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## ABSTRACT

### **Capital-Labor Substitution, Structural Change and the Labor Income Share<sup>\*</sup>**

Recent work has documented declines in the labor income share in the United States and beyond. This paper documents that these trends differ between manufacturing and services in the U.S. and in a broad set of other industrialized economies, and shows that a model where the degree of capital-labor substitutability differs across sectors is consistent with these trends. We calibrate the model exploiting additional information on the pace of structural change from manufacturing to services, on which the model also has predictions. We then conduct a decomposition to establish the relative importance of several potential drivers of changes in factor income shares and structural change that have been proposed in the literature. This exercise reveals that differences in productivity growth across sectors, combined with differences in substitution possibilities, have been the main driver of both changes in the labor income share and structural change.

JEL Classification: O40, O41, O30

Keywords: structural change, labor income share, capital-labor substitution

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# 1 Introduction

In two recent contributions, Elsby, Hobijn and Şahin (2013) and Karabarbounis and Neiman (2014) have documented the decline in the labor income share in the United States and in other countries. In this paper, we document that this decline was much more pronounced in manufacturing than in services, and propose an explanation that is consistent with these sectoral differences and with observed structural change from manufacturing to services over the period 1960 to 2005. The key element for explaining the observed differential evolution of sectoral labor income shares are differences in technical change across sectors, combined with sectoral differences in the substitutability of capital and labor.

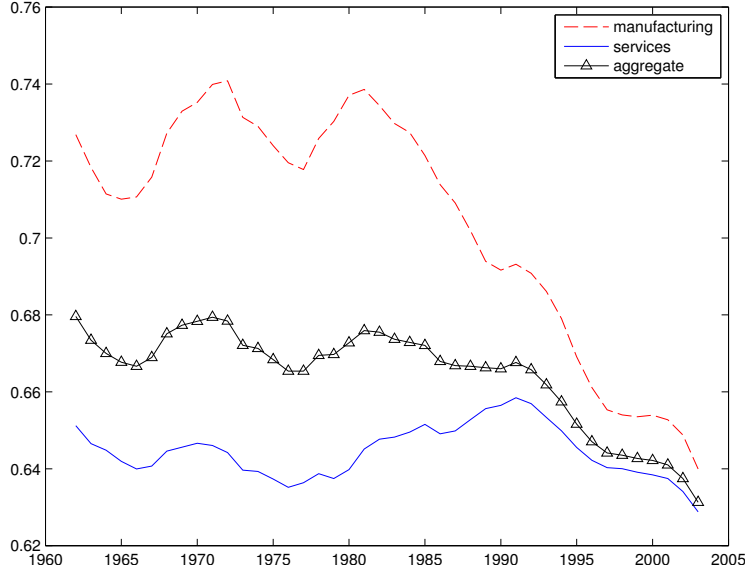
While aggregate factor income shares have long been thought to be constant, recent work has demonstrated that in the last few decades, the labor income share has declined substantially. This is illustrated in Figure 1 using U.S. data from Jorgenson (2007). The trend shown here is in line with the highly detailed analysis by Elsby et al. (2013) for the U.S., and with the findings by Karabarbounis and Neiman (2014) for a broad set of countries. Figure 1 also shows that the aggregate pattern is driven by differential developments at the sectoral level: Jorgenson’s KLEM data reveal that while in 1960, the labor share of income in manufacturing exceeded the aggregate labor income share, whereas the reverse was true for services, this pattern has changed substantially over the last 45 years. In this time, the labor share of income in manufacturing has declined substantially, and the one in services slightly. The decline has been pronounced; the labor income share in manufacturing has declined on average by 2.1 percentage points per decade, and the aggregate one by 1.2 percentage points. Section 2 describes the data underlying this pattern in more detail and shows that correcting income shares following Valentinyi and Herrendorf (2008) leaves these broad patterns intact. More than that, we show using EUKLEMS data that the labor income share has declined in all but three of a set of 16 industrialized economies, with the labor income share in manufacturing declining by even more in most of them.<sup>1</sup>

Unfortunately, most popular multi-sector models – for instance those used for the analysis of structural change in Kongsamut, Rebelo and Xie (2001), Ngai and Pissarides (2007) and Acemoglu and Guerrieri (2008) – are unable to account for such changes in sectoral income shares. The key reason for this is that they assume that sectoral production functions are

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<sup>1</sup>Karabarbounis and Neiman (2014) have shown some similar results. However, they provide less country-level detail and do not contrast the evolution of the labor income share in manufacturing with that in services. Similarly, using BEA industry accounts, Elsby et al. (2013) show that the labor income share in U.S. manufacturing declined severely in the period 1987-2011. In earlier work, Blanchard (1997) and Caballero and Hammour (1998) have documented medium term variation in the labor income share in some continental European economies and linked them to labor market rigidities.

Figure 1: The labor income share in the U.S.



Sources: Jorgenson's (2007) 35-sector KLEM data base. The figure shows 5-year moving averages.

Cobb-Douglas.<sup>2</sup> In this case, as is well known, sectoral labor income shares are forced to be constant if factor markets are competitive. If the elasticity of output with respect to capital differs across sectors, the aggregate labor income share may still change with structural change. However, as we show below, this channel can only account for a small fraction of the observed change in the aggregate labor income share, although structural change in the United States since 1960 was substantial.<sup>3</sup> These theories thus have no chance of replicating the observed changes in sectoral and aggregate labor income shares.

To address this issue, we propose a theory where the elasticity of substitution between capital and labor is different from one (i.e. production functions are not Cobb-Douglas), and potentially differs between manufacturing and services. Assuming that the elasticity of substitution within each sector is constant, this implies that sector  $i$  output is produced using the production function

$$Y_{it} = \left[ \alpha_i (B_{it} K_{it})^{\frac{\sigma_i - 1}{\sigma_i}} + (1 - \alpha_i) (A_{it} L_{it})^{\frac{\sigma_i - 1}{\sigma_i}} \right]^{\frac{\sigma_i}{\sigma_i - 1}}, \quad (1)$$

where  $\alpha_i$  governs the relative importance of the two inputs,  $A_{it}$  and  $B_{it}$  are the time- $t$  levels

<sup>2</sup>Kongsamut et al. (2001) assume instead that sectoral production functions are proportional. This is similarly restrictive, as it does not allow for sectoral differences in the level or the evolution of factor income shares.

<sup>3</sup>The value added share of manufacturing declined by 25 percentage points, or about half, over the sample period. See Section 2 for more background on structural change in the United States.

of labor- and capital-augmenting productivity, respectively,  $K_{it}$  and  $L_{it}$  are capital and labor used in sector  $i$  at time  $t$ , and  $\sigma_i \in (0, \infty)$  is the elasticity of substitution between these two inputs.<sup>4</sup> In this case, if factor markets are competitive and firms choose inputs optimally, the ratio of factor income shares in sector  $i$  is given by

$$\frac{KIS_{it}}{LIS_{it}} = \frac{R_t K_{it}}{w_t L_{it}} = \frac{\alpha_i}{1 - \alpha_i} \left( \frac{B_{it} K_{it}}{A_{it} L_{it}} \right)^{\frac{\sigma_i - 1}{\sigma_i}}, \quad (2)$$

where  $KIS_{it}$  and  $LIS_{it}$  stand for the capital and labor income shares in sector  $i$  at time  $t$ , and  $R_t$  and  $w_t$  for the rental rate and the wage rate, respectively. This expression is potentially consistent with observed trends: if capital and labor are gross complements, i.e.  $\sigma_i < 1$ , as most empirical evidence suggests (see León-Ledesma, McAdam and Willman (2010) for a recent review of empirical estimates), then the labor income share in sector  $i$  decreases (increases) as long as the amount of effective capital per unit of effective labor in sector  $i$  decreases (increases). The intuition is simple: If capital and labor are gross complements in production, a decrease in effective capital per unit of effective labor induces a more than proportional increase in the ratio of the rental rate to the wage, and therefore a reduction in the sectoral labor income share.<sup>5</sup> In this case, the aggregate labor income share may decrease not only as a result of structural change, but also due to declining labor income shares within sectors.<sup>6</sup>

The evolution of sectoral labor income shares in this setting thus depends on the elasticity of substitution between capital and labor in the sector,  $\sigma_i$ , on the “bias” of productivity growth in the sector (the growth of  $A_{it}/B_{it}$ ), and on capital accumulation in the sector (the growth of  $K_{it}/L_{it}$ ). While the former two drivers are primitives in macroeconomic models with exogenous growth, the latter is determined endogenously, by two margins: the

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<sup>4</sup>This elasticity was first introduced by Hicks in *The Theory of Wages* (1932) in an attempt to explore the functional distribution of income in a growing economy. It is defined as the elasticity of the capital-labor ratio in sector  $i$ ,  $k_i$ , to the ratio of factor prices, the wage to rental rate ratio,  $\omega \equiv \frac{w}{R}$ :

$$\sigma_i \equiv \frac{\partial k_i}{\partial \omega} \frac{\omega}{k_i}$$

Here, we assume that  $\omega$  is common across sectors.

<sup>5</sup>A similar mechanism is at work in the literature on capital-skill complementarity, for instance Krusell, Ohanian, Ríos-Rull and Violante (2000). In that context, the gross complementarity (substitutability) between equipment and skilled (unskilled) labor leads to an increase in the skill premium in response to capital accumulation.

<sup>6</sup>A falling labor income share is also consistent with  $\sigma_i > 1$  and rising  $B_{it}K_{it}/(A_{it}L_{it})$ , as in Karabarbounis and Neiman (2014), and as argued for by Piketty (2014). However, this scenario runs counter to the fact that virtually all estimates of aggregate  $\sigma$  in the recent literature lie below 1 (see Table 1 in León-Ledesma et al. (2010) and the review of some more recent contributions at the end of this section). See also footnote 29.

accumulation of capital in the economy as a whole, and the allocation of capital and labor across sectors. We therefore embed the sectoral production structure in (1) in a dynamic general equilibrium model with two sectors, and quantitatively analyze the dynamics of this model. Due to the potential impact of structural change on the aggregate labor income share, we also allow for the most prominent additional drivers of structural change proposed in the literature, i.e. non-homothetic preferences and cross-sectoral differences in capital-intensity in addition to differences in productivity growth and in factor substitutability. We then calibrate the model to the experience of changing labor income shares and structural change of the United States over the period 1960 to 2005 to draw lessons on the determinants of changing factor income shares over this period.

This quantitative exercise yields rich results. Firstly, the calibration exercise points to a strong bias of productivity growth ( $g(A_i/B_i) > 0$ ) in both sectors as the main driver of the decline in sectoral labor income shares. Secondly, the larger decline in the labor income share in manufacturing relative to that in services is driven to a similar extent by a larger bias in productivity growth and by a larger degree of flexibility (larger  $\sigma_i$ ) in manufacturing. Hence, differences across sectors in both of these features are key for understanding the evolution of sectoral labor income shares.<sup>7</sup> Thirdly, differences in productivity growth across sectors (notably, much faster growth of labor-augmenting productivity in manufacturing) clearly are the main determinant of structural change. Finally, non-homotheticities in preferences and differences in capital intensity hardly affect the evolution of factor income shares. Their effect on structural change is similarly small.<sup>8</sup>

It appears that an alternative to our analysis would have been to use (2) or the first order conditions for firms' choice of capital and labor from which it is derived to estimate  $\sigma_i$  and sectoral productivity growth rates, and to use parameter estimates to draw inference on the evolution of factor income shares. Our approach has two important advantages compared to this alternative. The first advantage is related to the identification of model parameters, in particular the elasticity of substitution and the bias of technical change. Our calibration exercise uses not only both first order conditions, but also exploits the additional restrictions imposed by the production function, just as recommended by Klump, McAdam and Willman (2007) and León-Ledesma et al. (2010) for the estimation of production systems. These authors argue, and illustrate with extensive Monte Carlo simulations, that single equation

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<sup>7</sup>While according to equation (2), a difference in either one of these two factors is sufficient for generating differential changes in factor income shares, the factor allocations generated by the model in this case are not consistent with the data.

<sup>8</sup>This may well be different for structural change out of agriculture in an earlier period.

approaches are largely unsuitable for jointly uncovering the elasticity of substitution and the bias of technical change. They stress the superiority of a multi-equation approach in terms of robustly capturing production and technical parameters.

The second advantage is conceptual and arises in the decomposition exercises. In addition to ignoring links across sectors, the alternative type of analysis proposed above would ignore the endogeneity of the evolution of the capital-labor ratio, and therefore would tend to attribute insufficient importance to productivity dynamics (which are the fundamental determinant of capital accumulation and allocation) as a driver of labor income shares. This is akin to a growth accounting exercise that ignores that capital accumulation is ultimately driven by productivity growth.

Apart from the recent work on the labor income share cited above, our paper is also closely related to a number of recent papers estimating the elasticity of substitution between capital and labor, in particular Herrendorf, Herrington and Valentinyi (2013), Oberfield and Raval (2014) and León-Ledesma, McAdam and Willman (2015). All of these authors focus on the estimation, and find similar substitution elasticities to the ones that result from our calibration exercise. None of them conducts a counterfactual decomposition exercise aiming to identify fundamental driving forces of changes in sectoral labor income shares. León-Ledesma et al. (2015) estimate an aggregate elasticity of substitution around 0.7 for the U.S. economy applying the technique they advocate in León-Ledesma et al. (2010) to aggregate U.S. time series. Herrendorf, Herrington and Valentinyi (2013) estimate CES production functions for the aggregate U.S. economy and for the sectors agriculture, manufacturing and services. Differently from our approach, they do so by sector, not jointly, and do not spell out the implications for the evolution of aggregate or sectoral factor income shares. They do however share our conclusion that productivity growth differences across sectors are the key driver of structural change. Finally, Oberfield and Raval (2014) estimate the elasticity of substitution in U.S. manufacturing by applying a novel identification strategy to plant-level data. They find a value of 0.71, close to the number in our calibration exercise. While this is not the focus of their paper, they also show that, given this estimate, observed changes in factor prices (an increasing wage to rental ratio) predict an increasing labor income share. This is of course generally the case when  $\sigma_i < 1$ . As a consequence, these authors attribute most of the observed decline in the labor income share to a residual catch-all “bias” term within manufacturing industries. This conclusion is consistent with our finding regarding the relevance of productivity growth as an important driver of changes in the labor income



share.<sup>9</sup>

Finally, since our paper also provides a quantitative analysis of structural change, it is also related to recent work on that topic, in particular Buera and Kaboski (2009, 2012*a*, 2012*b*), and Świecki (2014). Buera and Kaboski (2009) conduct a first quantitative evaluation of two potential drivers of structural change: productivity growth differences and non-homothetic preferences. Buera and Kaboski (2012*a*, 2012*b*) also analyze the “Rise of the Service Economy,” but focus on skill differences across different segments of the service sector and on differences in scale across sectors, respectively. Świecki (2014) quantitatively analyzes a set of four drivers of structural change that partly overlaps with those we consider. In line with our results, he stresses the importance of differences in sectoral growth rates for structural change. In addition, he concludes that non-homotheticities matter mostly for structural change out of agriculture. Trade, a factor that we abstract from, matters only for some individual countries. Overall, our work appears to be the first contribution linking the evolution of factor income shares and structural change.

