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ABSTRACT

A Reappraisal of the Inflation-Unemployment Tradeoff

This paper offers a reappraisal of the inflation-unemployment tradeoff, based on “frictional growth” describing the interplay between nominal frictions and money growth. When the money supply grows in the presence of price inertia (due to staggered wage contracts with time discounting), the price adjustments to each successive change in the money supply are never able to work themselves out fully. In this context, monetary shocks have a gradual and delayed effect on inflation, and these shocks also generate plausible impulse-responses for unemployment. Although our theory contains no money illusion, no permanent nominal rigidities, and no departure from rational expectations, there is a long-run inflation-unemployment tradeoff.

JEL Classification: E2, E3, E4, E5, J3

Keywords: inflation, unemployment, Phillips curve, nominal inertia, wage-price staggering, monetary policy, business cycles, forward-looking expectations

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1. Introduction

The in°ation-unemployment tradeo® is central to our understanding of the business cycle and, especially, the e®ectiveness of monetary policy; yet macroeconomists have yet to come up with a satisfactory explanation of it. Much of the recent literature in this area incorporates time-contingent nominal contracts in a dynamic general equilibrium framework to generate the \New Phillips Curve."[1] These models are widely used in the analysis of monetary policy.[2] Nevertheless, it is widely recognized that the models' predictions do not accord with some important empirical regularities. In particular, the models have trouble accounting for in°ation persistence. They also have di±culty explaining why monetary shocks have such a delayed and gradual e®ect on in°ation. Another well-known criticism is that since the New Phillips curve is forward-looking, credible disin°ations announced beforehand give rise to booms rather than recessions. The traditional Keynesian expectations-augmented Phillips curve does not of course su®er from these de¯ciencies, but it has received no proper microfoundations. In recent years various attempts have been made to bring the predictions of the New Phillips curve more closely into line with the traditional one, but no consensus on the nature of the Phillips curve has yet been reached.

What is generally ignored in the recent literature, however, is that both types of Phillips curves share a major de¯ciency. It is that if the natural rate of unemployment - or its empirical counterpart, the nonaccelerating in°ation rate of unemployment (NAIRU) - is taken to be reasonably stable through time, then in°ation must fall (rise) without limit when unemployment is high (low). This prediction is blatantly counterfactual.

This paper proposes a reappraisal of the in°ation-unemployment tradeo®, one that avoids the di±culties above. Our theory is based on a phenomenon we call \frictional growth," growth in the presence of frictions. We focus on nominal frictions arising from time-contingent staggered nominal contracts,[3] and on growth of the money supply. In this context, frictional growth describes the movements of real and nominal variables as the outcome of the interactions between the nominal frictions and money growth.

From the microfoundations of staggered nominal contracts under time discounting,[4] it is now well known that, when the temporal discount rate is positive,

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[1] It is also known as the \New Keynesian Phillips Curve" or the \New Neoclassical Synthesis." For surveys see, for example, Gali (2002), Goodfriend and King (1997), Mankiw (2001), and Roberts (1995).

[2] See, for example, Clarida, Gali, and Gertler (1999).

[3] State-contingent nominal contracts (menu costs) need not imply nominal inertia at the aggregate level, as shown by Caplin and Spulber (1987).

[4] Regarding Taylor contracts, see for example Helpman and Leiderman (1990), Ascari (2000),

current nominal values are influenced more strongly by the past than by the future nominal values. Specifically, current wages are a weighted average of past and future prices, with future prices receiving less weight. We will show that this asymmetry generates inflation inertia.

This source of nominal inertia cannot be dismissed as a fine point of high theory. The usual argument - that the relevant time discount rate is close to zero and thus, as a first approximation, the backward- and forward-looking determinants can be weighted equally - turns out to be seriously misleading. On the contrary, as we will show below, the weights are very sensitive to small variations in the time discount rate and, over the empirically reasonable ranges of the relevant parameters, the resulting asymmetry can have dramatic implications for the long-run relation between inflation and unemployment.

Our analysis indicates that when the money supply grows in the presence of inflation inertia, the price level chases after a moving target. This "target price level" is what the price level would be in the absence of nominal frictions (instantaneous price adjustment). Since the money supply keeps rising from period to period, the price adjustments never work themselves out fully. By the time the current price level has begun to respond to the current increase in the money supply, the money supply rises again, prompting a new round of price adjustments.

