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ABSTRACT

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The paper analyzes the effect of human-capital investments of heterogeneous individuals on the dynamics of the wage structure within a neoclassical growth model. The accumulation of physical capital changes relative factor prices and thus incentives to acquire skills, thereby altering the composition of the labor force. Without relying on exogenous shocks, our framework generates dynamics that resembles several important observations on wage inequality (e.g., the non-monotone evolution of the skill premium). Additional incorporation of wage rigidities emphasizes the trade off between residual wage inequality and employment opportunities for unskilled labor that is consistent with country-specific evidence.

JEL Classification: I21, J31, O15

Keywords: wage inequality, human capital investment, transitional adjustment dynamics, skill-specific unemployment

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1 Introduction

The development of wage inequality and employment prospects by skills has been the subject of considerable theoretical and empirical research in recent years. The great interest in this question is mainly due to two reasons. First, labor earnings constitute the major part of most households' incomes, and hence a change in the wage structure and/or employment chances has important implications for overall income inequality. Second, the behavior of some variables measuring wage dispersion was quite surprising and hardly to reconcile with conventional explanations.

The most important facts about the evolution of wage inequality can be summarized as follows:

a) Although the fraction of skilled workers increased during the past decades, wage differentials by skill did not behave monotonically; while many countries experienced a narrowing of the skill differentials in the 70s, the trend reversed in the 80s and 90s: the skill premium has been steadily increasing since then.[1]

b) Empirical studies unanimously find increasing inequality within the group of skilled workers.[2] The picture is less homogeneous for wage dispersion in the unskilled group. While many countries (esp. the US and the UK) experienced more unequal wages for the unskilled workers, other economies (among them Belgium and Germany[3]) went through a period of unskilled wage compression.

c) The rise in the wage dispersion observed in countries like the US and the UK was associated with a decline in unskilled unemployment, whereas wage compression in Belgium and Germany lead to higher unemployment rates of the unskilled workers.

Traditional explanations of these facts refer to trends in returns to education and ability. In this paper, we argue that changes in the composition of the

[1] See the summary of stylized facts in Katz and Autor (1999), and the references therein. Particularly helpful and illustrative is Table 2.1 in Freeman and Katz (1994).

[2] Cf. Katz and Autor (1999) for the United States and, e.g., OECD (1993) for other OECD countries.

[3] Cf. Steiner and Wagner (1998), Fitzenberger (1999), or Möller (1999).

skilled and unskilled labor force, that occur along the development path of an economy, can play an important role in determining the wage structure. We observe a growing number of people that upgrade their skills. Owing to a positive correlation between education and ability, this rise will typically be associated with human–capital investments of ability groups that previously did not opt for higher education. As a consequence, the sorting of abilities into skill groups is not time–invariant. Since earning capacities depend on both accumulated skills and innate abilities, a changing ability structure affects inequality between and within groups.

To characterize this relationship in a formal way, consider (following Taber, 2001) the earnings regression function

$$Y_t = \alpha_t(s) + \gamma_t a + \epsilon_t$$

where Y_t, a variable measuring earnings, is explained by years of schooling s, ability a, and a zero–mean stochastic disturbance ϵ. The wage gap between groups with different educational attainments s_1 and s_2 can then be described by

$$\begin{aligned} \mathrm{E}(Y_t|s=s_1) - \mathrm{E}(Y_t|s=s_2) = {} & \alpha_t(s_1) - \alpha_t(s_2) \\ & + \gamma_t \left[\mathrm{E}(a|s=s_1) - \mathrm{E}(a|s=s_2)\right]. \end{aligned} \tag{1}$$

In (1), $\alpha_t(s_1) - \alpha_t(s_2)$ measures earning differences arising from different skills, while $\gamma_t \left[\mathrm{E}(a|s=s_1) - \mathrm{E}(a|s=s_2)\right]$ is the wage differential generated by differences in abilities across groups. This 'ability bias' depends on the returns to ability γ_t and the distribution of abilities within skill groups. Changes in (1) can thus result from variations in the returns to education or to ability, and from alterations of $\mathrm{E}(a|s=s_1) - \mathrm{E}(a|s=s_2)$, for which the selection of abilities into skill groups is crucial. Our paper focuses on this sorting effect which, moreover, impacts on the extent of inequality within skill groups through its influence on the degree of heterogeneity of the members.

To investigate composition effects in a general equilibrium framework, we incorporate heterogeneous abilities into an otherwise standard Solow model where physical capital is used in the production of goods and human capital. In the course of economic growth, the accumulation of physical capital alters factor prices and thus incentives to acquire human capital. As successively less talented individuals are inclined to upgrade their inherent abilities, the fraction of skilled workers rises. The changing ability structure within skill groups triggers a non–monotone development of the wage differential by skills. Furthermore, wage dispersion within the skilled group increases, whereas the unskilled workers become more homogeneous. When we integrate wage rigidities into this basic model, the evolution of residual inequality and employment resembles the development that is observed in countries such as the US or the UK. If wage floors do not keep up with the increase in labor productivity, workers that, owing to their low productivity, could not find a job successively become employed. Thus, heterogeneity among the unskilled workers tends to increase while at the same time the unemployment rate goes down.[4]

While our analysis relies on composition effects, another strand of literature studies time–trends in returns to education to explain the size of the skill premium and its dynamics. These contributions attribute rising $\alpha_t(s_1) - \alpha_t(s_2)$ since the 80s to shifts in labor demand in favor of skilled workers that were caused by increased integration of international markets (cf., e.g., Wood, 1994, Dinopoulos and Segerstrom, 1999) or skill–biased technological change (cf., e.g., Mincer, 1991, Autor et al., 1998). In this context, Acemoglu (1998) and Kiley (1999) assert that non–monotonicities can arise if the bias of technological progress depends on the availability of both types of labor. They argue that an exogenous shock (educational reforms, attempts to avoid participation in the Vietnam war) raised the relative supply of high–skilled workers and drove down the relative skilled wage in the 70s. As a consequence, technological progress switched to skill complementarity, thus generating the increasing skill premia in the 80s.

A distinctive but related 'supply creates demand' story that explains various aspects of the wage structure is put forward in search models such as Acemoglu (1999) and Albrecht and Vroman (2002). These papers argue (i) that jobs differ in their skill requirements, and (ii) that the skill composition of the labor force affects the types of jobs that firms create. An exogenous increase in the relative supply of skilled labor can alter the qualitative composition of jobs thus leading to a segmentation of the labor market that involves an increase in between–group wage inequality. As long as the labor market is not completely segmented, some skilled

[4]Some other authors (cf., e.g., Card, 1996, DiNardo et al., 1996, and Freeman, 1996) refer to wage floors that do not keep up with the increase in labor productivity to explain residual wage inequality. These papers, however, do not stress the link between wage floors and the composition of labor supply.

workers hold unskilled jobs which gives rise to residual wage inequality even with homogeneous abilities. Another reason why search frictions may lead to differing wages across identical workers is proposed by Postel–Vinay and Robin (2002). In their model, employers are compelled by workers' on–the–job search activities to raise wages randomly so that wage inequality across identical employer–employee pairs follows. The main focus of their work is on equilibrium wage dispersion as opposed to the dynamic issues which our research is concerned with. Unlike this literature on search frictions, our paper endogenizes human–capital formation and explains residual wage inequality by the sorting of heterogeneous ability types into skill groups that results from the educational choice.