In the next section, we provide more detail on the evolution of sectoral factor income shares. In Section 3, we describe our model. In Section 4, we evaluate the power of the mechanism quantitatively. Finally, Section 5 concludes, while the appendices contain additional derivations and information on data sources.

## 2 The evolution of factor income shares in manufacturing and services

Recent literature has documented in detail the decline in the aggregate labor income share in the United States (Elsby et al. 2013) and across countries (Karabarbounis and Neiman 2014). In this section, we document that this decline was to a very large extent driven by a decline of sectoral labor income shares, in particular of that in manufacturing. We start by showing this for the U.S., and then add evidence for a cross-section of 16 other developed countries.<sup>10</sup> The section concludes with some background on structural change in the U.S..

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<sup>9</sup>Choi and Ríos-Rull (2009) document the evolution of the labor income share over the business cycle and evaluate the ability of a variety of real business cycle models to account for its response to productivity shocks. As in our case, their preferred specification includes a CES technology where capital and labor are gross complements.

<sup>10</sup>Elsby et al. (2013) draw a similar conclusion for the U.S. since 1987. Karabarbounis and Neiman (2014) also show cross-industry developments (their Figure 5), but provide less country-level detail and no value for a broad service sector.

## 2.1 Sectoral labor income shares in the United States

Figure 1 showed the evolution of the labor income share in manufacturing and services in the United States from 1960 to 2005. Over this period, the aggregate labor income share declined by 1.2 percentage points per decade, the one in manufacturing by 2.1 percentage points per decade, and that in services by 0.5.

The measure of the labor income share shown in Figure 1 is computed as total labor compensation in a sector divided by the sum of the value of capital services and labor compensation in that sector. We call this measure “naive” because it ignores links across industries. For example, food manufacturing uses inputs from the transportation industry which in turn are produced using capital, labor and other intermediate inputs. Therefore, the true labor income share in the production of manufacturing value added also depends on “naive” labor income shares in sectors producing intermediate inputs used in manufacturing. We thus also compute a measure that takes these links into account, following Valentinyi and Herrendorf (2008) (see the Appendix A.1 for details).

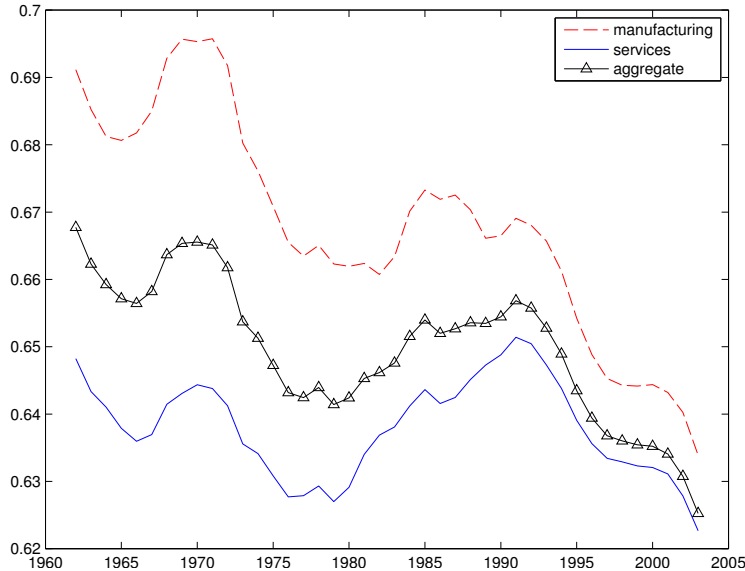
To do so, we use data from Jorgenson’s (2007) 35-sector KLEM database. These data are based on a combination of industry data from the BEA and the BLS and are described in detail in Jorgenson, Gollop and Fraumeni (1987), Jorgenson (1990) and Jorgenson and Stiroh (2000). They cover 35 sectors at roughly the 2-digit SIC level from 1960 to 2005. The raw data are accessible at <http://hdl.handle.net/1902.1/10684>. The data contain, for each industry and year, labor compensation, the value of capital services, and the value of intermediate inputs by source industry. These add up to the value of gross output in that industry. Data on capital and labor are carefully adjusted for input quality differences across sectors, allowing us to treat them as homogenous inputs in the following. Knowing the input-output structure allows computing the labor income share in production of sectoral value added.<sup>11</sup>

Results for the labor income share in value added are shown in Figure 2. Results using this measure are different in details, but the overall patterns remain unchanged. The main difference is that by this measure, the labor income share in manufacturing, while still exceeding that in services, is not as high as by the naive measure, since it takes into account that manufacturing value added also uses inputs from other sectors, with lower naive labor income shares. By this measure, the labor income share in manufacturing has fallen by 1.4 percentage points per decade, the one in services by 0.6 percentage points per decade,

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<sup>11</sup>For the definitions of our sectors manufacturing and services, see Appendix A.2. Numbers reported here for the aggregate economy refer to the aggregate of manufacturing and services. Results are not sensitive to details of sector definition like the treatment of utilities, government or mining.

Figure 2: Non-naive labor income share, by sector, 1960-2005



Sources: Jorgenson's (2007) 35-sector KLEM data base. The figure shows 5-year moving averages.

and the aggregate one by 1 percentage points per decade. The last two values are very close to the counterparts for the naive labor income share. Only the manufacturing number changes somewhat. However, the qualitative pattern is clearly maintained, with a decline in the aggregate labor income share driven largely by a decline in the labor income share in manufacturing.

Could the change in the aggregate labor income share be purely due to structural change? After all, the labor income share in manufacturing is higher than that in services, so that structural change from manufacturing to services will reduce the aggregate labor income share. A simple calculation shows that this channel is quantitatively minor. The aggregate labor income share is a value-added weighted average of the sectoral labor income shares. The share of manufacturing in value added has declined by almost 25 percentage points over the sample period (see Figure 4). Given a gap of 4.3 percentage points between the initial labor income shares in manufacturing and in services, this implies that structural change could account for a change in the aggregate labor income share of 1.075 percentage points over the period 1960 to 2005. This amount corresponds approximately to the average change in the labor income share over a single decade. Thus, structural change on its own can account for less than a quarter of the observed change in the aggregate labor income share.

## 2.2 Other countries

Changes in the labor income share have not been limited to the United States. In this section, we provide evidence from 16 other developed economies. We do so using EU KLEMS data. The March 2011 data release, available at <http://www.euklems.net>, contains data up to 2007 for 72 industries. We again aggregate those up to manufacturing and services. While this data contains a lot of industry detail, it does not contain an input/output structure, so we are limited to computing the naive labor income share, computed as compensation of employees over value added. However, we have seen above that developments in the U.S. have been qualitatively and even quantitatively similar for both measures of the labor income share. Karabarounis and Neiman (2014) have also used this data to document industry-level changes in the labor income share, but have not studied the manufacturing-services division.

We define sectors as for the U.S.. We use countries with at least 15 observations.<sup>12</sup> This leaves us with 16 countries.

The evolution of the LIS in these countries is shown in Table 1. The aggregate labor income share declines in all but three countries, by 2.1 percentage points per decade on average. The labor income share in manufacturing declines in all but 4 countries, by 2.5 percentage points per decade on average. The labor income share in services declines in all but five of the countries in our sample, by 1.5 percentage points per decade on average.<sup>13</sup> Overall trends are thus similar to those observed in the U.S., and even somewhat stronger. The overall ranking of sectors is also similar, with substantially stronger declines of the labor income share in manufacturing.

The same pattern is visible more formally in Figure 3, which shows the common component of the labor income share in 17 economies. It is obtained from these two regressions:

$$LIS_{cit} = D_{it} + D_{ic} + \epsilon_{cit} \quad (3)$$

$$LIS_{cit} = \beta_0 year_t + \beta_1 D_i year_t + D_{ic} + \epsilon_{cit}, \quad (4)$$

where  $c$ ,  $i$  and  $t$  index countries, sectors (manufacturing or services) and time, respectively.  $D_i$  are sector dummies,  $D_{it}$  sector  $\times$  year dummies, and  $D_{ic}$  sector  $\times$  country dummies. From the first specification, we obtain estimates of  $D_{it}$ , or annual sectoral labor income shares

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<sup>12</sup>This essentially implies excluding transition economies. We also exclude Korea, because it starts the manufacturing to services transition only part-way through the sample. We also exclude Ireland and Luxembourg due to data problems.

<sup>13</sup>These trends are statistically significant at the 5% level in all, all but one, and all but two of the cases, respectively.

Table 1: The labor income share by sector and country, 1970-2007

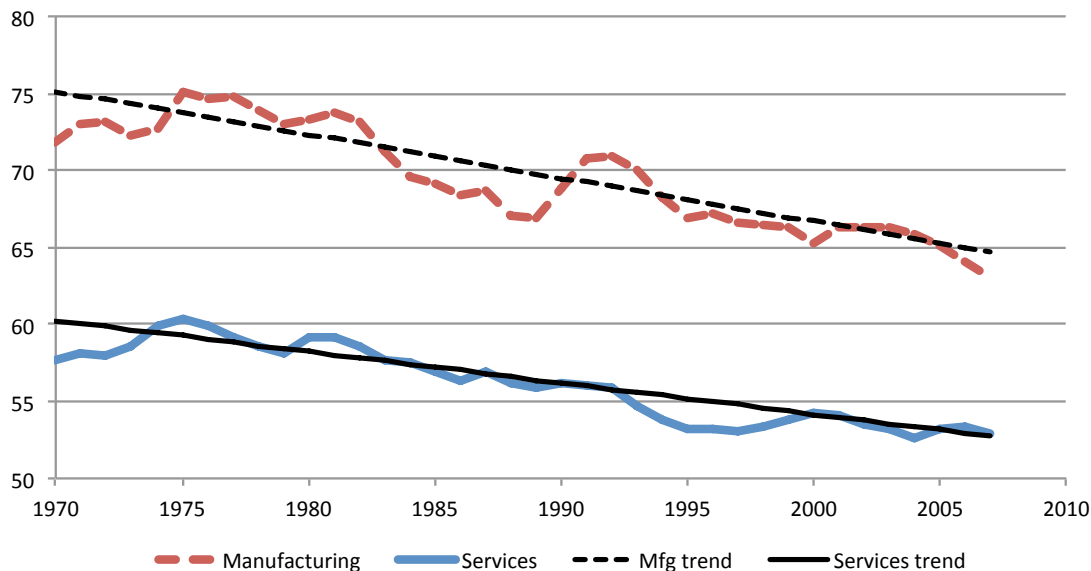
country	Manufacturing		Services		Aggregate		SC	N
	level	change	level	change	level	change		
AUS	0.707	-2.5	0.561	-0.8	0.601	-1.9	-0.8	38
AUT	0.719	-6.0	0.589	-4.1	0.633	-5.0	-0.5	38
BEL	0.673	1.0	0.537	3.2	0.581	2.0	-0.7	38
DNK	0.751	-1.7	0.561	2.3	0.615	0.8	-0.4	38
ESP	0.609	0.5	0.584	-4.6	0.587	-3.0	-0.2	38
FIN	0.645	-5.3	0.568	-2.9	0.597	-4.0	-0.2	38
FRA	0.700	0.7	0.611	-4.9	0.632	-3.7	-0.4	38
GER	0.755	-0.9	0.551	-1.1	0.629	-1.8	-1.0	38
GRC	0.748	-3.4	0.410	0.8	0.505	-1.7	-1.6	38
HUN	0.612	-10.9	0.570	2.4	0.584	-2.0	-0.0	13
ITA	0.720	-0.7	0.625	-3.6	0.655	-3.0	-0.4	38
JPN	0.578	-0.1	0.562	-5.1	0.566	-3.1	-0.1	37
NLD	0.680	-2.0	0.644	-2.0	0.654	-2.0	-0.2	38
PRT	0.681	-2.4	0.466	4.5	0.523	2.8	-0.1	37
SWE	0.756	-7.9	0.559	-1.0	0.625	-3.6	-0.4	38
UK	0.758	1.3	0.659	-0.7	0.691	-1.2	-0.8	38

Note: N denotes the number of observations. The level is the sample average for a country. Changes are in units of percentage points per decade. SC = change in the value added share of manufacturing  $\times$  (average LISM – average LISS), also in percentage points per decade. This is how much change in the aggregate labor income share could be explained by structural change alone. The series for Hungary excludes the first three observations (1992-1994), over which the labor income share in manufacturing collapses from 82 to 66% in three years.

net of country-specific effects, which are captured by  $D_{ic}$ . From the second specification, we obtain a time trend for the labor income share in each sector. Figure 3 displays both the series of estimates of  $D_{it}$  and the estimated time trends. It is evident from the figure that labor income shares in both sectors feature a negative trend in our sample, with some cyclical variation. The estimate of the trend coefficient  $\beta_0$  ( $\beta_1$ ) is  $-0.2$  ( $-0.08$ ), with a robust standard error of 0.024 (0.034).<sup>14</sup> The estimated trend thus implies that on average, the labor income share in services has declined by two percentage points per decade. In manufacturing, it has declined by an additional 0.8 percentage points, or almost 50% more.