In this setting, we will show that an increase in money growth causes the actual price level to lag further behind the target price. Specifically, suppose that the economy is initially in a long-run steady state, with the money supply growing at a constant rate and the price level rising in proportion. Next, suppose that there is a permanent, positive shock to money growth. Since the current price level depends more heavily on the past price than on the expected future price, the price level now falls further behind its target. Whereas the target price increases proportionately to the money supply (in the absence of money illusion), the actual price level - continually lagging behind - increases less than proportionately.[5] Thus, in the long run, real money balances rise and unemployment falls. In short, the long-run Phillips curve is downward-sloping - even though there is no permanent nominal rigidity or any departure from rational expectations.

By analogy, consider a running child, clutching a rubber band attached to a helium balloon. The faster the child runs, the greater will be the distance between the balloon and the child. Like the price level, the balloon is chasing a moving target, and the faster the target moves, the further the balloon will fall behind it.

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and Graham and Snower (2002); for Calvo contracts, see for example Bernanke, Gertler and Gilchrist (2000) and Gali (2002).

[5] In both the initial and final steady states, the price level is chasing after its moving target and the distance between the actual and target price levels remains constant. But in the final steady state this distance is larger than in the initial steady state.

Our analysis generates plausible impulse responses to shocks in money growth. Unemployment responds quickly, but the unemployment e®ect dies down with the passage of time. The in°ation response is more delayed and gradual. The only non-standard feature is that, in the long run, an increase in money growth leads to an equal increase in in°ation and a fall in the unemployment rate.

Thus far, downward-sloping long-run Phillips curves have been considered unacceptable (even heretical) on theoretical grounds. In the absence of money illusion - so the conventional argument goes - real economic activities do not depend on the unit of account and, by implication, monetary policy can have no long-term e®ect on unemployment. Our analysis calls this argument into question. In the absence of money illusion, money is neutral in the sense that a change in the money supply leads to a proportional change in the \target values" of all nominal variables (i.e. the values of these variables under instantaneous adjustment). But under in°ation inertia and money growth, as noted, the actual nominal variables lag behind their target values and never catch up with them. Thus the absence of money illusion does not imply money super-neutrality; and when money is not super-neutral an increase in money growth can have a long-run e®ect on unemployment. In short, under the standard classical principles, in which all demand and supply functions are homogeneous of degree zero in all nominal variables, it is still possible for monetary shocks to generate a long-run tradeo® between in°ation and unemployment.

The paper is organized as follows. In Section 2 we relate our analysis to the existing literature. Section 3 describes our underlying model. Section 4 derives the associated forward-looking short-run Phillips curve, which di®ers signi¯cantly from the standard speci¯cation of the New Phillips curve. Given that we do not have much accurate data on in°ation expectations, the forward-looking Phillips curve has little observational content without a theory of expectations formation. Under rational expectations, future expected in°ation depends on agents' information about the current and past macroeconomic variables and about the underlying stochastic processes. Having speci¯ed their information sets, we then derive a closed-form expression of our short-run Phillips curve by expressing the expectation of future in°ation in terms of current and past macroeconomic variables. The resulting Phillips curve looks remarkably like the traditional backward-looking Keynesian Phillips curve. We will argue that the critical di®erence between the forward-looking New Phillips curve and the traditional backward-looking one does not hinge - as much of the existing literature suggests - on whether current in°ation depends on future in°ation or on past in°ation. Rather, the forward-looking Phillips curve satis¯es a set of parameter restrictions (determined by the microfoundations of the model) that the backward-looking one is not subject to.

In Section 5 we derive the long-run Phillips curve. It turns out that, for reasonable parameter values, this curve may be quite °at (although the short-run Phillips curve is of course °atter). In Section 6 we link the short- and long-run Phillips curves by examining the impulse-response functions of in°ation and unemployment to monetary shocks. We ¯nd that the lower is the discount rate, the steeper is the associated long-run Phillips curve (ceteris paribus), but the longer it takes for unemployment, in°ation, and the slope of the Phillips curve to converge to their long-run values. Thus, observationally, it may make little di®erence whether the long-run Phillips curve is °at (so that a money growth shock has a permanent e®ect on unemployment) or near-vertical (so that the e®ect is not permanent, but very prolonged).