As the appearance of γ_t in (1) suggests, our paper also relates to research investigating the pay–off to ability. Empirical as well as theoretical work identified technological progress as the principal source of changes in the return to ability (cf. Bartel and Sicherman, 1999, Galor and Tsiddon, 1997, or Hassler and Mora, 2000). The idea is that coping with a rapidly progressing environment requires mental flexibility and thus cognitive ability as opposed to education. Note that, from an econometric point of view, returns to ability and the composition effect are ultimately related since time effects such as γ_t and cohort effects cannot be identified separately.

Galor and Moav (2000) study composition effects when economic growth impacts on returns to ability. In their work, technological progress is biased towards skilled labor and raises incentives to acquire skills. As a consequence, residual inequality in both skill groups as well as the skill premium increase over time.[5] To explain a decreasing college wage gap in the 70s they resort to an exogenous removal of barriers to higher education. Thus, a declining degree of capital market imperfection facilitated access to student loans and expanded the skilled workforce which drove down the relative skilled wage. We differ from Galor and Moav (2000) in several points. First, by studying an augmented Solow model without technological change, we show that skill–biased technological change is not a prerequisite of composition effects. Second, our analysis suggests that non–monotonicities can arise even without exogenous shocks that change the qualitative features of the dynamical system. Explaining phenomena that are coherently

[5] A similar setting is used by Meckl and Zink (2002). In their partial–equilibrium framework, a sequence of exogenous productivity shocks is the driving force behind changes in the wage gap. This work does not address residual inequality, however.

observed in many countries by invoking variations of exogenous variables that are to some extent country–specific (see also Acemoglu, 1998, or Kiley, 1999) seems problematic, to say the least. Third, our mechanism provides some explanation for the differences in residual inequality and employment between, for instance, the US and some European countries.

The paper proceeds as follows. Section 2 introduces our basic neoclassical growth model with heterogeneous agents. After a description of the static equilibrium in Section 3, we analyze the dynamics of the model in Section 4. While Section 5 presents the development of wage inequality in the basic model, Section 6 treats some extensions. Having discussed robustness of the results with respect to changes in our basic assumptions in Section 7, we close with some concluding remarks (Section 8).

2 The Model

2.1 Factor Supplies

Our economy is populated by a continuum of individuals of mass one. Each resident lives for one period and gives birth to an off–spring at the end of his life so that the population is constant over time. Agents are heterogeneous with respect to their innate abilities, a, and their wealth endowments, x, inherited from the parent generation.

Assumption 1 *Inherent abilities are distributed uniformly over* $[\underline{a}, \overline{a}]$ *(*$\underline{a} > 1$*). They are independent across generations and across individuals in one cohort.*

Let Φ denote the distribution function of inherent abilities. Using uniform abilities enables us to obtain explicit solutions so that the basic intuition can be presented with maximum simplicity but it is not crucial for our results. Section 7 will comment on the robustness with respect to changes in the ability distribution.

Having realized the amount of wealth inherited from their parents and their inherent ability at the outset of their lives, the individuals acquire human capital and earn income. At the end of their lives, they allocate their total lifetime wealth between consumption and transfers to their children. An individual's preferences can be characterized by a homothetic utility function with consumption and bequest to their off–spring as arguments. As usual, utility is strictly increasing and concave in both arguments. This specification of the utility function implies that a constant fraction $1-s$ of end–of–life wealth is spent on consumption, where we assume that $0 < s < 1$.[6] The agents maximize utility, iff they maximize the amount of resources they possess at the end of their lives.

Concerning the educational attainment, the individuals have two options. They can refrain from education (i.e., work as unskilled) thereby supplying a units of effective labor and invest their wealth endowment at the capital market. Given the wage rate w of effective labor and the interest rate r, total lifetime wealth amounts to $aw + x(1+r)$. Alternatively, they can upgrade their abilities at some fixed cost I and thus supply a^2 units of effective labor.[7] End–of–life income of a skilled individual is then given by $a^2w + (x-I)(1+r)$. Individuals that do not have sufficient wealth to cover upfront schooling costs can borrow at a perfect capital market.

An agent chooses education, iff total lifetime wealth resulting from human–capital accumulation is larger than that for unskilled work, i.e. whenever

$$a(a-1) \geq I\frac{1+r}{w}. \tag{2}$$

Without credit frictions, the investment decision solely depends on ability for given values of w, r and I. Obviously, the following result holds:

Lemma 1 *The function $a_0 : \mathbb{R}_+^2 \to \mathbb{R}_+$ which maps every pair $(w,r) \in \mathbb{R}_+^2$ to*

$$a_0(w,r) := \frac{1+\sqrt{1+4I\frac{1+r}{w}}}{2}$$

describes minimum ability for which investment in education is profitable, i.e. every individual with ability of at least a_0 finds it attractive to become skilled.[8] *We have: $\partial a_0/\partial w < 0$ and $\partial a_0/\partial r > 0$.*

[6] Assuming a constant bequest share simplifies the analysis considerably, as otherwise the aggregate amount of bequests to the next generation would depend on the distribution of parents' wealth and not only on aggregate wealth.

[7] The assumption on increasing returns to education may be interpreted as a shortcut for the fact that the more able individuals are more efficient *per se* and that they have a comparative advantage in applying or acquiring human capital. Suppose, for instance, that agents need not only incur I, but they also have to spend time on education where this time input diminishes working time. Combining larger productivity with less schooling requirements can yield increasing returns to ability. While a quadratic education reward function may seem plausible, it is not necessary for our results as Section 7 will show.

[8] Note that we do not presume the image of a_0 to be a subset of $[\underline{a}, \overline{a}]$.

The sign of the partial derivatives of a_0 reflects the fact that for increasing w and decreasing r investment becomes attractive for less talented individuals due to increasing returns to and decreasing costs of human–capital investment, respectively. Using the properties of a_0, we can easily calculate the fraction of skilled workers in the economy.

