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ABSTRACT

How Falsifiable is the Collective Model? A New Test with an Application to Monogamous and Bigamous Households in Burkina Faso*

Collective rationality is seldom if ever rejected in the literature, raising doubt about its falsifiability. We show that the standard approach to test the collective model with distribution factors may yield misleading inference. We generalize the model and provide an appropriate test procedure to assess its validity. Our new approach extends to households that include more than two decision-makers (*e.g.*, polygamous households, adult children). We investigate household consumption decision-making within monogamous and bigamous households in Burkina Faso. Using the standard testing approach, collective rationality within monogamous households is not rejected. Using our proposed test procedure, collective rationality is however rejected for monogamous households. Furthermore, our test also rejects collective rationality for bigamous households. We conclude that the household efficiency does yield empirically falsifiable restrictions despite being scarcely rejected in the literature.

JEL Classification: D1, D7, J12

Keywords: collective model, distribution factors, rationality, efficiency, polygamy

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1. Introduction

The collective household model, based on Pareto efficiency, has become the main paradigm through which much family microeconomics research is now conducted. The reasons for its widespread use are twofold. First, the model rests upon a very small set of assumptions and yet still provides testable restrictions (Browning and Chiappori, 1998; Chiappori and Ekeland, 2006). Second, under reasonable conditions, the fundamentals of the model (typically individual preferences and the decision process) can be partly identified (Chiappori and Ekeland, 2009). Therefore, intrahousehold distributional impacts of various policies can be thoroughly investigated.

Because of its fundamental importance, the efficiency hypothesis has been scrutinized empirically in a variety of contexts. Two broad sets of falsifiable restrictions have been proposed in the theoretical literature to test whether household outcomes are indeed efficient. The first focuses on the price effects in a given household demand system. The second is based on the effects of the so-called *distribution factors*.[1] The latter have received more attention in the empirical literature presumably because quality data on prices are hard to come by and because they may be endogenous at least in non-experimental settings.[2]

The theoretical literature provides three distinct ways distribution factors can be used to test the efficiency hypothesis within two-member households. First, the *proportionality condition* states that the ratio of the marginal effect of two distribution factors must be equal across demand equations (Bourguignon et al., 1993; Browning et al., 1994; Bourguignon et al., 2009). Second, the *z-conditional demand condition* requires the effects of the remaining distribution factors to vanish once the demand equations are conditioned on the demand for some other good and upon substituting out one of the distribution factors (Bourguignon et al., 2009). Finally, the *rank condition* posits that the impact of the distribution factors must be at most of size one (Chiappori and Ekeland, 2006).

The collective model is hardly ever rejected when using tests based on distribution factors, assuming given price levels. Some have thus been led to question the restrictive nature of the constraints imposed by the efficiency hypothesis as compared with those of a non-cooperative (Nash) approach (Naidoo, 2015). Others have raised concern over the statistical validity or power of the statistical tests they imply. For instance, the *proportionality condition* implies a nonlinear restriction across equations which is generally tested by means of a Wald test (Bourguignon et al., 1993; Browning and Chiappori, 1998; Quisumbing and Maluccio, 2003; Bobonis, 2009). Yet, Wald tests are not invariant to algebraically equivalent nonlinear parameterization of the null hypothesis (Dagenais and Dufour, 1991;

[1] Distribution factors are variables, such as the state of the marriage market, as proxied by the sex ratio, that influence the decision process within the household but neither individual preferences nor the household budget set. Papers that use distribution factors include Bourguignon et al. (1993), Browning et al. (1994), Browning and Chiappori (1998), Chiappori et al. (2002), Quisumbing and Maluccio (2003), Bobonis (2009), Dauphin et al. (2011), Attanasio and Lechene (2014), while papers that use price effects include Browning and Chiappori (1998) and Dauphin et al. (2011).

[2] There is a burgeoning literature which attempts to test collective rationality based on a non-parametric revealed-preference approach. In these studies, the collective model is generally not rejected by the data (*e.g.,* Vermeulen et al., 2009).

Agüero, 2008). Tests based on *z-conditional demands* are potentially more powerful because they boil down to testing single equation exclusion restrictions. Nevertheless, *z-conditional demand* equations include endogenous right-hand side variables and are typically estimated using an instrumental variables approach. The omitted distribution factors are natural instruments, but can prove to be weak. Finally, it is well-known that rank condition tests may suffer from poor statistical power in small samples (Cambda-Mendez and Kapetanios, 2009).

Obviously, the restrictiveness of the constraints and the statistical apparatus used to investigate them are intimately related. Yet, irrespective of the above issues, under-rejection of the efficiency hypothesis may arise for a more practical matter. Indeed, in their seminal theoretical contribution, Bourguignon, Browning and Chiappori (2009) (henceforth BBC2009) assume that *at least one distribution factor (locally) affects each demand equation*.[3] This assumption is hardly ever satisfied empirically. Yet, the standard testing approach used in the literature to investigate the efficiency hypothesis based either on the proportionality or the z-conditional demand condition requires the latter assumption to hold. Such inconsistency between the statistical procedures and their underlying assumptions, we argue, is a plausible candidate to explain under-rejection of the collective model.

The assumption of BBC2009 is perhaps too restrictive since distribution factors, by definition, need only affect two (or more) demand functions.[4] This is the starting point of our paper. We propose a falsifiable restriction of the efficiency hypothesis which extends BBC2009's approach insofar as it does not require a distribution factor to (locally) affect each demand equation. The basic intuition is that, even under collective rationality, it is possible that the demands for a subset of goods (*e.g.*, heating, electricity, lodging) are not affected by the relative bargaining power of the members, at least locally. In the case of two-member household, expenditures on these goods will then be independent from all distribution factors. On the other hand, the demands for those goods that are influenced by the spouses' bargaining power must depend on all the distribution factors. This provides an alternative *all or nothing* restriction which, we think, is more realistic than assuming that a distribution factor affects each demand equation. We derive a set of testable conditions that take this restriction into account and fully characterize collective rationality. This means that any given behavior is compatible with collective rationality if and only if these conditions are satisfied, absent price variations.

As with z-conditional demands, the new falsifiable restriction boils down to testing an exclusion restriction in each single equation. Because it rests upon unconditional demand functions, endogeneity of right-hand side variables is not an issue for testing this constraint. Furthermore, we show how our approach can be extended to households comprising potentially more than two decision-makers.

We illustrate our test procedure using a field survey conducted by one of the co-authors of this paper to collect information on the decision process in very poor households from rural Burkina Faso. The social and customary environments in which these households evolve are likely to impede enforcement of efficient marriage contracts as these are deeply rooted in traditions that dictate expected behaviour

[3]See the first sentence of their Proposition 2.

[4]A distribution factor cannot only influence a single demand however since, by definition, it cannot affect the household budget constraint.

from both spouses. We investigate the efficiency of outcomes both in monogamous and bigamous households. The latter face the same potential sources of inefficiency as monogamous households in addition to facing jealousy or rivalry between co-wives. Our analysis focuses on consumption inefficiency. In a somewhat loosely related paper, Udry (1996) investigated production outcomes of rural Burkina Faso households and strongly rejected efficiency.

The empirical analysis is based upon the widely used and flexible QUAIDS demand system. For both monogamous and bigamous households our data clearly reject the efficiency assumption using our test procedure. We also compute the *rank condition* of Chiappori and Ekeland (2006) which is asymptotically equivalent to our (simpler) test procedure. Concern with sample size leads us to bootstrap the rank test using a recent procedure proposed by Portier and Delyon (2014). The Chiappori and Ekeland (2006) rank test is consistent with our own test as it rejects efficiency for both monogamous and bigamous households. To illustrate how under-rejection of efficiency may arise, we next test the *proportionality condition* despite the fact that the condition of BBC2009 is not satisfied, as is customarily done in the literature. Our results show that collective rationality is (wrongly) not rejected for monogamous households. Finally, the z-conditional demands approach cannot be implemented to test the efficiency of our Bukinabé households by lack of proper (strong) instruments.

Our paper offers four contributions. First, we generalize the collective model proposed in BBC2009 by relaxing the assumption that at least one distribution factor affects each demand equation. We also provide an appropriate procedure to test the extended model. Second, we generalize our approach to the case of households comprising potentially more than two decision-makers (*e.g.*, polygamous households, extended families, adult children, multi-generational households). Our test is equivalent to the Chiappori and Ekeland (2006) rank test but is presumably statistically more powerful in small samples. Third, we show that the standard testing approach which usually overlooks the BBC2009 assumption is likely to lead to under-rejection of the collective model. Finally, using data from a survey we have conducted in Burkina Faso, we investigate efficiency in allocation of consumption within both monogamous and bigamous households. To our knowledge, this paper is the first to perform an econometric analysis of rational collectivity within bigamous families based on consumption decisions.

The rest of the paper is organized as follows. The next section presents our generalization of BBC2009's collective model to households comprising respectively two and potentially more potential decision-makers. We discuss our new procedure to test the model in each of these cases. We also illustrate how the standard approach to test BBC2009's model may yield misleading inference. Section 3 describes the socio-economic specificities of monogamous and polygamous households in Burkina Faso and discusses the choice of our distribution factors. We also present the design of our survey and the main samples characteristics. Section 4 presents our estimation results and provides various tests of our generalized model for both monogamous and polygamous households. Section 5 concludes.

2. Theoretical restrictions of the collective model based on distribution factors

Our theoretical approach is based on the collective model in the absence of price variation as in BBC2009. We generalize their approach in two directions. First, we relax one of their crucial assumptions, namely that at least one distribution factor (locally) influences all the demand functions. Second, given that part of our empirical analysis focuses on bigamous households, we allow households to comprise more than two adult members.[5]

Consider a household with $I + 1$ members. Each member i draws his/her well-being from the consumption of N market commodities, which we represent by the vector $\mathbf{x}$. Each commodity may be consumed privately or publicly by household members. All prices are normalized to 1 so that the household budget corresponds to $\boldsymbol{\iota}'\mathbf{x} = m$, where $\boldsymbol{\iota}$ is a unit vector of dimension N and m is the level of exogenous household expenditures.[6]

Each member i has his own preferences $U^i(\mathbf{x})$ over (private and public) goods consumed in the household. No restrictions are imposed on the nature of the preferences. They can be egotistic or altruistic and may involve externalities or other types of preference interactions. We assume that $U^i(\mathbf{x})$ is strongly concave, twice differentiable in $\mathbf{x}$ and increasing in each of its arguments.

Under rationality, the outcomes of the household decision process are assumed to be Pareto-efficient. This means that the household chooses a vector $\mathbf{x}$ such that no other feasible vector could make all members at least as well off and at least one member strictly better off. The collective model also allows the possibility for exogenous variables, called *distribution factors*, to influence the household's decisions. These variables are denoted by the vector $\mathbf{z}$ of dimension K.

The influence of these factors can be understood within a bargaining framework where each member has an outside option, that is, an option he can resort to in case of disagreement over a proposed consumption vector $\mathbf{x}$. The poorer his/her outside options, the more he/she will be willing to compromise and thus the lower will be his/her bargaining power. As a result, the less the consumption decisions will correspond to his/her preferences.

Outside options can vary across individuals and cultures. For example, members could behave non-cooperatively in case of minor disagreements (Lundberg and Pollak, 1993; Chen and Woolley, 2001) and eventually separate in case of major disagreements (Manser and Brown, 1980; McElroy, 1990). In the latter case, the state of the marriage market (Becker, 1993) as proxied by the sex ratio (Chiappori et al., 2002), the nature of divorce laws (Gray, 1998; Chiappori et al., 2002) and the relative contribution of the spouse to the household income (Browning and Chiappori, 1998; Dauphin et al., 2011) have been considered as distribution factors in the literature. In the context of developing countries, Haddad and Kanbur (1992) stress the possibility for women to return to their native families in case of disagreement, and discrimination against women in the market place as potential distribution factors.

[5]Note that Dauphin et al. (2011) investigate the efficiency of households comprising up to three potential decision-makers (couples with an adult child) using price-based statistical tests.

[6]This assumes that the household does not produce any of these N goods, or that the goods produced within the household can be freely sold and purchased on the market.