Section 7 provides an illustrative empirical analysis of the U.S. in°ation-unemployment tradeo®, allowing for frictional growth. We show that the resulting impulse-response functions are in broad accord with the stylized facts, but the long-run Phillips curve is not vertical. Finally, Section 8 concludes with some thoughts on the role of monetary policy and productivity growth in accounting for the U.S. trajectories of in°ation and unemployment in the 1990s.

2. Relation to the Literature

The traditional Keynesian expectations-augmented Phillips curve - in its simplest form, $¼_t = ¼_{t¡1}$ ¡ $b(u_t$ ¡ $u^n) + "_t$, where ¼ is the in°ation rate, u is the unemployment rate, u^n is the natural rate of unemployment or NAIRU, b is a positive constant, and $"_t$ is white noise - has been called \a fact in search of a theory," since it has proved di±cult to rationalize it through microfoundations. The New Phillips curve - in its simplest form, $¼_t = E_t¼_{t+1}$ ¡ $b(u_t$ ¡ $u^n) + "_t$, where E_t denotes expectations set at time t - has been derived from microfoundations, but it is less successful in accounting for the stylized facts. (With a bit of exaggeration, it could be called \a theory in search of a fact.") In particular, the New Phillips curve runs into the following well-documented problems:

(i) It has di±culty accounting for in°ation persistence, with autocorrelations close to unity.[6]

(ii) It cannot explain why monetary shocks have a delayed, gradual e®ect on in°ation.[7]

(iii) Nor can it explain why monetary shocks give rise to hump-shaped unemployment responses.

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[6]Fuhrer and Moore (1995) have shown that although the Taylor model can account for slow adjustment of wages and prices, in°ation is a jump variable that can adjust instantly (much like the capital stock adjusts slowly even though investment can adjust instantly).

[7]See, for example, Mankiw (2001).

(iv) It has the counterfactual implication that announced, credible monetary contractions lead to "disinflationary booms" rather than recessions.[8]

In recent years various attempts have been made to rectify these problems. For example, Mankiw and Reis (2001) address them in a model where price information disseminates gradually among economic agents. Roberts (1997) constructs a model in which price expectations are not fully rational. Ball (1995) investigates the effects of monetary policy that is not fully credible. Fuhrer and Moore (1995) generate inflation persistence through staggered real (rather than nominal) wages. Gali (2002) and Gali, Gertler, and Lopez-Salido (2001) examine inflation persistence in terms of price staggering and the cyclical behavior of marginal costs. Lindbeck and Snower (1999) examine the real effects of monetary shocks in the presence of price precommitment and production lags. Huang and Liu (2002) show that wage staggering is more effective than price staggering in amplifying real persistence of monetary shocks. Helpman and Leiderman (1990) and Erceg, Henderson and Levin (2000) examine the interaction between price- and wage-staggering. Some authors, e.g. Estrella and Fuhrer (1998) focus on rigidities such as habit formation in consumption. Other contributors derive real and nominal persistence from complementarities between wage-price staggering and various real rigidities. For instance, Christiano, Eichenbaum, and Evans (2001) and Dotsey, King, and Wolman (1997) examine the interaction between nominal staggering and variable capital utilization. Jeanne (1998) examines the complementarity between price staggering and real wage rigidity. Bergen and Feenstra (2000) investigate the real effects of monetary shocks under staggered price setting in the context of a translog demand structure and roundabout input-output technologies. Kiley (1997) examines the interaction between price staggering and increasing returns in production. Huang and Liu (2001) analyze price staggering in a vertical input-output structure.

As noted, however, both the New and traditional (expectations-augmented) Phillips curves suffer from what may be called the "knife-edge problem": If the natural rate is assumed to be reasonably constant - and most estimates of the NAIRU are indeed quite stable through time - then inflation changes without limit for as long as the unemployment rate remains above or below this NAIRU.[9]

[8] See Ball (1994). When monetary policy is credible, the announcement of a monetary contraction leads firms to expect disinflation, and thus they moderate their price rises even before the money supply slows down. Consequently, real money balances rise, stimulating aggregate demand and reducing unemployment. Conversely, expansionary monetary policy has a contractionary effect on unemployment. In practice the opposite happens; for a recent appraisal, see for example Ball (1997, 1999).

