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The Return of the Long-Run Phillips Curve

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ABSTRACT

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This paper shows that the interaction between money growth and staggered nominal contracts gives rise to a long-run inflation-unemployment tradeoff.

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"Most economists who came to accept the view that there was no long-run trade-off between inflation and unemployment were more affected by a priori argument than by empirical evidence." Blanchard and Fisher (1989)

1 Introduction

The existence of a non-vertical long-run Phillips curve[1] has long been considered untenable on theoretical grounds. It is generally accepted that, in the absence of money illusion, an increase in the growth rate of the money supply can have real effects only in the short run. In the long run, according to the conventional wisdom, the only effect is an increase in inflation. Our analysis calls this argument into question, without positing any permanent nominal rigidity, any departure from rational expectations or any form of money illusion. We integrate microfoundations of wage staggering into a simple dynamic general equilibrium model with rational expectations. In this context we show that a permanent increase in money growth leads to a permanent increase in the rate of inflation and a *permanent* reduction in the level of unemployment. In short, we derive a microfounded long-run downward-sloping Phillips curve.

Let us examine the intuition behind this result. Staggered wage setting implies that the current nominal wage is a weighted average of past and expected future nominal wages. A standard result (e.g. Helpmann and Leidermann (1990)) is that the weights on these terms are not symmetric: future wages receive less weight than current ones. If the money supply is growing, this implies that the optimal wage always lags behind the wage that would be chosen under wage flexibility, because more weight is put on past (lower) wages than on current (higher) ones. With prices set as a markup over wages, real money balances are higher than they would be under full wage flexibility. If money growth increases, the optimal wage lags further behind the flexible wage, so that real money balances rise. This raises output and reduces unemployment.

Is this effect likely to be empirically important? We show that, given parameter values that are common in the literature, a 1% increase in the money growth rate can lead to a long-run decrease in unemployment from 15% to 0.5% below its steady state level. In the short and medium run, the effects can be large and have a long half-life.

These results are consonant with a significant body of empirical findings surveyed, for example, in Mankiw (2000) who writes that "..if one does not approach the data with a prior favoring long-run neutrality, one would not leave the data with that posterior. The data's best guess is that monetary shocks leave permanent scars on the economy".

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[1]By a long-run Phillips curve we mean the long-run relation between inflation and unemployment generated by permanent changes in money growth.

2 The Model

We use a dynamic general equilibrium model with the following simplifying assumptions: labor supply is fixed, the production function is linear in labor and there is no capital.

2.1 Households

A continuum of households indexed on [0,1] consume, hold money balances and nominal bonds and supply differentiated labor which is imperfectly substitutable (à la Dixit-Stiglitz (1977)) in the production of each firm's output. We make the standard assumption that first the nominal wage is set and then consumption is chosen taking the wage as predetermined.

Each nominal wage is set for two periods, and in each period half of households change their wage. A household h that can change it's wage in period t does so by solving the problem[2]:

$$
\max_{\mathsf{W}_t(h)} E_t \sum_{i=0}^{1} \beta^i \left[U\left(c_{t+i}(h), \frac{\mathsf{M}_{t+i}}{\mathsf{P}_{t+i}}(h) \right) - V\left(n_{t,t+i}^d(h) \right) \right] \tag{1}
$$

(where $\mathsf{W}_t(h)$ is the nominal wage, $c_{t+i}(h)$ is consumption, β is the time discount factor, and $U_1, U_2 > 0, U_{11}, U_{22} < 0, V_1, V_{11} > 0$) subject to its budget constraint and the firms' demand curve for the household's labor

$$
n_{t,t+i}^d(h) = \left(\frac{\mathsf{W}_t(h)}{\mathsf{W}_{t+i}} \right)^{-\theta} n_{t+i}^d \tag{2}
$$

Deriving first order conditions, assuming a symmetric equilibrium and linearizing,[3] we obtain the following equation for the optimal wage, $\hat{W}_t^*$, chosen by all households that change their price at time t:

$$
\hat{W}_t^* = \alpha \hat{P}_t + (1-\alpha)\hat{P}_{t+1} + (1-\alpha)\hat{\mu}_t + \gamma \left[\alpha \hat{y}_t + (1-\alpha)\hat{y}_{t+1} \right] \tag{3}
$$

where $\alpha = \frac{1}{1+\beta}$ is a discounting parameter, γ is a demand-sensitivity parameter, $\hat{\mu}_t$ is the growth rate of the money supply defined by:

$$
\hat{\mu}_t = \hat{M}_t - \hat{M}_{t-1} \tag{4}
$$

---

[2] In a steady state with money growth, all nominal variables grow at the same rate as money and all real variables are non-trended. We use X_t for a trended variable, X_t for a detrended variable, X for the steady state and $\hat{X}$ for the log-linearisation. Upper case letters denote nominal variables, lower case real.