<sup>14</sup>Labor income shares enter the regression as percentages. We use data on the 16 countries listed in Table 1 plus the United States. This results in a country by sector panel with 1190 observations and up to 38 observations per country-sector pair. The estimate of  $\beta_0$  is statistically significant at the 1% level, and that of the trend difference  $\beta_1$  at the 5% level.

Figure 3: The common component of the labor income share in 17 countries, 1970-2007



Note: The figure displays fitted values from the regressions in equations (3) and (4).

These large changes in sectoral labor income shares account for the bulk of changes in the aggregate labor income share. The penultimate column of Table 1 shows how much change in the aggregate labor income share could be accounted for if sectoral production technologies were Cobb-Douglas. In this case, sectoral factor income shares are fixed at their average value over the sample, and the aggregate labor income share can change only due to shifts in the value added shares of the two sectors, i.e. structural change. The table shows that structural change on its own cannot even account for 10% of the observed change in the labor income share in most countries. (See the next subsection for more information on structural change in the United States.) It could explain more than half of the change in the aggregate labor income share in only three countries, and more than a quarter in only four. Hence, changes in the aggregate labor income share are mostly driven by changes in sectoral labor income shares, and in particular by that in manufacturing, which on average declined about 40% more quickly than that in services.

### 2.3 Structural change in the United States

Structural change, i.e. the reallocation of economic activity across the three broad sectors agriculture, manufacturing and services, has long been known to accompany modern economic growth. (See e.g. Kuznets 1966.) As a consequence, agriculture now employs less than 2% of the workforce in the U.S., while the reallocation between manufacturing and services

Table 2: The joint evolution of the labor income share and the employment share of manufacturing, 1970-2007

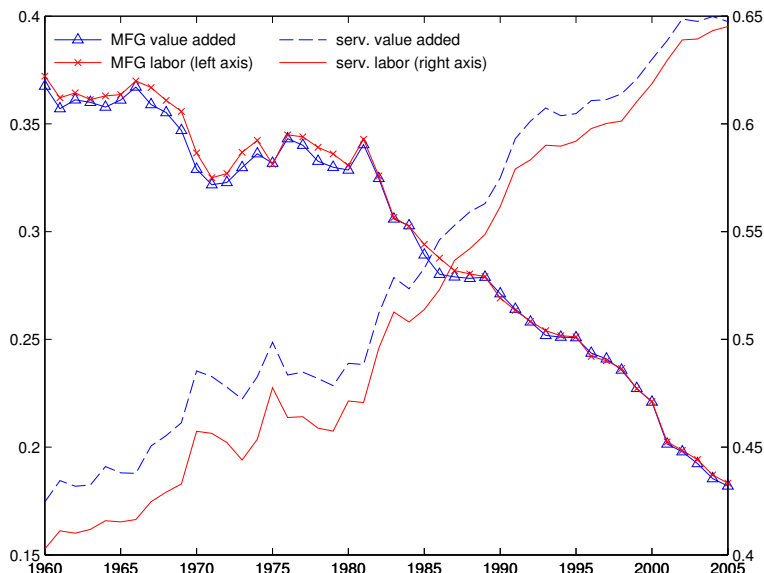
country	Change in... (pts per decade)				N
	$L_m/L$	LISM	LISS	LIS	
AUS	-6.3	-2.5	-0.8	-1.9	38
AUT	-5.7	-6.0	-4.1	-5.0	38
BEL	-6.8	1.0	3.2	2.0	38
DNK	-5.3	-1.7	2.3	0.8	38
ESP	-5.6	0.5	-4.6	-3.0	38
FIN	-3.6	-5.3	-2.9	-4.0	38
FRA	-6.1	0.7	-4.9	-3.7	38
GER	-6.8	-0.9	-1.1	-1.8	38
GRC	-6.0	-3.4	0.8	-1.7	38
HUN	-5.7	-10.9	2.4	-2.0	13
ITA	-5.3	-0.7	-3.6	-3.0	38
JPN	-4.8	-0.1	-5.1	-3.1	37
NLD	-6.0	-2.0	-2.0	-2.0	38
PRT	-3.3	-2.4	4.5	2.8	37
SWE	-5.8	-7.9	-1.0	-3.6	38
UK	-8.0	1.3	-0.7	-1.2	38

Note: N denotes the number of observations. All changes are in units of percentage points per decade, for the period 1970-2007. The series for Hungary excludes the first three observations (1992-1994), over which the labor income share in manufacturing collapses from 82 to 66% in three years.

is still in full swing. Figure 4 shows the evolution of the fraction of labor employed and the fraction of value added produced in the manufacturing and services sectors from 1960 to 2005 using data from Jorgenson’s (2007) 35-sector KLEM data base. Structural change is clearly evident.

Several theoretical channels have been proposed as drivers of structural change, the most prominent being non-homothetic preferences (Kongsamut et al. 2001) on the demand side and differences across sectors in productivity growth (Ngai and Pissarides 2007) or in capital intensity (Acemoglu and Guerrieri 2008) on the supply side. According to the first channel, the rise of the service sector may be due to a higher income elasticity of services demand: in a growing economy, consumers increase the share of their expenditure on services. The second channel, also related to “Baumol’s cost disease”, implies that sectors with rapid productivity growth can shed resources, which are then employed in slower-growing sectors.

Figure 4: The structural transformation in the United States, 1960-2005



Source: Jorgenson’s (2007) 35-sector KLEM data base.

The third channel is similar in spirit, but the difference lies in the cross-sectoral variation of the contribution of capital to output. Given the patterns of capital intensity and sector-specific technological change, the second channel requires manufacturing and services output to be gross complements in consumption, while the third requires the opposite.<sup>15</sup> All these channels can generate structural change in terms of both inputs and outputs. However, as discussed above, none of them can generate the observed movements in factor income shares.

Cross-sectoral differences in the substitutability of capital and labor also have implications for structural change. (We analyze this in detail in the companion paper Alvarez-Cuadrado, Long and Poschke (2015).) Notably, they affect the “shape” of structural change: the extent of capital versus labor reallocation. This occurs because with differences in the elasticity of substitution, the sector with higher substitutability (the “flexible sector”) moves towards using the factor that becomes more abundant more intensively. Hence, if for example effective labor becomes more abundant, as can occur if growth in the labor force together with labor-augmenting productivity growth outpace capital-augmenting productivity growth and capital accumulation combined, then the more flexible sector tends to become more labor-intensive.

<sup>15</sup>It is clear from the figures above that the capital income share in services exceeded that in manufacturing throughout the sample. With competitive factor markets, this implies a larger elasticity of output with respect to capital in services. Productivity growth since World War II, in turn, was larger in manufacturing.



### 3 Theoretical framework: A model with unequal sectoral substitution possibilities

The preceding section documented trends in factor income shares, and showed that changes in factor income shares within sectors, in particular within manufacturing, are key for understanding the decline in the aggregate labor income share. From (2), it is clear that differences in the evolution of sectoral factor income shares can come from differences in (the bias of) productivity growth, differences in factor allocation (differences in the growth of sectoral capital-labor ratios), or differences in the elasticity of substitution between capital and labor. Differences in their level can be due to differences in the capital-intensity of sectors, parameterized by  $\alpha_i$ . All of these factors would also lead to structural change across sectors (Ngai and Pissarides 2007, Acemoglu and Guerrieri 2008, Alvarez-Cuadrado et al. 2015). In addition, non-homothetic demand can also lead to structural change (Kongsamut et al. 2001) and thus to changes in the aggregate labor income share, as just illustrated.

We therefore conduct a joint quantitative analysis of the effect of cross-sectoral differences in capital-labor substitutability and of these additional factors (sectoral differences in productivity growth and in capital intensity, non-homothetic preferences) capable of driving joint changes in the labor income share and structural change. In the next Section, we then calibrate the model to the recent U.S. experience. Since our period of analysis is restricted by the availability of data on properly adjusted sectoral factor income shares from 1960 onwards, we conduct our analysis in an environment with two sectors, manufacturing and services, and abstract from agriculture, which already accounted for only 6% of employment and an even smaller fraction of value added in 1960.<sup>16</sup>

With quantitative results in hand, we can then conduct counterfactual exercises to determine the relative importance of these different factors for changes in the labor income share. As an added benefit, we can also quantify their importance for structural change.

#### 3.1 Model setup

We model a closed economy in continuous time. It is populated by a representative infinitely-lived household with instantaneous preferences given by<sup>17</sup>

$$v = \ln(u(c_m, c_s)) \quad \text{where} \quad u(c_m, c_s) = \left( \gamma c_m^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma)(c_s + s)^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}}.$$

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<sup>16</sup>In terms of sectoral reallocations, the contribution of agriculture over our sample period is similarly small, accounting for barely 10% of the reallocations of labor and 8% of the changes in sectoral composition of value added.

<sup>17</sup>Here and in the following, we omit time subscripts when there is no risk of confusion.

The subscripts  $m$  and  $s$  denote manufactures and services, respectively, so  $c_i$  stands for per capita consumption of value-added produced in sector  $i$ ,  $i \in \{m, s\}$ .  $\varepsilon$  is the elasticity of substitution between the two consumption goods. The term  $s$  introduces a non-homotheticity. The empirically relevant case is  $s > 0$ , which can be interpreted as households having an endowment of services. As a consequence, the income elasticity of demand for services is larger than that for manufactures. This introduces the possibility of demand-driven structural change, as in Kongsamut et al. (2001).

Sectoral outputs are produced according to general CES technologies.

$$Y_m = \left[ \alpha_m (B_m K_m)^{\frac{\sigma_m - 1}{\sigma_m}} + (1 - \alpha_m) (A_m L_m)^{\frac{\sigma_m - 1}{\sigma_m}} \right]^{\frac{\sigma_m}{\sigma_m - 1}} \quad (5)$$

$$Y_s = \left[ \alpha_s (B_s K_s)^{\frac{\sigma_s - 1}{\sigma_s}} + (1 - \alpha_s) (A_s L_s)^{\frac{\sigma_s - 1}{\sigma_s}} \right]^{\frac{\sigma_s}{\sigma_s - 1}} \quad (6)$$

This structure allows for a rich set of asymmetries. First, we allow for sector-specific elasticities of substitution and distributional parameters,  $\sigma_i$  and  $\alpha_i$  respectively. Second, we assume that sectoral labor- and capital-augmenting productivity,  $A_i$  and  $B_i$ , grow at constant exponential rates  $g(A_i)$  and  $g(B_i)$ , respectively, and allow both their initial levels and these growth rates to differ across sectors.<sup>18</sup> Third, non-homothetic preferences introduce an additional difference between the services and manufacturing sectors. Finally, we follow most of the literature on multi-sector models by assuming that capital is only produced in the manufacturing sector, so services are fully consumed. Therefore, using upper-case letters to denote aggregate variables,  $Y_s = C_s$  and  $Y_m = C_m + \dot{K} + \delta K$ , where a dot above a variable stands for its change and  $\delta > 0$  is the constant rate of depreciation of the capital stock.

Factors are fully utilized which, normalizing the labor endowment to one, implies

$$\frac{L_s}{L} + \frac{L_m}{L} \equiv l_m + l_s = 1 \quad (7)$$

$$l_m k_m + l_s k_s = l_m k_m + (1 - l_m) k_s = k, \quad \text{where } k_i \equiv \frac{K_i}{L_i}. \quad (8)$$

In the following, we will use lower-case variables to denote per capita quantities, except for  $k_i$ , which stands for the capital-labor ratio in sector  $i$ .

### 3.2 Model solution

We choose manufactures to be the numeraire and denote the price of services by  $p_s$ . Since markets are competitive, production efficiency requires equating marginal revenue products

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<sup>18</sup>Diamond, McFadden and Rodriguez (1978) show that in the presence of biased technical change, data on output, inputs, and input prices are not sufficient for the identification of the elasticity of substitution. In addition, functional restrictions on the rate of technical change are required. A constant exponential growth rate, as assumed here, is the standard identification restriction.

across sectors, so

$$\alpha_m B_m^{\frac{\sigma_m-1}{\sigma_m}} \left( \frac{Y_m}{K_m} \right)^{\frac{1}{\sigma_m}} = p_s \alpha_s B_s^{\frac{\sigma_s-1}{\sigma_s}} \left( \frac{Y_s}{K_s} \right)^{\frac{1}{\sigma_s}} = R \quad (9)$$

$$(1 - \alpha_m) A_m^{\frac{\sigma_m-1}{\sigma_m}} \left( \frac{Y_m}{L_m} \right)^{\frac{1}{\sigma_m}} = p_s (1 - \alpha_s) A_s^{\frac{\sigma_s-1}{\sigma_s}} \left( \frac{Y_s}{L_s} \right)^{\frac{1}{\sigma_s}} = w. \quad (10)$$

As a consequence, the following relationship between the sectoral capital-labor ratios emerges.

$$k_s = \left( \frac{1 - \alpha_m}{1 - \alpha_s} \frac{\alpha_s}{\alpha_m} \left( \frac{A_m}{B_m} \right)^{\frac{\sigma_m-1}{\sigma_m}} \right)^{\sigma_s} \left( \frac{A_s}{B_s} \right)^{1-\sigma_s} k_m^{\frac{\sigma_s}{\sigma_m}} \equiv \varrho k_m^{\frac{\sigma_s}{\sigma_m}}. \quad (11)$$

Using this notation, (9) implies that the relative price of services is given by

$$p_s = \frac{\alpha_m B_m^{\frac{\sigma_m-1}{\sigma_m}} \left[ \alpha_m B_m^{\frac{\sigma_m-1}{\sigma_m}} + (1 - \alpha_m) (A_m/k_m)^{\frac{\sigma_m-1}{\sigma_m}} \right]^{\frac{1}{\sigma_m-1}}}{\alpha_s B_s^{\frac{\sigma_s-1}{\sigma_s}} \left[ \alpha_s B_s^{\frac{\sigma_s-1}{\sigma_s}} + (1 - \alpha_s) (A_s/(\varrho k_m^{\sigma_s/\sigma_m}))^{\frac{\sigma_s-1}{\sigma_s}} \right]^{\frac{1}{\sigma_s-1}}}. \quad (12)$$

Household optimization in turn requires equating the marginal rate of substitution between the two consumption goods to their relative price in every period, implying relative demands given by

$$\frac{v_s}{v_m} = \frac{(1 - \gamma)}{\gamma} \left( \frac{c_s + s}{c_m} \right)^{-\frac{1}{\varepsilon}} = p_s. \quad (13)$$

Given a solution to the dynamic problem and state variables  $k, A_m, A_s, B_m, B_s$ , equations (11), (12) and (13) pin down  $p_s, k_m$  and  $l_m$  at each point in time. Their counterparts for services,  $k_s$  and  $l_s$ , then follow from equations (26) and (27).