Lemma 2 *The fraction of skilled (unskilled) workers n is $1 - \Phi(a_0)$ $(\Phi(a_0))$. As a result, n is non–increasing in a_0.*[9]

An individual works as skilled whenever the productivity gain from education compensates him for the fixed cost. For $a_0 \geq \overline{a}$ nobody wants to become educated, for $a_0 \leq \underline{a}$ everybody chooses to invest in education. For a given pair (w, r), demand for capital resulting from investment in education is nI. The relation between the total number of effective labor units and the threshold ability a_0 is described in

Lemma 3 *Effective labor supply as a function of the threshold ability a_0 can be characterized by the continuous mapping $H : \mathbb{R}_+ \to \mathbb{R}_+$ with*

$$H(a_0) := \begin{cases} \dfrac{\underline{a} + \overline{a}}{2} & \text{for} \quad a_0 > \overline{a} \\ \dfrac{1}{2}\dfrac{a_0^2 - \underline{a}^2}{\overline{a} - \underline{a}} + \dfrac{1}{3}\dfrac{\overline{a}^3 - a_0^3}{\overline{a} - \underline{a}} & \text{for} \quad a_0 \in [\underline{a}, \overline{a}] \\ \dfrac{\underline{a}^2 + \underline{a}\overline{a} + \overline{a}^2}{3} & \text{for} \quad a_0 < \underline{a}\,. \end{cases} \tag{3}$$

Proof. If a_0 is larger than $\overline{a}$, nobody wants to become skilled. Under a uniform distribution of abilities on $[\underline{a}, \overline{a}]$, effective labor supply is then given by

$$H = \int_{\underline{a}}^{\overline{a}} a \, d\Phi(a) = \frac{\underline{a} + \overline{a}}{2}\,.$$

For $a_0 < \underline{a}$ everybody becomes skilled implying an effective labor supply of

$$H = \frac{1}{\overline{a} - \underline{a}} \int_{\underline{a}}^{\overline{a}} a^2 \, da = \frac{\underline{a}^2 + \underline{a}\overline{a} + \overline{a}^2}{3}\,.$$

[9] We cannot claim n to be strictly decreasing in a_0, since $a_0(\mathbb{R}_+) \not\subset [\underline{a}, \overline{a}]$.

For $\underline{a} \le a_0 \le \overline{a}$, individuals whose ability exceeds a_0 choose to become skilled so that

$$H = \frac{1}{\overline{a}-\underline{a}} \left(\int_{\underline{a}}^{a_0} a\,da + \int_{a_0}^{\overline{a}} a^2\,da \right) = \frac{1}{2}\frac{a_0^2-\underline{a}^2}{\overline{a}-\underline{a}} + \frac{1}{3}\frac{\overline{a}^3-a_0^3}{\overline{a}-\underline{a}}\,.$$

Continuity is obvious. □

The function H maps every threshold ability to the corresponding amount of effective labor. Lemma 3 implies that, regardless of the pair (w, r), effective labor must lie in a compact interval. Specifically,

$$H \in [\underline{H}, \overline{H}] \quad \text{where} \quad \underline{H} := \frac{\underline{a}+\overline{a}}{2}\,, \overline{H} := \frac{\underline{a}^2+\underline{a}\overline{a}+\overline{a}^2}{3}\,,$$

with $\underline{H}$ being strictly larger than 0.

2.2 Production Decisions

There is a large number of firms acting competitively on goods and factor markets. The production technology can be described by some neoclassical production function F with physical capital and effective labor H as arguments. We denote capital used as a direct input to production by K_P. More specifically, we impose

Assumption 2 *The production function* $F : \mathbb{R}_+^2 \to \mathbb{R}_+$ *is twice continuously differentiable and has constant returns to scale:* $F(\lambda K_P, \lambda H) = \lambda F(K_P, H)$*. We have strictly positive, but diminishing returns to both factors:* $F_i > 0$ *and* $F_{ii} < 0$ *for* $i = K_P, H$*. The first derivatives satisfy the Inada–conditions (i.e.* $\lim_{i\to 0} F_i = +\infty$ *and* $\lim_{i\to+\infty} F_i = 0$ *for* $i = K_P, H$*), and the second derivatives are bounded.*

For any capital input $K_P > 0$, the marginal product of labor is bounded from above, since, owing to diminishing returns to H and $H \ge \underline{H}$, $F_H(K_P, H) \le F_H(K_P, \underline{H})$.

Making use of the homogeneity property, we obtain the usual per–capita version $f : \mathbb{R}_+ \to \mathbb{R}_+$ with $f(k) := F(k, 1)$ where $k := K_P/H$ denotes the capital intensity of production. Hence, $F(K_P, H) = Hf(k)$.

Under full depreciation of capital, profit–maximizing firms demand capital and effective labor according to

$$1 + r = f'(k) \tag{4}$$

$$w = f(k) - kf'(k). \tag{5}$$

3 The Static Equilibrium

For the economy to be in equilibrium, all markets must clear simultaneously. Due to Walras' law, we can concentrate on factor markets. At any instant, the available capital is given by the aggregate amount of bequests. Physical capital is used in the production of human capital and final goods, i.e. firms as well as households demand capital. As derived in the last section, capital demand for the accumulation of skills at factor prices (w, r) equals $nI = [1 - \Phi(a_0(w, r))]\, I$. Effective labor supply is then given by $H(a_0(w, r))$ (cf. (3)).

Equilibrium factor prices must satisfy (4) and (5). These inverse demand functions describe factor prices as a function of k: $w = w(k)$, $r = r(k)$. Clearly, $w'(k) > 0$ and $r'(k) < 0$. The static equilibrium is thus fully characterized by the equilibrium capital intensity. Given aggregate capital K, a capital–labor ratio k employed in production is compatible with static equilibrium iff the corresponding factor rates $w(k)$ and $r(k)$ generate investment incentives that reinforce k. This means that dividing capital not used for production of human capital, $K - [1 - \Phi(a_0(w, r))]\, I$, by the amount of physical labor, $H(a_0(w, r))$, must imply k. To make this equilibrium condition explicit, we introduce the function $h : \mathbb{R}_+^2 \to \mathbb{R}_+$ with

$$h(k, K) := \frac{K - I[1 - \Phi(a_0(w(k), r(k)))]}{H(a_0(w(k), r(k)))}\,.$$

For arbitrary positive K, the capital intensity k constitutes a static equilibrium iff

$$k = h(k, K) \tag{6}$$

holds.

The following proposition describes the set of equilibrium capital–labor ratios for a given aggregate capital stock.

Proposition 1 *For any $K > 0$, there is one and only one $k > 0$ which is compatible with static equilibrium.*

Proof. It suffices to prove that, for given K, there exists one and only one solution of the equation $k = h(k, K)$. To show this we examine the function h for fixed K: $h(\cdot, K)$. Using the firms' inverse factor demand functions, we can calculate critical values $\underline{k}$ and $\overline{k}$ from (2) such that $a_0(w(\underline{k}), r(\underline{k})) = \overline{a}$ and $a_0(w(\overline{k}), r(\overline{k})) = \underline{a}$. This means that no individual becomes skilled if $k \leq \underline{k}$, and all individuals become skilled if $k \geq \overline{k}$.