The setting of collective rationality is equivalent to stating that there exists a vector $\boldsymbol{\mu}(m, \mathbf{z})$ of I non-negative *Pareto weights* such that $\mathbf{x}$ is the solution to the following program:

$$\text{Max } \boldsymbol{\mu}(m, \mathbf{z})' \left[U^1(\mathbf{x}), ..., U^I(\mathbf{x})\right] + U^{I+1}(\mathbf{x})$$
$$\text{subject to } \boldsymbol{\iota}'\mathbf{x} = m.$$

Thus the household *pseudo-utility function*[7] to be maximized is a weighted sum of the individual utility functions. The Pareto weight associated with the preferences of member i (for $i \neq I + 1$) can be interpreted as the importance attached to these, relative to those of the $(I + 1)^{th}$ member, in the household decision process. If the Pareto weight of a given member is equal to zero, the household does not take into account that member's preferences in the decision process, other than via the possible caring preferences of the other members. Thus, it is as if the member has no influence or power over household choices. The I Pareto weights can therefore be viewed as the distribution of decision power within the household and the number of decision makers as the number of strictly positive Pareto weights plus one.

The Pareto weights might be functions of distribution factors and of household expenditures, in which case they are assumed to be twice continuously differentiable in $(m, \mathbf{z})$. It should be noted that some weights may be (locally) constant while others may respond to distribution factors. Furthermore, the non-constant weights may not all (locally) respond to the same distribution factors. When all the weights are constants, the household is said to behave rationally in a unitary way because the objective function can be interpreted as representing a unique utility function. When some of the weights are non-constants, the household is said to behave rationally in a collective way because the objective function cannot be interpreted as representing a unique set of preferences. If for example, two weights are non-constant and linearly independent, then the $I + 1$-person household behaves as if it had three distinct sets of preferences.

The demand system obtained from solving the above program for $\mathbf{x}$ can be written as: $\mathbf{x} = \widehat{\mathbf{x}}(m,\ \boldsymbol{\mu}(m, \mathbf{z}))$, with $\iota'\widehat{\mathbf{x}}(m,\ \boldsymbol{\mu}(m, \mathbf{z})) = m$ from the adding-up restriction. This shows that the distribution factors influence household consumption choices only through the non-constant Pareto weights entering the household pseudo-utility function, which are potentially as many as I. This follows from the fact that the distribution factors do not affect the Paretian frontier of the household consumption possibilities, but only the household's location on it. Clearly, since the Pareto weights are unobservable so is the structural demand system. Yet, it is still possible to test whether the reduced form of the latter, $\mathbf{x}(m, \mathbf{z})$, satisfies:

$$\mathbf{x}(m, \mathbf{z}) \equiv \widehat{\mathbf{x}}(m, \boldsymbol{\mu}(m, \mathbf{z})). \tag{1}$$

Proposition 2 of BBC2009 assumes that at least one distribution factor (locally) affects each demand function. Yet, distribution factors need not locally influence more than two of the latter to yield falsifiable restrictions. We thus start by relaxing this assumption and derive the appropriate test procedure. Since BBC2009 adopt a global approach, they assume that at least one good is strictly monotone

[7] Strictly speaking, the maximand is not a utility function since it depends on the total expenditures of the household.

in one observable distribution factor. We do not need this assumption because we adopt a local approach instead. Next, we generalize the test to the case where a household comprises more than two potential decision-makers.

2.1. Generalization of Proposition 2 of BBC2009 - Two Decison-Makers

Consider a partition $\mathbf{x} \equiv [\mathbf{x}'_J, \mathbf{x}'_{\text{-}J}]'$ of the demand system and a partition $\mathbf{z} \equiv [\mathbf{z}'_J, \mathbf{z}'_{\text{-}J}]'$ of the set of distribution factors, with $\mathbf{x}_J$ and $\mathbf{z}_J$ having the same dimension J. Given such a partition, (1) can be written as:[8]

$$\mathbf{x}_J = \mathbf{x}_J(\mathbf{z}_J, \mathbf{z}_{\text{-}J}) \equiv \widehat{\mathbf{x}}_J(\boldsymbol{\mu}(\mathbf{z}_J, \mathbf{z}_{\text{-}J})), \tag{2}$$
$$\mathbf{x}_{\text{-}J} = \mathbf{x}_{\text{-}J}(\mathbf{z}_J, \mathbf{z}_{\text{-}J}) \equiv \widehat{\mathbf{x}}_{\text{-}J}(\boldsymbol{\mu}(\mathbf{z}_J, \mathbf{z}_{\text{-}J})). \tag{3}$$

If the sub-system of reduced-form demand functions in (2) has continuous first partial derivatives and is such that $D_{\mathbf{z}_J}\mathbf{x}_J(\mathbf{z}_J, \mathbf{z}_{\text{-}J})$ is non-singular at a point $P = (\mathbf{z}_J, \mathbf{z}_{\text{-}J})$, then we can use the *Implicit Function Theorem* to invert $\mathbf{x}_J$ and $\mathbf{z}_J$ in some open neighborhood of P to get the following local inverse function:[9]

$$\mathbf{z}_J = \mathbf{z}_J(\mathbf{x}_J, \mathbf{z}_{\text{-}J}),$$

which has continuous first partial derivatives. Upon substituting the latter into (2) and (3) we get:

$$\mathbf{x}_J = \overline{\mathbf{x}}_J(\mathbf{x}_J, \mathbf{z}_{\text{-}J}) \equiv \mathbf{x}_J(\mathbf{z}_J(\mathbf{x}_J, \mathbf{z}_{\text{-}J}), \mathbf{z}_{\text{-}J}) \equiv \widehat{\mathbf{x}}_J(\boldsymbol{\mu}(\mathbf{z}_J(\mathbf{x}_J, \mathbf{z}_{\text{-}J}), \mathbf{z}_{\text{-}J})), \tag{4}$$
$$\mathbf{x}_{\text{-}J} = \overline{\mathbf{x}}_{\text{-}J}(\mathbf{x}_J, \mathbf{z}_{\text{-}J}) \equiv \mathbf{x}_{\text{-}J}(\mathbf{z}_J(\mathbf{x}_J, \mathbf{z}_{\text{-}J}), \mathbf{z}_{\text{-}J}) \equiv \widehat{\mathbf{x}}_{\text{-}J}(\boldsymbol{\mu}(\mathbf{z}_J(\mathbf{x}_J, \mathbf{z}_{\text{-}J}), \mathbf{z}_{\text{-}J})). \tag{5}$$

The (local) sub-system of demands $\mathbf{x}_{\text{-}J}$ in (5) is written as a function of the sub-system of (local) demands $\mathbf{x}_J$ and of the distribution factors $\mathbf{z}_{\text{-}J}$.[10] These are the so-called *z-conditional demands* proposed by BBC2009. The sub-system $\mathbf{x}_{\text{-}J}$ is said to be $\mathbf{z}_J$-conditional, since it is conditional on the inversion of $\mathbf{z}_J$. We further assume that $\boldsymbol{\mu}(\mathbf{z})$ and $\widehat{\mathbf{x}}(\boldsymbol{\mu}(\mathbf{z}))$ are differentiable at the point P. The first generalization is as follows:

[8] We omit m from the argument to simplify the notation.

[9] In what follows, we denote the jacobian of $\mathbf{x}$ with respect to $\mathbf{z}$ as $D_{\mathbf{z}}\mathbf{x}$.

[10] Note that a demand function that is insensitive to a distribution factor may respond to it once it is conditioned on $\mathbf{x}_J$ through the function $\mathbf{z}_J(\mathbf{x}_J, \mathbf{z}_{\text{-}J})$.

Proposition 1. *A system of $N \geq 2$ demand functions of a household with $I + 1 = 2$ members is locally compatible with* ***rationality*** *if, and only if, variables that do not enter the members' individual preferences or the overall household budget constraint,* $\mathbf{z}$, *either do not influence the demand system (**unitary rationality**):*

$$D_{\mathbf{z}} x_n(\mathbf{z}) = \mathbf{0} \quad \forall\, n\text{=1,...,}N, \tag{7a}$$

*or influence it in the following way when $K \geq 2$ (**collective rationality**):*[11]

$$D_{\mathbf{z}} x_n(\mathbf{z}) = \mathbf{0} \quad \text{or} \quad D_{\mathbf{z}} x_n(\mathbf{z}) \neq\neq \mathbf{0} \quad \forall\, n\text{=1,...,}N. \tag{7b}$$

Moreover, the demands for which $D_{\mathbf{z}} x_n(\mathbf{z}) \neq\neq \mathbf{0}$, denoted $x_m^*(\mathbf{z})$, also satisfy:

$$\frac{\partial x_m^*(\mathbf{z})/\partial z_1}{\partial x_m^*(\mathbf{z})/\partial z_k} = \frac{\partial x_1^*(\mathbf{z})/\partial z_1}{\partial x_1^*(\mathbf{z})/\partial z_k} \neq 0 \quad \forall\, k\text{=2,...,}K,\ m\text{=2,}\ldots,M \tag{7c}$$

and, equivalently,

$$D_{\mathbf{z}_{\text{-}1}} \overline{x}_m^*(x_1^*, \mathbf{z}_{\text{-}1}) = \mathbf{0} \quad \forall\ m\text{=2,}\ldots,M, \tag{7d}$$

where $2 \leq M \leq N$.

The proof is reported in Appendix A. The proposition states that the demand system of a two-person household is compatible with rationality *if and only if* it either complies with unitary rationality, (7a), or with collective rationality, (7b), (7c), and, equivalently to the latter, (7d). According to Restriction (7a), a demand system is compatible with *unitary rationality* if and only if none of its demand functions is influenced by distribution factors, *i.e.*, the existence of distribution factors is incompatible with unitary rationality ($K = 0$). Our new *all or nothing* Restriction (7b) stresses that a demand system that responds to at least two distribution factors is compatible with collective rationality if each of its demands either does not respond to any of the distribution factors or responds to all of the distribution factors. Equation (7c) further restricts the manner in which the distribution factors impact the demand functions that respond to all of the distribution factors (denoted x_m^*): the ratio of the marginal effects of any two distribution factors must be equal across the latter. Finally, Restriction (7d) is equivalent to (7c). It states that the demand functions x_m^* are compatible with collective rationality only if they no longer respond to the distribution factors once they are conditioned on any of them, *i.e.*, are transformed into their z_1-conditional form. Many empirical applications investigate the efficiency hypothesis using (7c). In most cases, the tests ignore Restriction (7b). Yet, the two go hand-in-hand. We will argue later on that focusing on (7c) while ignoring (7b) partly explains why the efficiency assumption is hardly ever rejected.

The intuition behind *Proposition 1* is the following. In a rational household composed of two members there is only one Pareto weight. The distribution factors, if they exist, can only exert their

[11] When $N \leq 2$ or $K = 1$, collective rationality imposes no restrictions on the demand system.

influence on consumption choices through the latter weight. If unitary rationality holds, then the Pareto weight is constant. Each of its demand function, x_n, must therefore satisfy (7a). On the other hand, if collective rationality holds and distribution factors do exist, then the single Pareto weight must respond to all of them. If a given demand function, x_n, say, does not locally respond to the Pareto weight, it will respond to none of the distribution factors ($D_{\mathbf{z}}x_n(\mathbf{z}) = 0$). Conversely, if x_n does respond to the Pareto weight, it will be sensitive to all the distribution factors ($D_{\mathbf{z}}x_n(\mathbf{z}) \neq\neq 0$). Each demand x_n stemming from a collectively rational household must therefore satisfy the *all or nothing* Restriction (7b). Furthermore, since distribution factors exert their effects through a single weight, the demand functions that respond to it (x_m^*) must be such that the ratio of the marginal effect of any two distribution factors is equal to the ratio of the marginal effect of the two distribution factors on the weight, and this ratio must be different from zero. Therefore, the ratio of the marginal effect of any two distribution factors is equal across x_m^* demands as stated by (7c). Finally, conditioning a given demand function, x_m^*, by another, x_1^* say, is equivalent to maintaining x_1^* constant. In order to maintain x_1^* constant, z_1 must compensate for the variations in $\mathbf{z}_{\text{-}1}$ in such a way that the variations in the weight cancel out. Restriction (7d) must thus hold for this z_1-conditional demand.

An important corollary to the *all or nothing* Restriction is that a system in which some demands respond to a subset of distribution factors while other demands respond to another subset of distribution factors is not compatible with collective rationality when $I + 1 = 2$. The *all or nothing* Restriction is absent from *Proposition 2* of BBC2009 because it is assumed at the outset that one of the distribution factors (locally) affects all the demand functions. Since there is a single Pareto weight, this amounts to assuming that all the demand functions respond to the weight and therefore that they all respond to all the distribution factors affecting the weight. This is equivalent to assuming that all the demand functions satisfy (7b), and more precisely its second part, that is $D_{\mathbf{z}}x_n(\mathbf{z}) \neq\neq \mathbf{0}$. Hence, in the BBC2009 framework our *Proposition 1* boils down to restriction (7c) and, equivalently to the latter, (7d).