The solution to the household's dynamic problem, stated in terms of choosing  $c_m$ , implies the Euler equation

$$-\hat{v}_m = \alpha_m B_m^{\frac{\sigma_m-1}{\sigma_m}} \left[ \alpha_m B_m^{\frac{\sigma_m-1}{\sigma_m}} + (1 - \alpha_m) A_m^{\frac{\sigma_m-1}{\sigma_m}} k_m^{\frac{1-\sigma_m}{\sigma_m}} \right]^{\frac{1}{\sigma_m-1}} - (\delta + n + \rho), \quad (14)$$

where  $\rho$  is the subjective discount factor and  $n$  is the rate of population growth. The law of motion of per capita capital is given by

$$\dot{k} = \left[ \alpha_m (B_m k_m)^{\frac{\sigma_m-1}{\sigma_m}} + (1 - \alpha_m) A_m^{\frac{\sigma_m-1}{\sigma_m}} \right]^{\frac{\sigma_m}{\sigma_m-1}} l_m - c_m - (\delta + n) k. \quad (15)$$

Combined with (27) and (31), this determines the law of motion of  $k_m$ .

An equilibrium in this economy consists of sequences of allocations  $\{c_m(t), c_s(t), k_m(t), k_s(t), l_m(t), l_s(t)\}_{t=0}^{\infty}$  and prices  $\{p_s(t), w(t), R(t)\}_{t=0}^{\infty}$  such that equations (9) to (15) hold and feasibility is satisfied.

For computational reasons, we solve a discrete-time version of this economy in the next Section. (See Appendix B.2 for details.) The discretized version of the dynamic problem can be represented as a system of three difference equations:

$$l_{m,t} = h_1(c_{m,t}, k_{m,t}), \quad (16)$$

$$\frac{c_{t+1}}{c_t} = h_2(k_{m,t+1}) \quad (17)$$

$$k_{m,t+1} = h_3(l_{m,t+1}, k_{m,t}, l_{m,t}, k_t, c_{m,t}) \quad (18)$$

where exogenous variables are omitted from the function arguments for the sake of clarity.  $h_1$  to  $h_3$  are complicated functions that are stated explicitly in equations (43), (44) and (45) in Appendix B.2.

Before proceeding to our quantitative results, three technical remarks are in order. First, since the marginal product of capital in manufacturing in the future depends on the labor allocation in that future period, the dynamic problem is not independent of the static problem.<sup>19</sup> Hence, we cannot separate the system of equations (16) to (18) into two blocks, but need to solve all three equations together.

Second, our model allows for capital-augmenting technical progress which, as is well-known, is not consistent with balanced growth. In general, many multi-sector models are not consistent with balanced growth. This is true for Kongsamut et al.’s (2001) model of demand-driven structural change, except in a special case, and also for Acemoglu and Guerrieri’s (2008) model of technology-driven structural change, except in the limit. This is no coincidence. In their chapter on structural change in the Handbook of Economic Growth, Herrendorf, Rogerson and Valentinyi (2014) state (p. 4): “It turns out that the conditions under which one can simultaneously generate balanced growth and structural transformation are rather strict, and that under these conditions the multi-sector model is not able to account for the broad set of empirical regularities that characterize structural transformation. ... we think that progress in building better models of structural transformation will come from focusing on the forces behind structural transformation without insisting on exact balanced growth.” As the quantitative results below show, our model features approximate balanced growth.

Third, the absence of exact balanced growth requires the imposition of a different terminal condition for the dynamic system given in equations (16) to (18). We proceed by a) choosing

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<sup>19</sup>This does not depend on whether we use the law of motion of  $k$  or  $k_m$  in the dynamic problem, or whether we use the continuous- or discrete-time version of the problem. Key is that the future marginal product of capital in manufacturing that appears in the Euler equation (14) depends on the future labor allocation.

a finite time horizon  $T$  (100 years in the results shown below), b) imposing that consumption growth is constant at the end of that horizon:  $c_{T+1}/c_T = c_T/c_{T-1}$  (this implies a solution to equation (16) for  $t = T$ , which in turn allows solving equations (17) and (18) for that period), and c) check that the specific horizon  $T$  that we chose does not affect results.

## 4 Quantitative analysis: Dissecting the U.S. experience

In this section, we describe the calibration of the model and present a set of counterfactual exercises decomposing model-predicted changes in sectoral labor income shares and in structural change into four components associated with the four fundamental drivers of sectoral differences in the model. These will show the relative contribution of the different fundamental forces to the observed declines in labor income shares.

### 4.1 Calibration

#### 4.1.1 Data

We calibrate the model to U.S. data over the period 1960-2005. The data we use to obtain factor income shares and allocations is from Jorgenson (2007) and has been discussed above. We convert capital services reported there into capital stock figures using the average long-run rental rate from the model. To calibrate preferences, we also require information on consumptions shares and the relative price, which we take from Herrendorf, Rogerson and Valentinyi (2013).

#### 4.1.2 Preferences

Herrendorf, Rogerson and Valentinyi (2013, Figures 9 and 10) show that despite an increase in the relative price of services, the ratio  $c_s/c_m$  has not fallen, but increased slightly over the last 65 years. With  $s \geq 0$ , Leontief preferences between manufacturing and services ( $\varepsilon = 0$ ) provide the best approximation to this trend. This is, of course, in line with Herrendorf, Rogerson and Valentinyi's (2013) estimates.<sup>20</sup>

Given  $\varepsilon = 0$ , equation (13) allows us to calibrate  $s$  using the growth rates of quantities and prices of manufacturing and services consumption, in value added terms, over the 1960

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<sup>20</sup>In Appendix C, we show that results are very similar when calibrating the model with a larger  $\varepsilon$  of 0.5; the route taken by Buera and Kaboski (2009) to avoid Leontief preferences.

to 2005 period. This results in a value for  $s$  of around 20% of first period services consumption. We then obtain a value of  $\gamma$  from the relative weight of manufacturing and services consumption in the initial period. Since the ratio of manufacturing to services consumption does not change much, this value is not very sensitive to the time period we use for calibrating it.

### 4.1.3 Technology

For reference, each sector's CES production function is given by

$$Y_{it} = \left[ (1 - \alpha_i)(A_{it}N_{it})^{\frac{\sigma_i-1}{\sigma_i}} + \alpha_i(B_{it}K_{it})^{\frac{\sigma_i-1}{\sigma_i}} \right]^{\frac{\sigma_i}{\sigma_i-1}}, \quad (19)$$

where  $\alpha_i, \sigma_i, A_{i0}, B_{i0}$ , and the growth rates of  $A_{it}$  and  $B_{it}$  are all allowed to differ across sectors. In general, changing  $\sigma$  in (19), as we do for counterfactuals below, does not only change the substitution elasticity (see e.g. Klump et al. 2007). We therefore work with a normalized version of the CES recommended by León-Ledesma et al. (2010):

$$Y_{it} = \bar{Y}_i \left[ \overline{1 - \theta_i} \left( \frac{A_{it}N_{it}}{\bar{A}_i\bar{N}_i} \right)^{\frac{\sigma_i-1}{\sigma_i}} + \bar{\theta}_i \left( \frac{B_{it}K_{it}}{\bar{B}_i\bar{K}_i} \right)^{\frac{\sigma_i-1}{\sigma_i}} \right]^{\frac{\sigma_i}{\sigma_i-1}}. \quad (20)$$

Here, variables with a bar denote the geometric sample average of the underlying series, and  $\bar{\theta}_i$  and  $\overline{1 - \theta_i}$  refer to the average sectoral capital and labor income shares, respectively. In terms of the production function above, using this normalization involves setting  $\alpha_i$  to  $\bar{\theta}_i$  and  $1 - \alpha_i$  to  $\overline{1 - \theta_i}$  (the two terms sum approximately to one even with geometric averages of the income shares). Since  $\alpha_i, A_{it}$  and  $B_{it}$  in (19) cannot be identified separately, choosing the value of  $\alpha_i$  in this way is a normalization. Herrendorf, Herrington and Valentinyi (2013) use the same normalization when estimating sectoral production functions.<sup>21</sup> The normalization chosen matters for exercises where the substitution elasticity is changed. The one we use implies that for all  $\sigma_i$ , isoquants are tangential at the average capital-labor ratio in the data, and output at that capital-labor ratio does not change with  $\sigma_i$ . With this normalization, changing  $\sigma_i$  (for example when computing a counterfactual to evaluate the effect of differences in  $\sigma$  across sectors) thus does not affect sectoral output at the sample average.

It is clear that even once the distributional parameter is fixed in this way, levels of  $A_{it}$  and  $B_{it}$  are not identified independently of the units of  $Y_{it}$ . We therefore normalize  $B_{i0}$  to 1 in each sector. We also set the predetermined initial capital-labor ratio in manufacturing

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<sup>21</sup>On normalization of CES production functions, see León-Ledesma et al. (2010) and Cantore and Levine (2012). Temple (2012) discusses the interpretation of the normalized production function in detail.

from the data.<sup>22</sup> We set the depreciation rate  $\delta$  to 5% per year and the discount factor  $\beta$  (the discrete time counterpart of  $\rho$ ) to 0.94.

At this point, eight parameters remain to be calibrated: the two elasticities of substitution between capital and labor,  $\sigma_m$  and  $\sigma_s$ , four growth rates of  $A_m, A_s, B_m, B_s$ , and initial levels  $A_{m0}$  and  $A_{s0}$ . We set their values to match eight data moments, all computed for the period 1960 to 2005: the change in the capital income share to labor income share ratio in each sector, the average labor income share in each sector, the change in the fraction of labor and capital (respectively) employed in manufacturing, the average fraction of labor employed in manufacturing, and the aggregate growth rate of output per worker, measured in units of the aggregate consumption good. All these data moments are computed from Jorgenson’s (2007) data.<sup>23</sup>

We choose these targets because of their information content regarding the model parameters. While all eight remaining parameters affect the values of all targets, some relationships are particularly strong. Thus, given  $\sigma_i, A_{m0}$  and  $A_{s0}$  strongly affect the average factor income shares in the two sectors.<sup>24</sup> The productivity growth bias or relative growth rates of  $A_i$  and  $B_i, g(A_i/B_i)$ , drive the change in relative income shares in a sector. The relative growth rate of  $A_m$  and  $A_s, g(A_m/A_s)$ , contribute strongly to the pace of structural change. The growth rate of  $A_m$  then determines overall output growth. The substitution elasticities drive the shape of structural change (strength of labor versus capital reallocation).<sup>25</sup>

## 4.2 Calibration results

Calibration results are shown in Table 3. The model can reproduce both the differential pattern in changes in the labor income share shown in Figure 2 and the amount and shape of structural change observed in the data. It also matches average levels of the labor income

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<sup>22</sup>One way to understand the need to impose initial conditions on the sectoral allocations is to contrast our solution method with that of a two sector model with a well-defined steady state. In this latter case, the initial conditions for sectoral allocations are determined in such a way that the system eventually converges to its steady state. In our case, in the absence of a steady state, we pin down the initial values of the sectoral allocations from the data. Thereafter, these allocations are endogenously determined. This is also the approach followed by Acemoglu and Guerrieri (2008).

<sup>23</sup>To avoid excessive influence of the first and last observation, we compute the growth rates between averages for the first and last 5 years.

<sup>24</sup>This is intuitive once you consider that an alternative normalization consisted in setting  $A_{i1}$  to 1 in each sector and calibrating  $\alpha_i$  to match the average labor income share in sector  $i$ . Moreover, it is important to realize that with CES production functions, setting  $\bar{\theta}_i$  to equal observed factor income shares does not on its own imply that the model will match these shares. For this to occur, it is necessary to set  $A_{i1}$  appropriately, given the chosen normalization of the distributional parameter.

<sup>25</sup>The Jacobian of model moments with respect to parameters, evaluated at our selected parameter values, has full rank.

shares and the labor allocation well, and fits aggregate output growth exactly. It does not fit the average labor income share in services exactly, as there is a tension between doing so and fitting the average labor allocation. In addition, the model replicates very closely the average and changes in the capital and output allocations, which were not targeted in the calibration. As a consequence, it also replicates the level and changes of the aggregate labor income share rather closely.

Table 3: Calibration: Targets and model moments and parameters.

<i>Panel A: Targets and model moments</i>				
	Data		Model	
	Manufacturing	Services	Manufacturing	Services
<i>calibration targets:</i>				
$\overline{LIS}_i$ (%)	66.7	63.8	66.4	65.1
$g(KIS_i/LIS_i)$ (% p.a.)	0.6	0.3	0.6	0.3
$d(L_m/L)$	-22.4		-23.9	
$d(K_m/K)$	-21.3		-20.7	
$\overline{L}_m/\overline{L}$	34.2		34.5	
$g(Y/L)$ (% p.a.)	1.8		1.8	
<i>not targeted:</i>				
$\overline{LIS}$ (aggregate, %)	64.8		65.6	
$g(KIS/LIS)$ (agg., % p.a.)	0.5		0.4	
$\overline{K}_m/\overline{K}$	33.2		33.3	
$\overline{Y}_m/\overline{Y}$	35.1		34.1	
$d(Y_m/Y)$	-24.5		-22.9	
<i>Panel B: Parameters</i>				
<i>sector-specific:</i>	Manufacturing	Services	<i>general:</i>	
$g(A_i)$ (% p.a.)	7.8	1.1	$\delta$	0.05
$g(B_i)$ (% p.a.)	-1.6	-5.9	$\beta$	0.94
$\sigma_i$	0.776	0.571	$\varepsilon$	0
$A_{i1}$	0.604	0.709	$\gamma$	0.314
$\overline{\theta}_i$ (normalization)	0.333	0.362		

Note: The data period used is 1960 to 2005. Bars indicate the average of a variable, which is computed as the geometric mean over the sample period. Absolute changes are percentage point changes over the entire period. Growth rates are computed using averages of the first and last 5 years. For readability, annual percentage changes are given for output growth and the growth in relative income shares.



