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Does Everyone Use Probabilities?
Intuitive and Rational Decisions about Stockholding

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ABSTRACT

Does Everyone Use Probabilities? Intuitive and Rational Decisions about Stockholding*

We investigate the relationship between subjective probabilities of future stock market returns and decisions about stockholding. Specifically, we examine whether acting upon subjective probabilities is confined to individuals with high cognitive skills. We explore this question using data from the U.S. Health and Retirement Study (HRS). Our empirical analysis is guided by a novel and simple model based on the dual-systems framework from psychology (Kahneman, 2003). In our model, individuals with low cognitive skills make decisions in an intuitive non-probabilistic way based on cues and feelings. Individuals with high cognitive skills make decisions akin to the expected utility model. As predicted by our model, in our empirical analysis we find that there is a significantly stronger association between subjective return probabilities and stockholding decisions for individuals with high cognitive skills, compared to individuals with lower cognitive skills. The paper contributes to a better understanding of the role of cognitive skills in decision making under uncertainty.

JEL Classification: D03, D81, D84, G11

Keywords: subjective expectations, probabilities, dual system decision making, cognitive skills, cognitive economics

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1 Introduction

Standard economic theory assumes that decision makers form beliefs about probabilities of uncertain future events, and that they act upon these beliefs (von Neumann and Morgenstern, 1947). In this study, we examine whether acting upon subjective probabilities is a universal pattern of decision making under uncertainty, or whether it is rather confined to particular subgroups of the population. Specifically, acting upon subjective probabilities may require substantial cognitive skills (Peters, 2008). As a result, subjective probabilities may be important determinants of decision making for individuals with high cognitive skills, but less so for individuals with low cognitive skills.

In our study we focus on the relationship between subjective probabilities of future stock market returns and decisions about stockholding. Specifically, we consider whether there is a stronger association between subjective probabilities and stockholding decisions for individuals with high cognitive skills, compared to those with lower cognitive skills. As a measure for subjective probabilities we use the answers to a survey question about the percent chance that returns for a broad stock index would be positive over the next 12 months. We do not assume that there are correct or incorrect views about future stock market returns. There is room for disagreement, even among experts.[1]

Previous studies show that there is substantial heterogeneity in beliefs about future stock market returns (Hurd, 2009; Hudomiet, Kézdi, and Willis, 2011). In two important studies, Hurd, van Rooij, and Winter (2011), and Kézdi and Willis (2011) find that subjective expectations about stock market returns are meaningfully correlated with stock holding decisions. Hurd and Rohwedder (2011) investigate how subjective expectations of stock market returns react to past movements in stock markets using monthly data over the course of the financial crisis. They also show that stock-trading tends to respond to changes in expectations. Similar results have been found in many other areas. Subjective

[1] For example, at the beginning of our study period renowned finance professors Jeremy Siegel and Robert Shiller offered very different outlooks about future stock returns in their bestselling books *Stocks for the Long Run* and *Irrational Exuberance,* respectively.

probabilities are related to actual decisions for saving and consumption (Gan, Hurd, and McFadden, 2004; Kapteyn, Kleinjans, and van Soest, 2009; Salm, 2010), claiming Social Security (Hurd, Smith, and Zissimopoulos, 2004; Delavande, Perry, and Willis, 2006), health behaviors (Fang, Keane, Khwaja, Salm, and Silverman, 2006; Carman and Kooreman, 2011), and committing crimes (Lochner, 2007).[2]

These findings seem to conflict with evidence from the psychological literature that individuals with low numeracy and cognitive ability often have difficulties in understanding probabilities (Bruine de Bruin, Fischhoff, Millstein, and Halpern-Felsher, 2000; Peters, 2008; Peters and Levin, 2008; Bruine de Bruin and Carman, 2012). For instance, women with low numeracy skills tend to misunderstand information about the reduction of mortality risk through mammograms (Schwartz, Woloshin, Black, and Welch, 1997). Similarly, individuals with low numeracy skills find it difficult to compare the relative merits of different medical treatments with varying survival probabilities (Zikmund-Fisher, Smith, Ubel, and Fagerlin, 2007).

In this paper we explore whether the correlation between behavior and subjective probabilities is mediated through individuals with high cognitive skills. We develop a novel and simple model of stockholding decisions. The model explicitly takes into account the role of cognitive skills for decision making under uncertainty.[3] Our model builds on the psychological literature on dual system processing (Stanovich and West, 2000; Kahneman, 2003, 2011; Kahneman and Frederick, 2005).[4] In our model, individuals with low cognitive skills make decisions in an intuitive non-probabilistic way based on cues and feelings, i.e. whether they feel that they "like" stocks. In the psychological literature, these are called "System 1" decisions. Individuals with high cognitive skills will scrutinize their choice, and they will make decisions akin to the expected utility model. These are called "System

[2] Surveys of the literature on subjective expectations can be found in Manski (2004), and Hurd (2009).

[3] We focus on stockholding in light of the fact that stockholding decisions are particularly suited for testing the model. In fact, the model can easily be generalized to describe any decision making under uncertainty.

[4] See Section 2 for a description of dual system processing.

2" decisions.

When asked about expectations on future stock market returns in a survey, "System-2" decision makers will respond with subjective probabilities. "System-1" decision makers, however, have not formed subjective probabilities. Instead, such respondents will "construct" a heuristic answer at the moment that the question is asked (Converse, 1964; Bishop, Tuchfarber, and Oldendick, 1986; Tourangeau and Rasinski, 1988; Zaller and Feldman, 1992). We define a heuristic answer as the result of a response strategy for guessing an acceptable answer if one does not know the actual answer.[5]

Heuristic answers are likely to be noisy. However, there may still be some correlation with actual stock holding since individuals may substitute the subjective probability question with an easier but related question such as: "How much do you like stocks, on a scale from 0 to 100" (Kahneman, 2003, 2011).

Our model predicts that subjective return probabilities are more strongly associated with stockholding for individuals with high cognitive skills than for those with lower cognitive skills. We find strong evidence for this hypothesis based on panel data for a large representative sample of the (near-) elderly U.S. population from the Health and Retirement Study (HRS) for the period from 2002 until 2010. Our analysis is based on pooled OLS models and models with individual fixed effects.

Our model also predicts that individuals with lower cognitive skills are more likely to give no answer or a heuristic answer to survey questions on subjective probabilities than individuals with high cognitive skills. Again, we find strong empirical evidence for this hypothesis. This is in line with the finding of Kleinjans and van Soest (2012) that rounding and nonresponse are more common among individuals with lower cognitive skills and a lower education level. Relatedly, Gouret and Hollard (2011) find that individuals with low education and income tend to give less coherent answers to probability questions.

Our results suggest that the relationship between economic behavior and subjective

[5]This definition follows Tversky and Kahneman (1974) and the literature on survey methodology.

probabilities is mediated through individuals with high cognitive skills. This is, on the one hand, good news for standard economic theory. We show that a part of the population behaves consistently with the prediction of expected utility theory. On the other hand, for individuals with lower cognitive skills, the effect of subjective probabilities on stockholding decisions is smaller or absent.

Overall, our model and our results may be consistent with rationality in a broader sense. For individuals with low cognitive skills, investing in "probability skills" may be very costly. As a result, it is "rational" for them to rely on their System 1 which provides a default decision in the absence of scrutinizing.

Our paper is organized as follows. Section 2 discusses the psychology of dual processing. Section 3 presents our simple model of stockholding. Section 4 discusses our empirical specification. In Section 5 we present the data. Section 6 presents our empirical results, and section 7 presents results on heuristic answers. Finally, Section 8 concludes.

2 Intuition vs. Reason: Dual Processing

In past decades, a large body of psychological research has established that decisions can be understood as governed by two distinct mental systems (Stanovich and West, 2000; Kahneman, 2003, 2011; Kahneman and Frederick, 2005; Evans, 2008). The first system (often called "System 1") is based on mental associations and intuition. The second system ("System 2") is based on logical reasoning. Kahneman (2003, p.1451) characterizes the two systems as follows.

> *The operations of System 1 are fast, automatic, effortless, associative, and often emotionally charged; they are also governed by habit, and are therefore difficult to control or modify. The operations of System 2 are slower, serial, effortful, and deliberately controlled; they are also relatively flexible and potentially rule-governed.*

System 1 provides the *default* system, and it is prompted involuntarily. It is based on the use of heuristics and on emotions (Tversky and Kahneman, 1974; Loewenstein et al., 2001; Kahneman, 2003). In particular, System 1 reacts strongly to "salient" features in a decision maker's environment, e.g. in the form of stories that easily come to mind.

An example about investing in stocks may illustrate the working of System 1. Without any conscious effort, System 1 may remind the decision maker of salient instances related to stock holding. For instance, assume that the decision maker has a friend who recently lost a lot of money in the stock market. In this case, System 1 is likely to "sample" the friend's story and associate it with a negative emotion of fear of loss. Based on this, System 1 may induce a feeling of "I don't like stocks" and the individual will not invest in stocks. For another individual, System 1 may "sample" the story of a successful investor and induce a feeling of "I like stocks". The important factor is that an individual following the suggestions of System 1 does not make decisions based on any scrutinizing reasoning, but on feelings that determine decisions in a very direct, "automatic" way.