It should be noted that while tests from *Proposition 1* are based on a static definition of rationality, they are also consistent with the *intra-household allocation stage* of any dynamic household decision process that assumes within-period efficiency. This includes limited-commitment and full-commitment intertemporal collective models (Chiappori and Mazzocco, 2014).[12] Besides, *Proposition 1* ignores the intertemporal stage of the household decision process under dynamic rationality (based on Euler equations). Therefore, in dynamic collective models the restrictions of *Proposition 1* are usually necessary but in general not sufficient for collective rationality.

2.2. Generalization of Proposition 2 of BBC2009 - Multiple Decision-Makers

Proposition 1 is valid for households in which it can legitimately be assumed that there are at most two decision-makers. Many household configurations (extended families, adult children, multi-generational households, polygamous households, *etc.*), though, may potentially have more than two

[12]In a full-commitment intertemporal model, it is assumed that household members can commit at the initial period to all future allocations of resources.

decision-makers.[13] It is relatively straightforward to extend *Proposition 1* to multiple potential decision-maker households.

Proposition 2. *A system of $N \geqslant I+1$ demand functions of a household with $I+1$ members is locally compatible with **rationality** if, and only if, variables that do not enter the members' individual preferences or the overall household budget constraint, $\mathbf{z}$, either do not influence the demand system (**unitary rationality**):*

$$D_{\mathbf{z}} x_n(\mathbf{z}) = \mathbf{0} \quad \forall\, n\text{=}1,...,N. \tag{8a}$$

*or influence it in the following way when $K \geq I+1$ (**collective rationality**): there exists a non-negative $J \leq I\text{-}1$ for which*[14]

$$\begin{aligned}&D_{\mathbf{z}_{\text{-}J}} \overline{x}_n(\mathbf{x}_J, \mathbf{z}_{\text{-}J}) = \mathbf{0} \quad \textit{or} \\ &D_{\mathbf{z}^*_L} \overline{x}_n(\mathbf{x}_J, \mathbf{z}_{\text{-}J}) \neq\neq \mathbf{0} \textit{ and } D_{\mathbf{z}^*_{\text{-}L}} \overline{x}_n(\mathbf{x}_J, \mathbf{z}_{\text{-}J}) = \mathbf{0} \quad n\text{=}J\text{+}1,...,N,\end{aligned} \tag{8b}$$

where $\mathbf{z}_{\text{-}J} \equiv [\mathbf{z}^{*\prime}_L, \mathbf{z}^{*\prime}_{\text{-}L}]'$ with $1 \leq L \leq K\text{-}J$. Moreover, when $2 \leq L$, the demand functions that satisfy $D_{\mathbf{z}^*_L} \overline{x}_n(\mathbf{x}_J, \mathbf{z}_{\text{-}J}) \neq\neq \mathbf{0}$, denoted $\overline{x}^*_m(\mathbf{x}_J, \mathbf{z}_{\text{-}J})$, must also satisfy:

$$\frac{\partial \overline{x}^*_m(\mathbf{x}_J, \mathbf{z}_{\text{-}J})/\partial z^*_1}{\partial \overline{x}^*_m(\mathbf{x}_J, \mathbf{z}_{\text{-}J})/\partial z^*_l} = \frac{\partial \overline{x}^*_1(\mathbf{x}_J, \mathbf{z}_{\text{-}J})/\partial z^*_1}{\partial \overline{x}^*_1(\mathbf{x}_J, \mathbf{z}_{\text{-}J})/\partial z^*_l} \neq 0 \quad \forall\, l\text{=}2,...,L \quad m\text{=}2,...,M \tag{8c}$$

and, equivalently,

$$D_{\mathbf{z}_{\text{-}(J+1)}} \overline{x}^*_m(\mathbf{x}_J, x^*_1, \mathbf{z}_{\text{-}(J+1)}) = \mathbf{0} \quad \forall\, m\text{=}2,...,M, \tag{8d}$$

where $2 \leq M \leq N\text{-}J$.

See Appendix A for the proof. This proposition states that the demand system of an $I+1$-person household is compatible with rationality if and only if it either complies with unitary rationality (8a) or with collective rationality (8b), (8c), and, equivalently to the latter, (8d). Restriction (8a) is identical to Restriction (7a). Restrictions (8b) and (8c) are equivalent to (7b) and (7c) but they involve z-conditional rather than unconditional demand functions. Restriction (8b) states that the demand system of an $I+1$-person household influenced by at least $I+1$ distribution factors is compatible with collective rationality if there is a non negative $J \leq I-1$ for which each demand contained in $\mathbf{x}_{\text{-}J}$, once conditioned on $\mathbf{x}_J$, either does not respond to any of the remaining distribution factors ($\mathbf{z}_{\text{-}J}$) or responds to a subset of them, denoted $\mathbf{z}^*_L$.[15] This subset must be the same for all the $\mathbf{z}_J$-conditional demand functions. In other

[13] An earlier extension of *Proposition 2.ii* of BBC2009 to multiple decision-makers can also be found in Dauphin and Fortin (2001).

[14] When $N \leq I+1$ or $K < I+1$, collective rationality imposes no restrictions on the demand.

[15] In the case where each Pareto weight depends on all distribution factors, one has $\mathbf{z}^*_L = \mathbf{z}_{\text{-}J}$.

words, those $\mathbf{z}_J$-conditional demands that still respond to distribution factors must respond to the same subset of distribution factors. The latter may include all the remaining distribution factors or a subset of them. Hence, having some $\mathbf{z}_J$-conditional demand functions responding to some distribution factors and other $\mathbf{z}_J$-conditional demand functions responding to other distribution factors is incompatible with collective rationality. Restriction (8c) further states that the $\mathbf{z}_J$-conditional demand functions ($\overline{x}^*_m(\mathbf{x}_J, \mathbf{z}_{\text{-}J})$) responding to distribution factors $\mathbf{z}^*_L$ are compatible with collective rationality only if the ratios of the marginal effect of any two distribution factors included in $\mathbf{z}^*_L$ are equal across them. Finally, and equivalently to Restriction (8c), Restriction (8d) stresses that the $\overline{x}^*_m(\mathbf{x}_J, \mathbf{z}_{\text{-}J})$ demand functions are compatible with collective rationality only if they no longer respond to the distribution factors once they are conditioned on one more demand influenced by $\mathbf{z}^*_L$ (say x^*_1).

Chiappori and Ekeland (2006) (henceforth CE2006) provide another generalization of *Proposition 2* of BBC2009, which we present here in a slightly modified manner.

Proposition 3. *(Chiappori and Ekeland (2006)) A system of $N \geqslant I+1$ demand functions of a household with $I+1$ decision-makers is compatible with* ***rationality*** *if, and only if, variables that do not enter the members' individual preferences or the overall household budget constraint, $\mathbf{z}$, either do not influence the demand system (**unitary rationality**):*

$$\textit{rank}\,[D_{\mathbf{z}}\mathbf{x}(\mathbf{z})] = \mathbf{0} \tag{9a}$$

*or satisfies the following condition whenever $K \geqslant I+1$ (**collective rationality**):*[16]

$$0 < \textit{rank}\,[D_{\mathbf{z}}\mathbf{x}(\mathbf{z})] \leq I. \tag{9b}$$

See CE2006 for the proof. Restriction (9a) is identical to our restrictions (7a) and (8a). Restriction (9b) states that if there exists $K \geqslant I+1$ distribution factors, then the rank of the matrix $D_{\mathbf{z}}\mathbf{x}(\mathbf{z})$ must be greater than zero, but no greater than I. Intuitively, there can be no more than I Pareto weights under collective rationality. Since the distribution factors only impact the demand system through the latter, if there are fewer weights than there are distribution factors, their effects on the demand functions must necessarily be linearly dependent. Propositions *2* and *3* are equivalent. If a demand system satisfies (8b), (8c) and (8d), it will also satisfy (9b) and vice versa. However, if (8b) is satisfied, but restrictions (8c) and (8d) do not apply because $L < 2$, then (9b) may not be satisfied. This is because (8b) is only necessary. However, under collective rationality, and as long as $K+1 \geq I$, it will always be possible to partition $\mathbf{x} \equiv [\mathbf{x}'_J, \mathbf{x}'_{\text{-}J}]'$ and $\mathbf{z} \equiv [\mathbf{z}'_J, \mathbf{z}'_{\text{-}J}]'$ in such a way that $L \geq 2$.[17]

The two propositions provide falsifiable restrictions for an overall test of rationality. In the case of Proposition *2*, the first step is to test whether all the demands satisfy Restriction (8a), *i.e.*, to test whether households behave rationality in a unitary way. If this hypothesis is rejected, the next step is to test collective rationality. The general formulation of the null hypothesis corresponding to collective

[16] When $N \leq I+1$ or $K < I+1$, collective rationality imposes no restrictions on the demand system.

[17] Note that just as for Proposition 2, Proposition 3 does not require any of the distribution factors to affect all the demands.

rationality is $H_0 : J \leq I$-1 versus $H_1 : J > I$-1. Since H_0 is a composite hypothesis, a sequential approach can be followed. One should thus start by testing whether all the demand functions satisfy Restriction (8b) when $J = 0$. If this hypothesis is rejected, the Restriction (8b) has to be tested for $J = 1$ and so on until it is not rejected for $J \leq I$-1. If the Restriction (8b) is not rejected for $J \leq I$-1, the restriction (8c) or (8d) should be tested for those demand functions that are influenced by distribution factors. If the latter hypothesis is not rejected, then testing stops and collective rationality is not rejected. Conversely, if restriction (8c) or (8d) is rejected, then Restriction (8b) should be tested again for a higher J. If the restrictions (8b) and (8c) or (8d) are successively rejected for all $J \leq I$-1, then collective rationality as well as overall rationality must be rejected. The same sequential approach must be used with Proposition *3*. The first step is to test whether the rank of $D_{\mathbf{z}}\mathbf{x}(\mathbf{z})$ is equal to zero. If not, then the next step is to test whether it is equal to 1 and so on until I is reached. If the rank is not found to be equal or inferior to I, then overall rationality is rejected. If the rank is found to be of any value greater than 1 but inferior or equal to I, then testing stops and collective rationality is not rejected.

2.3. Empirical Investigations of Household Collective Rationality

In this sub-section, we show and illustrate how our propositions partly explain why the standard testing approach is likely to under-reject the collective model with distribution factors. Household collective rationality is the object of much research in the empirical literature (see Chapter 5 of Browning et al., 2014). Most papers focus on households composed of two adults, which is the concern of our *Proposition 1*. Demand systems are estimated using data from developed as well as developing countries and are based on a variety of functional forms (AIDS, QUAIDS, EASI, *etc.*). Likewise, a rich set of distribution factors are used to proxy spouses' relative bargaining power (relative income, relative age, relative assets at marriage, *etc.*). It is thus rather surprising given such heterogeneous environments that collective rationality is seldom rejected.

As shown earlier, for collective rationality in households with two potential decision makers to be satisfied, the *all or nothing* Restriction (7b) needs to hold and restrictions (7c) and (7d) only apply to the subset of demand functions that are responding to all the distribution factors. Yet, it is customary in the literature to neglect Restriction (7b) and to test collective rationality by means of Restriction (7c) or Restriction (7d) over the full set of demands, thus including those that do not respond to distribution factors as well as those who respond to a subset of them. This, we argue, partly explains why collective rationality is likely to be under-rejected. For illustration, consider the paper by Quisumbing and Maluccio (2003). It uses data from various developing countries to estimate household demand systems. Assets of spouses at marriage are treated as distribution factors. Results for Bangladesh show that only the demand for food significantly responds to husband's assets while only the demand for education responds to the wife's assets. This is not compatible with collective rationality as it violates the *all or nothing* Restriction. Indeed, in a demand equation where one distribution factor is statistically significant, all other distribution factors should also be significant. Results for Ethiopia show that food responds to the two distribution factors, but alcohol and tobacco only respond to the second distribution factor. This is not compatible with collective rationality either. In both cases Restriction (7c) is tested and not rejected, and hence nor is collective rationality.

In Bobonis (2009), two distribution factors are considered. The first is a random dummy variable indicating whether the household benefited from the PROGRESA cash transfers in Mexico. The second distribution factor is the deviation from mean rainfall. Estimation results show that receiving cash transfers significantly affects female clothing, child clothing, expenditures on fruit and vegetable and expenditures on alcohol & tobacco. Deviation from mean rainfall further affects male clothing, expenditures on other food and expenditures on other household goods. These results violate the *all or nothing* Restriction. Despite this, Restriction (7c) is tested and is not rejected.