[3] We linearise around a steady state with no money growth.

and $\hat{P}_t$ is the linearized Dixit-Stiglitz aggregate price level:

$$\hat{P}_t = \frac{1}{2}\left(\hat{W}_{t-1}^* + \hat{W}_t^* - \hat{\mu}_t\right) \tag{5}$$

Substituting (5) into (3) gives:

$$\hat{W}_t^* = \alpha\hat{W}_{t-1}^* + (1-\alpha)\hat{W}_{t+1}^* - (2\alpha-1)\hat{\mu}_t + \gamma\left[\alpha\hat{y}_t + (1-\alpha)\hat{y}_{t+1}\right] \tag{6}$$

which is a microfounded version of equation (1) in Taylor (1980). Whereas Taylor assumed the coefficients on the backward-looking and forward-looking terms were the same, our derivation is in accord with the standard result that, if the discount factor is less than unity, the backward looking term is weighted more heavily than the forward-looking term. Observe that (6) contains a money growth term $(\hat{\mu}_t)$, which substantiates the intuition given above - namely, that a rise in money growth causes the wage to lag further behind the flexible wage (which is proportional to the money supply), and thus the detrended wage falls $(2\alpha - 1 > 0$ since $\alpha > \frac{1}{2})$.

Given this nominal wage, the representative household[4] chooses consumption, real money balances and bond holdings by solving the infinite horizon problem:

$$\max_{\{C_{t+j}, \mathsf{M}_{t+j}, \mathsf{B}_{t+j}\}_{j=0}^{\infty}} E_t \sum_{i=0}^{\infty} \beta^i \left[U\left(c_{t+i}, \frac{\mathsf{M}_{t+i}}{\mathsf{P}_{t+i}}\right) - V(n^s) \right] \tag{7}$$

subject to a series of budget constraints:

$$\mathsf{P}_t c_t + \mathsf{M}_t + \mathsf{B}_t = \mathsf{M}_{t-1} + R_t \mathsf{B}_{t-1} + \mathsf{W}_t n^s + \mathsf{T}_t, \quad \forall t \tag{8}$$

where R_t is the nominal interest rate on bond holdings B_t, and T_t is net lump-sum transfers from government.

For the utility function

$$U(c_t, \mathsf{M}_t) = \left[c_t^\gamma \left(\frac{\mathsf{M}_t}{\mathsf{P}_t}\right)^{1-\gamma} \right]^{1-\sigma} \tag{9}$$

the three resulting first-order conditions can be combined to give

$$c_t = \frac{\gamma}{1-\gamma}\left(1 - \frac{1}{\beta}\right)\frac{\mathsf{M}_t}{\mathsf{P}_t} \tag{10}$$

Detrending and linearizing gives:

$$\hat{c}_t = -\hat{P}_t \tag{11}$$

---

[4]Along standard lines, we assume complete insurance so that equilibrium decisions will be identical across households. For simplicity we have dropped the insurance transfer term from the budget constraint.

2.2 Firms

Firms face a production function linear in composite labor:

$$y_t = \left[\int_{h'=0}^{1} n_t \left(h' \right)^{\frac{\theta-1}{\theta}} dh' \right]^{\frac{\theta}{\theta-1}} \tag{12}$$

Maximizing profits subject to (12) gives the labour demand curve (2). Linearizing (12) yields $\hat{y}_t = \hat{n}_t$. Combining this with the assumption of fixed labor supply, n^s, we can write unemployment ($\hat{u}_t = -\nu \hat{n}_t$ where $\nu = \frac{n}{n^s - n}$) as

$$\hat{u}_t = -\nu \hat{y}_t \tag{13}$$

Then, evaluating the wage equation (6) in the long-run after a money growth shock and using (13), we obtain the long-run Phillips curve:

$$\hat{\pi}^{LR} = \hat{\mu}^{LR} = -\frac{\nu\gamma}{2\alpha - 1}\hat{u}^{LR} \tag{14}$$

where inflation is

$$\hat{\pi}_t = \hat{P}_t - \hat{P}_{t-1} + \hat{\mu}_t \tag{15}$$

2.3 General equilibrium

Since different points on the long-run Phillips curve correspond to different permanent money growth rates, we focus on permanent shocks to money growth. Accordingly, let the money growth rate be a random walk:

$$\hat{\mu}_t = \hat{\mu}_{t-1} + \varepsilon_t \tag{16}$$

where ε_t is a white noise shock.

