \geq 0$. Under Nash bargaining this is equivalent to the condition that $S(h,n)$ is non-negative.

Similarly, let γ denote the cost per unit of time of maintaining an unfilled job. The values of unfilled jobs are then given by:

$$rV(s) = -\gamma + \theta^{-\alpha}\left(1-\eta\right)\left[J(h,s) - V(s)\right] \tag{7}$$

$$rV(n) = -\gamma + \theta^{-\alpha}\left\{\eta\left[J(l,n) - V(n)\right] + (1-\eta)\max\left[J(h,n) - V(n), 0\right]\right\} \tag{8}$$

According to (8), firms with unskilled jobs agree to hire high-skill workers whenever $J(h,n) - V(n) \geq 0$. Given Nash bargaining, this is again equivalent to the condition that $S(h,n)$ is non-negative.

Finally, the three expressions for $J(i,j)$ satisfy:

$$rJ(h,n) = y\left(n\right) - w(h,n) - \delta\left[J(h,n) - V(n)\right] \tag{9}$$

$$rJ(l,n) = y\left(n\right) - w(l,n) - \delta\left[J(l,n) - V(n)\right] \tag{10}$$

$$rJ(h,s) = y\left(s\right) - w(h,s) - \delta\left[J(h,s) - V(s)\right] \tag{11}$$

Substituting equations (2) $-$ (11) into (1), and imposing the free entry conditions $V(j) = 0$ for $j \in (n, s)$, we obtain the following standard expression for wages:

$$w(i,j) = rU(i) + \beta \left[y(j) - rU(i) \right] \tag{12}$$

Notice that $rU(h) \geq rU(l)$ since high-skill workers have the option to work in skilled jobs. The wage of a mismatched high-skill worker, $w(h,n)$, therefore exceeds the wage of a low-skill worker.

3 Equilibrium

We now proceed with a brief discussion of the equilibrium allocations.[2]

3.1 Cross-skill Matching

We begin with the derivation of the equilibrium when the matches between high-skill workers and unskilled vacancies are mutually beneficial and, therefore, consummated. We refer to this situation as a "cross-skill matching" equilibrium. This equilibrium is characterized by a vector of endogenous variables $\{\theta, \phi, \eta, u\}$ satisfying: (i) two steady state conditions and (ii) two free entry conditions.

The steady state conditions require that the flows into and out of unemployment for both types of workers must be equal:

$$\phi\theta^{1-\alpha}\eta u = \delta \left[\mu - \eta u \right] \tag{13}$$

$$\theta^{1-\alpha} (1-\eta) u = \delta \left[1 - \mu - (1-\eta) u \right] \tag{14}$$

[2]This section is based on section 3 of AV.

where $\phi\theta^{1-\alpha}\eta u$ defines the flow of low-skill workers out of unemployment and $\delta\left[\mu-\eta u\right]$ denotes the corresponding flow into unemployment. Similarly, for high-skill workers, $\theta^{1-\alpha}\left(1-\eta\right)u$ denotes the flow of high-skill workers out of unemployment, and $\delta\left[1-\mu-\left(1-\eta\right)u\right]$ the corresponding flow into unemployment.

The two steady-state conditions can be solved for ϕ and u in terms of θ and η. This yields the first two equilibrium conditions:

$$\phi=\frac{\mu\left(1-\eta\right)\theta^{1-\alpha}+\left(\mu-\eta\right)\delta}{\left(1-\mu\right)\eta\theta^{1-\alpha}} \tag{15}$$

$$u=\frac{\delta(1-\mu)}{\left[\delta+\theta^{1-\alpha}\right]\left(1-\eta\right)}. \tag{16}$$

Notice that ϕ is decreasing in η and increasing in θ for $\eta>\mu$. This last condition is always satisfied. Under cross-skill matching, low-skill workers exit unemployment at a lower rate than high-skill workers. Thus, since δ is the same across jobs, low-skill workers are over-represented in the pool of unemployed workers.

The remaining equilibrium conditions follow from the free entry conditions. Using (7), (11) and (12), the free entry condition for skilled vacancies can be written as:

$$\gamma=\theta^{-\alpha}\left(1-\eta\right)\left(1-\beta\right)\left[\frac{\left[y(s)-rU(h)\right]}{r+\delta}\right]. \tag{17}$$

Similarly, substituting (9), (10) and (12) into (8), the free entry condition for unskilled vacancies becomes:

$$\gamma=\theta^{-\alpha}\left(1-\beta\right)\left\{\eta\left[\frac{\left[y(n)-rU(l)\right]}{r+\delta}\right]+\left(1-\eta\right)\left[\frac{\left[y(n)-rU(h)\right]}{r+\delta}\right]\right\}, \tag{18}$$

where the reservation wages $rU(h)$ and $rU(l)$ are given by the following expressions:

$$rU(h)=\frac{b\left(r+\delta\right)+\beta\theta^{1-\alpha}\left[\phi y(n)+(1-\phi)y(s)\right]}{r+\delta+\beta\theta^{1-\alpha}} \tag{19}$$

$$rU(l) = \frac{b(r+\delta) + \beta\phi\theta^{1-\alpha}y(n)}{r+\delta+\beta\phi\theta^{1-\alpha}}. \tag{20}$$

The third equilibrium condition, the so-called "equal-value condition", is obtained by equating the two free entry conditions

$$\left(r+\delta+\beta\phi\theta^{1-\alpha}\right) = \frac{\eta(r+\delta)(y(n)-b)}{(1-\eta)(y(s)-y(n))}, \tag{21}$$

while the fourth condition is obtained by substitution of (21) into the free entry condition for skilled vacancies:

$$\gamma\left(r+\delta+\beta\theta^{1-\alpha}\right) = (1-\beta)\,\theta^{-\alpha}(y(n)-b) \tag{22}$$

Given our Cobb-Douglas matching function, equation (22) defines a unique solution for θ. Furthermore, this solution depends on $y(n)$ but not on $y(s)$ and μ. The equilibrium labour market tightness is therefore invariant to shifts in the skill distribution and/or changes in the productivity of skilled jobs, $y(s)$.

Definition 1 *A cross-skill matching equilibrium can be summarized by a vector* $\{\theta_E, \phi_E, \eta_E, u_E\}$ *satisfying equations* (15), (16), (21) *and* (22).

Existence As mentioned above, a cross-skill matching equilibrium requires that $S(h,n) \geq 0$, so that high-skill workers are willing to accept unskilled jobs. From equation (18), it follows that $S(h,n) = [y(n) - rU(h)]/(r+\delta)$, and so we obtain the following condition:

$$y(n) - rU(h) \geq 0. \tag{23}$$

In other words, for a cross-skill matching equilibrium to exist, the flow output of an unskilled job must be greater or equal than the reservation wage of a high-skill worker. If not, high-skill workers reject unskilled jobs and we obtain an ex-post segmentation equilibrium.

Finally, before turning to the ex post segmentation equilibrium, we need to rule out the *corner solution* in which firms only supply unskilled jobs. The latter requires that the profits from skilled jobs, $V(s)$, are positive when $\phi = 1$. The parameter restriction that ensures an interior solution with $\phi < 1$ is given by:[3]

$$y(n) - b < (1-\mu)\left[y(s) - b + \frac{\beta\,(\theta^{*})^{1-\alpha}\,(y(s) - y(n))}{r+\delta}\right], \tag{24}$$

where θ^{*} is the unique value of θ that solves equation (22). Thus, in order to avoid the corner solution we need that skilled jobs are sufficiently more productive than unskilled jobs.

3.2 Ex Post Segmentation

We now proceed with a description of an ex-post segmentation equilibrium. In this case skilled jobs are so numerous, either because of a high productivity $y(s)$ or a large value of $1-\mu$, that high-skill workers reject unskilled jobs. Accordingly, the two steady-state conditions satisfy:

$$\phi\theta^{1-\alpha}\eta u = \delta(\mu - \eta u) \tag{25}$$

$$(1-\phi)\,\theta^{1-\alpha}\,(1-\eta)\,u = \delta\left[(1-\mu) - (1-\eta)\,u\right], \tag{26}$$

while the free entry conditions simplify to:

$$\gamma = \theta^{-\alpha}\eta(1-\beta)\left[\frac{[y(n) - b]}{r+\delta+\beta\phi\theta^{1-\alpha}}\right] \tag{27}$$

$$\gamma = \theta^{-\alpha}\,(1-\eta)\,(1-\beta)\left[\frac{[y(s) - b]}{r+\delta+\beta\,(1-\phi)\,\theta^{1-\alpha}}\right]. \tag{28}$$

[3] See Albrecht and Vroman (2002) for details.