We define the function $\psi : \mathbb{R}_+ \to \mathbb{R}_+$ by $\psi(k) := f'(k)/[f(k) - kf'(k)]$. Assumption 2 guarantees that $0 < \psi(k) < \infty$ and $\psi'(k) < 0$. Then $\underline{k}$ is the solution of

$$\overline{a}(\overline{a} - 1) = I\psi(k)\,,$$

while $\overline{k}$ solves

$$\underline{a}(\underline{a} - 1) = I\psi(k)\,.$$

With $\overline{a}(\overline{a} - 1) > \underline{a}(\underline{a} - 1) > 0$ from Assumption 1, there exist unique solutions $0 < \underline{k} < \overline{k} < \infty$. The preceding considerations show that $h(\cdot, K)$ is constant for $k \leq \underline{k}$ and $k \geq \overline{k}$:

$$h(k, K) = \begin{cases} \dfrac{K}{\underline{H}} & \text{for } k \leq \underline{k} \\[2ex] \dfrac{K - I}{\overline{H}} & \text{for } k \geq \overline{k}\,. \end{cases} \tag{7}$$

For $\underline{k} < k < \overline{k}$, $h(\cdot, K)$ is strictly decreasing: increasing k, raises w and lowers r which induces larger investment incentives. The identical function which maps k to k is strictly increasing while $h(\cdot, K)$ is non–increasing. This guarantees uniqueness of a solution of $k = h(k, K)$. Existence follows from the intermediate value theorem since the identical function and $h(\cdot, K)$ are continuous, and $\lim_{k\to 0} k < \lim_{k\to 0} h(k, K)$ and $\lim_{k\to +\infty} k > \lim_{k\to +\infty} h(k, K)$ hold. □

Figure 1 illustrates the determination of the equilibrium value k for a given capital stock K by the intersection of the graph of $h(k, K)$ and the 45–degree line. Specifically, it depicts the case of an interior solution $k \in (\underline{k}, \overline{k})$, where both skilled and unskilled labor exist.

Proposition 1 guarantees the existence of a well–defined function $g : \mathbb{R}_+ \to \mathbb{R}_+$ which assigns the unique equilibrium capital–labor ratio k to every aggregate capital stock K. The following corollary provides some useful properties of g.

Corollary 1 *The function g which maps every initial K to the equilibrium k is continuous and strictly increasing in K.*

Proof. Continuity follows from the corresponding properties of $(\Phi \circ a_0)$ and $(H \circ a_0)$. Since $\partial h(k, K)/\partial K > 0$, increasing K shifts up the point where $h(\cdot, K)$ and the 45–degree line intersect. □

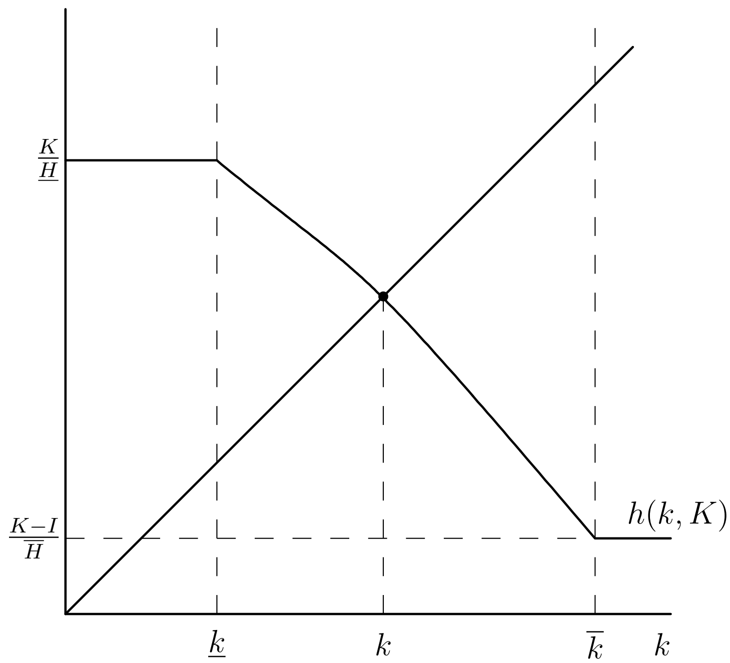

Figure 1: Determination of the equilibrium capital intensity

In the proof of Proposition 1, we defined $\underline{k}$ and $\overline{k}$ as the capital intensities for which the corresponding factor prices provide only the most able ($\overline{a}$) or even the least able ($\underline{a}$) with human capital investment incentives. That is, if the equilibrium capital–labor ratio lies below $\underline{k}$, no agent upgrades his innate ability. By contrast, all residents opt for additional qualification if the equilibrium k exceeds $\overline{k}$. We denote the capital stocks that are associated with $\underline{k}$ and $\overline{k}$ by $\underline{K}$ and $\overline{K}$, respectively: $\underline{K} := g^{-1}(\underline{k})$ and $\overline{K} = g^{-1}(\overline{k})$. A look at Figure 1 reveals that $g^{-1}(\underline{k}) = \underline{H}\,\underline{k}$ and $g^{-1}(\overline{k}) = \overline{H}\,\overline{k} + I$.

It follows from Proposition 1 that there is a well–defined function $a_s : \mathbb{R}_+ \to \mathbb{R}_+$ which assigns the corresponding threshold ability to every capital stock K. More precisely, it has the form

$$a_s = a_0 \circ (w, r) \circ g. \tag{8}$$

The definition of $\underline{K}$ and $\overline{K}$ is such that $a_s(K) \geq \overline{a}$ for $K \leq \underline{K}$ and $a_s(K) \leq \underline{a}$ for $K \geq \overline{K}$. An aggregate capital stock $K \in (\underline{K}, \overline{K})$ implies a capital intensity $g(K) \in (\underline{k}, \overline{k})$ and a threshold ability in the interior of $(\underline{a}, \overline{a})$. For $\underline{K} < K < \overline{K}$, increasing the capital stock therefore lowers a_0 and raises the number of skilled workers. We summarize these results in the following corollary.

Corollary 2 *(i) There is a positive capital stock $\underline{K}$ below which no capital is employed in human–capital production ($n = 0$). (ii) There exists $\overline{K} > 0$ above which all agents become skilled ($n = 1$). (iii) For $\underline{K} < K < \overline{K}$, the fraction of skilled workers n is strictly increasing in K.*

The intuition of Corollary 2 is straightforward. In early stages of development ($K < \underline{K}$), human–capital investments are not profitable because of the high marginal product of capital in production. The marginal product of labor in production is bounded from above, since effective labor always exceeds $\underline{H}$. If a capital stock above $\underline{K}$ is available, investment becomes attractive for the most talented individuals. Eventually, all human–capital–investment opportunities are exploited, and excess capital $K - I$ is directly applied in the production of final goods.

We are now in the position to examine the relation between the aggregate capital stock and the equilibrium output. The amount of capital used as a direct input in production amounts to total capital, K, minus capital used for production of human capital, $n(a_s(K))I$.