Calibrated parameters are shown in Panel B of Table 3. Key parameters coming from the calibration are those for the sectoral substitution elasticities and the growth rates. The calibration yields substitution elasticities below but close to unity in manufacturing, and substantially below unity in services. Cross-sector differences in the rates of technological change are large. Our results suggest positive labor-augmenting technical change and negative capital-augmenting technical change. Both growth rates are larger in the manufacturing sector. There is a large bias of technical change,  $g(A_i/B_i)$ , in both sectors. As discussed below, to understand the implications of non-unitary substitution elasticities for factor income shares and structural change, it is not sufficient to know the growth rates; differences in the elasticities are also key.

Next, we briefly compare our calibrated parameter values to estimates from the literature. Estimates of substitution elasticities below 1 are in line with previous estimates at the aggregate level compiled in León-Ledesma et al. (2010). For the sectoral level, we are only aware of estimates from Herrendorf, Herrington and Valentinyi (2013). Like us, they obtain estimates of the elasticities below unity, and also estimate manufacturing to be the more flexible sector. Their estimate for  $\sigma_m$  is 0.8, very close to ours. Their estimate for services is 0.75, slightly higher than ours. Calibrating all parameters jointly, instead of estimating sector by sector, thus makes a difference.<sup>26</sup>

To compare our results to the broader literature estimating the elasticity of substitution at the aggregate level, we can compute an analogous object in the model economy. It is given by

$$\sigma = \gamma_0 \varepsilon + \gamma_1 \sigma_s + \gamma_2 \sigma_m + \frac{(\lambda - \kappa)}{(\hat{w} - \hat{R})} \left( \hat{Y}_m - n - \hat{c}_m \left( 1 + \frac{s}{c_s} \right) \right), \quad (21)$$

where

$$\begin{aligned} \gamma_0 &\equiv (\epsilon_m - \epsilon_s) (\lambda - \kappa) \left( 1 + \frac{s}{c_s} \right) \\ \gamma_1 &\equiv \lambda \epsilon_s + \kappa (1 - \epsilon_s) \\ \gamma_2 &\equiv (1 - \lambda) \epsilon_m + (1 - \kappa) (1 - \epsilon_m). \end{aligned}$$

(See Appendix B.3 for a derivation of these expressions.) The aggregate elasticity of substitution between capital and labor is a weighted average of production elasticities of substitution in the two sectors and the elasticity of substitution in consumption between manufacturing

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<sup>26</sup>When inserting their estimates in our model, the model predicts too much change in the labor income share in manufacturing, and too little (in fact, positive) change in services. The latter is due to the higher substitution elasticities in services estimated by Herrendorf, Herrington and Valentinyi (2013). The model also predicts almost no change in the share of capital employed in manufacturing.

and services output. The final term in (21) is due to the modeling assumption that investment goods are only produced in the manufacturing sector. As the weights in (21) depend on the allocation of inputs, the aggregate elasticity is time-varying. In our benchmark economy, it declines from 0.67 to 0.64 as the weight of manufacturing, the more flexible sector, declines. The sample average is 0.65. Hence, it is close to but slightly below estimates in the literature. For instance, León-Ledesma et al. (2010, Table 1) report many estimates between 0.5 and 0.8. Herrendorf, Herrington and Valentinyi (2013) find a value of 0.85.

Our calibrated parameters for technical change may at first sight appear unusual. In the next subsection, we discuss why these values are required to match target moments. For now, it is important to note two points. First, while the finding of negative capital-augmenting technical change is surprising at first sight, such estimates are not uncommon in the literature. For instance, both Antras (2004) and Herrendorf, Herrington and Valentinyi (2013) obtain negative estimates for  $g(B)$ . These authors attribute their findings to potential mismeasurement. Negative  $g(B)$  could be induced if the data used either understate increases in labor quality (Antras) or understate depreciation of capital (Herrendorf et al.).

Second, the numbers for technical change reported in Table 3 refer to sectoral production functions. We can also compute measures of aggregate technical change, using the aggregate elasticity of substitution just computed. We do so in three steps. First, using the aggregate version of equation (2) combined with model data on aggregate factor income shares, capital deepening, and the aggregate elasticity of substitution yields a measure of the aggregate bias of technical progress.<sup>27</sup> Second, following Diamond, McFadden and Rodriguez (1978), define aggregate technical progress as growth in output per worker net of the effect of capital deepening,  $T = g(Y/L) - KIS g(K/L)$ . Denoting growth in the level of aggregate labor- (capital-) augmenting technology by  $g(A)$  ( $g(B)$ ), this can be decomposed as  $T = KIS g(B) + LIS g(A)$ . Third, insert the measure of the bias in this expression to obtain measures of  $g(A)$  and  $g(B)$  in the benchmark economy. Doing so, we find average annual values of  $g(A)$  of 2.1% and of  $g(B)$  of -0.6%. The former is very close to estimates in Antras (2004), Herrendorf, Herrington and Valentinyi (2013) and León-Ledesma et al. (2015). Average  $g(B)$  in our model economy is slightly more negative than estimated by Herrendorf, Herrington and Valentinyi (2013), but substantially less so than found by Antras (2004). (León-Ledesma et al. (2015) find values just slightly above zero.)

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<sup>27</sup>Note that this exercise implies assuming that the aggregate model data is generated by an aggregate production function with a constant elasticity of substitution. This of course runs counter to the two-sector model we use. The numbers we give here therefore purely serve to illustrate what an econometrician who used our model data and made this assumption would conclude.

Overall, measures of aggregate technical change are less extreme than sectoral ones. (This is a pattern also visible in Herrendorf, Herrington and Valentinyi’s (2013) estimation results.) This arises for two reasons. First, aggregate growth rates combine changes in the two sectors. Second, they are in units of final output, whereas sectoral growth rates are in units of sectoral output.

#### 4.2.1 Understanding calibration results

To understand what drives the calibration results for the sectoral elasticities of substitution  $\sigma_i$  and for the sectoral rates of technical change, it is useful to consider how the model can match observed patterns in sectoral factor income shares and the shape of structural change. The key observations here are that in the data, the labor income share declines in both sectors, and more so in manufacturing, and that structural change looks similar, whether expressed in terms of the allocation of employment or that of capital. Given limited importance of non-homotheticity and  $\alpha$  differences (see the decomposition below), the amount and shape of structural change and the relative evolution of the labor income shares in the model depend on the substitution elasticities and on relative productivity growth in the two sectors.

Consider first the evolution of factor income shares. This is determined by equation (2), which is reproduced here for convenience:

$$\frac{KIS_{it}}{LIS_{it}} = \frac{\alpha_i}{1 - \alpha_i} \left( \frac{B_{it}K_{it}}{A_{it}L_{it}} \right)^{\frac{\sigma_i - 1}{\sigma_i}}$$

From here it is clear that if  $\sigma_i < 1$  and capital per worker grows in both sectors,  $A$  needs to grow faster than  $B$  in both sectors for the labor income share to decline in both of them. The decline in the labor income share thus is driven by the increasing scarcity of effective capital relative to effective labor.<sup>28</sup> The larger  $\sigma_i$ , the larger  $g(A_i/B_i)$  needs to be to induce a given change in the labor income share. In line with this,  $g(A_m/B_m) > g(A_s/B_s)$  in our calibration, given that  $\sigma_m > \sigma_s$ .

What would happen if  $\sigma_i$  and  $g(A_i/B_i)$  in the two sectors were more similar? To answer this question, we reduce  $\sigma_m$  and  $g(A_m/B_m)$  in such a way as to keep the change in the labor income share and the growth of output per worker in manufacturing unaffected. Given a background of increasing scarcity of effective capital relative to effective labor, as required to

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<sup>28</sup>Similar reasoning explains why we find negative  $g(B)$ . In our setting, with both sectors exhibiting substitution elasticities below unity, negative  $g(B)$  is necessary for matching the observed declining labor income shares, combined with the observed level of output growth. The reason is that given observed growth in capital intensity in both sectors, a substantial difference between  $g(A)$  and  $g(B)$  is required in both sectors. At the same time, these growth rates need to take levels consistent with output growth of 1.8%. It turns out that these requirements are only met with  $g(B) < 0$ .

match the evolution of factor income shares, reducing  $\sigma_m$  makes manufacturing less flexible and thus pushes firms in the sector to retain more capital. At the same time, a smaller difference between  $g(A_m)$  and  $g(B_m)$  reduces the speed at which capital becomes more scarce, and thus pushes firms in the other direction. Quantitatively, the first channel dominates, making manufacturing progressively more capital-intensive. This leads to a worse fit in terms of the shape of structural change, with too little change in the fraction of capital used in manufacturing. With our calibrated parameters, in contrast, these forces are balanced in the right way.<sup>29</sup> This example shows how information on structural change helps identify parameters of the model. By extension, it is relevant to our results on the drivers of changes in factor income shares given below.

#### 4.2.2 The benchmark time path

Figure 5 compares the time path of our simulated economy with the data. The top right panel shows output per capita, which grows at an average and almost-constant rate of 1.8% per year. The interest rate (top left panel) is also almost constant: after an initial transition of 18 years, during which it increases by 1.3 percentage points, it becomes almost constant, falling only 0.11 percentage points over the remaining 28 years of our simulated data. The economy thus exhibits “approximate balanced growth”.<sup>30</sup>

The two bottom panels show structural change and the change in factor income shares. The left panel shows that the labor income share declines in both sectors, but more sharply in manufacturing, driven by the larger discrepancy between the growth rates of  $A$  and  $B$  in that sector. Since this is a calibration target, the model reproduces the observed declines in the labor income share well and also matches their average level rather well. The aggregate labor income share declines from 68.2% to 63.8% in the model, compared to a fall from 68.4% to 63.0% in the data. While this moment was not targeted directly, it is clearly closely related to three of our calibration targets.

The right panel depicts structural change generated in the model. It shows that the calibrated model fits the data both in terms of the amount of structural change generated and in terms of its composition: the reallocation of labor and capital look strikingly similar

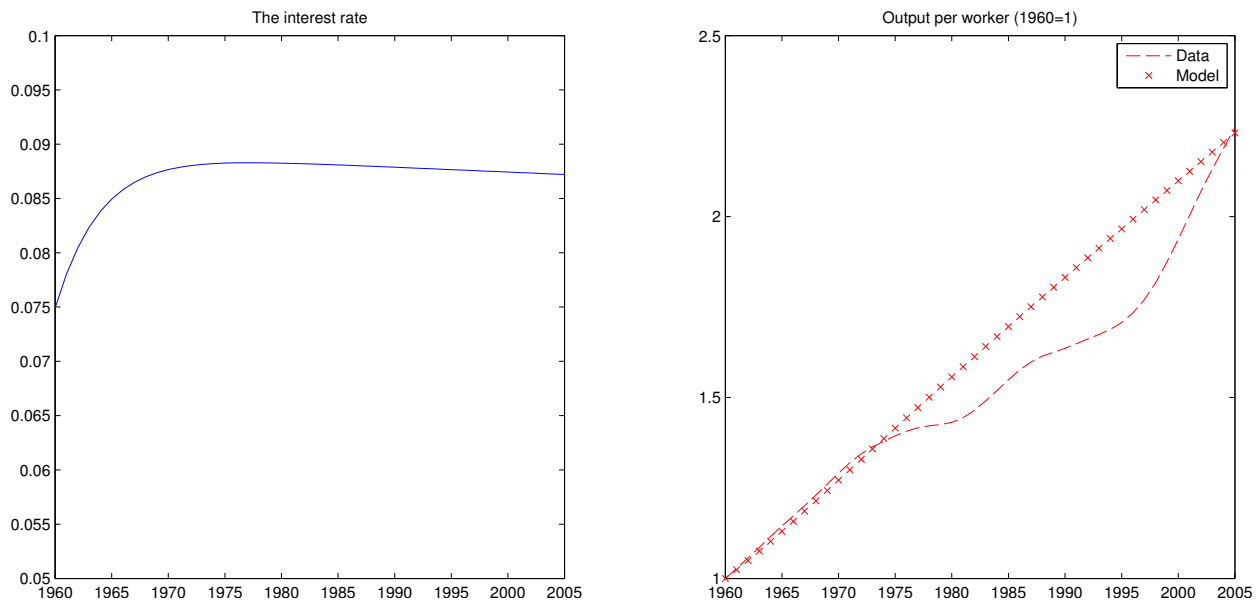
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<sup>29</sup>A rising capital income share is also qualitatively consistent with  $\sigma > 1$  and rising  $BK/(AL)$ , as in Karabarbounis and Neiman (2014). In our calibration, we also searched the region with  $\sigma_i > 1$  but could not find parameter combinations with a good fit to the data. In this region, it turns out to be much harder than in the benchmark to fit both the average labor income share in services and the average labor allocation. Parameterizations with a reasonable  $LIS_s$  feature far too little structural change, in terms of labor, capital and value added.