AV show that the equilibrium can be solved in two steps. In the first step we need to solve (25) and (26) for ϕ and u as functions of θ and η. Substituting the solution for $\phi(\theta, \eta)$ into the free entry conditions yields a system of two equations in two unknowns. The unique solution is illustrated in Figure 1. The free entry condition of unskilled vacancies is denoted by $V(n) = 0$. It has a positive slope in the positive quadrant (θ, η) since the value of $V(n)$ is increasing in η and decreasing in θ. Similarly, $V(s)$ is decreasing in both θ and η, resulting in a downward-sloping free entry condition $V(s) = 0$.

Thus, whenever an ex-post segmentation equilibrium exists it is unique. The same is also true in the case of a cross-skill matching equilibrium. Nonetheless, the equilibrium allocation need not be unique. For an intermediate range of parameter values for $y(s)$ and $1-\mu$ the two types of equilibrium may coexist. The origin of the multiplicity is a coordination externality (e.g. Sattinger, 1995). Imagine that high-skill workers reject unskilled jobs. In that case, firms will respond by creating many skilled jobs and relatively few unskilled jobs. A high-skill worker may therefore find it optimal to wait until she meets a skilled vacancy. By contrast, if high-skill workers accept unskilled jobs, firms will create many unskilled jobs and relatively few skilled jobs. For the same parameter values, a high-skill workers may now find it optimal to accept all offers.

4 Efficient Allocation

We now turn to the welfare properties of the model. The efficient allocation is derived using the construct of a "social planner" who chooses the time path of the masses of vacancies, $v(s)_t$ and $v(n)_t$, and unemployed workers, $u(h)_t$ and $u(l)_t$, to maximize the value of net output. Let $Y(i, j) = y(i, j)/(r+\delta)$ be the expected productivity of a match between a worker of type $i \in (l, h)$ and a job of type $j \in (n, s)$. Finally, let $R_t \in [0, 1]$ $\forall t$ define the matching rule for

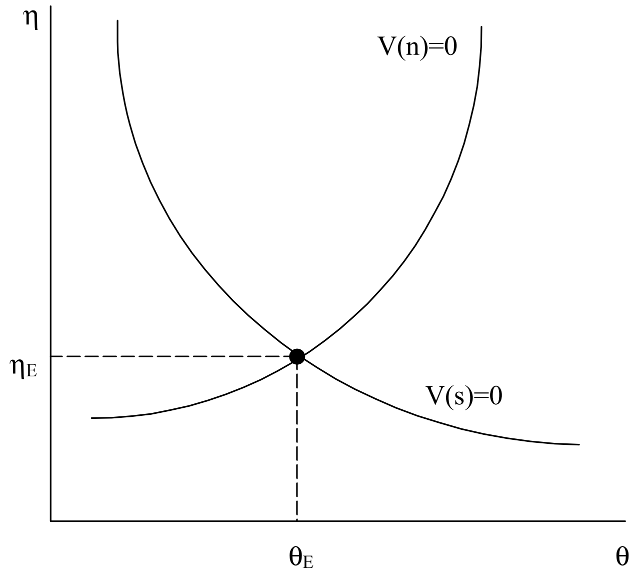

Figure 1: Equilibrium

high-skill workers and unskilled vacancies. If R_t takes value 1, then high-skill workers should always accept matches with unskilled vacancies (cross-skill matching), while a value of 0 means that high-skill workers should never accept unskilled job offers (ex-post segmentation). For any $R_t \in (0,1)$ we would have a mixed matching strategy.

The social planner's problem can succinctly be written as:

$$\max_{\theta_t,\phi_t,u_t,R_t} \int_0^\infty \{\theta_t^{1-\alpha}\left[u(h)_t\left[R_t\phi_t Y(h,n) + (1-\phi_t)\,Y(h,s)\right] + u(l)_t\phi_t Y(l,n)\right] + \\ +b\left(u(h)_t + u(l)_t\right) - \gamma\theta_t\left(u(h)_t + u(l)_t\right)\}e^{-rt}dt \tag{29}$$

s.t

$$\dot{u}(l)_t = \delta\left[\mu - u(l)_t\right] - \phi_t\theta_t^{1-\alpha}u(l)_t \tag{30}$$

$$\dot{u}(h)_t = \delta\left[1-\mu-u(h)_t\right] - \theta_t^{1-\alpha}u(h)_t\left[R_t + (1-R_t)(1-\phi_t)\right] \tag{31}$$

In Appendix A we solve for the efficient steady state allocations using optimal control theory. Henceforth, we therefore suppress all time indices. Furthermore, from the optimality condition for R, it follows immediately that the optimal matching rule depends on the sign of $y(n)-(r+\delta)\lambda(h)$, where $\lambda(h)$ is the endogeneous shadow value of a high-skill worker:

$$R = \begin{cases} 1 \quad \text{if} \quad y(n) > (r+\delta)\lambda(h) \\ \in [0,1] \text{ if } \ y(n) = (r+\delta)\lambda(h) \\ 0 \quad \text{if} \quad y(n) < (r+\delta)\lambda(h) \end{cases} \tag{32}$$

The above result implies that R is generically driven to the boundaries. Finally, for an intermediate range of parameters, cross-skill matching and ex-post segmentation both generate a value for $\lambda(h)$ that satisfies (32). This is somewhat similar to the potential of multiple equilibria discussed above. Nonetheless, the efficient allocation is generically unique since the planner will choose the candidate efficient allocation that generates the largest value of net output.

4.1 Cross-skill Matching

Let us start with the case of an efficient cross-skill matching allocation. When $y(n) > (r+\delta)\lambda(h)$ and $R=1$, the efficient allocation is fully characterized by the following four conditions:[4]

[4]Arrow's generalization of Mangasarian's sufficiency theorem (Kamien and Schwartz, 1991: 222) implies that equations (33) – (36) are the necessary and sufficient conditions

$$\phi = \frac{\mu\theta^{1-\alpha}(1-\eta) + (\mu-\eta)\delta}{\theta^{1-\alpha}(1-\mu)\eta} \tag{33}$$

$$u = \frac{\delta(1-\mu)}{\left(\delta + \theta^{1-\alpha}\right)(1-\eta)} \tag{34}$$

$$\left(r + \delta + \phi\theta^{1-\alpha}\right)\frac{\left(r + \delta + \alpha\theta^{1-\alpha}\right)}{\left(r + \delta + \theta^{1-\alpha}\right)} = \frac{\eta(r+\delta)(y(n) - b)}{(1-\eta)(y(s) - y(n))} \tag{35}$$

$$\gamma\left(r + \delta + \alpha\theta^{1-\alpha}\right) = (1-\alpha)\theta^{-\alpha}(y(n) - b) \tag{36}$$

Proposition 2 *An efficient steady state allocation with cross-skill matching can be summarized by the tuplet* $\{\theta_{SP}, \phi_{SP}, \eta_{SP}, u_{SP}\}$ *that solves conditions* $(33) - (36)$ *and is always unique.*

Proof: Appendix A

Equations (35) and (36) are, respectively, the efficient equal-value condition and the optimality condition for the mass of skilled jobs, while (33) and (34) coincide with eqs. (15) and (16). To characterize the efficiency properties of a cross-skill matching equilibrium we can therefore limit attention to eqs. (21) and (22) for the equilibrium and (35) and (36) for the efficient allocation. It is easy to show that these conditions never coincide.

First of all, from the conditions for the mass of skilled jobs, eqs. (22) and (36), it follows immediately that $\theta_E = \theta_{SP}$ whenever β is equal to α. In the matching literature this condition is commonly referred to as the Hosios'

for an efficient steady-state solution. Hence, starting from this stationary allocation, the planner does not want to deviate to an alternative (non-)stationary allocation.

condition. Thus, whenever the Hosios' condition is satisfied, the decentralized equilibrium generates the same number of jobs per unemployed worker as the optimal allocation. Nonetheless, plugging $\beta = \alpha$ and $\theta_E = \theta_{SP}$ into (21) and (35) shows that the equilibrium mix of jobs is not efficient. The latter would require that $\beta = \alpha = 1$.

Proposition 3 *A cross-skill matching equilibrium* $\{\theta_E, \phi_E, \eta_E, u_E\}$ *never coincides with an efficient allocation as defined in Proposition 2.*

Proof: Appendix B

Proposition 3 strengthens the well-known efficiency result of Hosios (1990). For an economy with homogenous agents, he shows that efficiency is attained if workers obtain a share $\beta = \alpha$ of the match surplus. The above result shows that this potential for efficiency is destroyed when agents are ex ante heterogenous.[5]

4.1.1 Over-creation of skilled jobs

In general, it is notoriously difficult to characterize the efficiency properties with heterogenous agents. Nonetheless, for the case in which the bargaining strength of workers satisfies Hosios' condition, we are able to provide a complete characterization. In particular,

Proposition 4 *Under Hosios' condition,* $\theta_E = \theta_{SP}$ *but firms create too few unskilled jobs as* $\phi_E < \phi_{SP}$.

Proof: Appendix B

[5] This statement implicitly assumes that the equilibrium exhibits cross-skill matching. The case in which the equilibrium features a different matching arrangement than the efficient allocation is treated in Section 4.3.