The government's budget constraint is:

$$\mathsf{G}_t = \mathsf{M}_t - \mathsf{M}_{t-1} + \mathsf{B}_t - R_t\mathsf{B}_{t-1} - \mathsf{T}_t \tag{17}$$

and we assume that government spending G_t is always zero.

Combining (8), (12), and (17) shows that the good market clears, $y_t = c_t$, and so an equilibrium for this economy is a collection of allocations for households and firms $\left\{ \hat{y}_t, \hat{M}_t \right\}$ together with prices $\left\{ \hat{P}_t, \hat{W}_t^* \right\}$ and a money supply process $\{\hat{\mu}_t\}$ that satisfy (4), (5), (6), (11), and (16). We can then use (13) and (15) to obtain unemployment and inflation. This system can be solved explicitly (Karanassou, Sala and Snower (2002)) or simulated using standard methods[5].

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[5]MATLAB programs available on request from the authors

3 Results

Figure 1 shows the response of the model economy to a change in the growth rate of money from zero to 1% . Calibrating on semi-annual data (so that our assumption of N=2 means wages are fixed for one year), we take the discount factor β to be 0.98. For the demand-sensitivity parameter γ, [6] we use a value of 0.1 and we assume a steady-state level of unemployment of 5%

We find that inflation initially overshoots its long run value (1% above its initial value) but falls back to this value within 2 years. Unemployment falls on impact by 8.6% (relative to its initial steady state) then gradually rises back to its long run level of 2% below its initial value. In short, unemployment (and of course output) responds strongly to an increase in the rate of money growth and in the long-run unemployment rate remains below its initial value. By contrast, if money were superneutral, the 1% rise in long-run inflation would leave long-run unemployment unaffected.

Table 1 shows how the magnitude of the real effects depends on the calibrated parameters. Whereas the impact response of unemployment is quite insensitive to plausible parameter variations, this is certainly not the case for the unemployment half-life[7] of the shock or the slope of the long-run Phillips curve. The higher the discount rate, the more asymmetric are the coefficients in the pricing equation, so the more pronounced is the non-neutrality we describe. As result, the monetary shock has a longer unemployment half-life as well as a larger long-run influence. Decreases in the demand sensitivity parameter γ have the same qualitative influence.

Finally, note that the standard New Keynesian Phillips curve, based on the Calvo pricing mechanism, has similar properties, provided that we take discounting seriously. In particular, if the curve is expressed in the standard way[8] as $\hat{\pi}_t - \beta E_t \hat{\pi}_{t+1} = - \hat{u}_t$, it is clear that a long-run Phillips curve exists here too and for similar reasons - the coefficient on future prices (embedded within future inflation) is larger than that on past prices (embedded within current inflation).

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[6]Values of between 0.01 and 1 are found in the literature, the bottom end of the range resulting from empirical studies, the top from microfounded models such as our own. Incorporating real rigidities in the microfounded models will, by making wages less responsive to output, bring them in closer agreement with the empirical work.

[7]The time taken for unemployment to reach half-way between the impact response and the new steady state.

[8]See, for example, Walsh (2000), p.219.

Figure 1 : Response of model economy to 1% increase in
rate of money growth
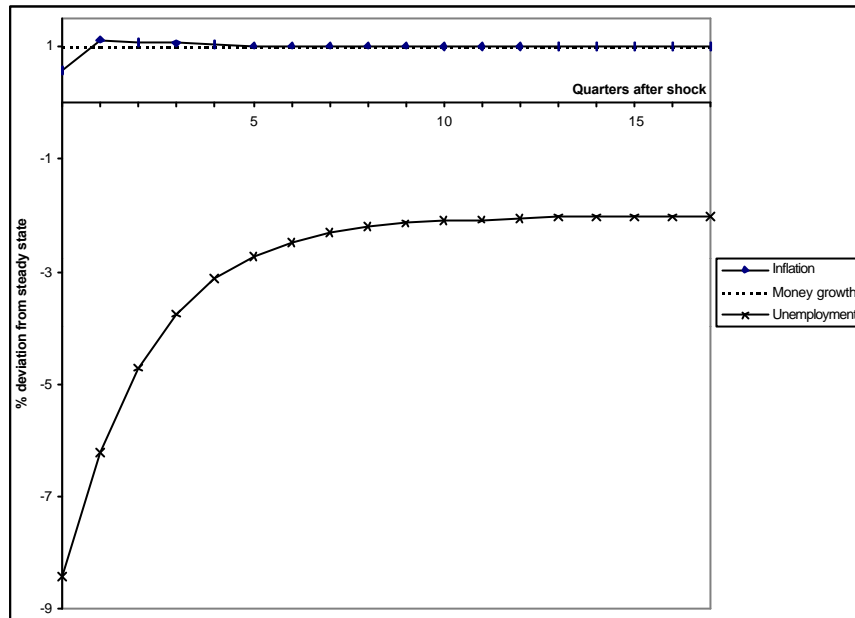
Table 1 : Sensitivity of model economy to calibration