Lemma 4 *The relationship between output in the competitive equilibrium and the available capital K can be characterized by a continuous, strictly increasing, and strictly concave function $Y : \mathbb{R}_+ \to \mathbb{R}_+$ where*

$$Y(K) := F\left[K - n(a_s(K))I, H(a_s(K))\right] . \tag{9}$$

Proof. See Appendix. □

Since there are no frictions in our economy, the equilibrium output effect of an additional marginal unit of capital in the economy is equal to the effect of marginally increasing the capital input in production. This implies that the derivative of equilibrium output as a function of K is continuous and strictly decreasing. Concavity seems logical, since we have a decreasing returns to scale production function and additional capital for schooling makes individuals invest whose ability is strictly smaller than that of previously educated.[10] Plotting the

[10]Note that we do not refer to particular persons or dynasties. Independence of ability across generations means that children from a dynasty which worked as skilled in the last period need not upgrade their abilities.

graph of Y as a function of K yields a figure which is very similar to that of an ordinary production function. This is precisely the reason why we can determine the dynamic structure of the economy without further complications.

4 Transitional Dynamics

Since capital depreciates fully during the production of human capital and final goods, the aggregate amount of wealth that is available in the economy at the end of a period equals total output. Owing to the constant bequest share s resulting from homothetic utility, the next generation's aggregate capital endowment reads

$$K_{t+1} = sY_t\,.$$

Making use of (9), we can fully characterize the dynamic evolution of an economy starting with K_0 in period 0 by the following first–order difference equation

$$K_{t+1} = sF\left[K_t - n(a_s(K_t))I, H(a_s(K_t))\right] \quad \text{for} \quad t \geq 0. \tag{10}$$

A capital stock K^* is a steady state of this dynamical system if K^* once reached will be preserved. That is, K^* must be a fixed point of the dynamic equation (10): $K^* = \alpha Y(K^*)$. Since the graph of Y as a function of K resembles that of an ordinary production function (cf. Lemma 4) the following dynamics obtains:

Proposition 2 *(i) The dynamical system given in (10) has exactly two steady states: $K = 0$ and $K = K^* > 0$. (ii) The no–capital equilibrium is unstable and, unless an economy starts in $K_0 = 0$, it converges to K^* (i.e. K^* is globally stable in this sense). (iii) The convergence obtained for $K_0 \neq 0$ in (ii) is monotone: for $K_0 < K^*$ it is strictly increasing, while for $K_0 > K^*$ it is strictly decreasing.*

Proof. (i) The existence of the steady state $K = 0$ is obvious, since an economy without capital can never produce anything.[11] The existence of a positive steady state $K^* > 0$ follows from the properties of Y as a function of K (Lemma 4), Corollary 2, and the intermediate value theorem. For $K < \underline{K}$ no human–capital

[11] The fact that, in our neoclassical setting, capital is essential for production follows from the Inada–conditions. A formal proof is provided, e.g., in Barro and Sala–i–Martin (1995).

investments take place. We have $Y(K) = F(K, \underline{H})$ with $\lim_{K\to 0} F_K(K, \underline{H}) = +\infty$ according to the first Inada–condition. For $K > \overline{K}$ we have $Y(K) = F(K - I, \overline{H})$, since all human–capital–investment opportunities are exploited. Hence, the second Inada–condition guarantees $\lim_{K\to+\infty} F_K(K - I, \overline{H}) = 0$. Hence, $sY(K)$ must exceed K for K close to zero, while $sY(K) < K$ for large K. Then an application of the intermediate value theorem proves the existence of a steady state $K^* > 0$. Strict concavity of Y (Lemma 4) makes sure that it is the only positive one.

The claims in (ii) and (iii) are simple consequences of Lemma 4, the Inada–conditions, and Corollary 2. □

The shape of the phase diagram for the path of aggregate capital which follows from Proposition 2 is depicted in Figure 2.

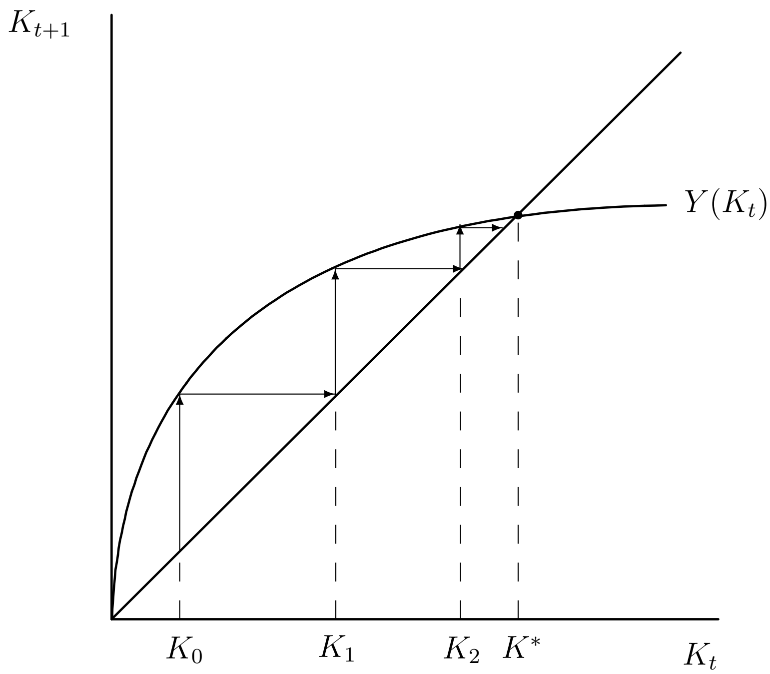

Figure 2: Phase diagram of the aggregate capital stock

The results of Proposition 2 do not come as a surprise, because our model uses a production technology with decreasing returns to scale with respect to both factors of production and which meets the Inada–conditions. In this purely neoclassical setup, long–run growth from factor accumulation cannot be expected.

The development of the economy is completely determined by the sequence of capital stocks $(K_t)_{t\geq 0}$. In the sequel, we will restrict ourselves to the case $0 < K_0 < K^*$. We can use properties of the functions g, a_s, and H to characterize the corresponding paths of human capital and the factor prices:

Corollary 3 *The sequences $(r_t)_{t\geq 0}$ and $(a_{s,t})_{t\geq 0}$ are strictly decreasing, while $(w_t)_{t\geq 0}$ is strictly increasing. The human–capital sequence $(H_t)_{t\geq 0}$ is non–decreasing.*

An economy starting from a positive amount of capital which is smaller than the steady state value will experience positive growth rates of K. Whether the long–run equilibrium induces a 100% share of skilled workers ($K^* \geq \overline{K}$) or whether there are any highly qualified workers at all ($K^* \leq \underline{K}$) is just a matter of the parameters and the production function. To be able to study effects of wage inequality between the high–skilled and the low skilled group we postulate

Assumption 3 *Parameters and the production function are chosen such that $K^* \geq \overline{K}$.*

Having determined the dynamical behavior of the economy, we now turn to the evolution of wage inequality along the adjustment path.

5 The Development of Wage Inequality

During the transition process to the steady state, the composition of the workforce changes since the threshold ability shifts downward. Let $(a_{s,t})_{t\geq 0}$ denote the sequence of threshold abilities implied by a some sequence of capital stocks K_t. Since there are no skilled workers for $K \leq \underline{K}$ and no unskilled workers for $K \geq \overline{K}$, we focus on that part of the sequence where $a_{s,t} \in (\underline{a}, \overline{a})$. To study the implications of this sorting effect for the wage structure we consider two aspects of wage inequality: first, the wage differential by skills (between–skill–group inequality), and second, the wage dispersion within the groups (within–group inequality).