The intuition behind Proposition 4 is that low-skill workers (high-skill workers) are over-valued (under-valued) in equilibrium, leading to an over-creation of skilled jobs and an under-creation of unskilled jobs. Thus, under Hosios' condition, we find that low-skill workers experience a suboptimally high rate of unemployment. Formally, let $\tilde{u}(l)_E$ and $\tilde{u}(l)_{SP}$ denote the unemployment rate of low-skill workers in equilibrium and in the social optimum. Then, if both exhibit cross-skill matching, we can conclude that:

Corollary 5 *Under Hosios' condition, $\tilde{u}(l)_E > \tilde{u}(l)_{SP}$.*

Similarly, for high-skill workers we find that $\tilde{u}(h)_E = \tilde{u}(h)_{SP}$ but a too large fraction of these workers is employed in skilled jobs.

The distortion of the relative wages can be understood by comparing the expressions for the outside options and the shadow values of both types of workers. For instance, in the case of high-skill workers, we obtain

$$rU(h) = b + \beta\theta^{1-\alpha}\left\{\phi Y(h,n) + (1-\phi)\,Y(h,s) - \frac{rU(h)}{r+\delta}\right\} \tag{37}$$

$$\begin{aligned}&(r+\delta)\,\lambda(h) = b + \theta^{1-\alpha}\left\{\phi Y(h,n) + (1-\phi)\,Y(h,s) - \lambda(h)\right\} - \\ &(1-\alpha)\,\theta^{1-\alpha}\{(1-\eta)\left[\phi Y(h,n) + (1-\phi)\,Y(h,s) - \lambda(h)\right] + \eta\phi\left(Y(l,n) - \lambda(l)\right)\},\end{aligned} \tag{38}$$

where $\lambda(l)$ denotes the endogeneous shadow value of a low-skill worker. According to (37), the reservation wage of a high-skill worker is equal to the flow value of leisure, b, plus the expected capital gain from employment in either a skilled or an unskilled job. The main difference with (38) is the negligence of the congestion effects. In equation (38), $(r+\delta)\,\lambda(h)$ is equal to the flow value of leisure, b, plus the expected capital gain from employment, minus the congestion externality of an additional high-skill unemployed on workers of the same type (high-skilled),

$$(1-\alpha)\,\theta^{1-\alpha}\,(1-\eta)\,[\phi Y(h,n)+(1-\phi)\,Y(h,s)-\lambda(h)]$$

and on workers of the other type (low-skilled),

$$(1-\alpha)\,\theta^{1-\alpha}\eta\phi\,(Y(l,n)-\lambda(l))\,.$$

In the absence of heterogeneity, this congestion externality corresponds to a share $1-\alpha$ of the expected capital gain from employment. Under Hosios' condition, the congestion externality is therefore perfectly internalized in the wage, leading to an efficient allocation.

In contrast, in our economy we have two types of workers and high-skill workers congest the market for some low-skill workers who are less-productive. The congestion externality is therefore *smaller* than $1-\alpha$ of the expected match surplus of a high-skill worker. Conversely, low-skill job seekers congest the market for some more productive high-skill workers. For this category of workers, the congestion externality is therefore *larger* than $1-\alpha$ of their expected match surplus. Hence, under Hosios' condition, the relative wage payments are distorted. All high-skill workers are under-valued (in the sense that their outside option is lower than their shadow value), while the opposite holds for low-skill workers.

Next, consider the effect on profits. In our economy, all jobs benefit in the same way from the under-valuation of high-skill workers, while the over-valuation of low-skill workers only hurts unskilled jobs. As a result, in equilibrium, firms will create too few unskilled jobs, since they have to pay low-skill workers more than their shadow value. Furthermore, since $\theta_E=\theta_{SP}$ this implies that firms create a suboptimally large share of skilled jobs.[6]

[6] The result that firms create exactly $\theta_E=\theta_{SP}$ jobs per unemployed worker is due to the fact that the expected wage costs of an unskilled job equals the expected marginal productivity of applicants. To obtain this result we need to take the weighted average

Implications of free entry Our explanation of the inefficient labour market outcome is entirely based on the distortion of the relative wage payments. It is easy to show that this feature is due to the assumption of free entry which ensures that the shadow value and the profits of the two types of vacancies are both equal to zero. Consider the optimality condition for skilled vacancies:

$$\begin{aligned}\gamma &= \theta^{-\alpha}(1-\eta)[Y(h,s)-\lambda(h)] - \alpha\theta^{-\alpha}[(1-\eta)\{\phi[Y(h,n)-\lambda(h)] \\ &\quad +(1-\phi)[Y(h,s)-\lambda(h)]\} + \eta\phi[Y(l,n)-\lambda(l)]] \qquad (39)\end{aligned}$$

Unlike firms, the planner takes into account the congestion effects from opening a vacancy. Moreover, as in the case of workers, this congestion externality corresponds to different shares of the expected output of the two types of jobs. Nonetheless, after equating the optimality conditions for $v(s)$ and $v(n)$ and solving for $\lambda(l)$, we obtain:

$$\gamma = (1-\alpha)\theta^{-\alpha}(1-\eta)\left[\frac{y(s)}{r+\delta} - \lambda(h)\right] \qquad (40)$$

Hence, under Hosios' condition the only difference with (17) is the appearance of $\lambda(h)$ rather than $rU(h)$ on the right-hand side. The over-creation of skilled vacancies is therefore entirely explained by the under-valuation of high-skill workers.[7]

of workers' outside options. When we repeat this calculation for workers' shadow values (using identical weights η and $1-\eta$) we see that the two quantities coincide. For details see Appendix B.

[7] In the proof of Proposition 4 we proof a similar result for unskilled vacancies.

Extensions Finally, it is easy to show that our results can be generalized to economies with more types of agents and/or weaker conditions on the production technology. Consider the case of $N > 2$ types of workers and firms. In this case, all workers with an above-average expected productivity will be under-valued, while all agents with a below-average expected productivity will be over-valued. Similarly, we could allow low-skill workers to perform both jobs. In that case we would need to consider more matching configurations. Nonetheless, as long as high-skill workers are relatively more productive in skilled jobs we would still obtain the same qualitative results for the distortion of the wage payments.

4.2 Ex post segmentation

We now proceed with a characterization of the efficient allocations with ex post segmentation. In this case, it is not efficient for high-skill workers to match with unskilled vacancies because $y(n) - (r + \delta)\lambda(h) < 0$.

An efficient ex post segmentation allocation is characterized by the vector of endogenous variables $\{\theta_{SP},\ \phi_{SP},\ \eta_{SP},\ u_{SP}\}$ satisfying the following equations:

$$\phi\theta^{1-\alpha}\eta u = \delta(\mu - \eta u) \tag{41}$$

$$(1-\phi)\,\theta^{1-\alpha}\,(1-\eta)\,u = \delta\left[(1-\mu) - (1-\eta)\,u\right] \tag{42}$$

$$\gamma = \theta^{-\alpha}\eta(1-\alpha)\left[\frac{[y(n)-b]}{r+\delta+[\phi-(1-\alpha)\,\eta]\,\theta^{1-\alpha}}\right] \tag{43}$$

$$\gamma = \theta^{-\alpha}\,(1-\eta)\,(1-\alpha)\left[\frac{[y(s)-b]}{r+\delta+[(1-\phi)-(1-\alpha)\,(1-\eta)]\,\theta^{1-\alpha}}\right] \tag{44}$$

The first two equations represent the laws of motion for the state variables $u(l)$ and $u(h)$ when $R = 0$. As in the decentralized equilibrium, the flow condition for low-skill workers is the same as under cross-skill matching. What changes is the flow equation for high-skill workers whose exit rate out of unemployment falls from $\theta^{1-\alpha}$ to $(1-\phi)\theta^{1-\alpha}$. Comparing the above conditions to the conditions for an ex-post segmentation equilibrium we immediately obtain the following result:

Proposition 6 *An ex post segmentation equilibrium never coincides with the efficient allocation* $\{\theta_{SP}, \phi_{SP}, \eta_{SP}, u_{SP}\}$ *as defined by* (41) − (44)*.*

Proof: Appendix A.

For the ex post segmentation case it is difficult to get analytical results for the efficiency properties of the equilibrium. Nonetheless, it is straightforward to show that the distortion of the relative wages is the same as under cross-skill matching (see Appendix). When high-skill workers only match with skilled jobs, we therefore again expect that firms create too few unskilled jobs.

4.3 Matching decisions

So far, we compared the equilibrium allocation to the corresponding efficient allocation of resource. Nonetheless, in our economy the matching rule of high-skill workers need not be efficient.