Value of γ	0.02	0.05	0.1	0.2	0.3
Long-run response of unemployment $\beta = 0.98$	-10.1%	-4.0%	-2.0%	-1.0%	-0.6%
Impact response of unemployment $\beta = 0.98$	-10.0%	-9.2%	-8.6%	-7.9%	-7.4%
Half-life (6-month periods) $\beta = 0.98$	5	4	3	2	1
Long-run response of unemployment $\beta = 0.97$	-15.0%	-6.1%	-3.0%	-1.5%	-1.0%
Impact response of unemployment $\beta = 0.97$	-10.4%	-9.5%	-8.8%	-8.0%	-7.5%
Half-life (6-month periods) $\beta = 0.97$	6	5	4	3	1

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[1] Blanchard, Olivier and Stanley Fisher, 1989, Lectures on macroeconomics, MIT Press, Cambridge, Mass.

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Derivations

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1 Notation

We use the following convention:

	nominal	real
trended	X_t	x_t
detrended	X_t	x_t
steady state	X	x
linearized	$\hat{X}_t$	$\hat{x}_t$

If the money supply is growing, the steady state will be one in which nominal variables grow at the same rate as money. In what follows we use both detrended variables, X_t and trended variable X_t related by:

$$X_t = \frac{\mathsf{X}_t}{\mathsf{M}_t} \tag{1}$$

2 The Household's Problem

- households supply differentiated labour and are Dixit-Stiglitz imperfect competitors

- households set N period wage contracts

- Continuum of households, indexed by h, divided into $\frac{1}{N}$ equal cohorts according to when they can change their wage

- prices are flexible

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Households choose their wage to maximize the present value of their expected future utility. Household h's problem is given by:

$$\max_{\mathsf{W}_t(h)} E_t \sum_{i=0}^{N-1} \beta^i \left[U\left(c_{t+i}\left(h\right)\right) - V\left(l_{t+i}^d\left(h\right)\right) \right] \qquad (2)$$

subject to series of budget constraints[1]

$$\mathsf{P}_{t+i} c_{t+i}\left(h\right) = \mathsf{W}_t\left(h\right) l_{t+i}^d\left(h\right) \qquad (3)$$

and subject to the demand curve for the household's differentiated labour

$$l_{t,t+i}^d\left(h\right) = \left(\frac{\mathsf{W}_t\left(h\right)}{\mathsf{W}_{t+i}}\right)^{-\theta_w} l_{t+i}^d \qquad (4)$$

where $\mathsf{W}_t\left(h\right)$ is the nominal wage chosen by the household, W_t is a nominal wage index (defined as in Dixit-Stiglitz), and l_t^d is aggregate labour demand.

Combining the two constraints gives:

$$c_{t+i}\left(h\right) = \frac{\mathsf{W}_t\left(h\right)}{\mathsf{P}_{t+i}} \left(\frac{\mathsf{W}_t\left(h\right)}{\mathsf{W}_{t+i}}\right)^{-\theta_w} l_{t+i}^d \qquad (5)$$

and substituting them into the objective

$$\max_{\mathsf{W}_t(h)} E_t \sum_{i=0}^{N-1} \beta^i \left[U\left(\frac{\mathsf{W}_t\left(h\right)}{\mathsf{P}_{t+i}} \left(\frac{\mathsf{W}_t\left(h\right)}{\mathsf{W}_{t+i}}\right)^{-\theta_w} l_{t+i}^d\right) - V\left(\left(\frac{\mathsf{W}_t\left(h\right)}{\mathsf{W}_{t+i}}\right)^{-\theta_w} l_{t+i}^d\right) \right] \qquad (6)$$