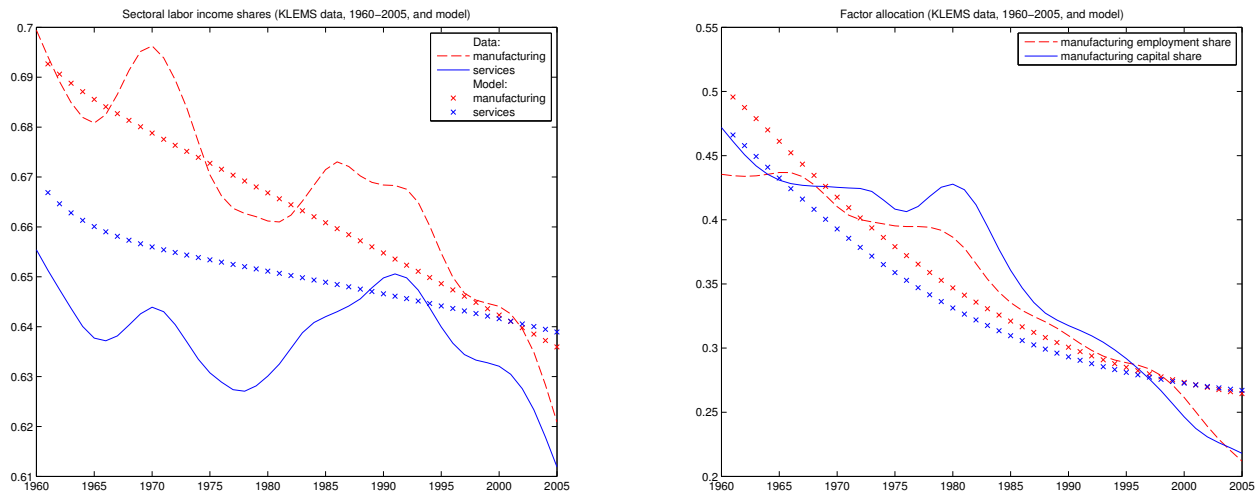
<sup>30</sup>The initial capital stock appears to be too high. This is similar to Acemoglu and Guerrieri (2008), who find a small decline in the interest rate in the first years, followed by stabilization.

both in the model and in the data, apart from some medium-run fluctuations which the model is not designed to capture. The model also fits the data well in terms of the level and change of the share of output in manufacturing; it falls by 22.9 percentage points, versus 24.5 in the data.<sup>31</sup>

Figure 5: The benchmark economy



(a) The rental rate and aggregate output per capita



(b) Factor income shares and structural change in the benchmark economy

Notes: Parameters are given in Table 3. Data is smoothed using an HP-Filter with smoothing parameter 6.25.

<sup>31</sup>Note that this was not targeted. Since the output share depends not only on the input shares but also on productivity and prices, it is not obvious that it should fit so well.

Overall, we take the fact that, despite some differences in the data used, our calibration results are close but not identical to findings in the related literature as a vindication of our approach. The literature on the estimation of CES production functions argues that using more than a single equation in estimation alleviates identification difficulties. It turns out that the effect on identified parameter values coming from the additional restrictions imposed by our approach compared to for instance the estimation in Herrendorf, Herrington and Valentinyi (2013) is moderate. The main benefit of using a full dynamic model comes into play in the decomposition exercise, to which we turn next.

### 4.3 Decomposition

In the following, we assess the relative importance of different drivers on changes in labor income shares. To do so, we explore how results change once we separately disable each one of the four fundamental drivers in the model.

To eliminate the non-homotheticity, we set  $s$  to zero. To eliminate differences in capital intensity, we set  $\bar{\theta}_m$  and  $\bar{\theta}_s$  to the average of the two, keeping  $A_{m0}$  and  $A_{s0}$  as in the benchmark. To eliminate differences across sectors in the bias of productivity growth, we set  $g(A_i) - g(B_i)$  in each sector to its average value in the two sectors, while keeping the average of  $g(A_i)$  and  $g(B_i)$  in each sector unchanged. To eliminate differences across sectors in the level of productivity growth, we set  $g(A_m) = g(A_s)$  and  $g(B_m) = g(B_s)$  and choose growth rates such that aggregate output growth and the mean difference between  $g(A_i)$  and  $g(B_i)$  remain unchanged. Finally, to eliminate differences in  $\sigma_i$ , we set  $\sigma$  in both sectors to the average, 0.673.

Decomposition results are reported in Table 4. The table shows the effect of these changes both on changes in sectoral and aggregate labor income shares and on structural change. All changes worsen the fit of the model, though only slightly for setting  $s$  to zero.

It is clear that two channels, non-homothetic preferences and differences in capital intensity, hardly matter. Setting  $s$  to zero hardly affects structural change, and therefore hardly affects labor income shares. The reason for this is that with low  $s$ , the importance of this channel is minor in any case. Eliminating differences in  $\alpha$  only affects the difference between sectoral factor income shares, but not their changes.

Differences in  $\sigma$  and in growth rates clearly have effects of a different calibre. Line 5 of the table reports results for an economy where the bias of technical change,  $g(A_i) - g(B_i)$ , is the same in both sectors. It is clear from equation (2) that this bias is a key contributor to changes in factor income shares. In the benchmark calibration, the bias in manufacturing exceeds that

Table 4: Counterfactuals

	dLIS <sub>m</sub>	dLIS <sub>s</sub>	dLIS (aggregate)	d(L <sub>m</sub> /L)	d(K <sub>m</sub> /K)	d(Y <sub>m</sub> /Y)
1. data	-5.7	-2.5	-5.4	-22.4	-21.3	-24.5
2. benchmark	-6.0	-3.1	-4.4	-23.9	-20.7	-22.9
3. $s = 0$	-6.1	-3.2	-4.5	-23.3	-20.2	-22.3
4. common $\bar{\theta}$	-6.1	-3.1	-6.0	-25.4	-22.7	-24.5
5. common growth bias	-4.3	-10.8	-9.2	-17.0	-22.0	-19.3
6. common $g(A), g(B)$	-16.0	-31.9	-22.9	10.5	-4.8	2.2
7. common $\sigma$	-8.9	-2.5	-4.1	-22.9	-16.3	-20.8

Notes: LIS stands for “labor income share” and d for the absolute change between the first and last years of the sample. All changes are in percentage points. Except for the parameters that are equated, all parameters as in the benchmark. Line 4:  $\bar{\theta}_i$  in both sectors is set to the average of  $\bar{\theta}_m$  and  $\bar{\theta}_s$  in Table 3, keeping  $A_{m1}$  and  $A_{s1}$  unchanged. Line 5:  $g(A_i) - g(B_i)$  is set to 0.0817, its average value in the calibration, for both sectors, keeping the average of  $g(A_i)$  and  $g(B_i)$  in each sector unchanged. Then, the levels of all four growth rates are shifted by the same amount to ensure that the growth rate of  $y$  is the same as in the benchmark economy. This implies  $g(A_m) = 0.0707$ ,  $g(B_m) = -0.0110$ ,  $g(A_s) = 0.0156$  and  $g(B_s) = -0.0661$ . Line 6:  $g(A_m) = g(A_s) = 0.0385$  and  $g(B_m) = g(B_s) = -0.0435$ . This value implies that the growth rate of  $y$  is the same as in the benchmark economy, and the average distance between  $g(A_i)$  and  $g(B_i)$  is preserved. Line 7:  $\sigma_m = \sigma_s = 0.673$ . This is the average of  $\sigma_m$  and  $\sigma_s$ .

in services by slightly more than 2 percentage points. Since the capital income share increases in the bias, equating the bias in the two sectors, i.e. reducing it in manufacturing and raising it in services, then implies that the change in the capital income share in manufacturing is reduced, while that in services is amplified. This occurs to a point where the change in income shares in services exceeds that in manufacturing, exactly as predicted by equation (2) for the case of equal bias and different  $\sigma_i$ . Given the increasing importance of services, this change also implies a larger decline in the aggregate labor income share.

Structural change is also affected by the change in the bias of technical change. In general, two forces determine the evolution of manufacturing capital intensity. First, in the benchmark economy, effective labor becomes relatively more abundant, and thus relatively cheaper, over time. Since the elasticity of substitution in manufacturing exceeds that in services, producers in manufacturing are better placed to take advantage of this price. As a consequence, labor intensity in manufacturing tends to increase. Second, for a given elasticity of substitution, increasing the bias of technical change towards labor in any sector requires increasing the capital intensity of production in that sector, given that capital and labor are gross complements. Hence, the stronger bias of technical change in manufacturing tends to increase capital intensity in that sector. This effect counteracts the first force. In the benchmark economy, the interaction of both forces leads to a minor increase in manufacturing capital intensity over our sample period. Equating the bias of technical change across sectors in the counterfactual implies disabling the second channel. As a result, manufacturing becomes less capital-intensive in this counterfactual.

Line 6 reports results for a counterfactual economy where not only the bias, but all growth rates are equated across sectors. The effects on changes in the labor income share are qualitatively similar to those of changing only the bias, but they are magnified. The equalization of growth rates across sectors, however, leads to a reduction in capital accumulation (which in the benchmark is boosted by strong productivity growth in manufacturing). With  $\sigma < 1$  in both sectors, this raises the capital income share, and more so in the sector that is further away from Cobb-Douglas (services). Eliminating growth rate differences also essentially stops structural change. The slowdown of capital accumulation implies that compared to the previous line, labor becomes more abundant in the economy. As a result, the more flexible sector, manufacturing, becomes more labor-intensive. Since overall there is hardly any structural change in this case, this means that the share of labor employed in manufacturing increases, while its share of capital declines.

The difference in  $\sigma$  also has a substantial effect both on changes in factor income shares



and on the shape of structural change. In the benchmark, services are further away from Cobb-Douglas, implying that for a given bias in productivity growth, the labor income share in services changes more. (Recall that this is why the benchmark calibration features a larger bias in manufacturing.) Eliminating the difference in  $\sigma$  implies making the manufacturing sector less flexible, so that its labor income share declines even more. Services, in contrast, become more flexible, leading to a smaller decline in the labor income share in that sector.

Finally, while differences in the elasticity of substitution are not the main driver of structural change, they are key for getting the shape of structural change right. Without  $\sigma$  differences, manufacturing, with its larger difference between  $g(A)$  and  $g(B)$  and its larger  $g(A)$ , shifts towards more capital-intensive production. Combined with an overall movement out of manufacturing, this implies that manufacturing sheds capital substantially more slowly than it sheds labor – differently from the data.

Observed movements in labor income shares and in sectoral allocations thus depend crucially on growth rate differences across sectors, and on differences in substitution possibilities. First, matching changing sectoral factor shares in a model with competitive labor markets requires non-unitary substitution elasticities. Second, matching different rates of change in these shares across sectors requires either differences in the bias of productivity growth or differences in substitution elasticities. Finally, lines 5 and 7 of Table 4 indicate that just one difference is not enough: matching the faster decline in the labor income share in manufacturing requires either a common bias and a higher  $\sigma$  in services (supposing that  $\sigma < 1$  in both sectors), or a common  $\sigma$  and a larger bias in manufacturing. In both cases, the shape of structural change would be different from the data: given that overall, effective labor becomes more abundant relative to effective capital, the relative capital-intensity of manufacturing would increase in both cases.<sup>32</sup> This contrasts with the data, where it is virtually constant. Hence, the joint analysis of the evolution of sectoral factor income shares and structural change leads us to the conclusion that sectoral differences both in factor substitutability and in factor-specific rates of technical change are required to match observed trends.

Returning to the trends in the data, our results suggest that declines in sectoral labor income shares have been driven by effective labor becoming more abundant relative to effective capital in a world where the two inputs are gross complements. This feeds through from sectors to the aggregate labor income share. The decline in the manufacturing labor

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<sup>32</sup>In the first case, services – here hypothetically the more flexible sector – would increase intensity of use of the factor that becomes more abundant: labor. In the second case, as labor becomes more abundant in manufacturing more quickly than in services, manufacturing would increase its capital intensity.

income share is particularly severe because the gap between labor- and capital-augmenting technical progress has been larger in that sector.

## 5 Conclusion

In this paper, we trace the decline in the aggregate labor income share, which has occurred in several developed countries, to developments at the sectoral level. In line with this, we establish a connection between this decline and another important development that has been taking place over the last half century: large reallocations of resources and production from manufacturing to services.

We show that to account for both developments, a simple dynamic two-sector model needs to feature cross-sectoral differences both in capital-labor substitutability and in the “bias” of productivity growth, i.e. in labor- versus capital-augmenting technical change. Overall, the key driver of the decline in the labor income share is the speed of labor-augmenting technical change, which outweighs economy-wide growth in effective capital. When capital and labor are complements in production, this increase in the relative abundance of effective labor reduces the labor income share.

This mechanism also affects structural change, in an intuitive way. (See Alvarez-Cuadrado et al. (2015) for more detail.) As the relative abundance of inputs of production changes with the process of economic growth, so does their relative price. In the presence of sectoral differences in the elasticity of substitution, this induces a process of factor reallocation with relatively abundant factors moving towards relatively flexible sectors, i.e. sectors with a relatively high elasticity of substitution. While our quantitative exercise clearly points to differences in productivity growth rates across sectors as the main driver of structural change, this new mechanism is required to match observed patterns of structural change in terms of capital and labor.

In this context, it is natural to wonder what kept the U.S. labor income share roughly constant for most of the last century. One possibility is that, as a result of structural change, sectoral changes canceled out in the aggregate. The careful estimates of sectoral labor income shares constructed by Valentinyi and Herrendorf (2008) show that in 1997, the labor income share in non-agriculture was 50% larger than that in agriculture, a difference of 21 percentage points. Caselli and Coleman II (2001) document a decrease in U.S. agricultural employment of 40 percentage points between 1880 and 1960. Over this period, the fraction of value added produced in agriculture declined by 21 percentage points. Combining these pieces of evidence, structural change out of agriculture by itself would generate an increase

in the aggregate labor income share of the order of 4.5 percentage points. This development would have counteracted a decline in the labor income share driven by manufacturing. In view of these calculations, it is possible that at the aggregate level, changes in the labor income shares in manufacturing and agriculture cancelled each other. This would be in line with Keynes's (1939) observation (cited in Elsby et al. 2013, p. 13) that the "remarkable constancy" of the aggregate labor share is "a bit of a miracle".

Another possibility, along the lines suggested by Acemoglu (2002), is that the mix of capital- and labor-augmenting technological change was different in the past, leading sector-level changes in income shares to balance at the aggregate level. At this stage, these are conjectures; they constitute interesting topics for future work.

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# Appendix

## A Data

### A.1 Computation of labor income shares

This section closely follows Valentinyi and Herrendorf (2008). Let the number of commodities be  $M$  and the number of industries be  $I$ . Index commodities by  $m$  and industries by  $i$ . From the 35-sector KLEM data, we compute the following objects:

- The “use matrix”  $\mathbf{B}$ : this is an  $M \times I$  matrix with representative element  $(m, i)$  that states how much output of commodity  $m$  is required to produce 1\$ of output of industry  $i$ .
- The “make matrix”  $\mathbf{W}$ : this is an  $I \times M$  matrix with representative element  $(i, m)$  that states for industry  $i$  which share of commodity  $m$  it produces.
- The final expenditure vector  $\mathbf{y}$ : this length- $I$  column vector states the amount of output of each industry  $i$  that serves as final expenditure.