A first reason why high-skill workers may adopt inefficient matching rules is related to the distortion of the relative wages. For instance, under Hosios' condition, we found that high-skill workers are under-valued in equilibrium. Moreover, $U(h)$ and $\lambda(h)$ are both increasing in $y(s)$ (and $1-\mu$). The latter implies that there exists a non-empty set of parameters for which a cross-skill matching equilibrium exists, while the efficient allocation exhibits ex-post segmentation. In this case, the relative productivity of skilled jobs is so high that the planner prefers a separation of the two types of workers.

Nonetheless, the equilibrium wage of high-skill workers is too low and they continue to accept unskilled jobs.

The opposite also seems possible, *i.e.* an ex-post segmentation equilibrium when a cross-skill matching allocation is efficient. The latter occurs when the economy exhibits multiple equilibria and high-skill workers reject unskilled jobs, while the planner prefers cross-skill matching. In this case, the labour market will respond by creating many more skilled jobs which may raise $U(h)$ to a level at which $rU(h) > y(n)$ while $y(n) > (r+\delta)\lambda(h)$. Hence, due to the coordination externality, the economy may be locked in the candidate equilibrium that generates the lowest value of net output.

In sum, in our economy three cases may occur. An inefficient cross-skill matching equilibrium, an inefficient ex-post segmentation equilibrium or an equilibrium with an inefficient matching set for high-skill workers. These cases correspond, respectively, to relatively low, relatively high and intermediate values for $y(s)$ and $1-\mu$. Inefficient matching sets are the only source of inefficiency in models with a fixed pool of agents like Sattinger (1995). In contrast, with free entry of firms the resource allocation is never efficient. Even if the matching decision of high-skill workers is efficient, the distortion of the wage payments will induce inefficient entry of firms.

4.4 Example

In this section we present a numerical example to illustrate the three different cases. The baseline parameters are virtually the same as in AV.[8] The results are summarized in Figure 2. It illustrates the effects of a gradual increase in $y(s)$ when workers' bargaining strength satisfies Hosios' condition. The line SP illustrates the sequence of efficient steady state allocations. This allo-

[8] Parameter configuration: $r = 0.02$; $\delta = 0.20$; $b = 0.10$; $\gamma = 0.40$; $y(n) = 1$; $\alpha = \beta = 0.50$; $\mu = 2/3; y(s) \in [1.5, 2.5]$

cation is generically unique since the planner chooses the candidate efficient allocation that generates the highest value of net output. Moreover, beyond a certain threshold the efficient matching rule switches from cross-skill matching to ex-post segmentation. The second line, EQ, denotes a sequence of equilibria in which the switch to ex-post segmentation occurs at the point where the cross-skill matching equilibrium ceases to exist. Hence, whenever the model generates multiple equilibria, we have depicted the resource allocation under cross-skill matching.

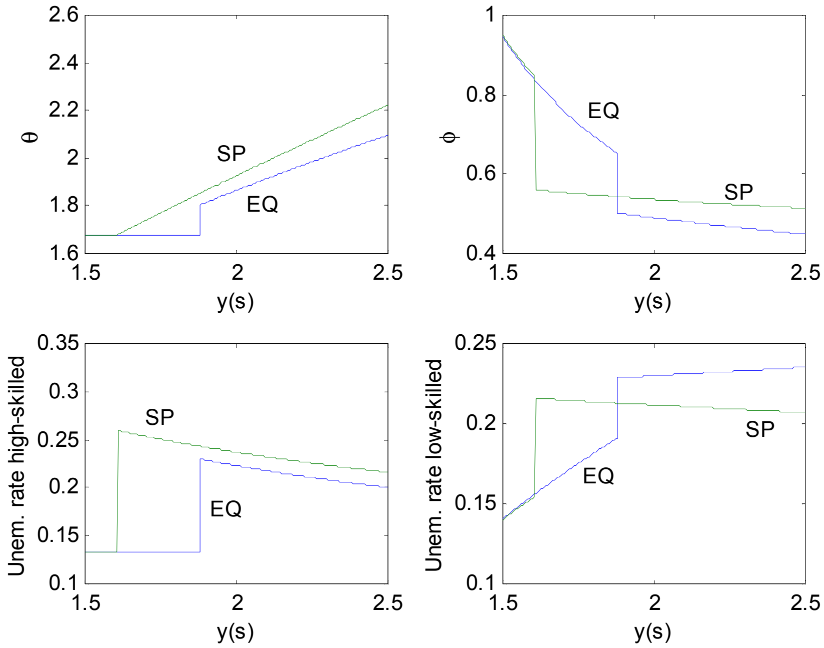

Figure 2: Efficient vs decentralized allocations

Three things are noteworthy. First, in the case of cross-skill matching, the results corroborate our predictions. The labour market tightness θ_E is equal to θ_{SP} and does not react to changes in $y(s)$, while the share of unskilled jobs is too low in equilibrium. Second, when both the equilibrium and the efficient allocation exhibit ex-post segmentation, we obtain the same distortion of the job distribution. However, in this case the negative effects for unskilled workers are more pronounced. Furthermore, $\theta_{SP} > \theta_E$ and both are increasing in $y(s)$. The fact that the unemployment rate of low-skill workers jumps up after the switch to ex-post segmentation is due to the discrete fall in ϕ. More surprising is the increase in the unemployment rate of high-skill workers. The latter is due to two opposing forces. On the one hand, $\widetilde{u}(h)$ tends to increase because high-skill workers no longer match with unskilled jobs. But, on the other hand, firms are willing to open more skilled jobs and in our example this offsets part of the increase in $\widetilde{u}(h)$. Finally, the distortion of the resource allocation is particularly strong when high-skill workers use an inefficient matching rule. In this case most of the results are reversed. Namely, the equilibrium generates a higher share of unskilled jobs, less unemployment among low-skilled and more unemployment among high-skilled than the efficient allocation.

5 Extensions

In the previous sections we showed that the labour market generates an excessive supply of skilled jobs under Hosios' condition. This achieved our first goal. Namely to show that the competition from "over-qualified" workers may harm low-skill workers. However, the over-creation of good jobs seems to contradict the concern of policy makers that labour markets generate too many low-quality jobs.

The aim of this section is twofold. First of all we demonstrate that efficiency can be restored by taxation. And, secondly, we show that the market can generate too many unskilled jobs when workers' bargaining strength violates Hosios' condition.

5.1 A simple tax experiment

In this section we perform a simple public choice exercise first proposed by Shimer *et al.* (2001). Namely, we try to determine whether there exists a tax scheme such that the steady state equilibrium coincides with the efficient steady state allocation. For convenience we consider the case of cross-skill matching. The instrument that we consider is a lump sum tax or subsidy on unemployed workers. Other more realistic tax schemes might achieve the same result.[9] However, the taxation scheme that we illustrate below reinforces the intuition on the undervaluation (overvaluation) of high-skill (low-skill) workers that we provided in Section 4.

Formally, let $\tau(h)$ and $\tau(l)$ denote, respectively, the taxes on unemployed high-skill and low-skill workers, which in principle may be positive (taxes) or negative (subsidies). To simplify the analysis we ignore the use of the net tax proceeds.[10] The new expressions of the expected lifetime income of unemployed workers then become as follows:

$$rU(h) = b - \tau(h) + \theta^{1-\alpha} \left\{ \phi \left[W(h,n) - U(h) \right] + (1-\phi) \left[W(h,s) - U(h) \right] \right\}$$

$$rU(l) = b - \tau(l) + \theta^{1-\alpha} \phi \left[W(l,n) - U(l) \right]$$

The decentralized equilibrium can be solved in the same way as in the model without taxes. We make use of the condition, $V(s) = V(n) = 0$ to

[9] One example is a hiring tax for skilled jobs. This tax can restore efficiency as it reduces the expected profits from skilled jobs. Furthermore, the hiring tax would have different distributional implications than the search taxes and subsidies considered here.

[10] It is straightforward to introduce a balanced government budget. To avoid distortions the government should redistribute the tax proceeds through a lump sum payment of size $\overline{\tau} = u(l) \cdot \tau(l) + u(h) \cdot \tau(h) \lesseqgtr 0$ to all workers.

derive the first two equations that characterize the solution, and we obtain the remaining two equations from the flow conditions.

Proposition 7 *For $\beta = \alpha$ the cross-skill matching equilibrium is efficient under the tax scheme $\{\tau^*(l), \tau^*(h)\}$ that satisfies*

$$\tau^*(l) = \frac{(1-\alpha)(1-\phi_{SP})(\theta_{SP})^{1-\alpha}(1-\eta_{SP})(y(s)-y(n))}{\eta_{SP}\left[r+\delta+(\theta_{SP})^{1-\alpha}\right]} \tag{45}$$

$$\tau^*(h) = -\frac{(1-\alpha)(1-\phi_{SP})(\theta_{SP})^{1-\alpha}(y(s)-y(n))}{\left[r+\delta+(\theta_{SP})^{1-\alpha}\right]} \tag{46}$$

Proof: Appendix B.