The first-order condition is:

$$E_t \sum_{i=0}^{N-1} \beta^i \left[(1-\theta_w)\frac{l_{t,t+i}^d\left(h\right)}{\mathsf{P}_{t+i}} U_c\left(c_{t+i}\left(h\right)\right) + \theta_w \frac{l_{t,t+i}^d\left(h\right)}{\mathsf{W}_t\left(h\right)} V_l\left(l_{t,t+i}^d\left(h\right)\right) \right] = 0 \qquad (7)$$

Rearranging:

$$E_t \sum_{i=0}^{N-1} \beta^i \left[(1-\theta_w)\frac{\mathsf{W}_t^*\left(h\right)}{\mathsf{P}_{t+i}} l_{t,t+i}^d\left(h\right) U_c\left(c_{t+i}\left(h\right)\right) + \theta_w l_{t,t+i}^d\left(h\right) V_l\left(l_{t,t+i}^d\left(h\right)\right) \right] = 0 \qquad (8)$$

_____

[1] These combine the household and government budget constraints.

If we wish to linearize around a stationary steady state we need all variables to be stationary. Using (1) we can express the detrend the trended part of this equation as follows:

$$\frac{\mathsf{W}_t^*(h)}{\mathsf{P}_{t+i}} \quad = \quad \frac{W_t^*(h)}{P_{t+i}}.\frac{\mathsf{M}_t}{\mathsf{M}_{t+i}} \tag{9}$$

$$= \quad \frac{W_t^*(h)}{P_{t+i}}.\frac{1}{\mu_t^{t+i}} \tag{10}$$

where we define the growth rate of money as:

$$\mu_t = \frac{\mathsf{M}_t}{\mathsf{M}_{t-1}} \tag{11}$$

and write the cumulative growth rate between period t and period t+i as

$$\mu_t^{t+i} = \frac{\mathsf{M}_{t+i}}{\mathsf{M}_t} = \frac{1}{\prod_{j=0}^{i-1}\mu_{t+j}} \tag{12}$$

and we assume that if i=0, $\prod_{j=1}^{i}\mu_{t+j} = 1$.With rational expectiation, if money supply is a random walk we can write:

$$E_t\mu_{t+j} = \mu_t \tag{13}$$

so

$$E_t\mu_t^{t+i} = \mu_t^{-i} \tag{14}$$

and

$$\frac{\mathsf{W}_t^*(h)}{\mathsf{P}_{t+i}} = \frac{W_t^*(h)}{P_{t+i}}\mu_t^{-i} \tag{15}$$

Similarly,

$$\frac{\mathsf{W}_t^*(h)}{\mathsf{W}_{t+i}} = \frac{W_t^*(h)}{W_{t+i}}.\mu_t^{-i} \tag{16}$$

So we can rewrite the FOC in terms of detrended variables:

$$E_t\sum_{i=0}^{N-1}\beta^i \quad (1-\theta_w)\frac{W_t^*(h)}{P_{t+i}}.\mu_t^{-i}l_{t,t+i}^d(h)\,U_c\left(c_{t+i}(h)\right) + \theta_w l_{t,t+i}^d(h)\,V_l\left(l_{t,t+i}^d(h)\right)\Bigg] = 0 \tag{17}$$

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and rearranging gives:

$$W_t^*(h) = \frac{\theta_w}{\theta_w - 1} \frac{E_t \sum_{i=0}^{N-1} \beta^i \left(-V_l \left(l_{t,t+i}^d(h)\right)\right) l_{t,t+i}^d(h)}{E_t \sum_{i=0}^{N-1} \beta^i \frac{\mu_t^{-i}}{P_{t+i}} U_c \left(c_{t+i}(h)\right) l_{t,t+i}^d(h)} \tag{18}$$

In symmetric equilibrium all households able to change their wage at time t choose the same wage so dropping the household index gives W_t^* as the optimal wage choice of all households changing their wage at time t and $l_{t,t+i}^d$ the corresponding aggregate labour demand. Then using the production function $y_t = l_t^d$ and goods market clearing $y_t = c_t$. we can rewrite the FOC as

$$W_t^* = \frac{\theta_w}{\theta_w - 1} \frac{E_t \sum_{i=0}^{N-1} \beta^i \left(-V_l \left(y_{t,t+i}\right)\right) y_{t,t+i}}{E_t \sum_{i=0}^{N-1} \beta^i \frac{\mu_t^{-i}}{P_{t+i}} U_c \left(y_{t,t+i}\right) y_{t,t+i}} \tag{19}$$