In our setting, we have no commodity level information and have to assume that each industry produces a single commodity. Therefore,  $M = I$ .

The final expenditure vector  $\mathbf{y}$  can be computed by subtracting each industry’s output that is used as an intermediate input in another industry from the total value of its output. The use matrix  $\mathbf{B}$  can be computed by dividing the value of intermediate inputs from industry  $M$  used in industry  $I$  by the value of gross output in industry  $I$ . When each industry produces a single, unique commodity, the make matrix  $\mathbf{W}$  simply is the  $I \times I$  identity matrix.

Let the column vector of sectoral shares of labor income in the value of gross output be  $\alpha_l$  and the column vector of the sectoral shares of capital income in the value of gross output be  $\alpha_k$ . Let the  $I \times 1$  sector identification vector with elements  $\mathbb{1}_{i=j}$  for a sector  $j$  be  $\mathbf{1}_j$  and let  $\mathbf{y}_j \equiv \mathbf{y}'\mathbf{1}_j$ . Following Valentinyi and Herrendorf (2008), we then obtain the share of labor income in value added in sector  $j$  as

$$\frac{\alpha_l' \mathbf{W} (\mathbf{I} - \mathbf{B}\mathbf{W})^{-1} \mathbf{y}_j}{(\alpha_l' + \alpha_k') (\mathbf{I} - \mathbf{B}\mathbf{W})^{-1} \mathbf{y}_j}, \quad (22)$$

### A.2 Sector classification.

**U.S. data:** Using data from Jorgenson’s (2007) 35-sector data base, we construct sectors as follows:

**Manufacturing:** Sectors 7-27: Food and kindred products, Tobacco, Textile mill products, Apparel, Lumber and wood, Furniture and fixtures, Paper and allied, Printing, publishing and allied, Chemicals, Petroleum and coal products, Rubber and misc plastics, Leather, Stone, clay, glass, Primary metal, Fabricated metal, Non-electrical machinery, Electrical machinery, Motor vehicles, Transportation equipment & ordnance, Instruments, Misc. manufacturing

**Services:** Transportation, Communications, Electric utilities, Gas utilities, Trade, Finance Insurance and Real Estate, Other Services

Results from Section 2 are not sensitive to excluding Utilities from Services or to including Mining and Construction with Manufacturing, or Government in Services.

**Cross-country data:** Using EU-KLEMS data from <http://www.euklems.net>, we define manufacturing analogously to the U.S. data. Again, as there, we include utilities (sector E), wholesale and retail trade (G), hotels and restaurants (H), transport, storage and communication (I), financial intermediation (J), real estate etc. (K) in services. We exclude public administration and defence (L), education (M), health and social work (N), other community services (O), private households (P).

### **A.3 The measurement of industry factor income shares.**

Jorgenson et al. (1987) contains a detailed description of measurement of industry-level factor income shares. The following are some key features. Labor compensation is measured as wage and salary income plus supplements, including employers' contribution to Social Security and unemployment compensation contributions by employers. Annual measures are computed using time actually worked. Earnings of the self-employed are split between capital and labor compensation assuming an after-tax rate of return that matches that of corporate businesses.



## B Details on Section 3

### B.1 Details on the model solution

Time is continuous. We model a closed economy populated by a representative households with preferences given by,

$$v = \ln(u(c_m, c_s)) \quad \text{where} \quad u(c_m, c_s) = \left( \gamma c_m^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma)(c_s + s)^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}} \quad (23)$$

where the variables and parameters are described in the body of the paper and instantaneous utility is discounted at a rate  $\rho$ . Sectoral outputs are produced according to the general CES technologies

$$Y_m = \left[ \alpha_m (B_m K_m)^{\frac{\sigma_m-1}{\sigma_m}} + (1-\alpha_m) (A_m L_m)^{\frac{\sigma_m-1}{\sigma_m}} \right]^{\frac{\sigma_m}{\sigma_m-1}} \quad (24)$$

$$Y_s = \left[ \alpha_s (B_s K_s)^{\frac{\sigma_s-1}{\sigma_s}} + (1-\alpha_s) (A_s L_s)^{\frac{\sigma_s-1}{\sigma_s}} \right]^{\frac{\sigma_s}{\sigma_s-1}}. \quad (25)$$

Factors are fully utilized. Normalizing the labor endowment to one, this implies

$$\frac{L_m}{L} + \frac{L_s}{L} \equiv l_m + l_s = 1 \quad (26)$$

$$l_m k_m + l_s k_s = l_m k_m + (1-l_m) k_s = k, \quad \text{where } k_i \equiv \frac{K_i}{L_i}. \quad (27)$$

We choose manufactures to be the numeraire and denote the price of services by  $p_s$ . Since markets are competitive, production efficiency requires equating marginal revenue products across sectors, so

$$\alpha_m B_m^{\frac{\sigma_m-1}{\sigma_m}} \left( \frac{Y_m}{K_m} \right)^{\frac{1}{\sigma_m}} = R = p_s \alpha_s B_s^{\frac{\sigma_s-1}{\sigma_s}} \left( \frac{Y_s}{K_s} \right)^{\frac{1}{\sigma_s}} \quad (28)$$

$$(1-\alpha_m) A_m^{\frac{\sigma_m-1}{\sigma_m}} \left( \frac{Y_m}{L_m} \right)^{\frac{1}{\sigma_m}} = w = p_s (1-\alpha_s) A_s^{\frac{\sigma_s-1}{\sigma_s}} \left( \frac{Y_s}{L_s} \right)^{\frac{1}{\sigma_s}}. \quad (29)$$

As a consequence, the following relationship between the sectoral capital-labor ratios emerges.

$$k_s = \left( \frac{1-\alpha_m}{1-\alpha_s} \frac{\alpha_s}{\alpha_m} \left( \frac{A_m}{B_m} \right)^{\frac{\sigma_m-1}{\sigma_m}} \right)^{\sigma_s} \left( \frac{A_s}{B_s} \right)^{1-\sigma_s} k_m^{\frac{\sigma_s}{\sigma_m}} \equiv \varrho k_m^{\frac{\sigma_s}{\sigma_m}}, \quad (30)$$

where  $\varrho \equiv \left( \frac{1-\alpha_m}{1-\alpha_s} \frac{\alpha_s}{\alpha_m} \left( \frac{A_m}{B_m} \right)^{\frac{\sigma_m-1}{\sigma_m}} \right)^{\sigma_s} \left( \frac{A_s}{B_s} \right)^{1-\sigma_s}$ . Then, using (27), the economy-wide capital-labor ratio can be written as

$$k = l_m k_m + l_s k_s = l_m k_m + (1-l_m) \varrho k_m^{\frac{\sigma_s}{\sigma_m}}. \quad (31)$$

Using this notation, the relative price is given by

$$p_s = \frac{\alpha_m B_m^{\frac{\sigma_m-1}{\sigma_m}} \left[ \alpha_m B_m^{\frac{\sigma_m-1}{\sigma_m}} + (1 - \alpha_m) (A_m/k_m)^{\frac{\sigma_m-1}{\sigma_m}} \right]^{\frac{1}{\sigma_m-1}}}{\alpha_s B_s^{\frac{\sigma_s-1}{\sigma_s}} \left[ \alpha_s B_s^{\frac{\sigma_s-1}{\sigma_s}} + (1 - \alpha_s) (A_s/(\varrho k_m^{\frac{\sigma_s}{\sigma_m}})) \right]^{\frac{1}{\sigma_s-1}}}. \quad (32)$$

On the household side, the representative agent equates the marginal rate of substitution between the two goods to their relative price, so

$$\frac{v_s}{v_m} = \frac{1 - \gamma}{\gamma} \left( \frac{c_s + s}{c_m} \right)^{-\frac{1}{\varepsilon}} = p_s. \quad (33)$$

since  $v_s = \frac{u_s}{u} = u^{\frac{1-\varepsilon}{\varepsilon}} (1 - \gamma) (c_s + s)^{-\frac{1}{\varepsilon}}$ . Define the following variables.

$$x_s \equiv \frac{p_s c_s}{c_m} \quad \text{and} \quad x_m \equiv \frac{c_m}{c_m} = 1. \quad (34)$$

Then, total consumption expenditure  $c = p_s c_s + c_m$  relative to consumption expenditure on manufactures is given by

$$X = x_s + x_m = \frac{p_s c_s + c_m}{c_m} \equiv \frac{c}{c_m}. \quad (35)$$

Now we are in a position to determine the static allocation of resources across sectors. Combining (32), (34), and (35) we can express consumption of services as

$$c_s = \frac{x_s c_m}{p_s} = x_s \frac{c}{X} \frac{\alpha_s B_s^{\frac{\sigma_s-1}{\sigma_s}} \left[ \alpha_s B_s^{\frac{\sigma_s-1}{\sigma_s}} + (1 - \alpha_s) (A_s/(\varrho k_m^{\frac{\sigma_s}{\sigma_m}})) \right]^{\frac{1}{\sigma_s-1}}}{\alpha_m B_m^{\frac{\sigma_m-1}{\sigma_m}} \left[ \alpha_m B_m^{\frac{\sigma_m-1}{\sigma_m}} + (1 - \alpha_m) (A_m/k_m)^{\frac{\sigma_m-1}{\sigma_m}} \right]^{\frac{1}{\sigma_m-1}}}. \quad (36)$$

Since service output is fully consumed,  $c_s$  is also given by

$$c_s = y_s = \left[ \alpha_s \left( B_s \varrho k_m^{\frac{\sigma_s}{\sigma_m}} \right)^{\frac{\sigma_s-1}{\sigma_s}} + (1 - \alpha_s) A_s^{\frac{\sigma_s-1}{\sigma_s}} \right]^{\frac{\sigma_s}{\sigma_s-1}} l_s. \quad (37)$$

Combining the previous two expressions, we can solve for  $l_m$  as a function of current values of the endogenous variables,  $k_m$  and  $c_m$ , given the exogenous levels of labor- and capital-augmenting productivity.

$$l_m = 1 - \frac{x_s c}{X} \frac{\alpha_s B_s^{\frac{\sigma_s-1}{\sigma_s}} \left[ \alpha_s (B_m k_m)^{\frac{\sigma_m-1}{\sigma_m}} + (1 - \alpha_m) A_m^{\frac{\sigma_m-1}{\sigma_m}} \right]^{\frac{1}{1-\sigma_m}}}{\alpha_m B_m^{\frac{\sigma_m-1}{\sigma_m}} \left[ \alpha_s B_s^{\frac{\sigma_s-1}{\sigma_s}} \varrho k_m^{\frac{\sigma_s-1}{\sigma_m}} + (1 - \alpha_s) A_s^{\frac{\sigma_s-1}{\sigma_s}} \varrho^{\frac{1}{\sigma_s}} \right]} \quad (38)$$

This labor allocation together with the full employment condition for capital given by (31) determines the current sectoral allocation of resources.

Now we turn to the dynamic evolution of the economy. The Euler equation for this problem is given by

$$-\hat{v}_m = \alpha_m B_m^{\frac{\sigma_m-1}{\sigma_m}} \left[ \alpha_m B_m^{\frac{\sigma_m-1}{\sigma_m}} + (1 - \alpha_m) A_m^{\frac{\sigma_m-1}{\sigma_m}} k_m^{\frac{1-\sigma_m}{\sigma_m}} \right]^{\frac{1}{\sigma_m-1}} - (\delta + n + \rho). \quad (39)$$

Since preferences are homogeneous of degree 1,

$$u = u_m c_m + u_s (c_s + s) = u_m c_m + p_s u_m (c_s + s) = (c_m + p_s (c_s + s)) u_m = (c + p_s s) u_m, \quad (40)$$

where the second equality uses  $v_m = \frac{u_m}{u}$  and (33) and the last one uses (35). As a result,

$$v_m = \frac{1}{c + p_s s}. \quad (41)$$

The per capita capital stock evolves according to

$$\dot{k} = \left[ \alpha_m (B_m k_m)^{\frac{\sigma_m-1}{\sigma_m}} + (1 - \alpha_m) A_m^{\frac{\sigma_m-1}{\sigma_m}} \right]^{\frac{\sigma_m-1}{\sigma_m}} l_m - c_m - (\delta + n)k. \quad (42)$$

Combined with (31), this determines the law of motion of  $k_m$ .