Crucially for our study, System 1 is not sensitive to probabilities. If the idea of stockholding being associated with a loss is salient, then System 1 does not qualify the loss with "but this may happen only with a probability of x percent." In the words of Kahneman (2003, p. 1454):

> *Ambiguity and uncertainty are suppressed in intuitive judgment. [...] Doubt is a phenomenon of System 2.*

System 2 refers to logical reasoning, based on the laws of probability. System 2 may override System 1's default decision. However, the activation of System 2 is associated with costs since its activity is effortful and slow. Activation of System 2 requires a sufficient amount of cognitive skills (Stanovich and West, 2000; Shafir and LeBoeuf, 2002), and it is encouraged by previous exposure to statistical thinking (Nisbett et al., 1983; Agnoli and Krantz, 1989; Agnoli, 1991).

3 A Simple Model of Cognitive Skills, Subjective Probabilities, and Stock Holding

In this section, we consider a simple and stylized model of how cognitive skills determine the decision to invest in stocks. In particular, the model highlights the interaction of cognitive skills and the use of probabilities in investment decisions. The model is based on the dual system framework described in Section 2 above.

3.1 The Decision Problem

An individual has wealth w. The individual can invest in risk free bonds or in risky stocks. Bond returns are denoted by r^f. Stock returns are denoted by r. For simplicity, we assume that r takes on either of two values, $\bar{r}$ or $\underline{r}$, where $\bar{r} > r^f > \underline{r}$. If the individual participates in the stock market, she invests a fraction $\alpha > 0$ of her wealth in stocks. At the end of the investment period, this results in wealth $(1-\alpha)wr^f + \alpha wr$. If the individual does not participate in the stock market, wealth at the end of the investment period is wr^f. For our stylized model, we are interested in characterizing the decision of whether or not to invest in stocks (rather than the choice of α).[6]

In our model, the investment decision consists of three distinct stages and a reporting stage, which are described below.

3.2 Decision Process

First Stage

In the first stage, System 1 provides the individual with a message that determines how much she "likes" stocks. We denote the message by s^*, which takes on either the value

[6]The model can easily be extended to a characterization of equity shares. The determinants of the conditional means of equity shares, given reported subjective probabilities, are very similar to the ones indicated in Proposition 1 below.

0 (“don’t like stocks”) or 1 (“like stocks”). The fraction of individuals who receive the message $s^* = 1$ is π.

We denote actual stock holdings by s, which takes on either the value 0 (“no stock holdings”) or 1 (“invest in stocks”). If an individual does not scrutinize System 1’s suggestion (see below), then $s = 1$ if and only if $s^* = 1$. Note that the first stage decision does not rely on probabilities in any way.

Second Stage

In the second stage of the decision problem, the individual decides whether to distrust System 1 and to scrutinize the investment decision. Scrutinizing requires a costly investment in “probability skills.” The costs of this investment depend on an individual’s general cognitive skills (fluid and crystallized intelligence; see Horn and McArdle, 2007, and McArdle et al., 2009).

Formally, an individual has gross benefit $B > 0$ of scrutinizing. The net benefit of scrutinizing is equal to

$$B - k(c), \tag{1}$$

where k denotes costs of scrutinizing, and $c \in \mathbb{R}^+$ denotes cognitive skills. We make the natural assumptions that k is continuous and strictly decreasing in c; in particular, $\lim_{c\to 0} k = \infty$, and $\lim_{c\to\infty} k = 0$.

Costs k reflect that scrutinizing is not just a matter of a few seconds of thinking. Rather, someone unfamiliar with statistical thinking, or someone unfamiliar with stock return data, will need to spend time on acquiring the information and statistical skills that are required for a rationally informed choice about stockholding. For someone with high cognitive skills who has already a thorough understanding of probabilities and statistics, the costs of scrutinizing are low, and, at the limit, may become negligible. For someone with a low level of cognitive skills and low education, these costs may be prohibitively high.

A decision maker scrutinizes the investment decision and invests in statistical skills if and only if the net benefit (1) is positive. Since B is finite and $\lim_{c \to 0} k = \infty$, this is the case if and only if cognitive skills c exceed some critical threshold $\bar{c} > 0$.

Third Stage

Upon scrutinizing, the decision maker learns a probability p^* of the "good outcome" of stock returns, i.e. $r = \bar{r}$. Different individuals may learn a different value of p^*. For simplicity, and since this case is particularly insightful, we assume that p^* is a random variable that takes on only two values, either $\bar{p}$ or $\underline{p}$, where $\bar{p} > \underline{p}$. Let u denote a (Bernoulli) utility function with standard properties, such that the maximization problem in (2) below is well defined. We assume that the two values of p^*, $\bar{p}$ and $\underline{p}$, are such that

$$\max_{\alpha > 0} p^* u((1-\alpha)wr^f + \alpha w\bar{r}) + (1-p^*)u((1-\alpha)wr^f + \alpha w\underline{r}) > u(wr^f) \qquad (2)$$

holds for $p^* = \bar{p}$, but does not hold for $p^* = \underline{p}$. I.e. the individual is better off holding stocks if $p^* = \bar{p}$, but better off holding only bonds if $p^* = \underline{p}$.[7]

At stage 3, a scrutinizing individual maximizes expected utility Eu, given $\Pr[r = \bar{r}] = p^* \in \{\underline{p}, \bar{p}\}$. Let s again denote a binary indicator for stock holding. Given our assumptions about $\underline{p}$, $\bar{p}$, we have $s = 1$ if and only if $p^* = \bar{p}$.

Reporting Stage

Eventually, an interviewer drops by and asks the decision maker about her "subjective probability" that stocks take on the good outcome $\bar{r}$.

Denote the answer to the interviewer by p, which can take on the values $\underline{p}$ and $\bar{p}$, i.e. the same two values as the actual "decision" probabilities p^* used by scrutinizers. For scrutinizers, the answer to the subjective probability question is arguably straightforward and they report $p = p^*$.

[7] Note that we assume that individuals cannot short-sell stocks.

For non-scrutinizers, the answer to the probability question is less straightforward. They have not formed subjective probabilities. Instead, they will construct a heuristic answer at the moment the question is asked (Converse, 1964; Bishop et al., 1986; Tourangeau and Rasinski, 1988; Zaller and Feldman, 1992). Among non-scrutinizers who receive signal $s^* = 0$, we assume that fraction $1 - \mu$ reports $p = \underline{p}$. This answer can be understood as an *ex post* justification of their actual stock holding decision, in the sense that they match no stockholding to a low number on the probability scale. A fraction μ reports $p = \bar{p}$. Correspondingly, among non-scrutinizers who receive signal $s^* = 1$, we assume that fraction $1 - \mu$ reports $p = \bar{p}$, and a fraction μ reports $p = \underline{p}$. μ parameterizes the *noisiness* of non-scrutinizers' answer to the subjective probability question. For simplicity, we assume that the degree of noisiness does not depend on s^*.

3.3 The Relationship between Stated Probabilities and Stockholding

In this study, we are interested in how subjective probabilities are related to decision making for different levels of cognitive skills. To this end, we derive conditional expectations for stock holding, given reported subjective probabilities.

Let E denote the expectation operator. For scrutinizers, we have

$$E[s|p = \underline{p}, c > \bar{c}] = 0 \tag{3}$$

$$E[s|p = \bar{p}, c > \bar{c}] = 1 \tag{4}$$

For non-scrutinizers, the relationship between p and s is nondeterministic. Using Bayes rule, it directly follows

$$E[s|p = \underline{p}, c \leq \bar{c}] = \frac{\pi\mu}{(1-\pi)(1-\mu) + \pi\mu} \tag{5}$$

$$E[s|p = \bar{p}, c \leq \bar{c}] = \frac{\pi(1-\mu)}{\pi(1-\mu) + (1-\pi)\mu} \tag{6}$$

Our main result is then contained in the following proposition.

Proposition 1 *Stock holdings react more strongly to changes in reported probabilities for scrutinizers than for non-scrutinizers. In particular,*

$$E[s|p = \bar{p}, c > \bar{c}] - E[s|p = \underline{p}, c > \bar{c}] = 1, \tag{7}$$

whereas, if $\mu > 0$, *then*

$$E[s|p = \bar{p}, c \leq \bar{c}] - E[s|p = \underline{p}, c \leq \bar{c}] < 1. \tag{8}$$

According to our model, individuals with lower cognitive skills provide noisy answers to subjective probability questions since they struggle with making sense of these questions. This results from the fact that investing in "probability skills" does not pay off if cognitive skills are relatively low. As a consequence, the association between reported subjective probabilities and stock holdings is stronger for individuals with high cognitive skills than for those with low cognitive skills.

The same logic applies when the stockholding decision and probabilities are continuous variables. Again, individuals with high cognitive skills will find it beneficial to invest in "probability skills" and, upon scrutinizing, draw a probability about positive stock market returns. Everything else equal, individuals who draw higher probabilities will invest a higher share of their wealth in stocks. The same relationship holds for reported probabilities. In contrast, for individuals with low cognitive skills, there is only a noisy relationship between reported probabilities and equity shares. In this case, higher reported probabilities lead to a stronger increase in conditional expectations for equity shares for individuals with high cognitive skills than for those with low cognitive skills.[8]

4 Empirical Specification

In the remaining sections of this paper, we present an empirical analysis of the relationship between subjective probabilities and stockholding. We are not in a position to provide direct evidence about whether individuals use System 1 or System 2 in their investment decisions. Nor do we attempt to estimate the "structural" parameters of the model. Our aim is more modest: we investigate whether the behavioral patterns predicted by our model can be observed in the data. Put differently, the model serves as a useful guideline for an informed look at the data.

We will look at two different pieces of evidence. First, we investigate the relationship between stockholding decisions and reported subjective probabilities for individuals with low and high cognitive skills (this Section and Sections 5 and 6). Furthermore, we examine whether individuals with lower cognitive skills are more likely to give "heuristic" answers about subjective probabilities (Section 7). We consider such answers as hints towards System 1 decision-making, which does not rely on probabilities.