In the steady -state (dropping time subscripts)

$$\frac{W^*}{P} = \frac{\theta_w}{\theta_w - 1} \frac{\sum_{i=0}^{N-1} \beta^i}{\sum_{i=0}^{N-1} \beta^i \mu_t^{-i}} \cdot \left(\frac{-V_l(y)}{U_c(y)}\right) \tag{20}$$

If we consider the case of no steady state money growth this reduces to the familiar markup equation:

$$\frac{W^*}{P} = \frac{\theta_w}{\theta_w - 1} \left(\frac{-V_l(y)}{U_c(y)}\right) \tag{21}$$

i.e. the real wage is set as a markup over the marginal disutility of labour expressed in terms of consumption, and the markup decreases as the labour types become better substitutes. For perfect substitutes ($\theta_w = \infty$) the markup is unity.

3 Linearization

Rewrite (19) as (dropping the expectations operator to save notation)

$$W_t^* \sum_{i=0}^{N-1} \beta^i \frac{\mu_t^{-i}}{P_{t+i}} U_c \left(y_{t,t+i}\right) y_{t,t+i} = \frac{\theta_w}{\theta_w - 1} \sum_{i=0}^{N-1} \beta^i \left[-V_l \left(\left(\frac{W_t^*}{W_{t+i}} \mu_t^{-i}\right)^{-\theta_w} y_{t+i}\right)\right] y_{t,t+i} \tag{22}$$

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Linearizing the LHS:

$$\frac{W^* y U_c(y)}{P} \sum_{i=0}^{N-1} \beta^i \mu^{-i} \left[\hat{W}_t^* - \hat{P}_{t+i} - i\hat{\mu}_t + \hat{y}_{t,t+i}(1+\xi_c) \right] \tag{23}$$

Linearizing the RHS:

$$\cdot \frac{\theta_w}{\theta_w - 1} y\left(-V_l(y)\right) \sum_{i=0}^{N-1} \beta^i \left[\hat{y}_{t,t+i} + \xi_l \left(\hat{y}_{t,t+i} - \theta_w \left(\hat{W}_t^* - \hat{W}_{t+i} + i\hat{\mu}_t \right) \right) \right] \tag{24}$$

where

$$\xi_c = \frac{c U_{cc}(c)}{U_c(c)} \text{ and } \xi_l = \frac{l V_{ll}(l)}{V_l(l)} \tag{25}$$

Equating (23) with (24) and using (21) to simplify

$$\sum_{i=0}^{N-1} \beta^i \mu^{-i} \left[\hat{W}_t^* - \hat{P}_{t+i} - i\hat{\mu}_t + \hat{y}_{t,t+i}(1+\xi_c) \right] =$$
$$\cdot \sum_{i=0}^{N-1} \beta^i \hat{y}_{t,t+i}^i \left[\hat{y}_{t,t+i} + \xi_l \left(\hat{y}_{t,t+i} - \theta_w \left(\hat{W}_t^* - \hat{W}_{t+i} + i\hat{\mu}_t \right) \right) \right] \tag{26}$$

Rearranging

$$\sum_{i=0}^{N-1} \beta^i \mu^{-i} \hat{W}_t^* = \sum_{i=0}^{N-1} \beta^i \left[\hat{y}_{t,t+i} + \xi_l \left(\hat{y}_{t,t+i} - \theta_w \left(\hat{W}_t^* - \hat{W}_{t+i} + i\hat{\mu}_t \right) \right) \right] +$$
$$\sum_{i=0}^{N-1} \beta^i \mu^{-i} \left[\hat{P}_{t+i} + i\hat{\mu}_t - \hat{y}_{t,t+i}(1+\xi_c) \right] \tag{27}$$

$$\sum_{i=0}^{N-1} \beta^i \left(\mu^{-i} + \xi_l \theta_w \right) \hat{W}_t^* = \sum_{i=0}^{N-1} \beta^i \left[\hat{y}_{t,t+i} + \xi_l \left(\hat{y}_{t,t+i} + \theta_w \left(\hat{W}_{t+i} + i\hat{\mu}_t \right) \right) \right] +$$
$$\sum_{i=0}^{N-1} \beta^i \mu^{-i} \left[\hat{P}_{t+i} + i\hat{\mu}_t - \hat{y}_{t,t+i}(1+\xi_c) \right] \tag{28}$$