Proposition 7 implies that governments should impose a tax on low-skill job seekers. The size of this tax is proportional to the reduction in the mass of skilled jobs due to an additional low-skill job seeker times the productivity differential $y(s) - y(n)$. Conversely, high-skill job seekers should receive a subsidy proportional to the reduction in the matching rate of workers due to an additional low-skill job seeker times the share of skilled jobs $(1 - \phi_{SP})$ times the productivity differential $y(s) - y(n)$. When the tax scheme satisfies (45) and (46), the congestion externalities are therefore perfectly internalized.

5.2 Changes in bargaining power

To complete our analysis, we now consider the effect of a change in the bargaining strength of workers. In our model, this change has two effects. First of all, an increase in the bargaining power of workers reduces the expected profits of firms. The equilibrium labour market tightness is therefore negatively correlated with β. Second, changes in β alter the relative profits of both types of jobs and this results in changes in ϕ_E. We can prove that this relation between ϕ_E and β is non-linear:

Proposition 8 *The share of skilled jobs $1-\phi_E$ follows an inverted U pattern in β. It achieves a maximum value $1-\phi_E > 1-\phi_{SP}$ when workers' bargaining strength satisfies Hosios' condition.*

Corollary 9 *Over-creation of unskilled jobs ($\phi_E > \phi_{SP}$) may occur for sufficiently low or high values of β.*

The intuition behind Corollary 9 is easy for the case in which $\beta > \alpha$. In this case, workers appropriate a suboptimally large share of the match surplus. Given that skilled jobs are more productive than unskilled jobs, an increase in β reduces the relative profits of skilled jobs and this leads to a reduction in $1 - \phi_E$.

Table 1. Comparative statics of changes in β

	θ	ϕ	η	$\widetilde{u}$	$\widetilde{u}(h)$	$\widetilde{u}(l)$
	1.68	0.948	0.676	0.138	0.133	0.140
$\beta = 0.30$	3.82	0.989	0.669	0.093	0.092	0.093
$\beta = 0.40$	2.50	0.955	0.675	0.115	0.112	0.116
$\beta = 0.50$	1.68	0.944	0.677	0.138	0.133	0.140
$\beta = 0.60$	1.11	0.953	0.675	0.163	0.159	0.165
$\beta = 0.70$	0.70	0.988	0.668	0.194	0.193	0.195

To obtain the overall effect on the unemployment rates of both types of workers we need to combine the changes in θ_E and ϕ_E. For high-skill workers this results in a positive relationship between $\widetilde{u}(h)$ and β as $\partial\theta_E/\partial\beta < 0$. Instead, for low-skill workers the effect is ambiguous. Nonetheless, our numerical results seem to indicate that $\partial\widetilde{u}(l)/\partial\beta > 0$.

Table 1 summarizes the comparative static effects of changes in β that are illustrated in Figure 3.[11] The first row represents the efficient allocation. Inspection of the table shows that this allocation is almost identical to

[11]Except for the value of β, the baseline parameters are the same as before.

the decentralized equilibrium for $\beta = 0.5$. The only difference is the small over-creation of skilled jobs ($\phi_E = 0.944$ versus $\phi_{SP} = 0.948$). When Hosios' condition is satisfied, the decentralised equilibrium is therefore nearly efficient. In contrast, for values of β above (below) α, we obtain an over-creation of unskilled jobs combined with a suboptimally low (high) value for θ. Furthermore, for low-skill workers the fall in θ dominates the increase in ϕ_E. At a higher value of β low-skill workers therefore experience a higher unemployment rate $\widetilde{u}(l)$.

Our numerical results yield a clear prediction about the relationship between the unemployment rate, the labour share and the degree of skill-mismatch. That is, in economies with a high labour share we should observe a high unemployment rate, a low share of skilled jobs and a large amount of high-skill workers employed on unskilled jobs.[12] Over-education is documented to be a prominent feature in European labour markets (*e.g.* Eurostat (2003)). However, so far this phenomenon has not been related to the size of the labour share or more accurately workers' marginal surplus share.

6 Concluding Remarks

In this paper we characterized the efficiency of the resource allocation in a matching model with heterogeneous agents. The conceptual framework is taken from Albrecht and Vroman (2002) and we show that the equilibrium allocation with ex post bargaining is never efficient. In particular, under Hosios' condition, we show that low-skill workers are overvalued. Consequently, the labour market tends to generate too few unskilled jobs.

[12] According to our results overeducation should also be prominent in economies with a very low labour share. In this case the explanation is less straightforward. Apparently, in economies with an over-creation of jobs, $\theta_E > \theta_{SP}$, firms prefer to create unskilled jobs to limit the risk of skill-mismatch.

The comparison between workers' shadow value and their expected lifetime income is the main innovation of this paper. It provides an intuitive explanation for the inefficiency with ex post bargaining. Moreover, the results generalize to environments with more than two types of workers and firms and different production technologies.

7 Appendix

7.1 Appendix A (Efficient Allocation)

Proof Proposition 2: Social Planner Problem

Let $H_c \equiv He^{rt}$ be the *"Current-value Hamiltonian"* associated with maximization problem (29).

$$H_c = \theta_t^{1-\alpha} \left[u(h)_t \left[R_t \phi_t Y(h,n) + (1-\phi_t) Y(h,s) \right] + u(l)_t \phi_t Y(l,n) \right] +$$
$$+ b\left(u(h)_t + u(l)_t\right) - \gamma \theta_t \left(u(h)_t + u(l)_t\right) + \lambda(l)_t \left[\delta\left(1-\mu-u(l)_t\right) - \phi_t \theta_t^{1-\alpha} u(l)_t \right] +$$
$$+ \lambda(h)_t \left[\delta\left(\mu - u(h)_t\right) - \theta_t^{1-\alpha} u(h)_t \left(1-\phi_t + R_t\phi_t\right) \right]$$

Henceforth, we concentrate on the steady state, suppressing the time dependence of variables. There are three types of necessary first order conditions. The first one is the optimality condition for matching between high-skill workers and unskilled vacancies, given by $\partial H_c / \partial R$. Taking derivatives of H_c with respect to R, we have:

$$\frac{\partial H_c}{\partial R} = \phi \theta^{1-\alpha} u(h) \left[Y(h,n) - \lambda(h) \right]$$

The optimal matching rule, therefore, depends on the sign of $[y(n) - (r+\delta)\lambda(h)]$, as specified in condition (32) in the main text.

The remaining conditions that characterize the steady state solution of this problem are:

$$\frac{\partial H_c}{\partial u(l)} = r\lambda(l)\,; \; \frac{\partial H_c}{\partial u(h)} = r\lambda(h)\,; \; \frac{\partial H_c}{\partial v(n)} = 0; \; \frac{\partial H_c}{\partial v(s)} = 0 \tag{47}$$

$$\frac{\partial H_c}{\partial \lambda(l)} = \dot{u}(l) = 0; \quad \frac{\partial H_c}{\partial \lambda(h)} = \dot{u}(h) = 0 \tag{48}$$

1. Cross-skill matching (R=1)

Under cross-skill matching, the conditions in (47) become:

$$
\begin{aligned}
(r+\delta)\lambda(l) = b + \phi\theta^{1-\alpha}\left[Y(l,n) - \lambda(l)\right] - \\
-(1-\alpha)\theta^{1-\alpha}\{(1-\eta)\left[\phi\left[Y(h,n)-\lambda(h)\right] + (1-\phi)\left[Y(h,s)-\lambda(h)\right]\right] + \\
+\eta\phi\left[Y(l,n)-\lambda(l)\right]\}
\end{aligned} \tag{49}
$$

$$
\begin{aligned}
(r+\delta)\lambda(h) = b + \theta^{1-\alpha}\left[\phi\left[Y(h,n)-\lambda(h)\right] + (1-\phi)\left[Y(h,s)-\lambda(h)\right]\right] - \\
-(1-\alpha)\theta^{1-\alpha}\{(1-\eta)\left[\phi\left[Y(h,n)-\lambda(h)\right] + (1-\phi)\left[Y(h,s)-\lambda(h)\right]\right] + \\
+\eta\phi\left[Y(l,n)-\lambda(l)\right]\}
\end{aligned} \tag{50}
$$

$$
\begin{aligned}
\gamma = \theta^{-\alpha}(1-\eta)\left[Y(h,s)-\lambda(h)\right] - \\
-\alpha\theta^{-\alpha}\{(1-\eta)\left[\phi\left[Y(h,n)-\lambda(h)\right] + (1-\phi)\left[Y(h,s)-\lambda(h)\right]\right] + \\
+\eta\phi\left[Y(l,n)-\lambda(l)\right]\}
\end{aligned} \tag{51}
$$

$$
\begin{aligned}
\gamma = \theta^{-\alpha}\left[(1-\eta)\left[Y(h,n)-\lambda(h)\right] + \eta\left[Y(l,n)-\lambda(l)\right]\right] - \\
-\alpha\theta^{-\alpha}\{(1-\eta)\left[\phi\left[Y(h,n)-\lambda(h)\right] + (1-\phi)\left[Y(h,s)-\lambda(h)\right]\right] + \\
+\eta\phi\left[Y(l,n)-\lambda(l)\right]\}
\end{aligned} \tag{52}
$$