We base our empirical analysis on the following linear model which we estimate alternatively by pooled ordinary least squares estimation (OLS) and by fixed-effects estimation:

[8] In our model we have assumed that individuals with high cognitive skills perfectly report their true subjective probabilities at the interview stage. As pointed out by Kézdi, and Willis (2011), even individuals with high cognitive skills may make reporting errors. It is of interest to see how this would change the predictions of our model. If an individual with high cognitive skills makes a reporting error, then she may answer $p = \underline{p}$ although $p^* = \bar{p}$, or $p = \bar{p}$ whereas $p^* = \underline{p}$. As a result, the reported probabilities become noisy signals for the probabilities of the decision stage, whereas in the model above they are perfect signals. Consequently, the slope of the regression line, i.e. $E[s|p = \bar{p}, c > \bar{c}] - E[s|p = \underline{p}, c > \bar{c}]$ becomes less than one. Suppose, however, that the degree of noisiness is small relative to the noisiness of the probabilities reported by individuals with low cognitive skills. In this scenario – which we view as plausible – it remains true that the slope of the regression line is steeper for individuals with high cognitive skills.

$$s_{it} = p_{it}\beta + c'_{it}\gamma + p_{it}c'_{it}\delta + X'_{it}\lambda + \omega_i + \varepsilon_{it}. \tag{9}$$

Here, s_{it} is a measure for stockholding; p_{it} denotes subjective stock market return probabilities; the vector c_{it} includes measures for cognitive ability; the vector $p_{it}c_{it}$ includes interaction terms of subjective return probabilities with variables for cognitive ability; X_{it} is a vector of additional explanatory variables. The index i refers to individuals, and t is a time index; β is a parameter, and γ, δ and λ are vectors of parameters. A person-specific unobserved fixed effect is represented by ω_i. ε_{it} is an unobserved time-varying component for person i in period t.

Parameter β determines the relationship between subjective return probabilities and stockholding if all variables for cognitive ability are set to zero. Crucially, equation (9) also includes interaction terms for subjective probabilities with measures of cognitive ability. This allows the effect of subjective return probabilities to vary according to cognitive ability.

Based on our model, our empirical hypothesis is that the effect of subjective return probabilities on stockholding is stronger for persons with high cognitive ability than for persons with low cognitive ability. We test this hypothesis based on our estimates for δ. According to our model, δ should be positive.

For pooled OLS models, parameters can be estimated consistently under the following exogeneity assumption:

$$E\left[\omega_i + \varepsilon_{it} | p_{it}, c_{it}, X_{it}\right] = 0. \tag{10}$$

It is tempting to conclude that if equation (10) were to hold, then β and δ in equation (9) would reflect the causal effect of reported probabilities on stockholding; and if (10) were to fail, we could not draw any reliable conclusions about our hypothesis from OLS estimates. This argument, however, is misleading in our context. In our study, we do not aim to estimate the treatment effect of a policy intervention.

Instead, our empirical hypothesis is based on Proposition 1, which is stated in terms of conditional expectations $E[s|p, c]$. Estimation equation (9) is the empirical counterpart of Proposition 1. Linear regression gives the best (minimum mean squared error) linear approximation of the conditional expectation function.[9] This property of linear regression holds even if the exogeneity assumption in equation (10) fails. We can therefore view OLS estimation results as a test of Proposition 1 conditional on a set of observed individual characteristics X_{it}, since OLS best approximates $E[s|p, c, X]$.

In addition to OLS models, we also examine fixed-effects models, which control for unobserved time-invariant characteristics ω_i. We can view fixed-effects estimation results as a test of Proposition 1 conditional on all unobserved time-constant individual characteristics in addition to a set of observed individual characteristics. Fixed-effects estimation provides the best linear approximation of $E[s|p, c, X^*, \omega]$.[10]

According to our model, there is *no* causal relationship between reported probabilities and stockholding for individuals with low cognitive skills. Rather, any positive association between stockholding and reported probabilities should be understood as a correlation, reflecting cues that individuals use for making sense of the probability question.

For individuals with high cognitive skills, our model does impose a causal relationship between reported probabilities and stockholding. Our model predicts that, due to this causal relationship, there is a higher correlation between probabilities and stockholding for individuals with high cognitive skills than for individuals with low cognitive skills.

In the following, we discuss how a violation of exogeneity assumption (10) could affect the interpretation of our estimation results. We first consider the case where explanatory variables are related to unobserved individual characteristics in ω_i, for example interest in money and finance.[11] Interest in money and finance could have a direct effect on stock holdings, and it could at the same time also be correlated with cognitive ability. In this

[9] See Wooldridge 2010, Ch. 2

[10] X^* includes the time-varying components of X.

[11] To be precise, what we mean is the time-persistent part of interest in money and finance.

case the exogeneity assumption (10) would indeed be violated.

The baseline version of our theoretical model does not take into account interest in money and finance. However, it can easily be extended to incorporate this trait. A high interest in money and finance can be modeled as leading to a general upward-shift of equity shares, everything else equal. In this case, it would still be true that an increase in probabilities would have a stronger effect on stock holdings for individuals with high cognitive skills than for those with low cognitive skills.

In models with individual fixed-effects, a correlation between explanatory variables and time-invariant unobserved characteristics would have no impact on the estimation results. Fixed-effect models control for unobserved time-invariant characteristics by examining the effect of changes in subjective probabilities on changes in stockholding.

We next consider the case where explanatory variables are related to unobserved time-varying characteristics in ε_{it}. This correlation could, for example, be caused by fixed costs of investing in the stock market (Haliassos, 2002; Gomes and Michaelides, 2005). Fixed costs of entering the stock market can explain why some people may not buy stocks even if they have high return expectations.[12] This pattern may occur more frequently for people with low cognitive skills and lead to a violation of (10). Fixed costs may then provide an alternative explanation for a positive interaction effect δ.

However, fixed costs of entering the stock market cannot explain a positive interaction effect if the estimation sample is restricted to stockholders only.[13] In our study, we examine three different measures of stockholding: a binary indicator of whether a household holds any stocks or mutual funds; the equity share in relation to total financial wealth; and the equity share in relation to total financial wealth for those households with positive stockholding. These different measures allow us to estimate the effect of subjective probabilities on stockholding both at the extensive margin and at the intensive margin.

[12] For instance, an individual's return expectation may be particularly high in one year, whereas the equity share stays at zero. For that year, ε_{it} would then be particularly low.

[13] Table 2 shows that, conditional on stockholding, equity shares are almost the same for individuals with high and low cognitive skills.

Clearly, our measures of stockholding take on values only within a limited range. This notwithstanding, our estimation is based on linear models. Compared with nonlinear models, linear models require (far) less restrictive assumptions in the case of fixed-effects specifications.[14] In our study, we are interested in the average partial effect of subjective probabilities on stock holding. In linear models, estimation coefficients can approximately be interpreted as average partial effects.[15] Furthermore, in linear models, the coefficients of interaction terms (i.e. δ in (9)) can be directly interpreted as interaction effects. In nonlinear models, the coefficients for interaction terms cannot be directly interpreted in this way (Ai and Norton, 2003), and they may not have a meaningful interpretation at all (Greene, 2010).

5 Description of Data

We use data from waves six to ten of the Health and Retirement Study (HRS) which were collected biannually between the years 2002 and 2010. The HRS is a national study which is representative for the elderly and near-elderly population in the United States. In addition to detailed information on wealth, assets and demographic characteristics, the HRS also includes questions on expectations about future events, numeracy, and preferences. The choice of our study period from 2002 until 2010 was motivated by the availability of questions on subjective probabilities about stock market returns.

We restrict our sample to respondents who indicate that they are responsible for financial matters within a household. As a result, we have no more than one respondent per household in each wave. Inclusion in our regression sample requires an answer to the subjective probability question about stock returns, information on numeracy, education, as well as on standard control variables such as income, age, marital status etc. In total,

[14]E.g., average partial effects cannot be estimated without strong distributional assumptions about the fixed effects in fixed-effects logit models (Wooldridge 2010, p. 625).

[15]See Wooldridge (2010, p. 579). Stoker (1986) shows that, under some assumptions, the linear probability model exactly estimates the average partial effect.

our baseline sample consists of 53,139 observations for 18,337 individuals.

Cognitive ability has multiple dimensions. In our study we focus on measures of cognitive ability that are most likely to have an impact on the use of probabilities in stockholding decisions, such as education level and numeracy. Formal schooling is an important source of knowledge about the rules of probabilities, and higher-educated people typically have also taken more courses in mathematics and statistics. Education level is measured by two binary indicators for respondents who have a high school degree (but no college degree) and for respondents who have a college degree.

Numeracy measures the current ability to solve problems in mathematics and statistics. The indicator for high numeracy is based on the following three questions: 1) "If the chance of getting a disease is 10 percent, how many people out of 1000 would be expected to get the disease?" 2) "If 5 people all have the winning numbers in the lottery and the price is two million dollars, how much will each of them get?" 3) "Let's say you have \$200 in a savings account. The account earns 10 percent interest per year. How much would you have in the account at the end of two years?" The indicator for high numeracy takes on a value of one if a respondent answers at least two of the above three questions correctly. Questions about numeracy were asked in 2002, 2006, and 2010. If an answer is missing, we use the most recent available answer. Summary statistics are shown in Table 1. 50.2 percent of respondents have a high school degree as their highest degree, and 29.3 percent have a college degree. 44.1 percent have high numeracy.