In a recent paper, Attanasio and Lechene (2014) use the same dataset as Bobonis (2009). Random assignment to treatment is also used as a distribution factor. Deviation from mean rainfall is omitted and is replaced by each spouses' relative size and wealth of own family networks. Estimation results show that expenditures on pulses significantly respond to the PROGRESA dummy variable but not to the size of the family network. Based on the *all or nothing* Restriction, collective rationality should have been rejected.[18] Nevertheless, Restriction (7d) is tested and cannot be rejected.

The above examples are representative of how most papers test collective rationality. In what follows, we use data from rural Burkina Faso to investigate rationality within monogamous and bigamous households. *All or nothing* Restriction (7b) (monogamous) or (8b) (bigamous) is first tested. If satisfied we next move on to test (7c) or (8c) as the case may be. We also test collective rationality using the asymptotically equivalent CE2006 rank test. We next use the same approach as in the above papers and test Restriction (7c) irrespective of whether the *all or nothing* restriction (7b) holds.

3. The Burkinabé Family

Burkina Faso is one of poorest countries in the world. In 2014, the country ranked 181^{th} out of 185 countries, with a life expectancy of 56.3 years, an adult literary rate of 28.7% and a GDP per capita of 1 602 PPP US$ (UNDP, 2014). Over 83% of the population lives in rural areas and traditional agriculture, the mainstay of the economy, accounts for roughly 90% of total employment (Jeune Afrique, 1998). In 2013, the total population was estimated at 16.9 millions (UNDP, 2014) and comprises around 60 different ethnic groups of different sizes, although the Mossis account for nearly half of the total (Jeune Afrique, 1998). Animism, the traditional religion, is gradually abandoned in favor of Muslim and Catholic religions.

Polygamy is quite prevalent in Burkina Faso. It is estimated that up to 22% of men and 42% of women are in a polygamous union (INSD, 2010). The proportion of women living in a polygamous union increases with the age of the women. From 24% for the age group 15-19, it gradually increases to 30% for the age group 20-24 and then to 55% for those aged between 40-44 (INSD, 2010). Marriage is first and foremost an agreement between two families. Once married, divorce may not occur without both their consent. Girls have little saying on their family's decision. Marriages are sometimes agreed

[18] One could argue that individual Student-t tests for each distribution factor does not take into account the covariance between the distribution factors. Since one of the distribution factors used by Bobonis (2009) and Attanasio and Lechene (2014) is the random allocation of a cash transfer to women, its covariance with the other distribution factor should be zero.

upon when the girls are still in their infancy. Usually, though, girls marry at the age of 18 and boys around the age of 25 (INSD, 2010). Three types of marriage are possible: traditional, religious and civil. Each type involves different rights and obligations. Traditional marriages, by far the most common, do not restrict the number of spouses a man may have. Muslim marriages restrict the number of wives to four, while catholic and civil marriages prohibit polygamy altogether. In practice, however, it is not infrequent to encounter polygamous catholic men married under more than one matrimonial regime.

3.1. Distribution Factors

According to the anthropological literature,[19] in the Mossi society, and in Burkina Faso in general, spouses tend to behave non-cooperatively when a disagreement arises, at least initially. The husband "refuses to give cereals, money and gifts to his wife and will favor another wife. The wife, in return, will refuse to carry out her domestic and conjugal duties [...] The wife may thus refuse to fetch water from the well for him, to heat it up for him, to wash his clothes and give him food that she has herself produced or bought".[20] The more financially independent the wife is the greater will be her bargaining power. The wife's contribution to the household income could thus qualify as a distribution factor.

As time passes, the husband may gradually accumulate enough wealth to pay for the dowry of an additional wife. The husband's threat of a co-wife may thus become more and more credible over time, thereby gradually reducing the bargaining power of the wife, *ceteris paribus*. The duration of the marriage should thus qualify as a distribution factor in monogamous households. In the case of polygamous households, the anthropological literature has also highlighted that a wife's bargaining power depends on the number of years since her marriage and on her rank. The wives "must submit to an internal hierarchy conditioned by age and the length of the marriage: although negligible when less than a decade separates their birth or their union, it is perceptible beyond that. [...] Furthermore, the first wife has authority on the other wives".[21] This suggests to use the duration of the first wife's marriage relative to the duration of the second's wife marriage.

Here, an important remark is in order. While our model assumes that the marriage status of the household is exogenous, the above discussion suggests that it could be endogenous as it may partly be determined by the relative bargaining power of the (first) wife. Indeed, the higher the bargaining power of the wife is, the more likely she will be able to impose her (presumed) preference for monogamy. This may be the source of a selection bias in our estimators as our econometric analysis is conditional on the household marriage contract. Thus, omitting polygamous households when analyzing rationality of monogamous households and vice versa may imply that the average bargaining power of the first wife is overestimated in monogamous households and underestimated in polygamous households. A natural approach to address this potential selection bias would involve modeling the marital status of the household. This however would greatly complicate the analysis while being peripheral to the main

[19] The literature on Mossi families, on which this section is based, dates back to the seventies and eighties. The main references are Lallemand (1977), Rookhuizen (1986) and Rohatynskyj (1988).

[20] Rookhuizen (1986), p. 59, free translation.

[21] Lallemand (1977), p. 263, free translation.

point we wish to underline in this article. It must also be recognized that such a selectivity problem applies to almost all the empirical literature on collective rationality since the formation and dissolution of households is itself endogenous.

3.2. The Survey

With an aim at testing household efficiency in Burkina Faso, Anyck Dauphin, one co-author of this paper, conducted a field survey between January and March 1999 under the auspices of the Centre canadien d'étude et de coopération internationale (CECI). Its primary purpose was to collect information on the household decision process pertaining to consumption spending, time allocation, fertility, as well as potential distribution factors. The information on the income of the different spouses was collected indirectly. Since most households live out of agriculture and since agricultural production survey are very complex, the survey focused on household expenditures which can be considered as a good indicator of their permanent income. For each spouse data were collected on expenditures on food and non-food products, durable goods and self-consumption. Expenditures on so-called "assignable" goods were also collected such as household expenditures on clothes and hairdressing for the husband, the wives and their respective children.

The survey was conducted in the Province of Passoré which has a population of approximately 322 000[22], primarily because the CECI had been involved in the region for a long time and had established close links with the local institutions. The province is divided into nine administrative regions. In order to minimize cost, the survey was limited to the five regions that were deemed the most representative of the economic and social fabric of the province. These include Dakiégré, Pelegtanga, Rallo, and Sectors 1 and 5 of the City of Yako (Yako-1, Yako-5).

To be included in the sample a household had to meet the following two conditions: (1) The (male) household head as well as his spouse(s) had to be less than 70 years of age and; (2) They all had to live permanently on the same compound. Prior to sampling, a census was conducted in each of the five regions to identify married households and to determine eligibility. Over 125 married households were then randomly selected among those eligible in each region, except for the village of Dakiégré where all 111 households were selected.

Table 1 indicates the number of potential households as well as the number of households who were present at the time of the survey and agreed to answer the detailed questionnaire. Overall, as many as 551 households out of 611 were interviewed (response rate = 90%). The questionnaires were pre-tested during a period of two weeks by local trained investigators. For each household, the head and each of his spouses were interviewed individually and separately using a "female" and "male" version of the questionnaire. Heads were interviewed by a male investigator and each spouse was interviewed by a female investigator.

TABLE 1 ABOUT HERE

[22] According to the National 2006 Census.

3.3. Sample Characteristics

Polygamy is more prevalent in rural areas. Indeed, Table 2 shows that the average number of wives per head is lower in Yako-1 and Yako-5, the only two urban areas in our sample, and is highest in the village of Dakiégré. The table also shows that 71.1% of households are monogamous.

TABLE 2 ABOUT HERE

Sample size for monogamous and bigamous households are 392 and 117, respectively. The main characteristics of these samples are presented in Table 3, which is divided into three separate panels. The first shows that more than 30% of monogamous households are Muslim, a percentage that increases to 44% in bigamous households. Monogamous husbands are on average 42 years old, that is 7 years younger than polygamous males. Wives from monogamous households are on average 33 years old, somewhat in-between the age of first and second wives of bigamous households. Not surprisingly then, monogamous wives with an average of 3.5 children have fewer (more) children than the first (second) wife of bigamous households. The estimated budget for monogamous and bigamous households over a two month period covered by the survey are respectively 117 620 CFA and 216 743 CFA.

TABLE 3 ABOUT HERE

The ability to assess the impact of distribution factors on expenditures is greatly enhanced if the survey focuses on assignable goods. These may be consumed by more than one household member but individual consumption is observed in the data. *A priori*, distribution factors that favour a particular household member should have a noticeable impact on his/her share of a given assignable good. The field survey was thus designed to collect information on the main assignable goods consumed by each member of the household. Survey pretesting indicated that clothing and hairdressing were the two most important items that could qualify as assignable goods in the rural Burkinabé context. Expenditures on these two goods were aggregated into a single category which we refer to as "Personal Care". Each spouse in the household was thus surveyed about the expenditures made on these goods for his/her own purpose, and for those of the other spouses and their children. The second panel of the table reports the average share of the household budget devoted to the clothing and hairdressing of the husband (*PC-Husband*), his wives (*PC-Wife1* and *PC-Wife2*) and their respective children (*PC-Child1* and *PC-Child2*). The shares of personal care accruing to the wives are larger than those of husbands and children in both monogamous and bigamous households. In monogamous households the wives' share amounts to 9.95% while it represents about 4% for both wives in bigamous households. As in most poor countries, expenditures on food is the single most important item. The main staple food, millet, represents between 7.3% and 10.5% of the household budget whereas the remaining food items (*O-Food*) account for slightly more than 40%.

The last panel of the table focuses on distribution factors. In our data, a monogamous wife contributes on average to approximately 23% of total household income. In bigamous households, the first and second wives' shares are 17% and 14%, respectively. We may thus expect husbands in bigamous households to have less overall bargaining power as their share of total income is less. Yet, because both wives individually hold a smaller share of income than wives in monogamous households, it is not clear how bargaining power differs across the two types of households. Finally, monogamous households had lasted approximately 14 years and bigamous households had formed 22 years prior to our survey. The time lapse between first and second marriages is about eleven years.

4. Estimation Results

Our estimation strategy is threefold. For both monogamous and bigamous households we estimate a QUAIDS demand system.[23] We first test rationality with *Proposition 1* for monogamous households and with *Proposition 2* for bigamous households. Second, we use the rank test proposed by CE2006. Recall that this test is asymptotically equivalent to ours. Finally, we test collective rationality for monogamous households using the test procedure proposed by Bourguignon et al. (2009), irrespective of whether one of the distribution factors locally affects each demand of the system.

4.1. Monogamous Households

The demand system is composed of six non-durable goods, of which three are assignable : *PC-Husband*, *PC-Wife*, *PC-Children*, *Millet*, *Other Foods* and expenditures on remaining nondurable goods. Only the first five demands are estimated due to the adding-up constraint. Individual shares are regressed against the log of total expenditures on non-durable goods (ln*TotExp*) and its square (ln*TotExp2*). The two distribution factors are the log of the wife's share of household income (*SIncomeW*) and the duration of marriage (*DMarriage*). We also control for location, religion, age of spouses and the number of children under 16 years of age (**X**). The budget shares functions are written as:

$$w_n = \mathbf{X}'_n \alpha_n + \beta_n \ln TotExp + \theta_n (\ln TotExp)^2 + \delta_n SIncomeW + \gamma_n DMarriage + \varepsilon_n \quad (6)$$

Table 4 reports the OLS estimation results. Several parameter estimates are statistically significant. Ethnic groups and location appear to be important determinants of expenditure shares. Husbands' age is negatively related to both *PC-Husband* and *PC-Wife* but positively related to *Other Foods*. Likewise, the number of children has a negative impact on *PC-Husband* and *PC-Wife* but a positive one on *PC-Children*, as expected. The log of expenditures and its square are not individually significant for any share, but are jointly significant for *PC-Children*, *Millet* and *Other Foods*, as shown at the bottom of the table.[24]

[23]The QUAIDS model has the advantage of being a flexible functional form that accommodates quadratic nonlinearities in the Engel curves. More specifically, it is a rank three demand system in the sense of Lewbel (1991). Also, it has been validated empirically on many occasions (*e.g.,* Banks et al., 1997; Blundell and Robin, 1999; Browning et al., 2013). Given the relatively small size of our samples, we thought this parametric model would be a good approximation to a full-fledged non-parametric approach.

[24]Total expenditures are usually instrumented with income in demand systems as they may be endogenous. Yet, households in rural Burkina Faso have next to no savings and spend very little on durable goods.