Combining eqs. (49) – (52) we obtain:

$$
\left(r+\delta+\phi\theta^{1-\alpha}\right)\frac{\left(r+\delta+\alpha\theta^{1-\alpha}\right)}{\left(r+\delta+\theta^{1-\alpha}\right)} = \frac{\eta(r+\delta)(y(n)-b)}{(1-\eta)(y(s)-y(n))} \tag{53}
$$

$$
\gamma\left(r+\delta+\alpha\theta^{1-\alpha}\right) = (1-\alpha)\theta^{-\alpha}(y(n)-b) \tag{54}
$$

From the conditions in (48) we obtain

$$
\phi = \frac{\mu\theta^{1-\alpha}(1-\eta) + (\mu-\eta)\delta}{\theta^{1-\alpha}(1-\mu)\eta} \tag{55}
$$

$$u=\frac{\delta\left(1-\mu\right)}{\left(\delta+\theta^{1-\alpha}\right)\left(1-\eta\right)} \tag{56}$$

Ex post segmentation (R=0)

Similarly, under ex post segmentation, the conditions in (47) become:

$$(r+\delta)\lambda\left(l\right)=b+\phi\theta^{1-\alpha}\left[Y(l,n)-\lambda\left(l\right)\right]-$$
$$-\left(1-\alpha\right)\theta^{1-\alpha}\left\{\left(1-\eta\right)\left(1-\phi\right)\left[Y(h,s)-\lambda\left(h\right)\right]+\eta\phi\left[Y(l,n)-\lambda\left(l\right)\right]\right\} \tag{57}$$

$$(r+\delta)\lambda\left(h\right)=b+\theta^{1-\alpha}\left(1-\phi\right)\left[Y(h,s)-\lambda\left(h\right)\right]-$$
$$-\left(1-\alpha\right)\theta^{1-\alpha}\left\{\left(1-\eta\right)\left(1-\phi\right)\left[Y(h,s)-\lambda\left(h\right)\right]+\eta\phi\left[Y(l,n)-\lambda\left(l\right)\right]\right\} \tag{58}$$

$$\gamma=\theta^{-\alpha}(1-\eta)\left[Y(h,s)-\lambda(h)\right]-$$
$$-\alpha\theta^{-\alpha}\left\{\left(1-\eta\right)\left(1-\phi\right)\left[Y(h,s)-\lambda\left(h\right)\right]+\eta\phi\left[Y(l,n)-\lambda\left(l\right)\right]\right\} \tag{59}$$

$$\gamma=\theta^{-\alpha}\eta\left[Y(l,n)-\lambda\left(l\right)\right]-\alpha\theta^{-\alpha}\{\left(1-\eta\right)\left(1-\phi\right)\left[Y(h,s)-\lambda\left(h\right)\right]+$$
$$+\eta\phi\left[Y(l,n)-\lambda\left(l\right)\right]\} \tag{60}$$

Combining eqs. (57) – (60) we get:

$$\frac{\gamma}{\theta^{-\alpha}}=\eta\left[\frac{\left(1-\alpha\right)\left[y(n)-b\right]}{r+\delta+\phi\theta^{1-\alpha}-\left(1-\alpha\right)\eta\theta^{1-\alpha}}\right] \tag{61}$$

$$\frac{\gamma}{\theta^{-\alpha}}=\left(1-\eta\right)\left[\frac{\left(1-\alpha\right)\left[y(s)-b\right]}{r+\delta+\left(1-\phi\right)\theta^{1-\alpha}-\left(1-\alpha\right)\left(1-\eta\right)\theta^{1-\alpha}}\right] \tag{62}$$

And, from the conditions in (48) we have:

$$\phi=\frac{\mu\left(1-\eta\right)\theta^{1-\alpha}+\left(\mu-\eta\right)\delta}{\theta^{1-\alpha}\left(\eta+\mu-2\eta\mu\right)} \tag{63}$$

$$u=\frac{\delta\left(1-\mu\right)}{\left(\delta+\left(1-\phi\right)\theta^{1-\alpha}\right)\left(1-\eta\right)} \tag{64}$$

Arrow's Sufficiency Theorem:

Arrow's generalization of Mangasarian's sufficiency theorem (Kamien and Schwartz, 1991: 222) states that conditions (53) – (56) are necessary and sufficient for $\{\theta, \phi, \eta, u\}$ to be the steady-state solution to the dynamic optimization problem (29), when $R = 1$, if the maximized Hamiltonian function H^0 (the Hamiltonian evaluated along ϕ^* and θ^*, where ϕ^* and θ^* are given by eqs. (33) and (36) respectively) is concave in the variables $u(h)$ and $u(l)$ for given $\lambda(h)$ and $\lambda(l)$. This implies that the Hessian matrix specified below must be negative semidefinite:

$$HES = \begin{pmatrix} \frac{\partial^2 H^0}{\partial^2 u(l)} & \frac{\partial^2 H^0}{\partial u(l)\partial u(h)} \\ \frac{\partial^2 H^0}{\partial u(h)\partial u(l)} & \frac{\partial^2 H^0}{\partial^2 u(h)} \end{pmatrix}$$

Taking derivatives with respect to $u(h)$ and $u(l)$ in H^0 we obtain:

$$\frac{\partial^2 H^0}{\partial^2 u(l)} = \frac{2\mu \left(u(h)\right)^2 \left(\delta + (\theta^*)^{1-\alpha}\right)\left(y(n) - y(s)\right)}{(1-\mu)\left(u(l)\right)^3 (r+\delta)} < 0$$

$$\frac{\partial^2 H^0}{\partial^2 u(h)} = \frac{2\mu \left(\delta + (\theta^*)^{1-\alpha}\right)\left(y(n) - y(s)\right)}{(1-\mu) u(l) (r+\delta)} < 0$$

$$\frac{\partial^2 H^0}{\partial u(l)\partial u(h)} = \frac{\partial^2 H^0}{\partial u(h)\partial u(l)} = \frac{-2\mu u(h) \left(\delta + (\theta^*)^{1-\alpha}\right)\left(y(n) - y(s)\right)}{(1-\mu)\left(u(l)\right)^2 (r+\delta)} > 0$$

Operating with the previous expressions we obtain that $|HES| = 0$. Thus, since $\partial^2 H^0/\partial^2 u(l) < 0$, $\partial^2 H^0/\partial^2 u(h) < 0$ and $|HES| = 0$, the Hessian matrix is negative semidefinite which gives us the necessary and sufficient conditions for $\{\theta, \phi, \eta, u\}$ to be a steady-state solution for the dynamic optimization problem (29) when high-skill workers accept unskilled job offers[13].

[13] The Hessian matrix is not very difficult to be computed for the cross-skill matching case. However, applying Arrow's generalization of Mangasarian's sufficiency theorem to the dynamic optimization problem (29) when $R = 0$ (ex post segmentation case), would require computing the Hessian matrix numerically.

Nonetheless, since the Hessian matrix is negative semidefinite, this does not guarantee uniqueness.

Uniqueness of the cross-skill matching efficient allocation

From equation (54), we know that $\partial\theta_{SP}/\partial\mu = 0$. Using this result, we establish uniqueness by showing that $d(1-\phi)/d(1-\mu) < 0$.

From equation (55) we obtain:

$$\frac{\eta}{1-\eta} = \frac{\mu\left(\delta+\theta^{1-\alpha}\right)}{(1-\mu)\left(\delta+\phi\theta^{1-\alpha}\right)}$$

Substituting into condition (53) we have:

$$1-\mu = \frac{\left(\delta+\theta^{1-\alpha}\right)(r+\delta)\left(r+\delta+\theta^{1-\alpha}\right)(y(n)-b)}{\Delta} \tag{65}$$

where:

$$\Delta = \left(r+\delta+\phi\theta^{1-\alpha}\right)(y(s)-y(n))\left(r+\delta+\alpha\theta^{1-\alpha}\right)\left(\delta+\phi\theta^{1-\alpha}\right)+ \\ \left(\delta+\theta^{1-\alpha}\right)(r+\delta)\left(r+\delta+\theta^{1-\alpha}\right)(y(n)-b)$$

The existence of the cross-skill matching efficient allocation implies that there exist a solution for (65). Furthermore equation (65) is continuously differentiable, allowing the use of the Implicit Function Theorem. Thus, taking the total derivative in (65):

$$\frac{d(1-\phi)}{d(1-\mu)} = \left\{-\left[\frac{\left(\delta+\theta^{1-\alpha}\right)(r+\delta)\left(r+\delta+\theta^{1-\alpha}\right)(y(n)-b)}{\Delta^2}\right]\frac{\partial\Delta}{\partial(1-\phi)}\right\}^{-1} \tag{66}$$

where:

$$\frac{\partial\Delta}{\partial(1-\phi)} = -(y(s)-y(n))\left(r+\delta+\alpha\theta^{1-\alpha}\right)\theta^{1-\alpha}\left[r+2\left(\delta+\phi\theta^{1-\alpha}\right)\right] < 0 \tag{67}$$