Our measure of subjective return probabilities is based on the following question:

> *By next year at this time, what is the percent chance that mutual fund shares invested in blue chip stocks like those in the Dow Jones Industrial Average will be worth more than they are today?*

The average subjective probability of positive stock market returns is 46.5 percent (see Table 2, where this probability is denoted as p_0). This is below long-run historical averages, in which returns for the Dow Jones Industrial Average were positive in 68 percent of years in the post-war period until 2002 (Kézdi and Willis, 2011). However, stock returns were low during the first decade of the 21st century, which started with unusually high stock market valuation levels; and respondents might have anticipated this. Return expectations were higher for respondents with high numeracy as opposed to respondents with low numeracy (50.9 percent vs. 43 percent), and for respondents with higher education (52.5 percent for respondents with a college degree, 45.3 percent for respondents with a high school degree, and 40.7 percent for respondents without high school degree). Figure 1 shows the distribution of subjective return probabilities for the full sample and, in separate graphs, according to numeracy and education. By far the most common answer in all groups is 50 percent. Answers are concentrated at 10-percent intervals and a sizeable share of respondents give answers of 0 percent and 100 percent. In Section 7, we discuss to what degree respondents with low cognitive ability are more likely to struggle to give meaningful answers to probability questions, compared to those with high cognitive skills. Important for our estimation, however, there is substantial variation in subjective probabilities for *all* cognitive skill groups.

As mentioned in the previous section, we use three alternative measures of stockholding: a binary indicator of whether a household holds any stocks or mutual funds; the equity share in relation to total financial wealth; and the equity share in relation to total financial wealth for those households with positive stockholding.[16] In our main analysis we focus on stockholding outside pension accounts. We show estimation results for stockholding decisions in pension accounts in the Appendix.

Table 2 shows summary statistics for our measures of stockholding. 31.6 percent of households in our sample hold any stocks or mutual funds; the average equity share is 19.2 percent; and the average equity share for households owning stocks is 57.2 percent. Stockholding is more common for respondents with high numeracy than for respondents

[16]Total financial wealth includes stocks and mutual funds, checking, saving, and money market accounts, CDs, government saving bonds and T-bills, bonds and bond funds. We use the measures of the amount of stocks held and of total financial wealth computed by RAND.

with low numeracy (41.7 percent vs. 23.7 percent); stockholding is also more common for respondents with higher education (48.2 percent with college degree, 30 percent with high school degree, and 11.9 percent with no degree). Conditional on positive stockholding, the equity share does not vary much according to numeracy and education. For some fixed-effects estimations, our estimation strategy relies on changes in stock ownership. On average, 17.3 percent of respondents moved in or out of the stock market during the study period from 2002 until 2010. This implies that there is substantial variation in stock ownership over time.

Further variables included in our analysis are socio-economic and demographic characteristics such as age, gender, race and ethnic group, marital status, income and financial wealth. Summary statistics for these variables are shown in Table 1. On average, respondents are 68 years old, average household income is around \$66,200, and average household financial wealth around \$153,600. 55.5 percent of respondents are female, 59.8 percent are married, 12.8 percent are black, and 6.7 percent are Hispanic. We also employ a binary measure for risk tolerance. This measure is based on a question about a lifetime income gamble.[17] Risk tolerance is set to zero if respondents always choose the safe option, and it is set to one if respondents were willing to accept at least one of the lifetime income gambles. Answers to questions on risk tolerance are available for only about half of the respondents in our sample.[18] We include a binary indicator for missing observations on risk tolerance.

[17] See Barsky, Juster, Kimball, and Shapiro (1997) and Kimball, Sahm, and Shapiro (2008) for a detailed discussion of this risk aversion measure.

[18] In 2002 and 2006 the question on risk tolerance was asked only to respondents younger than age 65, and in 2004 the question on risk tolerance was asked only to respondents in the Early Baby Boomers' cohort born between 1948 and 1953. The question on risk tolerance was not asked in 2008 and 2010. Our indicator for risk tolerance is constant over time, and it is based on the average of available answers.

6 Empirical Results

6.1 OLS Estimation Results

Table 3 shows OLS estimation results. The first two columns show regression results with a binary indicator for stock ownership as dependent variable. Column 1 includes only the simple non-interacted form of the subjective probability of positive stock returns as an explanatory variable. We henceforth denote this probability as p_0. An increase in p_0 by 10 percentage points is associated with an increase in the probability of stock ownership by about 1.4 percentage points. This coefficient is strongly significant. In the second column, we also include interaction terms of p_0 with cognitive skills, which reduces the size of the baseline effect of p_0 by 40 percent to 0.8 percentage points. The effects of interactions with high school and college are positive and significant, as predicted by our model. An increase in p_0 by 10 percentage points leads to an extra increase in the probability of stock ownership of 0.8 percentage points for high school, and 0.5 percentage points for college. The interaction with numeracy is not significant.

Columns 3 and 4 of Table 3 show results for equity shares, including equity shares of zero. Results are very similar to specifications with a binary indicator for stock ownership. The coefficient of p_0 decreases substantially when interaction terms with cognitive skills are included. The coefficients for interaction terms with high school and college are both positive and significant.

Columns 5 and 6 of Table 3 show results for equity shares for a sample that is restricted to stockholders only. While individual coefficients for the interaction terms are not significant, F-tests at the bottom of the table show that the total effect of subjective probabilities on equity shares is significantly positive for individuals with high cognitive skills. This effect is significant at the 5-percent level for high school, and at the one percent level for college. The combined effect of high school and numeracy, and college and numeracy is also significant at the 1-percent level.[19]

Our alternative measures of stockholding allow us to estimate the effect of subjective probabilities on stockholding both at the extensive and at the intensive margin. There may be fixed costs associated with accessing the stock market. Fixed costs can explain why some people may not buy stocks even if they have high return expectations. This pattern may occur more frequently for people with low cognitive skills. This could explain why the estimated effect of subjective probabilities on stock ownership is larger for individuals with higher cognitive skills than for individuals with lower cognitive skills. However, this explanation does not hold at the intensive margin if we restrict our sample to stockholders only.[20]

6.2 Estimation Results with Individual Fixed-Effects

Table 4 shows the results for fixed-effects estimations. These are qualitatively very similar to the OLS results. Three patterns prevail. First, the coefficients on the non-interacted probabilities drop substantially when interaction terms with cognitive skills are included. In particular, the former are never significant when the interaction terms are included. Second, one or several of the coefficients for the interaction terms with cognitive skills are positive and individually significant in all three models. Third, the F-tests at the bottom of the table indicate that the joint effect of subjective probabilities are positive and significant for at least one group with high cognitive skills, for all three measures of stockholding. The results in column 6 of Table 4 provide particularly strong evidence for the predictions of our model. Since these results are obtained for a sample of stockholders only, they cannot be affected by the fact that individuals with low cognitive skills are less likely to invest in stocks for reasons that are unrelated to subjective probabilities.

[19] All results shown have been estimated including all answers to subjective probability questions. When reestimating the results for a sample excluding individuals who always answer "50" to the probability question, the results are basically unchanged (not shown).

[20] Conditionally on owning stocks the average equity share is almost constant across cognitive skill groups, and varies only between 55.5 percent and 58.8 percent (see Table 2).

Once potential concern may be that the interaction terms of p_0 with cognitive skills capture some non-linear effects of p_0. For example, the interaction terms of p_0 with cognitive skills may be a proxy for p_0^2 if cognitive skills are positively correlated with the level of p_0. A similar concern may be that cognitive skills can be positively correlated with wealth or income. In that case, the interaction terms of p_0 with cognitive skills could act as a proxy for an interaction term of p_0 with wealth or income. We address these concerns by including additional interaction terms to our fixed-effect models.

Table 5 shows estimation results for fixed-effects models that include additional interaction terms of p_0 with risk tolerance, missing risk tolerance, age, income and wealth.[21] A crucial finding is that coefficients for the interaction terms of p_0 with cognitive skills are essentially unaffected by the inclusion of additional interaction terms. For all specifications, at least one of the interaction terms of p_0 with cognitive skills is significantly positive, including for the specification based on stockholders only. In sum, the fixed-effects estimation results clearly support the prediction of our model.[22]

As a further test, we apply the same specifications as discussed above to stockholding in company pension accounts (such as 401(k) accounts). This is of interest since arguments that individuals with low cognitive skills systematically have too little wealth to invest in stocks is unlikely to be an important concern for money saved in pension accounts. Our sample of individuals with pension accounts is much smaller. The results are shown in the Appendix (Tables 8 to 10). In spite of the small sample, we find that results are broadly consistent with our results for stockholding outside pension accounts.

[21] We use income and wealth in the *first* wave that an individual appears in our data, in order to avoid possible endogeneity problems arising from the fact that holding stocks may have a direct effect on income and wealth in a subsequent wave.

[22] The results also remain virtually identical when excluding individuals who always answer "50" (not shown).

6.3 Discussion

Our empirical results clearly support the hypothesis that there is a stronger association between subjective probabilities and stockholding for individuals with high cognitive skills, compared to individuals with low cognitive skills. Consequently, they provide evidence for our theoretical model. In light of this, it is interesting to ask whether our findings could also be explained differently.

We have discussed some alternative explanations that would explain our estimation results as statistical artifacts. For example, unobserved individual characteristics such as interest in money and finance could be related to subjective probabilities; or widespread absence of stockholding among individuals with lower cognitive skills might explain positive interaction effects. However, these explanations are not applicable for the estimation results with individual fixed-effects, and for the sample of stockholders only, respectively.