First, we investigate how the wage gap evolves along the adjustment path to the steady–state equilibrium. Since agents with the same educational attainment but heterogeneous abilities differ in their labor earnings, we need to choose a measure of location. For the sake of simplicity, we use the mean income for both groups. To guarantee scale invariance, between–group wage inequality is then measured by the ratio of the average wage income of the two groups which we denote by ω. That is,

$$\omega := \frac{\text{average labor earnings of the skilled workers}}{\text{average labor earnings of the unskilled workers}}.$$

Since ω does not depend on the wage rate w for an effective labor unit, changes in ω solely result from composition effects.

Our analysis starts by analyzing the relationship between ω and the threshold ability a_s. We define the function $w_l : (\underline{a}, \overline{a}) \to \mathbb{R}_+$ which assigns the mean wage of the low–skilled group to every threshold ability $a_s \in (\underline{a}, \overline{a})$. That is, $w_l(a_s)$ denotes average labor earnings of the unskilled workers given that people with at least a_s acquire education. Analogously, we introduce $w_h : (\underline{a}, \overline{a}) \to \mathbb{R}_+$, where $w_h(a_s)$ is the respective mean income of the skilled workers. Relative inequality is then measured by

$$\omega(a_s) := \frac{w_h(a_s)}{w_l(a_s)}.$$

The unskilled workers have abilities which are distributed uniformly over $[\underline{a}, a_s]$, while the ability distribution within the skilled group is uniform with support $[a_s, \overline{a}]$. Thus, the distribution of abilities conditional on the educational attainment clearly depends on the value of the threshold ability. Some simple calculations deliver the following functions w_l and w_h[12]

$$\begin{aligned} w_l(a_s) &= \frac{a_s + \underline{a}}{2} w(a_s) \\ w_h(a_s) &= \frac{\overline{a}^2 + \overline{a} a_s + a_s^2}{3} w(a_s). \end{aligned}$$

The relative wage reads

$$\omega(a_s) = \frac{2}{3} \frac{\overline{a}^2 + \overline{a} a_s + a_s^2}{\underline{a} + a_s}.$$

As a_s decreases, the ability structure within the skill groups changes. In particular, mean ability falls for both educational attainments: on the one hand, the most talented among the previously unskilled become skilled, thus reducing average ability in the unskilled group; on the other hand, their ability is smaller

[12]As the wage for an effective unit of labor changes for increasing human–capital–investment incentives, we let $w(a_s)$ denote this functional dependence.

than that of workers that opted for higher labor market qualification before, thus reducing mean ability in the skilled group. We find that under some simple condition the relative wage $\omega(a_s)$ is not monotone on $(\underline{a}, \overline{a})$:

Lemma 5 *For* $3\underline{a}^2 + \overline{a}\underline{a} - \overline{a}^2 < 0$ *there exists* $\widetilde{a} \in (\underline{a}, \overline{a})$ *so that* $\omega' > 0$ *for* $a_s > \widetilde{a}$ *and* $\omega' < 0$ *for* $a_s < \widetilde{a}$.

Proof. The derivative of ω with respect to a_s can be calculated as

$$\omega'(a_s) = \frac{2}{3} \frac{\underline{a}\overline{a} + 2a_s\underline{a} + a_s^2 - \overline{a}^2}{(\underline{a} + a_s)^2} \,. \tag{11}$$

The sign of (11) is equal to the sign of the numerator which is a quadratic function m in $a_s \in (\underline{a}, \overline{a})$. Obviously, the quadratic function is strictly increasing in a_s. We observe that $\lim_{a_s \to \overline{a}} m(a_s) > 0$. The above condition is sufficient and necessary to guarantee $\lim_{a_s \to \underline{a}} m(a_s) < 0$, so that continuity and monotonicity of m prove the claim by an application of the intermediate value theorem. □

According to Lemma 5, the relation between ω and a_s is U–shaped. Note that the condition guaranteeing non–monotonicity will always be satisfied, if $\overline{a} - \underline{a}$ is sufficiently large. Every sequence of capital stocks generated by (10) produces a sequence of threshold abilities $(a_{s,t})_{t \geq 0}$. The following theorem which is an immediate consequence of Lemma 5 provides a sufficient condition for a non–monotone evolution of the relative wage.

Proposition 3 *Let* $(a_{s,t})_{t \geq 0}$ *denote the sequence of threshold abilities implied by the sequence of capital stocks* $(K_t)_{t \geq 0}$. *Let, furthermore,* $N \subset \mathbb{N}$ *(*$M \subset \mathbb{N}$*) be the set of natural numbers for which* $a_{s,t} \in (\underline{a}, \widetilde{a}]$ *(*$a_{s,t} \in [\widetilde{a}, \overline{a})$*); i.e.* $N := \{t \in \mathbb{N} | a_{s,t} \in (\underline{a}, \widetilde{a}]\}$ *and* $M := \{t \in \mathbb{N} | a_{s,t} \in [\widetilde{a}, \overline{a})\}$. *If both of these sets have at least two elements, the economy experiences a period of decreasing between–group inequality followed by a phase of an increasing wage differential by skills.*[13]

Along adjustment paths to the steady–state equilibrium with an initial capital stock below K^*, the rise in the return to education (increasing w) and the simultaneous reduction in the cost of education (decreasing r) make more people invest in human capital. At the same time, the distribution of abilities within the skill groups changes its shape so that average productivities are not time–invariant.

[13]Both sets N and M depend on the initial capital stock.

An economy can then experience extended periods of decreasing between–group wage inequality followed by a phase of increasing inequality, given the difference between the least and the most talented individuals is sufficiently large. A path of threshold abilities $a_{s,t}$ which is sufficient for non–monotone behavior of the relative wage is depicted in Figure 3.