In the case of no steady state money growth $\mu = 1$ this reduces to

$$\hat{W}_t^* = \frac{1}{(1+\xi_l\theta_w) \sum_{i=0}^{N-1} \beta^i} \left[\sum_{i=0}^{N-1} \beta^i \left(\hat{P}_{t+i} + \xi_l\theta_w \hat{W}_{t+i} + (1+\xi_l\theta_w) i\hat{\mu}_t \right) + (\xi_l + \xi_c) \sum_{i=0}^{N-1} \beta^i \hat{y}_{t,t+i} \right]$$
$$\tag{29}$$

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Prices are flexible so are set as a constant markup over the wage which means $\hat{P}_{t+i} = \hat{W}_{t+i}$ and we can write

$$\hat{W}_t^* = \frac{1}{\sum\limits_{i=0}^{N-1} \beta^i} \left[\sum_{i=0}^{N-1} \beta^i \left(\hat{W}_{t+i} + i\hat{\mu}_t \right) + \gamma \sum_{i=0}^{N-1} \beta^i \hat{y}_{t,t+i} \right] \tag{30}$$

where

$$\gamma = \frac{\xi_l + \xi_c}{1 + \xi_l \theta_w} \tag{31}$$

And if we consider the case N=2 the household's wage setting equation is:

$$\hat{W}_t^* = \alpha \hat{W}_t + (1-\alpha)\hat{W}_{t+1} + (1-\alpha)\hat{\mu}_t + \gamma \left[\alpha \hat{y}_t + (1-\alpha)\hat{y}_{t+1} \right] \tag{32}$$

or in terms of the price level

$$\hat{W}_t^* = \alpha \hat{P}_t + (1-\alpha)\hat{P}_{t+1} + (1-\alpha)\hat{\mu}_t + \gamma \left[\alpha \hat{y}_t + (1-\alpha)\hat{y}_{t+1} \right] \tag{33}$$

The aggregate wage level is given by

$$\mathsf{W}_t = \left(\int_o^1 \mathsf{W}_t\,(h)^{1-\sigma}\,dh \right)^{\frac{1}{1-\sigma}} \tag{34}$$

With N cohorts of households each choosing their optimal wage this is:

$$\mathsf{W}_t = \left(\frac{1}{N} \sum_{i=0}^{N-1} \mathsf{W}_{t-i}^{*\,1-\sigma} \right)^{\frac{1}{1-\sigma}} \tag{35}$$

Writing this in terms of detrended wages gives:

$$W_t = \left(\sum_{i=0}^{N-1} \left(W_{t-i}^* \frac{\mathsf{M}_{t-i}}{\mathsf{M}_t} \right)^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \tag{36}$$

$$= \left(\sum_{i=0}^{N-1} \left(W_{t-i}^* \mu_{t-i}^t \right)^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \tag{37}$$

Consider case N=2:

$$W_t = \left(W_t^{*\,1-\sigma} + \left(\frac{1}{\mu_t} W_{t-1}^* \right)^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \tag{38}$$

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Linearizing:

$$\hat{W}_t = \frac{W^{1-\sigma}\left(\hat{W}_t^* + \mu^{\sigma-1}\left(\hat{W}_{t-1}^* - \frac{1}{\mu}\hat{\mu}_t\right)\right)}{W^{1-\sigma} + \left(\frac{1}{\mu}W\right)^{1-\sigma}} \tag{39}$$

$$= \frac{\hat{W}_t^* + \mu^{\sigma-1}\left(\hat{W}_{t-1}^* - \frac{1}{\mu}\hat{\mu}_t\right)}{1 + \mu^{\sigma-1}} \tag{40}$$

Around the steady state with no money growth $\mu = 1$ this simplifies to:

$$\hat{W}_t = \frac{1}{2}\left(\hat{W}_t^* + \hat{W}_{t-1}^* - \hat{\mu}_t\right) \tag{41}$$

Substituting this into (32) gives (dropping the terms in output for simplicity):

$$\hat{W}_t^* = \frac{\alpha}{2}\left(\hat{W}_t^* + \hat{W}_{t-1}^* - \hat{\mu}_t\right) + \frac{1-\alpha}{2}\left(\hat{W}_{t+1}^* + \hat{W}_t^* - \hat{\mu}_t\right) + (1-\alpha)\hat{\mu}_t \tag{42}$$

$$= \frac{1}{2}\hat{W}_t^* + \frac{1}{2}\left[\alpha\hat{W}_{t-1}^* + (1-\alpha)\hat{W}_{t+1}^* + (1-2\alpha)\hat{\mu}_t\right] \tag{43}$$

$$= \alpha\hat{W}_{t-1}^* + (1-\alpha)\hat{W}_{t+1}^* + (1-2\alpha)\hat{\mu}_t \tag{44}$$