An alternative to our model that is observationally equivalent would be that individuals with low and high cognitive skills do both have "probability skills." However, individuals with high cognitive skills do have better memories. At the interview stage, they report the probabilities underlying the stockholding decision with little reporting error. In contrast, individuals with low cognitive skills report those probabilities with substantial error. In this situation, the slope of the regression line would also be steeper for individuals with high cognitive skills than for individuals with low cognitive skills.

While individuals with high cognitive skills may indeed have better memories, we do not find this explanation entirely plausible. The psychological literature on probabilistic thinking provides evidence that individuals with lower cognitive skills do have difficulties in understanding the concept of probabilities even in situations where memory does not play any important role.[23]

[23]See the literature mentioned on p.2.

7 Heuristic Answers

We now examine patterns in subjective probabilities that can plausibly be explained by heuristic response behavior. We define answers to survey questions as heuristic if they are the result of a response strategy for guessing an acceptable answer when the actual answer is not known. This definition follows Tversky and Kahneman (1974) and the literature on survey methodology.

Individuals who have not formed subjective probabilities at the decision stage "construct" an answer at the moment the question is asked. Previous studies discuss several heuristic approaches to answering questions on unfamiliar topics (Converse, 1964; Bishop et al., 1986; Kleinjans and van Soest, 2012; Tourangeau and Rasinski, 1988; Zaller and Feldman, 1992). For example, respondents who have not formed subjective probabilities can refuse to answer the question; they can give a focal answer of 0, 50, or 100; or they can substitute difficult questions on subjective probabilities with easier, but related questions, e.g. on their attitude towards owning stocks (Kahneman, 2003, 2011).

In our data, 18.1 percent of respondents refuse to answer the question on the subjective probability of future stock market returns (see Table 6). We form a binary indicator for giving no answer, and we regress this indicator on cognitive ability and a list of control variables, using a linear probability model.[24] The estimation results are shown in column 1 of Table 7. Respondents with higher cognitive skills are much more likely to answer the question on subjective probability. Individuals with high numeracy are 6 percentage points less likely to refuse. For individuals with a high school as their highest degree the effect is 11 percentage points, and for those with a college degree the effect is 14 percentage points. This finding suggests that individuals with high cognitive ability have more often formed an opinion on the probability distribution of future stock market returns, consistent with our model.

[24] We use a linear probability model since coefficients can then be easily interpreted. The magnitude of marginal effects from a probit model are very similar.

Focal answers are also very common in our sample. 39.9 percent of respondents answer with subjective probabilities of 0, 50, or 100 (Table 6). Column 2 of Table 7 shows that respondents with higher cognitive abilities are significantly less likely to give focal answers. The high frequency of focal answers to questions on subjective probabilities has been noted before (Hurd, 2009; Kleinjans and van Soest, 2012). In previous studies, focal answers have frequently been explained by rounding behaviors. Instead, focal answers might also suggest that a substantial share of respondents may not have formed meaningful subjective probabilities. An answer of 50 percent to a probability question can mean "I don't know."

The HRS includes a question which allows us to directly test two alternative interpretations of focal answers against each other. Respondents who give a focal answer of 50 percent are asked whether they mean "equal chances of gains and losses" or "no one can know." 64.2 percent of respondents answer with "no one can know". This answer is significantly less common for respondents with high cognitive skills (column 3 of Table 7). This finding provides evidence that rounding alone cannot explain all focal answers, and that a significant share of respondents has not formed meaningful subjective probabilities.

A common approach to react to a difficult question is to substitute the difficult question with an easier, but related question (Kahneman, 2011). For example, respondents might substitute the question on subjective return probabilities with a question such as: "How much do I like stocks on a scale from 0 to 100". If respondents effectively answer questions that are different from those actually asked in the survey, then these answers need not confirm to the laws of probability. In the HRS, respondents are asked several questions about the subjective probabilities of stock returns. In addition to the subjective probability that stock returns are larger than 0 percent for a broad stock market index during the following twelve months, respondents are also asked about the subjective probabilities that returns are larger or smaller than 10 percent. Many answers seem inconsistent. 56 percent of answers violate monotonicity; 41 percent of respondents give the same answer to two probability questions (see Table 6). Respondents with higher cognitive skills are less likely to give inconsistent or identical answers to both questions (Column 4 and 5 of Table 7).

The "question substitution" heuristic provides a straightforward explanation for this pattern. Suppose that individuals with low cognitive skills effectively answer the probability questions with a number that indicates how much they *like* stocks. The answer to this substitute question is likely to be the same, independent of whether the actual question asks about an increase in the stock market of 0, or 10 percent. This directly leads to answers that are inconsistent with the laws of probability. The answers are not inconsistent, however, once the psychological meaning of the questions is taken into account.

In sum, the findings above suggest that many respondents struggle to give meaningful answers to questions on subjective probabilities. This finding is especially true for respondents with lower cognitive skills. While many answers seem to contradict the laws of probability, they are consistent with the dual-processing framework. Our findings confirm the prediction of that framework that respondents with lower cognitive skills are more likely to give heuristic answers to questions on subjective probabilities.

8 Conclusion

In this study, we investigate to what degree individuals use subjective probabilities in making decisions under uncertainty. We develop a novel and simple model of stockholding which is based on the dual processing framework from psychology (Kahneman, 2003). In our model, individuals with low cognitive skills make decisions in intuitive non-probabilistic ways. Individuals with high cognitive skills make decisions akin to the expected utility model. Our model predicts that the relationship between subjective probabilities of stock market returns and stockholding varies according to cognitive ability.

We find that higher subjective return probabilities are associated more strongly with stock holdings for individuals with high cognitive skills than for individuals with lower cognitive skills, as predicted by our model. Furthermore, we find complementary evidence that individuals with lower cognitive skills are more likely to give heuristic answers to questions on subjective probabilities. Specifically, respondents with lower cognitive skills are more likely to give no answer, or to give focal or inconsistent answers. While many answers to questions on subjective probabilities seem to contradict the laws of probability, they are consistent with the dual-processing framework.

Our study contributes to a understanding of decision making under uncertainty. Previous studies have found that subjective probabilities are strongly related to actual decisions in many important areas (Hurd 2009). Our study suggests that the relationship between subjective stock market return probabilities and stockholding may be mediated through individuals with high cognitive ability. A part of the population behaves consistently with the predictions of expected utility theory. For individuals with lower cognitive skills, the effect of subjective probabilities on stockholding is much smaller. Taking into account that individuals with lower cognitive skills behave differently is important for a realistic estimation of responses to policy changes, e.g. concerning saving for retirement.

For future research, our framework can also be applied to decisions other than stockholding. It is important to better understand how decision making under uncertainty depends on cognitive skills.

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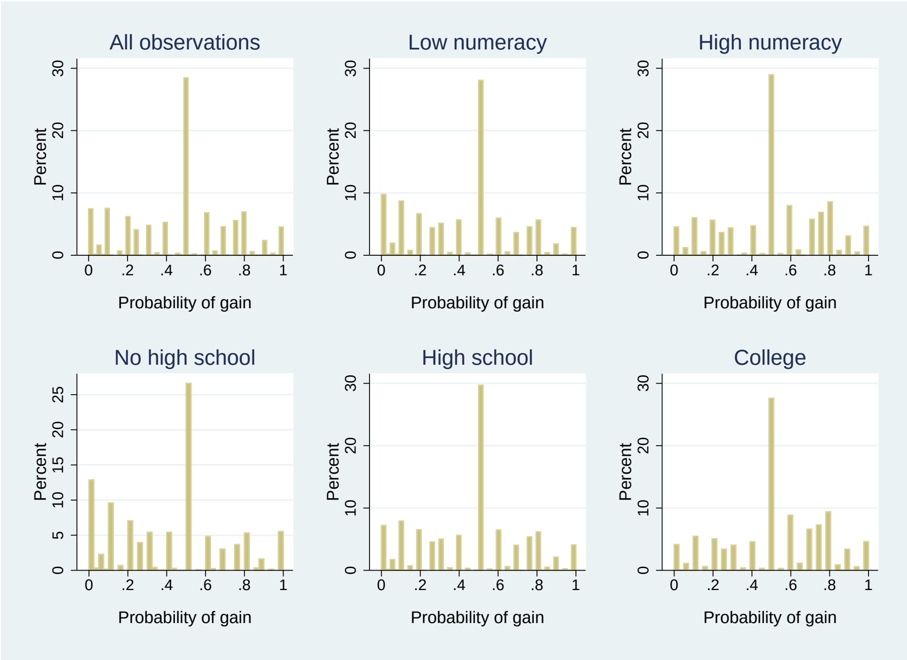
Figures

Figure 1: Probability of gain, by cognitive skills group



NOTE: "Probability of gain" refers to the subjective probability of positive returns for a broad stock index over the next 12 months, which we also denote by p_0*. The figure shows histograms for the distribution of this probability for all observations, and for different cognitive skills groups.*

Tables

Table 1: Summary statistics

	Mean	Standard deviation
Age	68.0	10.3
Income	66.2	308.6
Financial wealth	153.6	911.3
High school	0.502	0.500
College	0.293	0.455
High numeracy	0.441	0.496
Risk tolerant	0.397	0.489
Female	0.555	0.497
Married	0.598	0.490
Black	0.128	0.334
Hispanic	0.067	0.250
Observations	53,139	
Individuals	18,337	

NOTE: Risk tolerance is available only for 25,936 observations. In our regression analysis, we use a dummy variable for indicating missing risk tolerance. Income and wealth are measured in thousands of dollars.