[9] Specifically, the traditional Phillips curve implies that $\Delta\pi_t = -b(u_t - u^n) + \varepsilon_t$, so that inflation falls (rises) without limit when unemployment is high (low), relative to the NAIRU. By contrast, the New Phillips curve implies that $\Delta\pi_{t+1} = b(u_t - u^n) + \varepsilon_{t+1}$ (where $\varepsilon_{t+1} =$

Empirical support for such behavior is thin to non-existent; there is certainly no evidence of limitlessly large de°ation when unemployment is high ($u_t > u^n$ in the traditional Phillips curve) or low ($u_t < u^n$ in the New Phillips curve). In Europe the rise in unemployment over much of the 80's and early 90's despite stable in°ation is not in accord with this interpretation.[10] In the US, the fall in both in°ation and unemployment during much of the 90's does not ¯t it either.

There are two ways of avoiding the knife-edge problem. One is to assume that the NAIRU varies through time in agreement with the NAIRU hypothesis.[11] Then the NAIRU hypothesis becomes tautologous and thus lacks explanatory power. The charge of tautology can only be avoided if we provide convincing ex ante explanatory evidence for the predicted movements of the NAIRU. But such evidence is often hard to come by. For example, if the movements of the NAIRU relative to the actual unemployment rate are to be inversely related to movements in in°ation (according to the traditional Phillips curve), then the NAIRU must have been rising during the European stag°ation of the mid-70's and early 80's and during the climb of unemployment in the mid-80's and early 90's. But it is far from clear where these NAIRU movements could have come from. The large increases in union density, unemployment bene¯ts and bene¯t durations, and other welfare state entitlements, as well as the increased stringency of job security legislation, occurred primarily in the 60's and early 70's in Europe. By the 80's and 90's these trends had largely ceased and there were even important moves in the opposite direction.[12] The alleged fall in the U.S. NAIRU in the second half of the 90's is also not easy to explain.[13] With 20-20 hindsight, it is of course possible always to identify new constellations of economic variables that could plausibly have pushed the NAIRU in any direction required by the underlying theory. But the selective nature of this exercise has made a growing number of economists uncomfortable.

$\pi_{t+1} - E_t\pi_{t+1}$ is an expectational error), so that in°ation rises (falls) without limit when past unemployment is high (low).

[10] The rise of European in°ation and unemployment in the mid-70s and early 80s is not in agreement with the traditional Phillips curve, with a stable NAIRU.

[11] In other words, the variations in the NAIRU are such that the resulting di®erence between the NAIRU and the actual unemployment rate is always inversely proportional to variations in the in°ation rate, according to the traditional Phillips curve, or directly proportional to the in°ation variations, according to the New Phillips curve.

[12] Rising interest rates and tax rates may well have played a role in driving the NAIRU upwards over the 80's, but the timing of these factors does not always mesh well with the timing of the unemployment increases in various European countries. The relevant literature is voluminous and well-known; an impressive example is Phelps (1994, ch. 17).

[13] This literature is also well-known. See, for example, Phelps (1999) and Phelps and Zoega(2001).

The other way to avoid the knife-edge problem is to dispense with the NAIRU. Clearly, as the NAIRU hypothesis implies that in°ation keeps falling or rising when unemployment deviates from the the NAIRU, the way to avoid this knife-edge property is to drop the NAIRU hypothesis, which implies that the long-run Phillips curve is not vertical.

The existing empirical evidence on the NAIRU hypothesis and the slope of the long-run Phillips curve is distinctly mixed, and has led major contributors such as Mankiw (2001) to be \agnostic" on the issue. Given economists' predilection for the classical dichotomy, it is striking that a number of well-known recent studies reject it. King and Watson (1994) and Fair (2000) ¯nd a long-run in°ation-unemployment tradeo®. Ball (1997) shows that countries experiencing comparatively large and long declines in in°ation tend also to encounter comparatively large increases in their NAIRU's. Ball (1999) suggests that such a relationship may be due to monetary policy: countries with relatively contractionary monetary policy in the 1980s tended to have relatively large increases in their NAIRU's. In Bernanke and Mihov (1998) the estimated impulse-response functions of unemployment to monetary shocks do not go to zero (although the estimated in°uence is statistically insigni¯cant). Akerlof, Dickens and Perry (1996) ¯nd evidence of a long-term tradeo® between in°ation and unemployment at low in°ation rates. Dolado, Lopez-Salido and Vega (2000) ¯nd some evidence of such a tradeo® over the entire range of observations for Spain during 1964-1995.