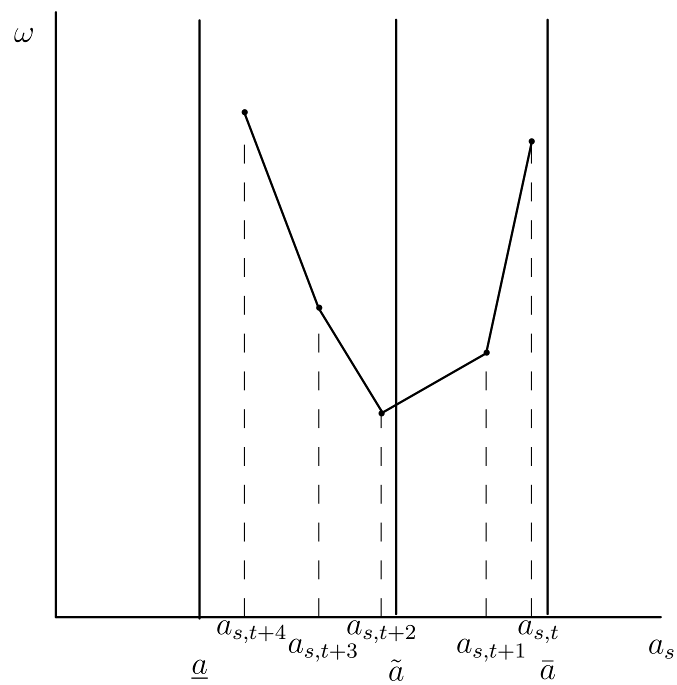

Figure 3: The U–shaped path of the relative wage

The rest of this section deals with residual inequality. Apart from the non–monotonicity observed for the skill premium, empirical studies obtain a rise in within–group inequality at least for the skilled, whereas evidence concerning inequality among the unskilled is mixed (cf. Introduction). The model, as it is, predicts increasing residual inequality for the skilled workers and decreasing wage dispersion among the unskilled.

To assess the change of inequality, we calculate the Gini coefficient at first for the educated class. For $a_s \in (\underline{a}, \overline{a})$ the Gini–coefficient for the skilled workers is given by

$$G_h(a_s) = \frac{\overline{a}^2 - a_s^2}{2(a_s^2 + \overline{a}a_s + \overline{a}^2)} \, ,$$

with $G_h'(a_s) < 0$. So any sequence of capital stocks $(K_t)_{t\geq 0}$ successively produces a more unequal group of skilled workers.[14]

Proposition 4 *Inequality among the skilled workers rises along the adjustment path.*

This result seems quite intuitive, since a larger capital stock provides people with lower abilities with investment incentives. That is, the skilled workers become more heterogeneous. The opposite holds true for the unskilled group. Computing their Gini–coefficient yields

$$G_l(a_s) = \frac{5}{3}\frac{a_s - \underline{a}}{a_s + \underline{a}} \, .$$

Obviously, $G_l'(a_s) > 0$, i.e. lower a_s implies less inequality among the unskilled.

Proposition 5 *The wage dispersion in the unskilled group measured by the Gini–coefficient is declining.*

That capital accumulation leads to decreasing inequality among the unskilled does not come as a surprise. As more and more people opt for qualified work, heterogeneity among the unskilled measured, for instance, by the variance of abilities within the unskilled group declines.

Hence, apart from consistence with the empirical evolution of the wage differential by skill, the predictions of the model resemble the observed development pattern of wage dispersion among the skilled (Proposition 4). Our theoretical results on inequality within the unskilled group are consistent with the data of at least some countries.

6 Wage Floors and Within–Group Inequality

While wage compression in the low–skilled group, as described in Proposition 5, is compatible with observations in countries such as, e.g., Belgium or Germany,

[14] Alternatively, one could have considered the evolution of various quantile coefficients, but the results are similar.

it is at odds with the empirical facts for other countries—most prominently, the US and the UK. To bring our model to these facts, we integrate an idea put forward by Card (1996), DiNardo et al. (1996), and Freeman (1996) into our basic framework. These authors attribute the rise in within–group wage inequality to the non–adjustment of some institutionally set wages floors (either minimum wages or wages bargained between unions and firms). As wage incomes *in general* rise with increasing productivity of labor, the incomes of workers that just receive the minimum wage income remain unchanged. Consequently, within–group wage inequality rises if wage floors do not keep up with increases in labor productivity. Card (1996) and DiNardo et al. (1996) claim this to be the case especially for the US and the UK, either because minimum wages did not keep up with the productivity increase (US) or because of declining bargaining power of unions (UK).

Integrating such an argument into our model makes our results consistent with these country–specific observations. Moreover, our analysis sheds light on the differences in the evolution of unemployment rates between the US or UK on the one hand, and Germany on the other.

We concentrate on the basic mechanism and present the argument in a rather informal way rather than adapting the complete general–equilibrium structure of our model. Suppose there is an institutionally set wage floor below which wage incomes paid in the economy must not lie. For simplicity, we take this minimum wage income, $\overline{w}$, as given exogenously. As a result of a binding wage floor, there are individuals with a productivity that is too low to guarantee them a wage income of $\overline{w}$. Consequently, these individuals are not employed by any of the firms. For a given effective wage w, the threshold ability to become employed is $\overline{w}/w$.[15]

As w rises along the adjustment path (we again assume $K_0 < K^*$), the threshold ability $\overline{w}/w$ falls over time if wage floors are not adjusted. In reaction to that, some individuals that were previously unemployed can now get a job. Additionally—as in the case without institutional wage rigidities—, more people acquire education, thus increasing the number of skilled workers. While these changes in the composition of both groups of labor do not affect inequal-

[15] We exclude for simplicity the possibility that the least able can alternatively acquire education thus raising their productivity above the threshold.

ity for the high–skilled, they have an important consequence for the low–skilled group. As the most able leave this group for skilled work and individuals with lower abilities enter the group, it is now by no means clear that within–group wage dispersion must be decreasing. Hence, increasing inequality for the low–earnings group may result if wage floors are not adjusted.

The change in the threshold ability that enables individuals to become employed also affects the rate of unemployment. As $\overline{w}/w$ declines along the adjustment path, a greater range of abilities becomes employed. Therefore, the unemployment rate should fall due to the non–adjustment of wage floors. On the other hand, if wage floors are adjusted and keep up with the rise in labor productivity, the unemployment rate will not be affected by labor–productivity gains. Thus, there is a trade off between wage inequality and employment opportunities for unskilled labor. In the US and the UK, the extraordinary reductions in the unemployment rates (esp. for unskilled labor) came at cost of a considerable rise in residual wage inequality. In contrast, the success of German unions to keep up bargained wages to the rise in productivity reduced residual inequality at cost of persistent high unemployment there.

7 Discussion of Basic Assumptions

The above analysis presumes a uniform distribution of abilities (Assumption 1) and increasing returns to education. While the assumption that education squares one's efficiency is plausible but not uncontroversial, specifying a uniform distribution hardly bears resemblance to real–world ability distributions. This section illustrates why, while being analytically convenient, both of these requirements are not crucial for the results, i.e. why the composition effect produces similar results under more general specifications. To this purpose assume abilities have a continuous distribution with a density that is strictly positive and differentiable in the interior of $[\underline{a}, \overline{a}]$ and education yields $e(a) > 0$ efficiency units with $e' > 0$. Moreover, take e to be such that there exists some threshold a_s that decreases over time.[16]

[16] An example would be $e(a) := ba$ where $b > 1$. Assuming there is no threshold value above which agents opt for higher education contradicts the observation of a positive correlation between ability and education.