Table 2: Summary statistics of probabilities and outcome variables

	All observations	Numeracy		Education		
		Low	High	No high school	High school	College
p_0						
Mean	0.465	0.430	0.509	0.407	0.453	0.525
Standard dev	0.272	0.275	0.261	0.288	0.266	0.258
Observations	53,139	29,729	23,410	10,884	26,695	15,560
Holding stocks						
Mean	0.316	0.237	0.417	0.119	0.300	0.482
Standard dev	0.465	0.425	0.493	0.323	0.458	0.500
Observations	53,139	29,729	23,410	10,884	26,695	15,560
Equity share, all						
Mean	0.192	0.148	0.243	0.083	0.173	0.286
Standard dev	0.328	0.300	0.352	0.239	0.315	0.367
Observations	48,728	26,145	22,583	8,643	24,969	15,116
Equity share, stockholders only						
Mean	0.572	0.566	0.576	0.580	0.555	0.588
Standard dev	0.323	0.327	0.320	0.336	0.328	0.315
Observations	16,371	6,847	9,524	1,238	7,769	7,364
Moving in/out of stockmarket						
Mean	0.173	0.154	0.193	0.095	0.174	0.213
Standard dev	0.378	0.361	0.395	0.293	0.380	0.409
Observations	32,065	16,910	15,155	5,593	16,161	10,311

NOTE: p_0 denotes the answer to the question about the subjective probability of positive stock market returns over the next 12 months. "Holding any stocks" refers to a dummy variable that takes on the value one if an individual holds any stocks outside retirement accounts, including indirectly held stocks via mutual funds. "Equity share, all" includes zero equity shares for non-stockholders. "Moving in/out of stockmarket" refers to a dummy variable that takes on the value one if an individual does not hold stocks in one wave but holds stocks in the next wave; or vice versa. See page 16 for the definition of high numeracy.

Table 3: OLS

	Any stocks		*Equity share: all*		*Equity share: stockholders*	
	(1)	(2)	(3)	(4)	(5)	(6)
p_0	0.1365**	0.0829**	0.1009**	0.0564**	0.0646**	0.0162
	(0.0078)	(0.0112)	(0.0058)	(0.0096)	(0.0107)	(0.0388)
$p_0\times$ high numeracy		0.0074		0.0165		0.0341
		(0.0164)		(0.0118)		(0.0212)
$p_0\times$ high school		0.0760**		0.0461**		0.0262
		(0.0163)		(0.0126)		(0.0421)
$p_0\times$ college		0.0492*		0.0473**		0.0371
		(0.0220)		(0.0165)		(0.0430)
High numeracy	0.0615**	0.0577**	0.0324**	0.0244**	0.0039	-0.0135
	(0.0054)	(0.0090)	(0.0038)	(0.0063)	(0.0064)	(0.0127)
High school	0.1127**	0.0809**	0.0582**	0.0391**	-0.0290*	-0.0407
	(0.0063)	(0.0084)	(0.0046)	(0.0063)	(0.0128)	(0.0235)
College	0.2232**	0.2038**	0.1291**	0.1087**	-0.0033	-0.0211
	(0.0089)	(0.0131)	(0.0063)	(0.0094)	(0.0132)	(0.0246)
Risk tolerant	0.0190*	0.0187*	0.0116*	0.0112*	0.0027	0.0024
	(0.0078)	(0.0078)	(0.0057)	(0.0057)	(0.0101)	(0.0101)
Risk tolerance missing	0.0047	0.0048	-0.0067	-0.0066	-0.0267*	-0.0267*
	(0.0082)	(0.0082)	(0.0060)	(0.0060)	(0.0109)	(0.0109)
Income	0.0002**	0.0002**	0.0001*	0.0001*	-0.0001**	-0.0001**
	(0.0001)	(0.0001)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
Income^2	-0.0000**	-0.0000**	-0.0000**	-0.0000**	0.0000*	0.0000*
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
Financial wealth	0.0002**	0.0002**	0.0001**	0.0001**	0.0000**	0.0000**
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
Financial wealth^2/100	-0.0000**	-0.0000**	-0.0000**	-0.0000**	-0.0000**	-0.0000**
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
Age	0.0029	0.0028	0.0017	0.0016	-0.0017	-0.0016
	(0.0024)	(0.0024)	(0.0018)	(0.0018)	(0.0035)	(0.0035)
Age^2/100	0.0013	0.0013	0.0012	0.0012	0.0016	0.0015
	(0.0018)	(0.0018)	(0.0013)	(0.0013)	(0.0025)	(0.0025)
Female	0.0203**	0.0206**	0.0113**	0.0117**	0.0006	0.0007
	(0.0057)	(0.0057)	(0.0042)	(0.0042)	(0.0071)	(0.0071)
Married	0.0750**	0.0749**	0.0320**	0.0320**	-0.0291**	-0.0293**
	(0.0061)	(0.0061)	(0.0045)	(0.0045)	(0.0075)	(0.0075)
Black	-0.1486**	-0.1480**	-0.0826**	-0.0821**	0.0168	0.0170
	(0.0065)	(0.0065)	(0.0048)	(0.0048)	(0.0169)	(0.0169)
Hispanic	-0.1431**	-0.1424**	-0.0732**	-0.0726**	0.0487*	0.0487*
	(0.0077)	(0.0077)	(0.0063)	(0.0063)	(0.0220)	(0.0220)
Constant	-0.1699*	-0.1459	-0.1128	-0.0932	0.6276**	0.6460**
	(0.0824)	(0.0825)	(0.0606)	(0.0607)	(0.1248)	(0.1260)
Year dummies	yes	yes	yes	yes	yes	yes
F-tests on p_0-terms						
$p_0 + p_0 \times num = 0$		0.0000**		0.0000**		0.2356
$p_0 + p_0 \times hs = 0$		0.0000**		0.0000**		0.0241*
$p_0 + p_0 \times coll = 0$		0.0000**		0.0000**		0.0127**
$p_0 + p_0 \times num + p_0 \times hs = 0$		0.0000**		0.0000**		0.0000**
$p_0 + p_0 \times num + p_0 \times coll = 0$		0.0000**		0.0000**		0.0000**
Observations	53,139	53,139	48,728	48,728	16,371	16,371
R^2	0.197	0.197	0.148	0.148	0.017	0.017

NOTE: Dependent variables are indicated in the column headers. "Any stocks" refers to a dummy variable that takes on the value one if an individual holds any stocks outside retirement accounts, including indirectly held stocks via mutual funds. "Equity share, all" includes zero equity shares for non-stockholders. p_0 denotes the answer to the question about the subjective probability of positive stock market returns over the next 12 months. See page 16 for the definition of high numeracy. Observations are pooled across different waves. Risk tolerance is available only for 25,936 observations. Missing information on risk tolerance is indicated by the dummy variable "risk tolerance missing." Standard errors, indicated in parentheses, are robust and clustered at the individual level. The numbers indicated under "F-tests" are p-values for the respective hypotheses. One, and two asterisks indicate a p-value less than .05, and .01, respectively. The abbreviations "num," "hs," and "coll" refer to "high numeracy," "high school," and "college," respectively.

Table 4: Fixed effects

	Any stocks		*Equity share: all*		*Equity share: stockholders*	
	(1)	(2)	(3)	(4)	(5)	(6)
p_0	0.0335**	0.0107	0.0231**	-0.0091	0.0168	-0.0866
	(0.0071)	(0.0104)	(0.0059)	(0.0107)	(0.0123)	(0.0496)
$p_0\times$ high numeracy		0.0179		0.0218**		0.0298*
		(0.0095)		(0.0076)		(0.0129)
$p_0\times$ high school		0.0333*		0.0378**		0.1060*
		(0.0146)		(0.0133)		(0.0528)
$p_0\times$ college		-0.0052		0.0091		0.0756
		(0.0196)		(0.0167)		(0.0530)
Age	0.0018	0.0017	0.0037	0.0035	-0.0026	-0.0036
	(0.0069)	(0.0069)	(0.0058)	(0.0058)	(0.0115)	(0.0115)
Age^2/100	-0.0042	-0.0040	-0.0033	-0.0030	-0.0005	0.0004
	(0.0034)	(0.0034)	(0.0028)	(0.0028)	(0.0057)	(0.0057)
Constant	0.4052	0.4061	0.1122	0.1129	0.7551	0.7804
	(0.3495)	(0.3495)	(0.2942)	(0.2941)	(0.5861)	(0.5852)
Year dummies	yes	yes	yes	yes	yes	yes
F-tests on p_0-terms						
$p_0 + p_0 \times num = 0$		0.0289*		0.2983		0.2591
$p_0 + p_0 \times hs = 0$		0.0000**		0.0008**		0.3125
$p_0 + p_0 \times coll = 0$		0.7478		0.9978		0.5852
$p_0 + p_0 \times num + p_0 \times hs = 0$		0.0000**		0.0000**		0.0105**
$p_0 + p_0 \times num + p_0 \times coll = 0$		0.1475		0.0885		0.2993
R^2	0.012	0.013	0.007	0.008	0.001	0.002
Observations	53,264	53,264	48,843	48,843	16,400	16,400
Number of individuals	18,369	18,369	17,378	17,378	7,230	7,230