Uniqueness of the cross-skill matching efficient allocation is proved recursively. First, from equation (54) we derive a unique value of θ. Second, from (66) and (67), $(1-\phi)$ defines a strictly increasing function of $(1-\mu)$. Thus, for any value of $(1-\mu)$ we get a unique value of $(1-\phi)$. Given unique values of θ and $(1-\phi)$, from equation (53) we obtain a unique value of η. Finally, given unique values of θ and η, equation (56) defines a unique value for u.■

7.2 Appendix B (Proofs of main results)

Proof Proposition 3: **Inefficient cross-skill matching equilibrium**
When $\beta = \alpha$, from eqs. (22) and (36) we have $\theta_E = \theta_{SP}$. Plugging this result into eqs. (21) and (35) we obtain:

$$\left(r+\delta+\beta\phi_E\theta_{SP}^{1-\alpha}\right) = \frac{\eta_E\left(r+\delta\right)\left(y(n)-b\right)}{\left(1-\eta_E\right)\left(y(s)-y(n)\right)} \tag{68}$$

$$\left(r+\delta+\phi_{SP}\theta_{SP}^{1-\alpha}\right)\frac{\left(r+\delta+\alpha\theta_{SP}^{1-\alpha}\right)}{\left(r+\delta+\theta_{SP}^{1-\alpha}\right)} = \frac{\eta_{SP}\left(r+\delta\right)\left(y\left(n\right)-b\right)}{\left(1-\eta_{SP}\right)\left(y\left(s\right)-y\left(n\right)\right)} \tag{69}$$

From (68) and (69) it follows that $\beta = \alpha = 1$ is needed for a cross-skill matching equilibrium to be efficient.■

Proof Proposition 4

1. Overcreation of skilled jobs:

Plugging $\beta = \alpha$, and $\theta_E = \theta_{SP}$ into (15), (16) and (21) we can derive the following expression:

$$\frac{\mu\left(r+\delta\right)\left(y\left(n\right)-b\right)}{\left(1-\mu\right)\left(y\left(s\right)-y\left(n\right)\right)} = \frac{\left(\delta+\phi_E\theta_{SP}^{1-\alpha}\right)}{\left(\delta+\theta_{SP}^{1-\alpha}\right)}\left(r+\delta+\alpha\phi_E\theta_{SP}^{1-\alpha}\right) \tag{70}$$

Next, proceeding in the same way for the efficient allocation, we combine eqs $(33), (34)$ and (35) which yields:

$$\frac{\mu (r+\delta)(y(n)-b)}{(1-\mu)(y(s)-y(n))} = \frac{\left(\delta+\phi_{SP}\theta_{SP}^{1-\alpha}\right)\left(r+\delta+\phi_{SP}\theta_{SP}^{1-\alpha}\right)\left(r+\delta+\alpha\theta_{SP}^{1-\alpha}\right)}{\left(\delta+\theta_{SP}^{1-\alpha}\right)\left(r+\delta+\theta_{SP}^{1-\alpha}\right)}$$
$$< \frac{\left(\delta+\phi_{SP}\theta_{SP}^{1-\alpha}\right)\left(r+\delta+\alpha\phi_{SP}\theta_{SP}^{1-\alpha}\right)}{\left(\delta+\theta_{SP}^{1-\alpha}\right)} \tag{71}$$

The term on the left-hand side of eqs. (70) and (71) is the same, therefore, we can derive the following inequality:

$$\left(\delta+\phi_{E}\theta_{SP}^{1-\alpha}\right)\left(r+\delta+\alpha\phi_{E}\theta_{SP}^{1-\alpha}\right) < \left(\delta+\phi_{SP}\theta_{SP}^{1-\alpha}\right)\left(r+\delta+\alpha\phi_{SP}\theta_{SP}^{1-\alpha}\right) \tag{72}$$

In order for (72) to hold, it must be the case that $\phi_E < \phi_{SP}$. As $\partial\phi/\partial\eta < 0$, then we have $\eta_E > \eta_{SP}$.

2. Undervaluation (overvaluation) of high-skill (low-skill) workers:
Substituting $\beta = \alpha$ $(\theta_E = \theta_{SP})$ and the values of $J(h,s)$ and $w(h,s)$ into the free entry condition for skilled vacancies we obtain:

$$\gamma = (1-\alpha)\,\theta_{SP}^{-\alpha}(1-\eta_E)\left[\frac{y(s)-rU(h)}{r+\delta}\right] \tag{73}$$

On the other hand, the optimality condition for $v(s)$ can be written as follows:

$$\gamma = (1-\alpha)\,\theta_{SP}^{-\alpha}(1-\eta_{SP})\left[\frac{y(s)}{r+\delta}-\lambda(h)\right] \tag{74}$$

Combining eqs. (73) and (74) we obtain:

$$(1-\eta_E)\left[\frac{y(s)-rU(h)}{r+\delta}\right] = (1-\eta_{SP})\left[\frac{y(s)}{r+\delta}-\lambda(h)\right] \tag{75}$$

When $\beta = \alpha$, $(1-\eta_E) < (1-\eta_{SP})$, then, in order for equation (75) to hold it must be the case that $rU(h) < (r+\delta)\,\lambda(h)$ (high-skill workers are undervalued).

Similarly, substituting $\beta = \alpha$ ($\theta_E = \theta_{SP}$) and the values of $J(h,n)$, $J(l,n)$, $w(h,n)$ and $w(l,n)$ into the free entry condition for unskilled vacancies, we have:

$$\gamma = (1-\alpha)\,\theta_{SP}^{-\alpha}(1-\eta_E)\left[\frac{y(s)-rU(h)}{r+\delta}\right] +$$
$$+(1-\alpha)\,\theta_{SP}^{-\alpha}\left\{\eta_E\left[\frac{y(n)-rU(l)}{r+\delta}\right]-(1-\eta_E)\left[\frac{y(s)-y(n)}{r+\delta}\right]\right\} \tag{76}$$

while the optimality condition for $v(n)$ can be written as follows:

$$\gamma = (1-\alpha)\,\theta_{SP}^{-\alpha}(1-\eta_{SP})\left[\frac{y(s)}{r+\delta}-\lambda(h)\right] +$$
$$+(1-\alpha)\,\theta_{SP}^{-\alpha}\left\{\eta_{SP}\left[\frac{y(n)}{r+\delta}-\lambda(l)\right]-(1-\eta_{SP})\left[\frac{y(s)-y(n)}{r+\delta}\right]\right\} \tag{77}$$

Combining eqs. (76) and (77), and plugging equation (75) into them, we have:

$$\frac{\eta_E}{1-\eta_E}\left[\frac{y(n)-rU(l)}{r+\delta}\right] = \frac{\eta_{SP}}{1-\eta_{SP}}\left[\frac{y(n)}{r+\delta}-\lambda(l)\right] \tag{78}$$

When $\beta = \alpha$, $\eta_E > \eta_{SP}$ then, in order for equation (78) to hold, it must be true that: $rU(l) > (r+\delta)\lambda(l)$ (low-skill workers are overvalued).

3. Identical labour market tightness ($\theta_E = \theta_{SP}$)

Making use of the expression of $rU(h)$, given by equation (37), and the analogous expression for $rU(l)$ given by:

$$rU(l) = b + \beta\phi\theta^{1-\alpha}\left[Y(l,n) - \frac{rU(l)}{r+\delta}\right], \tag{79}$$

the weighted average of workers' outside options can be written as:

$$\eta rU(l) + (1-\eta)rU(h) = b + \beta\theta^{1-\alpha}\{\eta\phi\left[Y(l,n) - \frac{rU(l)}{r+\delta}\right] +$$
$$(1-\eta)\left[\phi Y(h,n) + (1-\phi)Y(h,s) - \frac{rU(h)}{r+\delta}\right]\} \tag{80}$$

Following the same reasoning, we use the expression of $\lambda(h)$, given by equation (38), and the expression of $\lambda(l)$ given by:

$$\lambda(l) = b + \phi\theta^{1-\alpha}\left[Y(l,n) - \lambda(l)\right] -$$
$$(1-\alpha)\,\theta^{1-\alpha}\{(1-\eta)\left[\phi Y(h,n) + (1-\phi)\,Y(h,s) - \lambda(h)\right] + \eta\phi\left(Y(l,n) - \lambda(l)\right)\}, \tag{81}$$

to derive the following expression for the weighted average of workers' shadow values:

$$\eta(r+\delta)\lambda(l) + (1-\eta)(r+\delta)\lambda(h) = b + \alpha\theta^{1-\alpha}\{\eta\phi\left[Y(l,n) - \lambda(l)\right] +$$
$$(1-\eta)\left[\phi Y(h,n) + (1-\phi)Y(h,s) - \lambda(h)\right]\} \tag{82}$$

Thus, for given η (ϕ), the two quantities, (80) and (82) coincide when the Hosios' condition is satisfied.