Interestingly, the wife's share of total income (*SIncomeW*) impacts negatively *PC-Husband* and positively *PC-Children*. These results are consistent with a larger share of income translating into a larger bargaining power. Notice also that *DMarriage* is usually deemed unfavorable to the wife presumably because the likelihood of the husband contracting a new marriage increases. According to the parameter estimates, more years of marriage do indeed translate into larger and smaller shares of *PC-Husband* and *PC-Wife*, respectively.

TABLE 4 ABOUT HERE

4.1.1. Rationality Tests Based on Proposition 1

Proposition 1 provides restrictions that are gradually more restrictive which allows us to adopt a sequential approach. The first step is to test unitary rationality (7a), which assumes away the existence of distribution factors. A simple way to test this restriction when the covariance between the distribution factors is negligible is to use simple t-tests.[25] According to Table 4, this restriction must be rejected since the two distribution factors we consider are statistically significant in various demand functions.

We thus move on to test collective rationality using Restriction (7b). This *all or nothing* restriction is a necessary condition requiring that each demand function either do not respond to any of the distribution factors or respond to all of the distribution factors. Again, when the covariance between the distribution factors can be neglected, simple t tests of significance can be used. As shown in Table 4, the shares of *PC-Wife* and *PC-Children* respond significantly only to one out of two distribution factors. Therefore, collective rationality must be rejected.

4.1.2. Rationality Tests Based on CE2006

As stressed earlier, the CE2006 rank test is asymptotically equivalent to the test of our *Proposition 1*. Their test is carried out sequentially, starting with the Restriction (9a). The null hypothesis $H_0 : rank[D_{\mathbf{z}}\mathbf{x}(\mathbf{z})] = 0$ is computed using an F statistic to test that both distribution factors are simultaneously statistically significant in all the demand functions. The $F(10, 1870)$ statistic is equal to 4.28 and has an associated P-value of 0.00001. Clearly, the unitary rationality is rejected by our data. We thus move on and test $H_0 : rank[D_{\mathbf{z}}\mathbf{x}(\mathbf{z})] = 1$. Since our sample is relatively small, we implement a recent *constrained bootstrap method* proposed by Portier and Delyon (2014) to insure both our proposition and that of CE2006 have similar statistical properties.[26] More precisely, we use the test statistic

[25] The covariance between our two distribution factors is 0.26. When the covariance between the distribution factors is not negligible, a more complex test must be implemented.

[26] Portier and Delyon (2014) developed a so-called *constrained bootstrap method* which allows to compute the bootstrap distribution of three distinct rank-test statistics proposed in the literature under the null hypothesis that the rank of the matrix is of a given size. Among the three available statistics, we have chosen that of Li (1991) since the consistency of its associated constrained bootstrap test relies on less stringent conditions than the other two. The Li (1991) statistics is the following:

$$\widehat{\Lambda} = n \sum_{p=m+1}^{P} \widehat{\lambda}_p,$$

proposed by Li (1991) but do not use its asymptotic distribution because it is not pivotal. Instead we estimate its bootstrap distribution under the null hypothesis that $rank[D_{\mathbf{z}}\mathbf{x}(\mathbf{z})] = 1$ as suggested by Portier and Delyon (2014). The sampling value of the statistic is equal to 0.00577 and has a P-value of 0.0001. The CE2006 test is thus consistent with our own test in rejecting collective rationality.

4.1.3. Rationality Tests Based on BBC2009

Recall that *Proposition 2* of BBC2009 requires that all the demands function respond to at least one common distribution factor. As stressed earlier, this condition is hardly ever satisfied. It is certainly not in our data. We nevertheless omit this condition and carry on testing collective rationality. Restriction *ii* of *Proposition 2* of BBC2009 stipulates that for collective rationality to hold the ratio of the marginal effects of the two distribution factors must be equal across the demand functions, *i.e.*, H_0: $\delta_1/\gamma_1=\delta_2/\gamma_2=\delta_3/\gamma_3=\delta_4/\gamma_4=\delta_5/\gamma_5$. Based on the parameter estimates of Table 4, we get a test statistic of $\chi^2(4) = 7.02$ with an associated P-value of 0.135. In other words, the null assumption can not be rejected and so neither is collective rationality.

Obviously, this test is fundamentally flawed because it is based on a false premise. Indeed, both distribution factors are not statistically different from zero in three demand functions. Hence, the ratios of the marginal effects are essentially zero in most cases. As a matter of fact, a joint test that all the ratios are equal to zero, *i.e.*, $H_0 : \delta_1/\gamma_1=\delta_2/\gamma_2=\delta_3/\gamma_3=\delta_4/\gamma_4=\delta_5/\gamma_5=0$, yields a test statistic of $\chi^2(5) = 9.06$ with an associated P-value of 0.107. In other words, we can not reject the null assumption that all the ratios are equal to zero. A naive application of the BBC2009 test would thus lead us to (correctly) reject unitary rationality and (wrongly) not to reject collective rationality.

The *Proposition 2* of BBC2009 provides another restriction stated in terms of z-demand functions which is equivalent to the Restriction *ii*. This restriction, like our Restriction (7d), requires that a given demand function be inverted relative to one of its distribution factors and that the latter be substituted into the remaining demand functions. It states that the resulting z-conditional demand functions must no longer respond to the distribution factors. One difficulty with this approach is that the conditioning demand function must be instrumented to obtain consistent estimators. The literature suggests the substituted distribution factor be used as an instrument. We thus considered the four possibilities provided by Table 4, that is inverting *PC-Husband* or *PC-Children* on *SIncomeW* and inverting *PC-Husband* or *PC-Wife* on *DMarriage*. For each of these possibilities, we tested whether the distribution factor constituted an adequate instrument. In each case, the Cragg-Donald Wald F statistic, which is equal to the effective F statistic in the just-identified case, was found to be well below the three critical values suggested in the literature. These are the rule of thumb of Staiger and Stock (1997), the Stock and Yogo (2005) critical value for 10% maximal IV size distortion and the critical value associated to a bias

where n is the sample size, $(\widehat{\lambda}_1, ..., \widehat{\lambda}_P)$ are the singular values of the matrix $D_{\mathbf{z}}\mathbf{x}(\mathbf{z})$ arranged in descending order and $m > 0$ is the assumed rank of $D_{\mathbf{z}}\mathbf{x}(\mathbf{z})$. The null assumption is $H_0 : rank[D_{\mathbf{z}}\mathbf{x}(\mathbf{z})] = m$ against $H_1 : rank[D_{\mathbf{z}}\mathbf{x}(\mathbf{z})] > m$. Since this procedure cannot test whether $H_0 : rank[D_{\mathbf{z}}\mathbf{x}(\mathbf{z})] = 0$, we begin with an F test that all the distribution factors are simultaneously statistically significant in all the demand functions. If rejected, the next step is to test $H_0 : rank[D_{\mathbf{z}}\mathbf{x}(\mathbf{z})] = 1$ with a constrained bootstrap test of $\widehat{\Lambda}$, and so on until the maximum rank of $D_{\mathbf{z}}\mathbf{x}(\mathbf{z})$ (*i.e.* I) is reached.

of Nagar of 10% as proposed by Montiel and Pflueger (2013). Furthermore, despite our best efforts, we could not come up with a single set of satisfactory instruments in an overidentified context. All proved to be weak according to the Stock-Yogo weak-ID critical value for 5% maximal IV relative bias. Consequently, we elected not to investigate Restriction *iii* of BBC2009 further.[27]

4.2. Bigamous Households

The demand system includes the same items as with monogamous households but in addition includes personal care of the second wife (*PC-Wife2*) and her children (*PC-Children2*). The system is composed of eight non-durable goods, five of which are assignable. The first seven demand functions are estimated using the QUAIDS system. The three distribution factors include the log of the share of income of each wife relative to total household income as well as the share of the duration of the first wife's marriage relative to the total duration of the marriage of the two wives, *i.e.*, *SDMarriageW1* = *Years of marriage wife1/(Years of marriage wife1+Years of marriage wife2)*. The other explanatory variables are the same as with monogamous households but also include the age of the second wife as well as her number of children aged under 16. The budget shares functions are written as follows:

$$\begin{aligned} w_n = \mathbf{X}_n'\alpha_n + \beta_n \ln TotExp + \theta_n(\ln TotExp)^2 + \delta_n SIncomeW1 + \\ \rho_n SIncomeW2 + \gamma_n SDMarriageW1 + \varepsilon_n \end{aligned} \tag{7}$$

Table 5 reports the OLS estimation results. Several parameter estimates are statistically significant. As with monogamous households, ethnic origin and region of residence are important determinants of spending patterns. The log of total expenditures and its square are not statistically significant.

TABLE 5 ABOUT HERE

The distribution factors *SIncomeW1* and *SIncomeW2* are statistically significant in two demand functions and have the expected signs. The former proxies the bargaining power of the first wife and interestingly is shown to have a negative impact on both *PC-Husband* and *PC-Wife2*. The latter proxies the second wife's bargaining power and positively affects *PC-Wife2* and *PC-Children2*, as expected. Finally, *SDMarriageW1* proxies the seniority of the first wife and is associated with her having greater bargaining power. Results show it has a negative impact on *PC-Children2* and a positive one on *Millet* and *O-Food*. In short, the marginal effects are intuitively appealing and are consistent across the demand system.

4.2.1. Rationality Tests Based on Proposition 2

Proposition 2 is a generalization of *Proposition 1* to multi-person households. Testing begins with Restriction (8a) which implies that the three variables *SIncomeW1*, *SIncomeW2* and *SDMarriageW1*

[27] Few, if any, papers ever report tests of weak instruments in the empirical literature.

must have no influence on the demand system. Because the distribution factors are statistically significant in various demand functions we reject that bigamous households behave in a unitary way.

TABLE 6 ABOUT HERE

We next investigate whether collective rationality holds using the necessary Restriction (8b). The first step is to test whether (8b) holds when $J = 0$, *i.e.* that each individual demand function either does not respond to any of the distribution factors or responds to all of them. This restriction is clearly rejected since none of the demand functions responds to all three distribution factors. We thus move on to test whether Restriction (8b) is satisfied when $J = 1$. According to Restriction (8b), any given demand function that responds to the distribution factors may be inverted relative to one of the latter. Upon substituting the distribution factor, the remaining conditioned demand functions must either all be insensitive to the remaining distribution factors, or only be sensitive to a common subset. From Table 5, there are six possible inversions: *PC-Husband* and *PC-Wife2* relative to *SIncomeW1*, *PC-Wife2* and *PC-Children2* relative to *SIncomeW2* and finally, *PC-Children2* and *O-Food* relative to *SDMarriageW1*. For each of these possibilities, we tested whether the distribution factor constituted a weak instrument. The Cragg-Donald Wald F statistic is reported in Table 6. Based on the rule of thumb of Staiger and Stock (1997), the variable *SIncomeW2* is not a weak instrument in *PC-Wife2* and *PC-Children2*. However, based on the Stock and Yogo (2005) critical value for the 10% maximal IV size (= 16.38), we do not reject that *SIncomeW2* is a weak instrument of *PC-Wife2*. This instrument is also weak accordingly to the critical value associated to a 10% bias of Nagar as suggested by Montiel and Pflueger (2013) (=23.1). Because of these mixed results, and in order to show how to implement Restriction (8b) with $J = 1$, we carry out the analysis and invert *SIncomeW2* and *PC-Wife2*. We next substitute out the latter in the unconditional demand functions that respond to the distribution factors, namely *PC-Husband*, *PC-Children2* and *O-Food*. The resulting z-conditional demand functions were estimated by 2SLS and the results are reported in Table 7.

TABLE 7 ABOUT HERE

The parameter estimates of the z_1-conditional demand functions are very similar to their unconditional counterpart. The distribution factor *SIncomeW1* remains statistically significant in *PC-Husband* while the same holds for *SDMarriageW1* in *PC-ChildrenW2* and *OFood*. This is at odds with Restriction (8b) when $J = 1$. Therefore, conditionally on the assumption that *SIncomeW2* is not a weak instrument of *PC-Wife2*, collective rationality is rejected.

4.2.2. Rationality Tests Based on CE2006

As with monogamous households, we begin by testing unitary rationality. Restriction (9a) posits that $rank[D_{\mathbf{z}}\mathbf{x}(\mathbf{z})]$ must be equal to zero for this to hold. The test yields $F(10, 1420) = 4.85$ with a very small P-value. Unitary rationality is thus strongly rejected. We next test $H_0 : rank[D_{\mathbf{z}}\mathbf{x}(\mathbf{z})] = 1$ using the bootstrapped Li (1991) statistic. The sample value of the statistic is 0.1018 with a P-value of about 0.000. Once again, the null assumption must be rejected. This is consistent with our previous result based on Restriction (8b) for $J = 0$. Finally, we move on to test $H_0 : rank[D_{\mathbf{z}}\mathbf{x}(\mathbf{z})] = 2$. This is equivalent to testing there are three decision-makers in the household. The sample statistic is equal to 0.017 and its P-value is about 0.000. Rejection of the hypothesis thus implies rejection of collective rationality, which is consistent with the result from our Proposition 2.