NOTE: Dependent variables are indicated in the column headers. "Any stocks" refers to a dummy variable that takes on the value one if an individual holds any stocks outside retirement accounts, including indirectly held stocks via mutual funds. "Equity share, all" includes zero equity shares for non-stockholders. p_0 *denotes the answer to the question about the subjective probability of positive stock market returns over the next 12 months. See page 16 for the definition of high numeracy. Income and wealth are omitted. Robust standard errors are indicated in parentheses. The numbers indicated under "F-tests" are p-values for the respective hypotheses. One, and two asterisks indicate a p-value less than .05, and .01, respectively. The abbreviations "num," "hs," and "coll" refer to "high numeracy," "high school," and "college," respectively.*

Table 5: Fixed effects with additional interactions

	Any stocks	*Equity share: all*	*Equity share: stockholders*
	(1)	(2)	(3)
p_0	0.0648	-0.1121	-0.4414
	(0.2469)	(0.2020)	(0.5414)
$p_0\times$ high numeracy	0.0181	0.0216**	0.0300*
	(0.0095)	(0.0076)	(0.0129)
$p_0\times$ high school	0.0359*	0.0388**	0.1062*
	(0.0148)	(0.0135)	(0.0527)
$p_0\times$ college	0.0006	0.0118	0.0792
	(0.0211)	(0.0179)	(0.0537)
p_0^2	0.0333	0.0351	0.0433
	(0.0219)	(0.0189)	(0.0386)
$p_0\times$ risk tol	0.0017	-0.0064	-0.0996*
	(0.0216)	(0.0178)	(0.0403)
$p_0\times$ risk tol miss	0.0175	-0.0053	-0.0638
	(0.0215)	(0.0180)	(0.0404)
$p_0\times$ income$_0$	-0.0002	-0.0002	-0.0000
	(0.0001)	(0.0001)	(0.0002)
$p_0\times$ income$_0^2/100$	0.0012**	0.0009**	-0.0007
	(0.0002)	(0.0002)	(0.0007)
$p_0\times$ wealth$_0$	0.0000	0.0000	-0.0000
	(0.0000)	(0.0000)	(0.0000)
$p_0\times$ wealth$_0^2/100$	-0.0000	-0.0000	0.0000
	(0.0000)	(0.0000)	(0.0001)
$p_0\times$ age	-0.0023	0.0028	0.0105
	(0.0071)	(0.0059)	(0.0151)
$p_0\times$ age$^2/100$	0.1351	-0.2244	-0.7191
	(0.5111)	(0.4274)	(1.0526)
Age	0.0028	0.0024	-0.0099
	(0.0076)	(0.0063)	(0.0139)
Age$^2/100$	-0.0048	-0.0021	0.0045
	(0.0041)	(0.0033)	(0.0078)
Constant	0.3675	0.1521	1.0259
	(0.3664)	(0.3049)	(0.6486)
Year dummies	yes	yes	yes
R^2	0.013	0.008	0.003
Observations	53,252	48,833	16,398
Number of individuals	18,362	17,372	7,228

NOTE: Dependent variables are indicated in the column headers. "Any stocks" refers to a dummy variable that takes on the value one if an individual holds any stocks outside retirement accounts, including indirectly held stocks via mutual funds. "Equity share, all" includes zero equity shares for non-stockholders. p_0 denotes the answer to the question about the subjective probability of positive stock market returns over the next 12 months. See page 16 for the definition of high numeracy. Income$_0$ and wealth$_0$ refer to the first survey wave that an individual appears in our data. Risk tolerance is available only for 25,936 observations. Missing information on risk tolerance is indicated by the dummy variable "risk tolerance missing." Robust standard errors are indicated in parentheses. One, and two asterisks indicate a p-value less than .05, and .01, respectively.

Table 6: Descriptive statistics for heuristic answers

	Mean	Standard deviation	Observations
No answer	0.181	0.385	60,230
Focal answer	0.399	0.490	53,139
"No one can know"	0.642	0.479	8,278
Violation of monotonicity	0.562	0.496	7,004
Equal answer	0.408	0.491	7,004

NOTE: "No answer" is an indicator variable for no response to the question about the probability that stock prices will increase ("p_0 question"). "Focal answer" is an indicator variable for an answer to the p_0 question of either 0, 50, or 100. "No one can know" is only defined for individuals who answer 50 to the p_0 question. It takes on a value of one if an individual indicates that 50 means "no one can know", rather than "equal chances of gains and losses." "Violation of monotonicity" indicates that the answer to a survey question about the probability that stock prices will increase more than 10 percent ("p_{10} question") is not (strictly) lower than p_0. "Equal answer" indicates an equal answer to the p_{10} and the p_0 question. All observations for the last two variables refer to the year 2002.

Table 7: Heuristic answers

	No answer	Focal answer	"No one can know"	Monotonicity	Equal answers
	(1)	(2)	(3)	(4)	(5)
High numeracy	-0.0577**	-0.0174**	-0.0740**	-0.0456**	-0.0283*
	(0.0036)	(0.0050)	(0.0115)	(0.0131)	(0.0130)
High school	-0.1077**	-0.0326**	-0.0425**	-0.0587**	-0.0384*
	(0.0056)	(0.0065)	(0.0137)	(0.0154)	(0.0159)
College	-0.1406**	-0.0674**	-0.1369**	-0.0889**	-0.0487*
	(0.0060)	(0.0075)	(0.0166)	(0.0188)	(0.0190)
Risk tolerant	-0.0207**	-0.0277**	0.0020	-0.0766**	-0.0672**
	(0.0047)	(0.0070)	(0.0149)	(0.0212)	(0.0209)
Risk tolerance missing	0.0275**	0.0015	0.0105	-0.0297	-0.0443*
	(0.0057)	(0.0076)	(0.0156)	(0.0221)	(0.0223)
Income	-0.0000	-0.0000	-0.0002**	-0.0003**	-0.0003*
	(0.0000)	(0.0000)	(0.0001)	(0.0001)	(0.0001)
Income^2	0.0000	0.0000	0.0000**	0.0000	0.0000
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
Financial wealth	-0.0000**	0.0000	-0.0001**	-0.0000	-0.0000
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
Financial wealth^2/100	0.0000**	-0.0000	0.0000*	0.0000	0.0000
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
Age	-0.0192**	-0.0111**	-0.0013	-0.0057	0.0014
	(0.0018)	(0.0024)	(0.0055)	(0.0077)	(0.0081)
Age^2/100	0.0161**	0.0089**	0.0035	0.0044	-0.0003
	(0.0014)	(0.0017)	(0.0041)	(0.0053)	(0.0055)
Female	0.0851**	0.0063	0.0641**	0.0906**	0.0646**
	(0.0038)	(0.0051)	(0.0111)	(0.0130)	(0.0130)
Married	-0.0306**	-0.0128*	0.0180	-0.0082	-0.0176
	(0.0041)	(0.0052)	(0.0114)	(0.0133)	(0.0134)
Black	0.0496**	-0.0125	0.0555**	0.0445*	0.0195
	(0.0062)	(0.0075)	(0.0171)	(0.0183)	(0.0190)
Hispanic	0.1286**	0.0074	-0.0135	0.0332	-0.0213
	(0.0082)	(0.0097)	(0.0225)	(0.0282)	(0.0287)
Constant	0.8036**	0.8077**	0.4794*	0.8198**	0.3964
	(0.0618)	(0.0817)	(0.1886)	(0.2724)	(0.2870)
Year dummies	yes	yes	yes	yes	yes
Observations	60,230	53,139	8,278	7,004	7,004
R^2	0.114	0.009	0.085	0.034	0.016

NOTE: The table shows results from OLS estimations. The dependent variables are indicated in the column headers. "No answer" is an indicator variable for no response to the question about the probability that stock prices will increase ("p_0 question"). "Focal answer" is an indicator variable for an answer to the p_0 question of either 0, 50, or 100. "No one can know" is only defined for individuals who answer 50 to the p_0 question. It takes on a value of one if an individual indicates that 50 means "no one can know", rather than "equal chances of gains and losses." "Violation of monotonicity" indicates that the answer to a survey question about the probability that stock prices will increase more than 10 percent ("p_{10} question") is not (strictly) lower than p_0. "Equal answer" indicates an equal answer to the p_{10} and the p_0 question. All observations for the last two variables refer to the year 2002. See page 16 for the definition of high numeracy. Standard errors, indicated in parentheses, are robust and clustered at the individual level. One, and two asterisks indicate a p-value less than .05, and .01, respectively.