## B.2 Discrete-time version of the model

The discrete time counterparts of (38), (39) and (42) are given by,

$$l_{m,t} = 1 - \frac{x_{s,t}}{X_t} c_t \frac{\alpha_s B_{s,t}^{\frac{\sigma_s-1}{\sigma_s}}}{\alpha_m B_{m,t}^{\frac{\sigma_m-1}{\sigma_m}}} \frac{\left[ \alpha_m (B_{m,t} k_{m,t})^{\frac{\sigma_m-1}{\sigma_m}} + (1 - \alpha_m) (A_{m,t})^{\frac{\sigma_m-1}{\sigma_m}} \right]^{\frac{1}{1-\sigma_m}}}{\left[ \alpha_s B_{s,t}^{\frac{\sigma_s-1}{\sigma_s}} \varrho_t k_{m,t}^{\frac{\sigma_s-1}{\sigma_m}} + (1 - \alpha_s) A_{s,t}^{\frac{\sigma_s-1}{\sigma_s}} \varrho_t^{\frac{1}{\sigma_s}} \right]} \quad (43)$$

$$c_{t+1} = (c_t + s p_t^s) \frac{\beta}{1+n} \left( \alpha_m B_{m,t+1}^{\frac{\sigma_m-1}{\sigma_m}} \left[ \alpha_m B_{m,t+1}^{\frac{\sigma_m-1}{\sigma_m}} + (1 - \alpha_m) A_{m,t+1}^{\frac{\sigma_m-1}{\sigma_m}} k_{m,t+1}^{\frac{1-\sigma_m}{\sigma_m}} \right]^{\frac{1}{\sigma_m-1}} + (1 - \delta) \right) - s p_{t+1}^s \quad (44)$$

$$k_{m,t+1} = \frac{1}{(1+n) l_{m,t+1}} \left[ y_{m,t} - \frac{c_t}{X_t} + (1 - \delta) \left( l_{m,t} k_{m,t} + (1 - l_{m,t}) \varrho_t k_{m,t}^{\frac{\sigma_s}{\sigma_m}} \right) - (1+n) (1 - l_{m,t+1}) \varrho_{t+1} k_{m,t+1}^{\frac{\sigma_s}{\sigma_m}} \right], \quad (45)$$

where  $\beta = e^{-\rho}$  is the discrete time subjective discount factor and

$$y_{m,t} = \left[ \alpha_m (B_{m,t} k_{m,t})^{\frac{\sigma_m-1}{\sigma_m}} + (1 - \alpha_m) A_{m,t}^{\frac{\sigma_m-1}{\sigma_m}} \right]^{\frac{\sigma_m}{\sigma_m-1}} l_{m,t}$$

$$p_{s,t} = \frac{\alpha_m B_{m,t}^{\frac{\sigma_m-1}{\sigma_m}} \left[ \alpha_m B_{m,t}^{\frac{\sigma_m-1}{\sigma_m}} + (1 - \alpha_m) (A_{m,t}/k_{m,t})^{\frac{\sigma_m-1}{\sigma_m}} \right]^{\frac{1}{\sigma_m-1}}}{\alpha_s B_{s,t}^{\frac{\sigma_s-1}{\sigma_s}} \left[ \alpha_s B_{s,t}^{\frac{\sigma_s-1}{\sigma_s}} + (1 - \alpha_s) (A_s/(\varrho_t k_{m,t}^{\frac{\sigma_s}{\sigma_m}}))^{\frac{\sigma_s-1}{\sigma_s}} \right]^{\frac{1}{\sigma_s-1}}}$$

$$x_{s,t} = \frac{\left( \frac{1-\gamma}{\gamma} \right)^\varepsilon p_{s,t}^{1-\varepsilon} - \frac{p_{s,t}^S}{c_t}}{1 + \frac{p_{s,t}^S}{c_t}}$$

$$X_t = x_{s,t} + 1$$

### B.3 The Aggregate Elasticity of Substitution

This proof follows Jones (1965) and Miyagiwa and Papageorgiou (2007). The dual relationship between sectoral prices and input prices and factor endowments and sectoral outputs are given by

$$C_s(w, R) = p_s \quad (46)$$

$$C_m(w, R) = p_m \quad (47)$$

$$Y_s C_{sw} + Y_m C_{mw} = L \quad (48)$$

$$Y_s C_{sR} + Y_m C_{mR} = K \quad (49)$$

where  $C_i(w, R)$  is the minimum unit cost function for sector  $i = s, m$ . It is linearly homogeneous.  $C_{ij}$  are its partial derivatives with respect to each factor price  $j = w, R$ . By Shepard's lemma,

$$C_{iw} = L_i/Y_i \quad (50)$$

$$C_{iR} = K_i/Y_i \quad (51)$$

are the unit factor demands.

Differentiating the previous expressions we reach the following relationships.

$$(1 - \epsilon_s) \hat{w} + \epsilon_s \hat{R} = \hat{p}_s \quad (52)$$

$$(1 - \epsilon_m) \hat{w} + \epsilon_m \hat{R} = \hat{p}_m \quad (53)$$

$$\lambda \left( \hat{Y}_s + \hat{C}_{sw} \right) + (1 - \lambda) \left( \hat{Y}_m + \hat{C}_{mw} \right) = \hat{L} \quad (54)$$

$$\kappa \left( \hat{Y}_s + \hat{C}_{sR} \right) + (1 - \kappa) \left( \hat{Y}_m + \hat{C}_{mR} \right) = \hat{K}, \quad (55)$$

where we use the fact that sectoral production functions are homogeneous of degree one.  $\epsilon_i$  denotes the capital income share in sector  $i = s, m$ , and  $\lambda \equiv L_s/L$  and  $\kappa \equiv K_s/K$  are the fractions of labor and capital used in the service sector, respectively.

Subtracting (52) from (53),

$$(\epsilon_m - \epsilon_s) (\hat{w} - \hat{R}) = \hat{p}_s - \hat{p}_m. \quad (56)$$

Rewriting the sector-specific elasticity of substitution  $\sigma_i$  as  $\frac{C_i C_{iwR}}{C_{iw} C_{iR}}$  and expressing the factor income shares as  $\epsilon_i = \frac{r C_{iR}}{C_i}$  and  $1 - \epsilon_i = \frac{w C_{iw}}{C_i}$ , we reach the following rates of change of partial derivatives of the unit cost functions.<sup>33</sup>

$$\begin{aligned} \hat{C}_{iw} &= \frac{C_{iww} dw + C_{iwr} dR}{C_{iw}} = \frac{-C_{iwr} \frac{R}{w} dw + C_{iwr} dR}{C_{iw}} = \\ &= -\frac{(C_{iwr} R \hat{w} - C_{iwr} dR)}{C_{iw}} = -\frac{C_{iwr} R}{C_{iw}} (\hat{w} - \hat{R}) = -\frac{C_i C_{iwr}}{C_{iw} C_{iR}} \frac{R C_{iR}}{C_i} (\hat{w} - \hat{R}), \end{aligned}$$

where the second equality uses the fact that  $C_{iw}(w, R)$  is homogeneous of degree 0. As a result,

$$\hat{C}_{iw} = -\sigma_i \epsilon_i (\hat{w} - \hat{R}) \quad (57)$$

$$\hat{C}_{iR} = \sigma_i (1 - \epsilon_i) (\hat{w} - \hat{R}) \quad (58)$$

Replacing (57) and (58) in (54) and (55) and subtracting them we reach,

$$(\lambda - \kappa) (\hat{Y}_s - \hat{Y}_m) = (\hat{L} - \hat{K}) + \Theta (\hat{w} - \hat{R}) \quad (59)$$

where  $\Theta \equiv \lambda \sigma_s \epsilon_s + (1 - \lambda) \sigma_m \epsilon_m + \kappa \sigma_s (1 - \epsilon_s) + (1 - \kappa) \sigma_m (1 - \epsilon_m)$ .

Then we combine (33) and (56) to reach

$$\frac{\dot{c}_s}{c_s + s} - \hat{c}_m = -\varepsilon (\epsilon_m - \epsilon_s) (\hat{w} - \hat{R}) \quad (60)$$

Since the aggregate elasticity of substitution is defined as  $\sigma \equiv -\frac{(\hat{L} - \hat{K})}{(\hat{w} - \hat{R})}$  we combine

the fact that services are fully consumed, so  $Y_s = Lc_s$ , (42), (59) and (60) to reach,

<sup>33</sup>The standard definition of the elasticity of substitution between capital and labor is given by  $\sigma_i \equiv \frac{d \ln(K_i/L_i)}{d \ln(w/R)} = \frac{d \ln(C_{iR}/C_{iw})}{d \ln(w/R)}$ , where the last expression uses (50) and (51). The numerator can be written as  $d \ln(C_{iR}/C_{iw}) = \frac{(C_{iRw} C_{iw} - C_{iww} C_{iR}) dw + (C_{iRR} C_{iw} - C_{iwr} C_{iR}) dR}{C_{iR} C_{iw}}$ . Since  $C_{ij}$  is homogeneous of degree zero,  $C_{iww} = -C_{iwr} \frac{R}{w}$  and  $C_{iRR} = -C_{iRw} \frac{w}{R}$ . Inserting these in the previous expression, we reach  $d \ln(C_{iR}/C_{iw}) = \frac{C_{iRw}}{C_{iR} C_{iw}} d \ln(w/R)$ , which leads to the definition in the text.

$$\sigma = \gamma_0 \varepsilon + \gamma_1 \sigma_s + \gamma_2 \sigma_m + \frac{(\lambda - \kappa)}{(\hat{w} - \hat{R})} \left( \hat{Y}_m - n - \hat{c}_m \left( 1 + \frac{s}{c_s} \right) \right) \quad (61)$$

where,

$$\begin{aligned} \gamma_0 &\equiv (\epsilon_m - \epsilon_s) (\lambda - \kappa) \left( 1 + \frac{s}{c_s} \right) \\ \gamma_1 &\equiv \lambda \epsilon_s + \kappa (1 - \epsilon_s) \\ \gamma_2 &\equiv (1 - \lambda) \epsilon_m + (1 - \kappa) (1 - \epsilon_m) \end{aligned}$$

## C Robustness: Quantitative results with $\varepsilon = 0.5$

We also compute results for an economy where preferences over manufacturing and services consumption are not Leontief. Just as Buera and Kaboski (2009), we consider a value of  $\varepsilon$  of 0.5. While far away from Leontief, this still implies that manufacturing and services output clearly are gross complements in consumption. We recalibrate the model in this setting, and conduct the same decomposition as above.

Table 5 shows calibration results. The calibration for higher  $\varepsilon$  clearly fits less well. While the model replicates changes in the factor income shares and structural change well, it proved impossible to get levels of factor income shares and the initial labor allocation to fit as closely as in our benchmark calibration. In particular, the average labor income share in services does not fit well. This also affects the fit of the aggregate labor income share. Note that changes in both the labor income share and in allocations do fit rather well, though. Parameters are overall similar; the manufacturing sector is more flexible, and growth rate patterns are as above, driven again by the same data patterns.

Broadly speaking, decomposition results are similar to those in the Leontief case. Non-homotheticity of preferences hardly affects changes in labor income shares or structural change. Common  $\alpha$  here has some effect, though this is also due to the fact that in this calibration, the average labor income share in services is off by 8 percentage points. Differences in  $\sigma$  and in growth rates again prove to be the most important determinants of changes in labor income shares and structural change. Eliminating differences in growth rates essentially eliminates structural change in terms of value added; the value added share of manufacturing falls by only half a percentage point. There is still substantial factor reallocation because of  $\sigma$  differences, though: Since overall, growth rates are such that efficiency units of labor become more abundant relative to capital, the more flexible sector (manufacturing) substitutes towards that more abundant input. Without structural change on average, this implies that labor flows into manufacturing, while the fraction of capital used in the sector declines.

Eliminating differences in  $\sigma$  in this setting affects not only the shape but also the amount of structural change. As above, a larger difference between  $g(A)$  and  $g(B)$  in manufacturing

Table 5: Calibration for  $\varepsilon = 0.5$ .

<i>Panel A: Targets and model moments</i>				
	Data		Model	
	Manufacturing	Services	Manufacturing	Services
<i>calibration targets:</i>				
$\overline{LIS}_i$ (%)	66.7	63.8	65.7	71.9
$g(KIS_i/LIS_i)$ (% p.a.)	0.6%	0.3%	0.60%	0.31%
$d(L_m/L)$	-22.4		-22.6	
$d(K_m/K)$	-21.3		-20.9	
$\overline{L_m/L}$	34.2		36.5	
$g(Y/L)$ (% p.a.)	1.8%		1.78%	
<i>not targeted:</i>				
$\overline{LIS}$ (aggregate, %)	64.8		69.5	
$g(KIS/LIS)$ (agg., % p.a.)	0.5		0.3	
$\overline{K_m/K}$	33.2		44.1	
$\overline{Y_m/Y}$	35.1		38.6	
$d(Y_m/Y)$	-24.5		-21.9	
<i>Panel B: Parameters</i>				
<i>sector-specific:</i>	Manufacturing	Services	<i>general:</i>	
$g(A_i)$ (% p.a.)	9.8	0.8	$\delta$	0.05
$g(B_i)$ (% p.a.)	-2.6	-6.9	$\beta$	0.94
$\sigma_i$	0.875	0.534		
$A_{i1}$	0.641	1.143		
$\bar{\theta}_i$ (normalization)	0.333	0.362		

Note: The data period used is 1960 to 2005. Averages are geometric means over this period. Absolute changes are percentage point changes over the entire period. Growth rates are computed using averages of the first and last 5 years. For readability, annual percentage changes are given for output growth and the growth in relative income shares.

implies that if  $\sigma$  is equal, labor moves towards services more quickly than capital does. With structural change taking place at the same time, this results in a decline of the fraction of capital used in manufacturing that is slower than the decline of the fraction of labor employed in manufacturing. The changes in sectoral labor income shares are in line with this. Differences in growth rates tend to lead to a faster decline of the labor income share in manufacturing. The higher substitution elasticity in manufacturing counteracts this; eliminating

Table 6: Counterfactuals ( $\varepsilon = 0.5$ )

	dLIS <sub>m</sub>	dLIS <sub>s</sub>	dLIS (aggregate)	d( $L_m/L$ )	d( $K_m/K$ )	d( $Y_m/Y$ )
data	-5.7	-2.5	-5.4	-22.4	-21.3	-24.5
best fit	-6.1	-2.8	-2.4	-22.6	-20.9	-21.9
$s = 0$	-6.1	-2.9	-2.7	-21.6	-19.6	-20.8
common $\bar{\theta}$	-5.0	0.3	-0.4	-24.5	-20.6	-23.4
common $g(A)$	-11.8	-44.1	-25.6	14.4	-18.9	-0.6
common $\sigma$	-15.2	-2.6	-6.0	-17.5	-6.0	-13.8

Notes: LIS stands for “labor income share” and d for the absolute change between the first and last years of the sample. All changes are in percentage points. Except for the parameters that are equated, all parameters as in Table 5. Line 4:  $\bar{\theta}_i$  in both sectors is set to the average of  $\bar{\theta}_m$  and  $\bar{\theta}_s$  in Table 5, keeping  $A_{m1}$  and  $A_{s1}$  unchanged. Line 5:  $g(A_m) = g(A_s) = 0.0462$  and  $g(B_m) = g(B_s) = -0.0543$ . This value implies that the growth rate of  $y$  is the same as in the benchmark economy, and the average distance between  $g(A_i)$  and  $g(B_i)$  is preserved. Line 6:  $\sigma_m = \sigma_s = 0.705$ . This is the average of  $\sigma_m$  and  $\sigma_s$ .

the  $\sigma$  difference then makes the labor income share in manufacturing decline much faster than that in services. The largest difference between this case and the benchmark is that equating  $\sigma$  affects not only the shape of structural change, but also how much structural change takes place overall: the decline in the share of value added produced in the manufacturing sector drops from 22.9 percentage points in the benchmark calibration to 13.8 percentage points here.

Overall, these results are in line with our benchmark results above. We can thus conclude that it is the combination of differences in  $\sigma$  and in growth rates across sectors that are key for understanding the evolution of factor income shares and structural change from manufacturing to services.