5. Conclusion

The collective household model has become the main paradigm to conduct empirical research for good reasons: It is based on relatively innocuous assumptions, and assuming these hold, allows investigating intrahousehold distributional impacts of numerous policies. Yet, one can not help but be concerned about the falsifiability of the model as it is seldom if ever rejected in the empirical literature. Such overwhelming evidence in favor of the collective model may eventually stray many from investigating the foundations of the model toward assuming it holds, irrespective of the environment under investigation.

We suspect the under-rejection of the collective model is primarily due to the manner in which its underlying *theoretical restrictions* are translated into *statistical restrictions*. Indeed, most papers investigate the collective model using a test procedure that was proposed by Bourguignon et al. (2009). Yet the procedure requires that in any given demand system all functions respond to at least one common distribution factor. This assumption is scarcely met in the empirical literature. Most papers simply ignore this and thus conduct tests based on a false premise.

In this paper we provide a new falsifiable restriction which extends Bourguignon et al. (2009)'s approach insofar as it does not require a distribution factor to affect each equation of a demand system. When there are potentially two decision-makers, our test procedure to assess the validity of the latter restriction is appropriate even in small samples. We derive a set of testable conditions that take this restriction into account and fully characterize collective rationality, assuming no variations in prices. Moreover, our approach is generalized to households comprising potentially more than two members.

We illustrate the usefulness of our approach by investigating efficiency in allocation of consumption within monogamous and bigamous households in rural Burkina Faso. Social and cultural environments as well as institutional arrangements are likely to impede the enforcement of efficient marriages. We thus do not expect, *a priori*, outcomes to be efficient. Based on our proposed test procedure and on Chiappori and Ekeland (2006)'s rank test (which is asymptotically equivalent to ours but more complicated to implement), rationality is found not to hold for monogamous and polygamous households alike. We next proceed to test rationality for monogamous households using the test procedure of Bourguignon et al. (2009) while neglecting the fact that no one distribution factor is statistically significant in every demand functions. Collective rationality is then (wrongly) found to hold for monogamous households.

In a Popperian sense, an important conclusion that can be drawn from this paper is that the collective model is clearly *falsifiable*: its underlying assumptions translate into non-trivial constraints.

Appropriately accounting for the latter may reveal that far fewer households behave efficiently than what the current literature suggests.

Recent work (*e.g.,* Lechene and Preston, 2011) has shown that non-cooperative models with public goods or externalities may impose restrictions on household behavior. A natural extension to our paper would be to develop and take to data a general model that has the collective and the non-cooperative (Nash) models as particular cases. Rigorous testing of competing sets of constraints would enhance our understanding of household behavior.[28]

Finally, our analysis is conditional on the marital status of the household: either monogamous or bigamous. Yet, since the seminal work of Becker (1993) on marriage markets, it is generally acknowledged that formation, dissolution and even the marital status of households are endogenous (see in particular his Ch. 3 on polygamy and monogamy). Another extension to our work would consist in introducing the marriage market into household behavior.[29]

[28] See Naidoo (2015) for an analysis of such possible extensions.

[29] There exists a recent (still unpublished) literature that attempts to jointly analyze household behavior as well as household formation and dissolution within a collective setting (Jacquemet and Robin, 2012; Mazzocco et al., 2013; Goussé, 2014). However, these studies, which all apply to developed countries, focus on savings, labor supply or household production, and not on consumption goods.

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Appendix A. Proofs of Propositions 1–2

Proof of Proposition 1

Necessity: In rational households composed of two members, $\mathbf{x}(\mathbf{z}) \equiv \widehat{\mathbf{x}}(\mu(\mathbf{z}))$ and there is a single Pareto weight. If the Pareto weight is constant, then $D_{\mathbf{z}}\mu(\mathbf{z}) = \mathbf{0}$ or, put differently, $K = 0$. Therefore, $D_{\mathbf{z}}\mathbf{x}(\mathbf{z}) \equiv D_{\mu}\widehat{\mathbf{x}}(\mu(\mathbf{z}))D_{\mathbf{z}}\mu(\mathbf{z}) = \mathbf{0}$ as stated by result (7a). Now, if there are $K \geq 2$ distribution factors, then $D_{\mathbf{z}}\mu(\mathbf{z}) \neq\neq \mathbf{0}$ since distribution factors can only impact the demand system through their effect on the one Pareto weight. Since $D_{\mathbf{z}}x_n(\mathbf{z}) \equiv \partial\widehat{x}_n(\mu(\mathbf{z}))/\partial\mu \cdot D_{\mathbf{z}}\mu(\mathbf{z})$ with $n = 1, ..., N$, if $\partial\widehat{x}_n(\mu(\mathbf{z}))/\partial\mu = 0$, then $D_{\mathbf{z}}x_n(\mathbf{z}) = \mathbf{0}$. Otherwise $D_{\mathbf{z}}x_n(\mathbf{z}) \neq\neq \mathbf{0}$. This shows that each demand stemming from collectively rational households composed of two members must satisfy (7b). Furthermore, since a distribution factor must not change the budget constraint, there must be at least two demands such that $D_{\mathbf{z}}x_n(\mathbf{z}) \neq\neq \mathbf{0}$ when (7b) holds. Denoting the number of demands such that $D_{\mathbf{z}}x_n(\mathbf{z}) \neq\neq \mathbf{0}$ when (7b) holds as M, we have $2 \leq M \leq N$. For these demands, denoted as $x^*_m(\mathbf{z})$, the ratio of the marginal effect of 1^{st} and k^{th} distributions factor is equal to:

$$\frac{\partial x^*_m(\mathbf{z})/\partial z_1}{\partial x^*_m(\mathbf{z})/\partial z_k} = \frac{\partial\widehat{x}^*_m(\mu(\mathbf{z}))/\partial\mu \cdot \partial\mu(\mathbf{z})/\partial z_1}{\partial\widehat{x}^*_m(\mu(\mathbf{z}))/\partial\mu \cdot \partial\mu(\mathbf{z})/\partial z_k} = \frac{\partial\mu(\mathbf{z})/\partial z_1}{\partial\mu(\mathbf{z})/\partial z_k} \neq 0 \quad \forall\, k = 2, ..., K,\ m = 1, ..., M,$$

which proves (7c). Finally, based on the *Implicit Function Theorem*, it is possible to invert any of these M demands with any of the K distribution factors at $P = (\mathbf{z})$. Arbitrarily selecting $x^*_1(\mathbf{z})$ and distribution factor z_1, one gets $z_1 = z_1(x^*_1, \mathbf{z}_{\text{-}1})$ which is substituted into the M demands:

$$\begin{aligned} \overline{x}^*_1(x^*_1, \mathbf{z}_{\text{-}1}) &\equiv x^*_1(z_1(x^*_1, \mathbf{z}_{\text{-}1}), \mathbf{z}_{\text{-}1}) \equiv \widehat{x}^*_1(\mu(z_1(x^*_1, \mathbf{z}_{\text{-}1}), \mathbf{z}_{\text{-}1})) \\ \overline{x}^*_m(x^*_1, \mathbf{z}_{\text{-}1}) &\equiv x^*_m(z_1(x^*_1, \mathbf{z}_{\text{-}1}), \mathbf{z}_{\text{-}1}) \equiv \widehat{x}^*_m(\mu(z_1(x^*_1, \mathbf{z}_{\text{-}1}), \mathbf{z}_{\text{-}1})) \quad \forall\, m = 2, ..., M. \end{aligned}$$

Deriving these demands with respect to $\mathbf{z}_{\text{-}1}$ at $P = (\mathbf{z})$ yields :

$$\mathbf{0} = \partial\widehat{x}^*_1(\mu(\mathbf{z}))/\partial\mu \cdot D_{\mathbf{z}_{\text{-}1}}\mu(z_1(x^*_1, \mathbf{z}_{\text{-}1}), \mathbf{z}_{\text{-}1}) \tag{A1}$$

$$D_{\mathbf{z}_{\text{-}1}}\overline{x}^*_m(x^*_1, \mathbf{z}_{\text{-}1}) \equiv \partial\widehat{x}^*_m(\mu(\mathbf{z}))/\partial\mu \cdot D_{\mathbf{z}_{-1}}\mu(z_1(x^*_1, \mathbf{z}_{\text{-}1}), \mathbf{z}_{\text{-}1}) \quad \forall\, m = 2, ..., M. \tag{A2}$$

Since $D_{\mathbf{z}}x^*_1(\mathbf{z}) \neq \mathbf{0}$, it follows that $\partial\widehat{x}^*_1(\mu(\mathbf{z}))/\partial\mu \neq 0$ in (A1). Therefore, $D_{\mathbf{z}_{\text{-}1}}\mu(z_1(x^*_1, \mathbf{z}_{\text{-}1}), \mathbf{z}_{\text{-}1}) = \mathbf{0}$ is the only possible solution to (A1), which implies that $D_{\mathbf{z}_{\text{-}1}}\overline{x}^*_m(x^*_1, \mathbf{z}_{\text{-}1}) = \mathbf{0}$ in (A2) and proves (7d). Accordingly, (7b), (7c) and (7d) are necessary conditions for collective rationality when $K \geq 2$ and $N \geq 2$.

Sufficiency: A proof of the sufficiency of the conditions (7c) and (7d) for collective rationality can be found in BBC2009 (see their Appendix A, Proof of proposition 2).■

Proof of Proposition 2

Necessity: In rational households composed of I+1 members, $\mathbf{x}(\mathbf{z}) \equiv \widehat{\mathbf{x}}(\boldsymbol{\mu}(\mathbf{z}))$ and there are I Pareto weights. If they are all constant, then $D_{\mathbf{z}}\boldsymbol{\mu}(\mathbf{z}) = \mathbf{0}$ or, put differently, $K = 0$. Therefore, $D_{\mathbf{z}}\mathbf{x}(\mathbf{z}) \equiv D_{\boldsymbol{\mu}}\widehat{\mathbf{x}}(\boldsymbol{\mu})D_{\mathbf{z}}\boldsymbol{\mu}(\mathbf{z}) = \mathbf{0}$ as stated by result (8a). Now, if there are $K \geq I$+1 distribution factors, then some Pareto weights are responding to some distribution factors, that is $D_{\mathbf{z}}\boldsymbol{\mu}(\mathbf{z}) \neq \mathbf{0}$. Let us denote the rank of $D_{\mathbf{z}}\boldsymbol{\mu}(\mathbf{z})$, a matrix of dimension $I \times K$, as I^*. Further denote the rank of $D_{\mathbf{z}}\mathbf{x}(\mathbf{z})$ as J+1, a $N \times K$ matrix with $N \geq I$+1 with $1 \leq J\text{+}1 \leq I^*$. Since $rank[D_{\mathbf{z}}\mathbf{x}(\mathbf{z})] = J\text{+}1$, we can arrange the N demands so that $D_{\mathbf{z}}\mathbf{x}_{\text{-(J+1)}}(\mathbf{z}) = \mathbf{C}D_{\mathbf{z}}\mathbf{x}_{J+1}(\mathbf{z})$ where $D_{\mathbf{z}}\mathbf{x}_{J+1}(\mathbf{z})$ is a matrix of dimension $(J\text{+}1) \times K$ which has a rank equal to J+1, and $\mathbf{C}$ is a matrix of coefficients of dimension $(N\text{-}J\text{-}1) \times (J\text{+}1)$. Without loss of generality, we can arrange the distribution factors so that $rank[D_{\mathbf{z}_J}\mathbf{x}_J(\mathbf{z})] = J$, which allows us to use the *Implicit Function Theorem* to invert $\mathbf{x}_J$ and $\mathbf{z}_J$ in order to get $\mathbf{z}_J = \mathbf{z}_J(\mathbf{x}_J, \mathbf{z}_{\text{-}J})$. Upon substituting in the demand system we get:

$$\begin{aligned}\overline{\mathbf{x}}_J(\mathbf{x}_J, \mathbf{z}_{\text{-}J}) &\equiv \mathbf{x}_J(\mathbf{z}_J(\mathbf{x}_J, \mathbf{z}_{\text{-}J}), \mathbf{z}_{\text{-}J}) \\ \overline{\mathbf{x}}_{\text{-}J}(\mathbf{x}_J, \mathbf{z}_{\text{-}J}) &\equiv \mathbf{x}_{\text{-}J}(\mathbf{z}_J(\mathbf{x}_J, \mathbf{z}_{\text{-}J}), \mathbf{z}_{\text{-}J}).\end{aligned}$$