i) Between–group inequality

The following ratio of average wages results for an interior a_s:

$$\omega(a_s) := \frac{w_h(a_s)}{w_l(a_s)} = \frac{F(a_s)}{1-F(a_s)} \cdot \frac{\int_{a_s}^{\overline{a}} e(a)f(a)\,da}{\int_{\underline{a}}^{a_s} af(a)\,da}$$

where F denotes the distribution function associated with f. Differentiating with respect to a_s yields

$$\omega'(a_s) \gtreqless 0 \quad \Longleftrightarrow \quad F(a_s)\frac{e(a_s)}{w_h} + (1-F(a_s))\,\frac{a_s}{w_l} \lesseqgtr 1\,.$$

The sign of ω' cannot be decided in general. Since ω' depends on both the distribution of abilities f and the reward for investment function e there is no reason why ω should be monotone over the whole range of a_s values. Uniform abilities and a quadratic function e are just one combination where this is not the case. Other specifications generating non–monotonicity, however, do not admit closed–form solutions.

ii) Within–group inequality

A downward–shift of the threshold level a_s makes abilities acquire higher education that were previously unskilled. That is, the most talented among the unskilled drop out of low–skilled work. Without minimum wages, this raises (lowers) heterogeneity in the skilled (unskilled) group through its influence on $\text{Var}(a|s = s_i)$ so that within–group inequality tends to increase (decrease). This effect plays a role in determining within–group inequality regardless of the particular specification of f. Heterogeneity in the unskilled cohort may increase if low abilities switch from unemployment to low–skilled work since the minimum wage ceases to be binding for them.

These considerations should illustrate that the composition effect induces similar results under more general ability distributions and education reward functions. Thus, making these simplifying assumptions facilitates the presentation without being really essential for the basic intuition.

8 Conclusions

This paper analyzed the development of wage inequality by skills within a simple neoclassical growth model with heterogeneous labor. Changes in the composition of skill groups imply a non–monotone time path of the wage differential by skills. The development of wage dispersion within skill groups is also consistent with empirical facts from a number of countries, esp. if we additionally incorporate some non–adjustment of wage floors—a feature that has been claimed to be important for the US and the UK. A model extended along these lines also accounts for observed differences in the behavior of the unemployment rates over time.

We employ a neoclassical model where the transitional dynamics is the driving force behind changes in the ability structure to show that a non–trivial evolution of the wage structure is inherent to macroeconomic models even if we neglect technological progress and other complications. Our findings suggest, for instance, that there is no necessity to refer to exogenous shocks to explain non–monotonicities. This is particularly important if, while shocks appear to be country–specific, a non–monotone evolution is observed in a broad cross–section of countries. We do not claim the mechanism to be a comprehensive explanation of the wage structure. By emphasizing the role of composition effects, it is complementary to analysis relying on time–trends in returns to education and ability. In fact, the kind of sorting described above amplifies demand side effects, since these impact on pay–offs to education and to ability which certainly affect incentives to acquire skills. An increasing skilled workforce is the simplest example of such a 'selection effect'.

Appendix

This appendix contains the proof of Lemma 4. The proof makes use of the following lemma:

Lemma A.1 *The outcome of the competitive economy is equivalent to the outcome in a command economy where a social planner chooses an allocation that maximizes aggregate output.*

We proof this equivalence first.

Proof. A social planner that tries to maximize aggregate output has to decide how to allocate given initial K between production and human–capital acquisition (i.e. he has to choose how much capital K_P to apply as a direct input to production and how much capital K_I to use for education). His problem is

$$\max_{K_I,K_P} \left\{ F(K_P,H) \;:\; K \geq K_P + K_I\,,\; K_I = (\overline{a}-a_1)/(\overline{a}-\underline{a})I\,,\right.$$
$$H = \int_{\underline{a}}^{a_1} \frac{a}{\overline{a}-\underline{a}}\,da + \int_{a_1}^{\overline{a}} \frac{a^2}{\overline{a}-\underline{a}}\,da\,,$$
$$\left. K_P \geq 0\,,\; 0 \leq K_I \leq I \right\},$$

where a_1 denotes the threshold ability chosen by the planner. That optimization problem can be recast as

$$\max_{a_1} \left\{ F\left(K - \frac{\overline{a}-a_1}{\overline{a}-\underline{a}}I, \frac{1}{2}\frac{a_1^2-\underline{a}^2}{\overline{a}-\underline{a}} + \frac{1}{3}\frac{\overline{a}^3-a_1^3}{\overline{a}-\underline{a}}\right) \;:\; \underline{a} \leq a_1 \leq \overline{a} \right\}.$$

Some simple calculations show that for $K \leq \underline{K}$ no physical capital is used for additional qualification of the workers, since in this case $F_K(K,\underline{H}) \geq F_H(K,\underline{H})(\overline{a}^2 - \overline{a})/I$ according to the definition of $\underline{K}$. This result is analogous to the corresponding competitive equilibria in which there are no skilled workers. For $K \geq \overline{K}$ all human–capital–investment opportunities are exploited and the dictator uses additional capital as a direct production input. For $K \in (\underline{K},\overline{K})$, the above maximization problem has an interior solution with a_1 implicitly given by

$$F_K\left(K - \frac{\overline{a}-a_1}{\overline{a}-\underline{a}}I, \frac{1}{2}\frac{a_1^2-\underline{a}^2}{\overline{a}-\underline{a}} + \frac{1}{3}\frac{\overline{a}^3-a_1^3}{\overline{a}-\underline{a}}\right) - F_H\left(K - \frac{\overline{a}-a_1}{\overline{a}-\underline{a}}I, \frac{1}{2}\frac{a_1^2-\underline{a}^2}{\overline{a}-\underline{a}} + \frac{1}{3}\frac{\overline{a}^3-a_1^3}{\overline{a}-\underline{a}}\right)(a_1^2-a_1)/I = 0$$

Obviously this corresponds to a competitive equilibrium with

$$k = \frac{K - \dfrac{\overline{a}-a_1}{\overline{a}-\underline{a}}I}{\dfrac{1}{2}\dfrac{a_1^2-\underline{a}^2}{\overline{a}-\underline{a}} + \dfrac{1}{3}\dfrac{\overline{a}^3-a_1^3}{\overline{a}-\underline{a}}}\,,$$

since $1+r = f'(k)$ and $w = f(k) - kf'(k)$ imply a threshold ability a_1. Owing to the uniqueness of competitive equilibria for given K, the equivalence is proven.

□

We are now able to prove Lemma 4.

Proof. Owing to the equivalence of the competitive and the command environment, the continuity between K and k (cf. Corollary 1) holds in the command economy, as well. For $K > 0$, the threshold ability above which the planner chooses to upgrade human capital is given by the $a_s(K)$ (the corresponding function a_s was introduced in (8)). An additional unit of capital yields more output according to $F_K(K - (\overline{a} - a_1)(\overline{a} - \underline{a})I, H(a_1))$ (the envelope theorem).[17] The equivalence between the environments implies that the marginal product equals $f'(g(K))$. Continuity of g yields continuity of dY/dK. Concavity of the per–capita production function f and the monotonicity of g prove the claim. □

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[17] For $a_1 \in [\underline{a}, \overline{a}]$ this is due to the fact that in the margin capital is equally useful in both applications.

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