Most of the recent literature on the Phillips curve ignores the knife-edge problem and is compatible with the NAIRU hypothesis. Notable exceptions are Akerlof, Dickens and Perry (1996, 2000), who show that the Phillips curve becomes downward-sloping at low in°ation rates when there are permanent downward wage rigidities or departures from rational expectations. Our theory also dispenses with the NAIRU hypothesis, but in contrast with other contributions, we show that the long-run Phillips curve is downward-sloping even in the absence of money illusion, permanent nominal rigidities or departures from rational expectations, and that this feature need not necessarily apply exclusively to low in°ation rates. The analysis presented here provides a theoretical foundation and empirical support for this view. We now present a theoretical model which formalizes our central ideas.

3. The Model

We construct a particularly simple macroeconomic model with the following salient features: (a) money illusion is absent, (b) the money supply grows, and (c) there is nominal inertia in the form of staggered wage contracts and time discounting. The dynamic general equilibrium model underlying our macro model is presented in Graham and Snower (2002). For brevity, we skip the standard microfoundations of our macro relations, but we will interpret our results in the light of these microfoundations.

All variables in our model - except the unemployment rate - are in logs . All uninteresting constants are ignored.

Aggregate product demand depends on real money balances:[14]

$$Q_t^D = M_t - P_t; \tag{3.1}$$

where M_t is the money supply and P_t is the price level. The aggregate production function exhibits constant returns to labor:[15]

$$Q_t^S = N_t; \tag{3.2}$$

where N_t is aggregate employment. The product market clears, so that

$$Q_t^D = Q_t^S: \tag{3.3}$$

The labor supply is constant:

$$L_t = L; \tag{3.4}$$

so that the unemployment rate (not in logs) can be approximated as

$$u_t = L - N_t: \tag{3.5}$$

Substituting equations (3.1)-(3.4) into (3.5), we obtain a simple unemployment equation:

$$u_t = L - (M_t - P_t) \tag{3.6}$$

Since we are interested in the long-run inflation-unemployment tradeoff, we need to consider permanent shocks to money growth, which move the economy along this tradeoff. Thus let the growth rate of the money supply be a random walk:

$$\Delta M_t \equiv \mu_t = \mu_{t-1} + \varepsilon_t; \tag{3.7}$$

[14] In the standard derivation of this demand function, households maximize a CES utility function, containing consumption and real money balances as arguments, and additively separable labor.

[15] Since we seek to derive the long-run inflation-unemployment tradeoff, this labor demand function is interpretted as a long-run relation.

where M_t is the log of the money supply and ε_t is a white-noise error term. We assume that rational agents at time t know the stochastic process generating money growth, and have information up to the shock ε_t, but do not know future realizations of the money growth shock.[16]

To close the model, we need to specify the relation between the price level and the money supply. We do this through wage and price setting equations, which depict sluggish nominal adjustment due to staggered wage contracts à la Taylor (1979, 1980a).[17] We make the standard assumption that there are two nominal wage contracts, each lasting for two periods[18] and evenly staggered. Let W_t be the log of the contract wage, set at the beginning of period t for periods t and $t+1$: The Taylor contract equation is[19]

$$W_t = \alpha W_{t-1} + (1-\alpha)\, E_t W_{t+1} + \gamma\, [c + \alpha \Gamma_t + (1-\alpha)\, E_t \Gamma_{t+1}] + \omega_t; \qquad (3.8)$$

where α and γ are positive constants, $0 < \alpha < 1$, E_t denotes expectations formed in period t, ω_t is a white noise process, and Γ_t is what Taylor calls "excess demand," i.e. the difference between actual output (Q_t) and full-employment output

[16] Although the random walk assumption receives some moderate support from the data (see Appendix 1a), our qualitative conclusions do not hinge on it. Appendix 1b shows how our central results can be derived from other money growth processes as well.