Deriving these demands with respect to $\mathbf{z}_{\text{-}J}$ at the point $(\mathbf{z})$ gives:

$$\mathbf{0} = D_{\mathbf{z}_{\text{-}J}}\overline{\mathbf{x}}_J(\mathbf{x}_J, \mathbf{z}_{\text{-}J}) \tag{A3}$$

$$D_{\mathbf{z}_{\text{-}J}}\overline{\mathbf{x}}_{\text{-}J}(\mathbf{x}_J, \mathbf{z}_{\text{-}J}) \equiv \begin{bmatrix} D_{\mathbf{z}_{\text{-}J}}\overline{x}_{J+1}(\mathbf{x}_J, \mathbf{z}_{\text{-}J}) \\ D_{\mathbf{z}_{\text{-}J}}\overline{\mathbf{x}}_{\text{-}J\text{-}1}(\mathbf{x}_J, \mathbf{z}_{\text{-}J}) \end{bmatrix} = \begin{bmatrix} D_{\mathbf{z}_{\text{-}J}}\overline{x}_{J+1}(\mathbf{x}_J, \mathbf{z}_{\text{-}J}) \\ \mathbf{C}D_{\mathbf{z}_{\text{-}J}}\overline{\mathbf{x}}_{J+1}(\mathbf{x}_J, \mathbf{z}_{\text{-}J}) \end{bmatrix} \tag{A4}$$

Using (A3) and the fact that $D_{\mathbf{z}_{\text{-}J}}\overline{\mathbf{x}}_{J+1} \equiv [D_{\mathbf{z}_{\text{-}J}}\overline{\mathbf{x}}'_J, D_{\mathbf{z}_{\text{-}J}}\overline{x}'_{J+1}]'$, we can rewrite (A4) as:

$$\begin{aligned} D_{\mathbf{z}_{\text{-}J}}\overline{\mathbf{x}}_{\text{-}J}(\mathbf{x}_J, \mathbf{z}_{\text{-}J}) &= \begin{bmatrix} D_{\mathbf{z}_{\text{-}J}}\overline{x}_{J+1}(\mathbf{x}_J, \mathbf{z}_{\text{-}J}) \\ \mathbf{C}[D_{\mathbf{z}_{\text{-}J}}\overline{\mathbf{x}}'_J(\mathbf{x}_J, \mathbf{z}_{\text{-}J}), D_{\mathbf{z}_{\text{-}J}}\overline{x}'_{J+1}(\mathbf{x}_J, \mathbf{z}_{\text{-}J})]' \end{bmatrix} \\ &= \begin{bmatrix} D_{\mathbf{z}_{\text{-}J}}\overline{x}_{J+1}(\mathbf{x}_J, \mathbf{z}_{\text{-}J}) \\ \mathbf{C}[\mathbf{0}', D_{\mathbf{z}_{\text{-}J}}\overline{x}_{J+1}(\mathbf{x}_J, \mathbf{z}_{\text{-}J})']' \end{bmatrix} = \begin{bmatrix} 1 \\ C_{J+1} \end{bmatrix} D_{\mathbf{z}_{\text{-}J}}\overline{x}_{J+1}(\mathbf{x}_J, \mathbf{z}_{\text{-}J}) \end{aligned} \tag{A5}$$

where $C_{J+1} \equiv [c_{1J+1}, ..., c_{N-J-1J+1}]'$ is a vector of dimension N-J-1 corresponding to the last column of the matrix $\mathbf{C}$. Clearly, $D_{\mathbf{z}_{\text{-}J}}\overline{x}_{J+1} \neq \mathbf{0}$ since $rank[D_{\mathbf{z}_J}\mathbf{x}_J(\mathbf{z})] = J$ and $rank[D_{\mathbf{z}}\mathbf{x}_{J+1}(\mathbf{z})] = J\text{+}1$. Let us thus partition $\mathbf{z}_{\text{-}J}$ as $\mathbf{z}_{\text{-}J}{\equiv}[\mathbf{z}^{*\prime}_L, \mathbf{z}^{*\prime}_{\text{-}L}]'$ with $1 \leq L \leq K\text{-}J$ so that $D_{\mathbf{z}^*_L}\overline{x}_{J+1} \neq\neq \mathbf{0}$ and $D_{\mathbf{z}^*_{\text{-}L}}\overline{x}_{J+1} = \mathbf{0}$. Since (A5) implies that $D_{\mathbf{z}_{\text{-}J}}\overline{x}_n = c_{nJ+1}D_{\mathbf{z}_{\text{-}J}}\overline{x}_{J+1}$ for $n = J\text{+}2, ..., N$, it follows that $D_{\mathbf{z}_{\text{-}J}}\overline{x}_n = \mathbf{0}$ if $c_{nJ+1} = 0$. Otherwise $D_{\mathbf{z}^*_L}\overline{x}_n \neq\neq \mathbf{0}$ and $D_{\mathbf{z}^*_{\text{-}L}}\overline{x}_n = \mathbf{0}$. This establishes (8b). Hence, each demand stemming from rational households composed of two members or more must satisfy either (8a) or (8b).

Denote the demand x_{J+1} as x^*_1 and any other demand such that $D_{\mathbf{z}^*_L}\overline{x}_n \neq\neq \mathbf{0}$ as x^*_m, where $m = 2, ..., M$ with $2 \leq M \leq N\text{-}J$. Assuming (8b) holds and that $L \geq 2$, equation (A5) implies that the ratio of the marginal effect of the 1^{st} and l^{th} distribution factors is equal across these conditional demands:

$$\frac{\partial\overline{x}^*_m(\mathbf{x}_J, \mathbf{z}_{\text{-}J})/\partial z^*_1}{\partial\overline{x}^*_m(\mathbf{x}_J, \mathbf{z}_{\text{-}J})/\partial z^*_l} = \frac{\partial\overline{x}^*_1(\mathbf{x}_J, \mathbf{z}_{\text{-}J})/\partial z^*_1}{\partial\overline{x}^*_1(\mathbf{x}_J, \mathbf{z}_{\text{-}J})/\partial z^*_l} \neq 0 \quad \forall\, l = 2, ..., L \quad m = 2, ..., M,$$

which establishes (8c). Finally, note that because $rank[D_{\mathbf{z}}\mathbf{x}_J(\mathbf{z})] = J\text{+}1$ there exists a z_{J+1} such that $rank[D_{\mathbf{z}_{J+1}}\mathbf{x}_{J+1}(\mathbf{z})] = J\text{+}1$. We can thus invert $\overline{x}_{J+1}(\mathbf{x}_J, \mathbf{z}_{\text{-}J})$, denoted as $\overline{x}^*_1(\mathbf{x}_J, \mathbf{z}_{\text{-}J})$, and z_{J+1} to get $z_{J+1} = z_{J+1}(\mathbf{x}_J, x^*_1, \mathbf{z}_{\text{-}(J+1)})$ which we substitute in $\mathbf{z}_J = \mathbf{z}_J(\mathbf{x}_J, z_{J+1}(\mathbf{x}_J, x^*_1, \mathbf{z}_{\text{-}(J+1)}), \mathbf{z}_{\text{-}(J+1)}) = \mathbf{z}_J(\mathbf{x}_J, x^*_1, \mathbf{z}_{\text{-}(J+1)})$. Upon substitution we get:

$$\overline{\mathbf{x}}_{J+1}(\mathbf{x}_J, x_1^*, \mathbf{z}_{\text{-}(J+1)}) \equiv \mathbf{x}_{J+1}(\mathbf{z}_{J+1}(\mathbf{x}_J, x_1^*, \mathbf{z}_{\text{-}(J+1)}), \mathbf{z}_{\text{-}(J+1)})$$
$$\overline{x}_m^*(\mathbf{x}_J, x_1^*, \mathbf{z}_{\text{-}(J+1)}) \equiv x_m^*(\mathbf{z}_{J+1}(\mathbf{x}_J, x_1^*, \mathbf{z}_{\text{-}(J+1)}), \mathbf{z}_{\text{-}(J+1)}) \quad \forall m = 2, ..., M.$$

Deriving the equations with respect to $\mathbf{z}_{\text{-}(J+1)}$ at $(\mathbf{z})$ gives:

$$\mathbf{0} = D_{\mathbf{z}_{\text{-}(J+1)}} \overline{\mathbf{x}}_{J+1}(\mathbf{x}_J, x_1^*, \mathbf{z}_{\text{-}(J+1)}) \tag{A6}$$
$$D_{\mathbf{z}_{\text{-}(J+1)}} \overline{x}_m^*(\mathbf{x}_J, x_1^*, \mathbf{z}_{\text{-}(J+1)}) = C_m' D_{\mathbf{z}_{\text{-}(J+1)}} \overline{\mathbf{x}}_{J+1}(\mathbf{x}_J, x_1^*, \mathbf{z}_{\text{-}(J+1)}) \quad \forall m = 2, ..., M, \tag{A7}$$

where C_m is a vector of dimension $(J+1)$ corresponding to the row associated with the demand x_m^* in the matrix $\mathbf{C}$. Substituting (A6) in (A7) gives $D_{\mathbf{z}_{\text{-}(J+1)}} \overline{x}_m^*(\mathbf{x}_J, x_1^*, \mathbf{z}_{\text{-}(J+1)}) = \mathbf{0}$ as stated in result (8d). This implies that (8b), (8c) and (8d) are necessary conditions for collective rationality when $K \geq I + 1$, $N \geq I + 1$ and $L \geq 2$. Note finally that it is always possible to partition $\mathbf{x} \equiv [\mathbf{x}_J', \mathbf{x}_{\text{-}J}']'$ and $\mathbf{z} \equiv [\mathbf{z}_J', \mathbf{z}_{\text{-}J}']'$ in such a way that $L \geq 2$. To see this, note that since $K \geq I + 1 > I^*$, where I^* represents the number of linearly independent weights, some of them must depend on more than one distribution factor. Therefore, it is possible to find partitions of $\mathbf{x}$ and $\mathbf{z}$ where the last remaining linearly independent weight in (A5) depends on more than one distribution factor.

Sufficiency: To prove sufficiency, we need to show that whenever $\mathbf{x}(m, \mathbf{z}) = \widehat{\mathbf{x}}(m, \boldsymbol{\nu}(m, \mathbf{z}))$ where $\boldsymbol{\nu}(m, \mathbf{z})$ is a vector of dimension $I^* \leq I + 1$, there exist $I^* + 1$ quasi-concave utility functions and I^* Pareto-weights such that the observed demands $\mathbf{x}(m, \mathbf{z})$ are solutions to the maximization program associated to collective rationality. Take I^* different arbitrary functions that are positive, increasing and quasi-concave in $\mathbf{x}$ and denote these by $G^i(\mathbf{x})$ with $i = 1, ..., I^*$ and define:

$$M^i(m, \boldsymbol{\nu}) \equiv G^i(\widehat{\mathbf{x}}(m, \boldsymbol{\nu})) \quad \forall \ i\text{=}1,\ldots,I^*. \tag{A8}$$

Let $V^i(\mathbf{X})$ with $i = 1, ..., I^* + 1$ be increasing and quasi-concave utility functions, where the vector $\mathbf{X}$ represents the public consumption of each commodity. Each utility function is obviously a particular case of the general utility function $U^i(\mathbf{x})$. The necessary and sufficient first-order conditions for collective rationality implied by these utility functions are:

$$\sum_{i=1}^{I^*} \mu^i D_{\mathbf{x}} V^i(\mathbf{x}) + D_{\mathbf{x}} V^{I^*+1}(\mathbf{x}) = \lambda \iota, \tag{A9}$$

where λ is the Lagrange multiplier associated with the budget constraint. Define:

$$V^i(\mathbf{x}) \equiv C^i(G^i(\mathbf{x})) \quad \forall \ i\text{=}1,\ldots,I^*, \tag{A10}$$
$$V^{I+1}(\mathbf{x}) \equiv A(\boldsymbol{\iota}'\mathbf{x}) + \sum_{i=1}^{I^*} B^i(G^i(\mathbf{x})), \tag{A11}$$

where A and the C^i are arbitrary increasing scalar functions, with B^i given by:

$$B^{i\prime} \equiv -G^i C^{i\prime} \quad \forall \ i\text{=}1,\ldots,I^*, \tag{A12}$$

and the function A is taken to be large enough with relative to B^i to ensure that V^{I+1} is increasing. Deriving (A10) and (A11) with respect to $\mathbf{x}$ yields:

$$D_{\mathbf{x}}V^i(\mathbf{x}) = C^{i\prime}D_{\mathbf{x}}G^i(\mathbf{x}) \quad \forall\ i\text{=}1,\ldots,I^*, \tag{A13}$$

$$D_{\mathbf{x}}V^{I+1}(\mathbf{x}) = A' + \sum_{i=1}^{I^*} B^{i\prime}D_{\mathbf{x}}G^i(\mathbf{x}). \tag{A14}$$

Substituting A(12) and (A13) into A(14) gives:

$$D_{\mathbf{x}}V^{I+1}(\mathbf{x}) = A' - \sum_{i=1}^{I^*} G^i(\mathbf{x})D_{\mathbf{x}}V^i(\mathbf{x}),$$

which shows that the observed demand functions $\mathbf{x}(m, \mathbf{z})$ are thus satisfy the first-order condition (A9) for the utility functions (A10) and (A11) when $\mu^i(m, \mathbf{z}) = M^i(m, \boldsymbol{\nu}(m, \mathbf{z}))$ with $i = 1, ..., I$. ■

TABLE 1: Sample Selection

Village	Population*	Number of married households	Number of eligible households	Number of households selected	Number of households who accepted
Dakiégré	1141	133	111	111	103
Pelegtanga	551	201	170	125	121
Rallo	1 053	207	162	125	108
Yako-1	856	221	128	125	117
Yako-5	1 311	246	236	125	102
Total	4 912	1 008	807	611	551

*According to the 1991 Demographic Census.