So the household's linearized wage setting equation is:

$$\hat{W}_t^* = \alpha\hat{W}_{t-1}^* + (1-\alpha)\hat{W}_{t+1}^* - (2\alpha-1)\hat{\mu}_t + \gamma\left[\alpha\hat{y}_t + (1-\alpha)\hat{y}_{t+1}\right] \tag{45}$$

4 Household consumption decision

The representative household chooses consumption, real money balances and bond holdings by solving the infinite horizon problem:

$$\max_{\{C_{t+j}, M_{t+j}, B_{t+j}\}_{j=0}^{\infty}} E_t \sum_{i=0}^{\infty} \beta^i \left[U\left(c_{t+i}, M_{t+i}\right) - V(L)\right] \tag{46}$$

subject to a series of budget constraints:

$$P_t c_t + M_t + B_t = M_{t-1} + i_t B_{t-1} + W_t L \tag{47}$$

Considering a utility function

$$U\left(c_t, M_t\right) = \left[c_{t+i}^{\gamma}\left(\frac{M_{t+i}}{P_{t+i}}\right)^{1-\gamma}\right]^{1-\sigma} \tag{48}$$

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The first order conditions are:

$$c_t \quad : \quad U_c = \mathsf{P}_t \lambda_t \tag{49}$$

$$\frac{\mathsf{M}_t}{\mathsf{P}_t} \quad : \quad U_{\frac{\mathsf{M}}{\mathsf{P}}} = \mathsf{P}_t \lambda_t - \mathsf{P}_{t+1} \lambda_{t+1} \tag{50}$$

$$\mathsf{B}_t \quad : \quad U_{\mathsf{B}} = \lambda_t - i_{t+1} \lambda_{t+1} \tag{51}$$

Considering a utility function

$$U\left(c_t, \mathsf{M}_t\right) = \left[c_{t+i}^{\gamma} \left(\frac{\mathsf{M}_{t+i}}{\mathsf{P}_{t+i}} \right)^{1-\gamma} \right]^{1-\sigma} \tag{52}$$

gives

$$c_t \quad : \quad \frac{\gamma.\left(1-\sigma\right)}{c_t} \left[c_t^{\gamma} \left(\frac{\mathsf{M}_t}{\mathsf{P}_t} \right)^{1-\gamma} \right]^{1-\sigma} = \mathsf{P}_t \lambda_t \tag{53}$$

$$\mathsf{M}_t \quad : \quad \frac{\left(1-\gamma\right).\left(1-\sigma\right)}{\mathsf{M}_t/\mathsf{P}_t} \left[c_t^{\gamma} \left(\frac{\mathsf{M}_t}{\mathsf{P}_t} \right)^{1-\gamma} \right]^{1-\sigma} = \mathsf{P}_t \lambda_t - \mathsf{P}_{t+1} \lambda_{t+1} \tag{54}$$

$$\mathsf{B}_t \quad : \quad 0 = \lambda_t - i_{t+1} \lambda_{t+1} \tag{55}$$

Using (55) to substitute for λ_{t+1} in (54) gives:

$$\frac{\left(1-\gamma\right).\left(1-\sigma\right)}{\mathsf{M}_t} \left[c_t^{\gamma} \left(\frac{\mathsf{M}_t}{\mathsf{P}_t} \right)^{1-\gamma} \right]^{1-\sigma} = \lambda_t \left(1 - \frac{\mathsf{P}_{t+1}}{\mathsf{P}_t} \frac{1}{i_{t+1}} \right) \tag{56}$$

and define the real interest rate:

$$r_{t+1} = \frac{\mathsf{P}_{t+1}}{\mathsf{P}_t} \frac{1}{i_{t+1}} \tag{57}$$

and substituting for λ_t from (53)

$$c_t = \frac{\gamma}{\left(1-\gamma\right)} \frac{\mathsf{M}_t}{\mathsf{P}_t} \left(1 - \frac{1}{r_{t+1}} \right) \tag{58}$$

Without capital the real interest rate will be constant and equal to $\frac{1}{\beta}$

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