Appendix: Stocks within Company Pensions

Table 8: OLS, company pensions

	Any stocks		*Equity share: all*		*Equity share: stockholders*	
	(1)	(2)	(3)	(4)	(5)	(6)
p_0	0.1074**	-0.0286	0.1156**	-0.1767	0.0561*	-0.1873
	(0.0282)	(0.1068)	(0.0304)	(0.1044)	(0.0284)	(0.0985)
$p_0\times$ high numeracy		0.0618		0.0885		0.0584
		(0.0564)		(0.0614)		(0.0597)
$p_0\times$ high school		0.1602		0.2436*		0.1690
		(0.1130)		(0.1129)		(0.1074)
$p_0\times$ college		0.0558		0.2983*		0.3026**
		(0.1160)		(0.1174)		(0.1114)
High numeracy	-0.0149	-0.0457	0.0137	-0.0319	0.0238	-0.0079
	(0.0144)	(0.0338)	(0.0157)	(0.0358)	(0.0147)	(0.0343)
High school	0.0329	-0.0449	-0.0071	-0.1235	-0.0364	-0.1154*
	(0.0311)	(0.0660)	(0.0323)	(0.0638)	(0.0302)	(0.0583)
College	0.0361	0.0111	-0.0143	-0.1611*	-0.0464	-0.1971**
	(0.0318)	(0.0673)	(0.0331)	(0.0662)	(0.0310)	(0.0607)
Risk tolerant	0.0253	0.0237	0.0490**	0.0473**	0.0372*	0.0363*
	(0.0141)	(0.0141)	(0.0160)	(0.0160)	(0.0150)	(0.0150)
Risk tolerance missing	0.0071	0.0062	0.0861**	0.0850**	0.0953**	0.0938**
	(0.0247)	(0.0246)	(0.0260)	(0.0261)	(0.0227)	(0.0228)
Income	0.0001	0.0001	0.0001	0.0001	0.0000	0.0000
	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)	(0.0001)
Income^2	0.0000	0.0000	-0.0000	-0.0000	-0.0000	-0.0000
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
Financial wealth	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
Financial wealth^2/100	-0.0000	-0.0000	-0.0000	-0.0000	0.0000	0.0000
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
Age	0.0193	0.0192	0.0080	0.0078	-0.0028	-0.0025
	(0.0130)	(0.0130)	(0.0137)	(0.0138)	(0.0119)	(0.0119)
Age^2/100	-0.0181	-0.0180	-0.0098	-0.0097	0.0003	-0.0001
	(0.0109)	(0.0110)	(0.0116)	(0.0116)	(0.0100)	(0.0101)
Female	0.0073	0.0072	-0.0337*	-0.0335*	-0.0433**	-0.0436**
	(0.0147)	(0.0146)	(0.0161)	(0.0161)	(0.0148)	(0.0147)
Married	0.0137	0.0145	-0.0020	0.0002	-0.0117	-0.0099
	(0.0167)	(0.0167)	(0.0179)	(0.0180)	(0.0164)	(0.0164)
Black	-0.0476	-0.0474	-0.0890**	-0.0882**	-0.0723**	-0.0706**
	(0.0251)	(0.0252)	(0.0255)	(0.0255)	(0.0251)	(0.0250)
Hispanic	0.0039	0.0019	0.0043	0.0047	-0.0036	-0.0011
	(0.0283)	(0.0283)	(0.0333)	(0.0335)	(0.0311)	(0.0310)
Constant	0.2359	0.3048	0.3511	0.5004	0.8068*	0.9187**
	(0.3841)	(0.3877)	(0.4091)	(0.4117)	(0.3531)	(0.3558)
Year dummies	yes	yes	yes	yes	yes	yes
F-tests on p_0-terms						
$p_0 + p_0 \times num = 0$		0.7691		0.4363		0.2336
$p_0 + p_0 \times hs = 0$		0.0046**		0.2009		0.7213
$p_0 + p_0 \times coll = 0$		0.6304		0.0639		0.0662
$p_0 + p_0 \times num + p_0 \times hs = 0$		0.0002**		0.0041**		0.4311
$p_0 + p_0 \times num + p_0 \times coll = 0$		0.0425*		0.0000**		0.0001**
Observations	3,024	3,024	3,000	3,000	2,615	2,615
R^2	0.024	0.026	0.036	0.040	0.032	0.038

NOTE: Information about employer-provided pension accounts is available only for the years 2006, 2008, and 2010 and comes from Section J of the HRS. We only take into account equity investments for accounts where individuals indicate that they have at least some choice about asset allocation. An individual may indicate up to 4 pension accounts. Dependent variables are indicated in the column headers. "Any stocks" refers to a dummy variable that takes on the value one if an individual holds any stocks within a retirement account. "Equity share, all" includes zero equity shares for non-stockholders. Observations are pooled across different waves. Standard errors, indicated in parentheses, are robust and clustered at the individual level. The numbers indicated under "F-tests" are p-values for the respective hypotheses. One, and two asterisks indicate a p-value less than .05, and .01, respectively. The abbreviations "num," "hs," and "coll" refer to "high numeracy," "high school," and "college," respectively.

Table 9: Fixed effects, company pensions

	Any stocks		*Equity share: all*		*Equity share: stockholders*	
	(1)	(2)	(3)	(4)	(5)	(6)
p_0	0.0177	-0.2861*	0.0040	-0.4637**	0.0095	-0.4422*
	(0.0404)	(0.1421)	(0.0472)	(0.1574)	(0.0470)	(0.1874)
$p_0\times$ high numeracy		-0.0430		-0.0041		0.0217
		(0.0329)		(0.0402)		(0.0402)
$p_0\times$ high school		0.3258*		0.4596**		0.4297*
		(0.1524)		(0.1700)		(0.1988)
$p_0\times$ college		0.3955*		0.5749**		0.5244**
		(0.1551)		(0.1708)		(0.1988)
Age	-0.0702	-0.0724	-0.0448	-0.0468	0.0111	0.0093
	(0.0519)	(0.0520)	(0.0505)	(0.0509)	(0.0517)	(0.0526)
$\text{Age}^2/100$	0.0587	0.0605	0.0228	0.0238	-0.0233	-0.0234
	(0.0391)	(0.0391)	(0.0373)	(0.0378)	(0.0393)	(0.0401)
Constant	2.9210	2.9817	2.3956	2.4718	0.7947	0.9014
	(1.8363)	(1.8439)	(1.8880)	(1.8969)	(1.8882)	(1.9104)
Year dummies	yes	yes	yes	yes	yes	yes
F-tests on p_0-terms						
$p_0 + p_0 \times num = 0$		0.0224*		0.0030**		0.0248*
$p_0 + p_0 \times hs = 0$		0.4757		0.9507		0.8583
$p_0 + p_0 \times coll = 0$		0.0907		0.1391		0.2757
$p_0 + p_0 \times num + p_0 \times hs = 0$		0.9569		0.9105		0.9011
$p_0 + p_0 \times num + p_0 \times coll = 0$		0.2772		0.1088		0.1073
R^2	0.010	0.017	0.017	0.028	0.016	0.028
Observations	3,040	3,040	3,016	3,016	2,628	2,628
Number of individuals	1,971	1,971	1,959	1,959	1,747	1,747

NOTE: Information about employer-provided pension accounts is available only for the years 2006, 2008, and 2010 and comes from Section J of the HRS. We only take into account equity investments for accounts where individuals indicate that they have at least some choice about asset allocation. An individual may indicate up to 4 pension accounts. Dependent variables are indicated in the column headers. "Any stocks" refers to a dummy variable that takes on the value one if an individual holds any stocks within a retirement account. "Equity share, all" includes zero equity shares for non-stockholders. Income and wealth are omitted. Robust standard errors are indicated in parentheses. The numbers indicated under "F-tests" are p-values for the respective hypotheses. One, and two asterisks indicate a p-value less than .05, and .01, respectively. The abbreviations "num," "hs," and "coll" refer to "high numeracy," "high school," and "college," respectively.

Table 10: Fixed effects with additional interactions, company pensions

	Any stocks (1)	*Equity share: all* (2)	*Equity share: stockholders* (3)
p_0	0.9338	3.2690	1.0257
	(1.7779)	(2.3122)	(2.0714)
$p_0\times$ high numeracy	-0.0444	-0.0091	0.0197
	(0.0329)	(0.0405)	(0.0399)
$p_0\times$ high school	0.3339*	0.5192**	0.5188**
	(0.1608)	(0.1613)	(0.1842)
$p_0\times$ college	0.3759*	0.6318**	0.6484**
	(0.1693)	(0.1720)	(0.1933)
p_0^2	-0.0274	-0.0940	-0.1771
	(0.1208)	(0.1376)	(0.1411)
$p_0\times$ risk tol	0.0213	-0.1354	-0.0819
	(0.0851)	(0.1018)	(0.1013)
$p_0\times$ risk tol miss	-0.1596	0.0933	0.2670
	(0.1046)	(0.1421)	(0.1472)
$p_0\times$ income$_0$	0.0008	0.0003	-0.0006
	(0.0007)	(0.0009)	(0.0008)
$p_0\times$ income$_0^2$/100	-0.0055	-0.0006	0.0054
	(0.0045)	(0.0059)	(0.0055)
$p_0\times$ wealth$_0$	-0.0006	-0.0001	0.0001
	(0.0004)	(0.0004)	(0.0004)
$p_0\times$ wealth$_0^2$/100	0.0040	-0.0004	-0.0014
	(0.0025)	(0.0026)	(0.0025)
$p_0\times$ age	-0.0427	-0.1214	-0.0459
	(0.0594)	(0.0782)	(0.0699)
$p_0\times$ age^2/100	3.7054	9.8924	3.8738
	(4.9367)	(6.5789)	(5.8974)
Age	-0.0518	0.0038	0.0237
	(0.0518)	(0.0669)	(0.0647)
Age^2/100	0.0446	-0.0168	-0.0367
	(0.0392)	(0.0520)	(0.0505)
Constant	2.2869	0.9022	0.5043
	(1.8284)	(2.3051)	(2.2261)
Year dummies	yes	yes	yes
R^2	0.022	0.038	0.041
Observations	3,037	3,013	2,626
Number of individuals	1,969	1,957	1,745

NOTE: Information about employer-provided pension accounts is available only for the years 2006, 2008, and 2010 and comes from Section J of the HRS. We only take into account equity investments for accounts where individuals indicate that they have at least some choice about asset allocation. An individual may indicate up to 4 pension accounts. Dependent variables are indicated in the column headers. "Any stocks" refers to a dummy variable that takes on the value one if an individual holds any stocks within a retirement account. "Equity share, all" includes zero equity shares for non-stockholders. Income$_0$ and wealth$_0$ refer to the first survey wave that an individual appears in our data. Robust standard errors are indicated in parentheses. One, and two asterisks indicate a p-value less than .05, and .01, respectively.