TABLE 2: Prevalence of Polygamy

	Average number of wife per household	Percentage of households with one wife	Percentage of households with two wives	Percentage of households with more than two wives
Dakiégré	1.7	56	26	18
Pelegtanga	1.4	69	26	5
Rallo	1.4	68	26	6
Yako-1	1.2	80	9	11
Yako-5	1.3	78	19	3
Total	1.4	71.1	21.2	7.7

TABLE 3: Sample Characteristics

	Monogamous Households		Bigamous Households	
	Mean	(Std error)	Mean	(Std error)
Muslim	32	-	44	-
Age Husband	42	(12.49)	49	(11.27)
Age Wife1	33	(11.84)	42	(11.07)
Age Wife2			31	(8.66)
Children Wife1	3.47	(2.33)	4.75	(2.36)
Children Wife2			3.16	(2.17)
Total Expenditures (CFA francs)	177 620	(111 803)	216 743	(100 045)
Budget Shares				
PC-Husband	0.0360	(0.0419)	0.0225	(0.0288)
PC-Wife1	0.0995	(0.0763)	0.0397	(0.0383)
PC-Wife2			0.0468	(0.0419)
PC-Child1	0.0534	(0.0462)	0.0313	(0.0285)
PC-Child2			0.0241	(0.0238)
Millet	0.0730	(0.1131)	0.1056	(0.1340)
Other Foods	0.4222	(0.1515)	0.4343	(0.1389)
Distribution factors				
Share Income Wife1	0.232	(0.157)	0.1680	(0.0895)
Share Income Wife2			0.1426	(0.0927)
Duration Marriage Wife1	14	(15.75)	22	(10.09)
Duration Marriage Wife2			11	(8.56)

Note: PC stands for Personal Care.

TABLE 4: OLS Estimation of the QUAIDS Demand System - Monogamous Households

Variables	PC-Husband	PC-Wife	PC-Children	Millet	Other Foods
Intercep	−0.133	−0.841	−0.421	−1.022**	−3.024
	(−0.188)	(−0.622)	(−0.598)	(−2.326)	(−0.392)
Dakiégré	0.178**	0.128	−0.148**	1.275***	−0.026
	(2.281)	(1.061)	(−2.422)	(6.901)	(−0.097)
Pelegtanga	0.173**	0.060	−0.032	−0.124	−1.069***
	(2.149)	(0.445)	(−0.434)	(−1.174)	(−4.205)
Rallo	0.025	−0.178*	−0.009	−0.094	−0.327
	(0.406)	(−1.712)	(−0.142)	(−0.986)	(−1.410)
Yako-1	−0.140**	−0.452***	−0.053	1.518***	−0.549*
	(−1.987)	(−3.484)	(−0.568)	(7.588)	(−1.826)
Muslim	0.056	0.056	−0.106**	0.149	0.384**
	(1.127)	(0.669)	(−2.324)	(1.540)	(2.265)
Age Husband	−0.126***	−0.119**	0.001	−0.036	0.270**
	(−4.327)	(−2.112)	(0.020)	(−0.454)	(2.142)
Age Wife1	−0.065	0.104	−0.127	0.091	−0.388
	(−1.329)	(0.935)	(−1.501)	(0.497)	(−1.642)
Children	−0.263***	−0.415**	0.542***	−0.230	0.359
	(−3.136)	(−2.264)	(4.181)	(−0.860)	(0.818)
ln(Total Expenditures)	0.338	1.712	0.668	1.897***	6.144
	(0.286)	(0.761)	(0.569)	(2.595)	(0.480)
ln(Total Expenditures)^2	−0.013	−0.071	−0.021	−0.086***	−0.273
	(−0.261)	(−0.759)	(−0.424)	(−2.821)	(−0.515)
Distribution Factors					
Share Income Wife1	−0.634**	0.832	0.731***	−0.340	−0.226
	(−2.308)	(1.578)	(2.882)	(−0.513)	(−0.207)
Duration Marriage Wife1	0.105**	−0.241**	0.137	0.015	0.307
	(2.366)	(−2.260)	(1.582)	(0.077)	(1.327)
Observations	392	392	392	392	392
R-squared	0.287	0.295	0.158	0.425	0.158
H_0: β_n=θ_n=0 (Eq. (11))					
F(2, 369)	0.240	0.290	6.340***	5.820***	3.650***
(P-value)	(0.787)	(0.749)	(0.002)	(0.003)	(0.027)

*Note: PC stands for Personal Care. t-statistics in parentheses *** p<.01, ** p<.05, * p<.1*

TABLE 5: OLS Estimation of the Demand System - Bigamous Households

Variables	PC-Husband	PC-Wife1	PC-Wife2	PC-Children1	PC-Children2	Millet	O-Foods
Intercep	−0.925	−1.779	−1.252	−0.991	−0.187	12.300	−3.001
	(−1.100)	(−1.205)	(−1.116)	(−0.826)	(−0.145)	(1.185)	(−0.332)
Diakégré	0.265***	0.123	0.203**	0.012	−0.001	−0.009	0.977*
	(3.261)	(1.032)	(2.163)	(0.155)	(−0.012)	(−0.019)	(1.912)
Pelegtanga	0.193**	0.026	0.338***	0.251**	0.127	−1.749***	0.452
	(2.134)	(0.204)	(2.846)	(2.439)	(1.381)	(−3.962)	(0.883)
Rallo	0.005	−0.057	0.080	0.114	0.062	−0.012	0.293
	(0.066)	(−0.453)	(0.764)	(1.259)	(0.786)	(−0.027)	(0.529)
Yako-1	0.118	−0.145	−0.042	0.125	0.033	−0.972**	0.461
	(1.325)	(−0.982)	(−0.322)	(1.232)	(0.401)	(−2.104)	(0.634)
Muslim	0.037	0.119	0.200**	0.187**	0.153**	−0.746***	0.741**
	(0.466)	(1.319)	(2.060)	(2.383)	(2.143)	(−3.244)	(2.454)
Age Husband	−0.010	−0.019	−0.047	−0.064	−0.076**	−0.193	0.127
	(−0.174)	(−0.313)	(−0.740)	(−1.559)	(−2.026)	(−1.179)	(0.609)
Age Wife1	−0.046	−0.039	0.007	0.050	0.103**	0.077	−0.149
	(−.835)	(−0.594)	(0.103)	(0.982)	(2.306)	(0.421)	(−0.557)
Age Wife2	−0.081	−0.071	−0.144*	−0.070	−0.044	0.206	0.246
	(−1.629)	(−1.006)	(−1.814)	(−0.140)	(−1.138)	(1.055)	(0.679)
Children Wife1	0.047	0.132	0.203	0.264*	0.079	0.808	−0.716
	(0.390)	(0.719)	(1.113)	(1.779)	(0.700)	(1.357)	(−1.004)
Children Wife2	−0.060	−0.169	−0.126	0.040	0.167	−0.605	0.835
	(−0.481)	(−0.700)	(−0.563)	(0.253)	(1.037)	(−0.880)	(0.668)
ln(Total Expenditures)	1.627	3.049	2.234	1.610	0.418	−19.190	4.729
	(1.157)	(1.253)	(1.188)	(0.808)	(0.200)	(−1.140)	(0.320)
ln(Total Expenditures)^2	−0.067	−0.123	−0.092	−0.064	−0.017	0.766	−0.182
	(−1.147)	(−1.225)	(−1.170)	(−0.775)	(−0.194)	(1.123)	(−0.302)
Distribution Factors							
Share Income Wife1	−1.034***	0.197	−1.158**	0.729*	−0.328	0.895	−0.702
	(−2.975)	(0.363)	(−2.200)	(1.740)	(−0.991)	(0.510)	(−0.293)
Share Income Wife2	−0.092	−0.215	1.867***	0.027	1.136***	−0.344	0.516
	(−0.242)	(−0.446)	(3.916)	(0.068)	(3.658)	(−0.228)	(0.247)
Share Duration Marriage Wife1	−0.316	−0.378	−0.019	0.140	−0.675***	−1.664*	3.253**
	(−1.175)	(−0.970)	(−0.049)	(0.618)	(−3.085)	(−1.839)	(2.061)
Observations	117	117	117	117	117	117	117
R-squared	0.432	0.257	0.387	0.223	0.367	0.494	0.175

*Note: PC stands for Personal Care. t-statistics in parentheses *** $p<.01$, ** $p<.05$, * $p<.1$*

TABLE 6: Tests Based on Proposition 2 – Bigamous Households

Demand	PC-Husband	PC-Wife1	PC-Wife2	PC-Children1	PC-Children2	Millet	O-Food
H_0: SIncomeW1 is a weak instrument							
Cragg-Donald Wald F statistic	8.390		4.605				
(10 % maximal IV size)	(16.38)		(16.38)				
(tau=10% with confidence level of 5%)	(23.11)		(23.11)				
H_0: SIncomeW2 is a weak instrument							
Cragg-Donald Wald F statistic			15.229		16.940		
(10 % maximal IV size)			(16.38)		(16.38)		
(tau=10% with confidence level of 5%)			(23.11)		(23.11)		
H_0: SDMarriageW1 is a weak instrument							
Cragg-Donald Wald F statistic					9.157		4.784
(10 % maximal IV size)					(16.38)		(16.38)
(tau=10% with confidence level of 5%)					(23.11)		(23.11)

TABLE 7: 2SLS estimation of z_1-Conditional Demands
Polygamous Households

Variables	PC-Husband	PC-Children2	O-Food
PC-Wife2	-0.049	0.609***	0.277
	(-0.256)	(3.103)	(0.274)
Intercep	-0.987	0.575	-2.655
	(-1.320)	(0.740)	(-0.579)
Dakiégré	0.275***	-0.124	0.921**
	(3.479)	(-1.422)	(2.008)
Pelegtanga	0.210*	-0.078	0.359
	(1.976)	(-0.783)	(0.707)
Rallo	0.009	0.014	0.271
	(0.131)	(0.168)	(0.542)
Yako-1	0.116	0.059	0.473
	(1.440)	(0.551)	(0.717)
Muslim	0.047	0.031	0.069**
	(0.719)	(0.400)	(2.224)
Age Husband	-0.012	-0.048	0.140
	(-0.240)	(-0.999)	(0.713)
Age Wife1	-0.045	0.099*	-0.151
	(-0.891)	(1.763)	(-0.613)
Age Wife2	-0.088	0.043	0.286
	(-1.558)	(0.702)	(0.802)
Children Wife1	0.057	-0.045	-0.772
	(0.465)	(-0.290)	(-1.153)
Children Wife2	-0.066	0.244	0.870
	(-0.551)	(1.366)	(0.772)
ln(Total Expenditures)	1.736	-0.942	4.110
	(1.356)	(-0.720)	(0.529)
ln(Total Expenditures)^2	-0.072	0.040	-0.157
	(-1.336)	(0.733)	(-0.485)
Share Income Wife1	-1.091***	0.377	-0.382
	(-2.630)	(0.992)	(-0.155)
Share Duration Marriage Wife1	-0.317	-0.664**	3.258**
	(-1.275)	(-2.599)	(2.291)
Observations	117	117	117

*Note: t-statistics in parentheses *** p<.01, ** p<.05, * p<.1*