4. Corollary 5: ($\beta = \alpha \Longrightarrow \widetilde{u}(l)_E > \widetilde{u}(l)_{SP}$)

From (36) we know that changes in μ do not affect θ_{SP}, so an increase in μ implies that, for a given ϕ_{SP}, $u(l)$ will increase. All else equal, this increases η_{SP}, so the right-hand side of equation (35) will increase, while the left-hand side will decrease. Thus, for equation (35) to hold, ϕ_{SP} has to increase. Summarizing, increasing μ makes the creation of low-skill vacancies more attractive, resulting in an increase of ϕ_{SP}. From this, it is easy to show that $\widetilde{u}(l)_{SP} = \delta/\left[\delta + \phi_{SP}\theta_{SP}^{1-\alpha}\right]$ is a decreasing function of μ. ■

Proof Proposition 6: Inefficient ex post segmentation equilibrium

Comparing eqs. (27) and (28) with eqs. (43) and (44), shows that we again need $\beta = \alpha = 1$ to get efficiency.

The distortion of the relative wages is also similar. This follows from a comparison of (57) and (58) with the asset value equations

$$rU(l) = b + \beta\phi\theta^{1-\alpha}\left[\frac{y(n) - rU(l)}{r+\delta}\right]$$

$$rU(h) = b + \beta(1-\phi)\theta^{1-\alpha}\left[\frac{y(s) - rU(h)}{r+\delta}\right]$$

■

Proof Proposition 7: Optimal tax scheme
Under the tax scheme $\{\tau(l), \tau(h)\} \in \Re^2$, the expressions for the outside options of both types of workers are the following:

$$rU(h) = b - \tau(h) + \theta^{1-\alpha}\left\{\phi\left[W(h,n) - U(h)\right] + (1-\phi)\left[W(h,s) - U(h)\right]\right\}$$

$$rU(l) = b - \tau(l) + \theta^{1-\alpha}\phi\left[W(l,n) - U(l)\right]$$

Substituting $rU(h)$ and $rU(l)$ into the free entry conditions for skilled and unskilled vacancies, and making use of the equal value condition $V(s) = V(n)$ we obtain the following expression:

$$(1-\eta_E)\left[r + \delta + \phi_E\theta_E^{1-\alpha}\right](y(s) - y(n)) = \eta_E(r+\delta)(y(n) - b + \tau(l)) \quad (83)$$

Plugging equation (83) into the free entry condition of skilled jobs we obtain:

$$\gamma\left[r + \delta + \beta\theta_E^{1-\alpha}\right] = (1-\beta)\theta_E^{-\alpha}\left[y(n) - b + \eta_E\tau(l) + (1-\eta_E)\tau(h)\right] \quad (84)$$

First, comparing eqs. (36) and (84), when $\beta = \alpha$, we need

$$\tau(l) = -(1-\eta_E)\tau(h)/\eta_E \quad (85)$$

so that $\theta_E = \theta_{SP} = \theta$. And second, by comparing eqs. (35) and (83) the optimal value, $\tau^*(l)$, that makes the cross-skill matching equilibrium efficient is given by:

$$\tau^*(l) = \frac{(1-\alpha)(1-\phi_{SP})\,\theta_{SP}^{1-\alpha}(1-\eta_{SP})(y(s)-y(n))}{\eta_{SP}\left[r+\delta+\theta_{SP}^{1-\alpha}\right]} \tag{86}$$

Substituting (86) into (85), we derive the following expression for $\tau^*(h)$:

$$\tau^*(h) = -\frac{(1-\alpha)(1-\phi_{SP})\,\theta_{SP}^{1-\alpha}(y(s)-y(n))}{\left[r+\delta+\theta_{SP}^{1-\alpha}\right]} \tag{87}$$

■

Proof Proposition 8: Overcreation of unskilled jobs

Taking derivatives with respect to β in both sides of equation (22) we obtain:

$$\frac{\partial\theta_E}{\partial\beta} = -\frac{\theta_E\left[r+\delta+\theta_E^{1-\alpha}\right]}{(1-\beta)\left[(r+\delta)\,\alpha+\beta\theta_E^{1-\alpha}\right]} < 0 \tag{88}$$

which shows that θ_E is monotonically decreasing in β.
Similarly, taking derivatives with respect to β in equation (21) we have:

$$\frac{\partial\eta_E}{\partial\beta} = \frac{\eta_E(1-\eta_E)\left[\phi_E\theta_E^{1-\alpha}+\beta\phi_E(1-\alpha)\,\theta_E^{-\alpha}\left(\frac{\partial\theta_E}{\partial\beta}\right)\right]}{r+\delta+\beta\phi_E\theta_E^{1-\alpha}}$$

Substituting equation (88), into the previous expression, we obtain:

$$\frac{\partial\eta_E}{\partial\beta} = \left[\frac{\eta_E(1-\eta_E)\,\phi_E\theta_E^{1-\alpha}}{r+\delta+\beta\phi_E\theta_E^{1-\alpha}}\right]\left[\frac{(\alpha-\beta)\left(r+\delta+\beta\theta_E^{1-\alpha}\right)}{(1-\beta)\left((r+\delta)\,\alpha+\beta\theta_E^{1-\alpha}\right)}\right] \tag{89}$$

Finally, for a given θ_E, taking derivatives with respect to β in equation (15) yields:

$$\frac{\partial\phi_E}{\partial\beta} = -\frac{\partial\eta_E}{\partial\beta}\left[\frac{\left(\delta+\theta_E^{1-\alpha}\right)\mu}{(1-\mu)\,\theta_E^{1-\alpha}\eta_E^2}\right] \tag{90}$$

Therefore, from eqs. (89) and (90) we can derive the following result:

$$\begin{aligned} \alpha &= \beta \Rightarrow \frac{\partial \eta_E}{\partial \beta} = 0; \quad \frac{\partial \phi_E}{\partial \beta} = 0 \\ \alpha &> \beta \Rightarrow \frac{\partial \eta_E}{\partial \beta} > 0; \quad \frac{\partial \phi_E}{\partial \beta} < 0 \\ \alpha &< \beta \Rightarrow \frac{\partial \eta_E}{\partial \beta} < 0; \quad \frac{\partial \phi_E}{\partial \beta} > 0 \end{aligned}$$

From this, it follows that $\phi_E > \phi_{SP}$ may occur for sufficiently low or high values of β (Corollary 10). ■

7.3 Appendix C

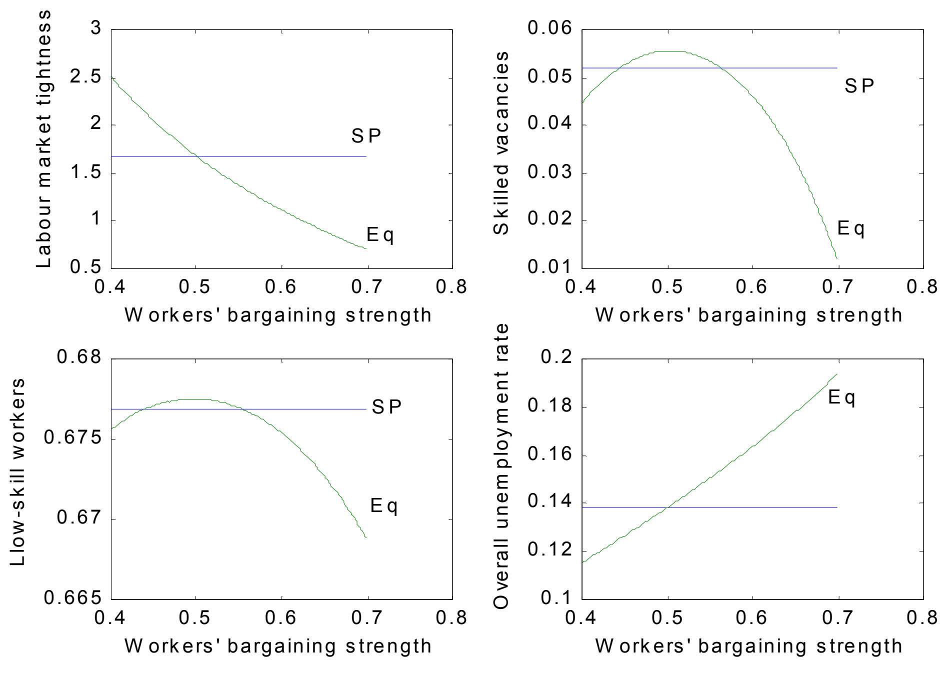

Figure 3: Comparative statics of changes in β

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