[17] The main alternative models of time-contingent contracts are (i) the Rotemberg (1982) model (in which each firm is assumed to face quadratic costs of price adjustment, which it minimizes) and (ii) the particularly popular Calvo (1983) model (in which each firm has to keep its price fixed until it receives a random "permission-to-adjust-price" signal, and the probability of receiving this signal remains constant through time). These alternatives however are problematic. In Rotemberg's approach, it is unclear why the cost of price change should be positively related to the magnitude of price change. In fact, the menu cost literature has been built up on the explicit assumption that no such relation exists. Regarding Calvo's approach, it is obviously far-fetched to assume that a firm's probability of price adjustment is independent of how long it has been since its last price adjustment. Nevertheless the Calvo model is commonly used as a convenient algebraic shorthand for the Taylor model. However, our analysis, like that of Kiley (2002), calls this presumption into question.

[18] For algebraic simplicity, we assume that the length of the wage contracts is constant through time. Romer (1990) and others provide models of endogenous frequency of nominal adjustment. Our model can be extended in this way, assuming that firms face a tradeoff between the costs of price adjustment and the loss from allowing prices to stray from their frictionless, profit-maximizing levels. However, it is easy to see why this extension makes no substantive difference to our qualitative conclusions: Since greater frequency of adjustment involves higher costs, an increase in money growth does not lead to a completely counterveiling change in contract length, and thus money is not superneutral.

[19] For brevity, once again, we skip the standard derivation of the microfoundations of this contract equation. See Ascari (2000); alternatively, see Huang and Liu (2002) and allow the discount factor to be less than unity.

($Q_t = L$, by the production function (3.2)):[20]

$$\Gamma_t = Q_t - L. \tag{3.9}$$

A well-known result from the microfoundations[21] of this contract equation is that α is a discounting parameter: $\alpha = \frac{1}{1+\delta}$, where δ is the time discount factor.[22] The "demand sensitivity parameter" γ describes how strongly wages are influenced by demand, and the "cost-push parameter" c gives the upward pressure on wages in the absence of excess demand. We assume that the wage setters have knowledge of nominal wages and excess demands up to period t, and of the contract shock up to period $t-1$, so that $E_t \omega_t = 0$.

Since there are constant returns to labor in the production function (3.2), the price level is a constant mark-up over the average wage:

$$P_t = \frac{1}{2}\left(W_t + W_{t-1}\right). \tag{3.10}$$

In sum, our model contains four basic building blocks: (i) the unemployment equation (3.6), (ii) the wage contract equation (3.8), (iii) the price equation (3.10), and (iv) the money supply equation (3.7). The supply and demand sides of the economy are equilibrated through the wage contract equation (3.8): if product supply rises relative to product demand (in period t), then excess demand Γ_t falls, putting downward pressure on the nominal wage W_t. The fall in the nominal wage, in turn, puts downward pressure on the price level (by eq. (3.10)). Thus, given the money supply (3.7), real money balances rise and aggregate demand is stimulated.

In the context of this model, we now proceed to derive the Phillips curve, first in the short-run and then in the long-run.

4. The Short-Run Phillips Curve

To derive the short-run Phillips curve, we substitute the wage contract equation (3.8) into the price mark-up equation (3.10) to obtain the following price equation:[23]

[20]Since employment cannot exceed the labor force, excess demand is always negative in our model.

[21]Helpman and Leiderman (1990), Ascari (2000), and Graham and Snower (2002).

[22]This interpretation of α holds exactly when the steady state money supply is constant. Thus our theoretical analysis applies to sufficiently small variations in money growth around this steady state. However, our empirical analysis below, as we will see, applies to larger variations, since the estimated behavioral equations are associated with the actual variations in money growth.

[23]To see this, substitute (3.8) into (3.10) and note that $\frac{1}{2}(E_t W_{t+1} + E_{t-1} W_t) = \frac{1}{2}(E_t W_{t+1} + W_t) + \frac{1}{2}(E_{t-1} W_t + W_{t-1}) - \frac{1}{2}(W_t + W_{t-1}) = E_t P_{t+1} + \gamma_t$.

$$
\begin{aligned}
P_t \quad = \quad & \alpha P_{t-1} + (1-\alpha)\left(E_t P_{t+1} + \nu_t\right) + \gamma c + \frac{1}{2}\left(\omega_t + \omega_{t-1}\right) \\
& + \frac{\gamma}{2}\left(\alpha \Gamma_{t-1} + \alpha \Gamma_t + (1-\alpha) E_{t-1}\Gamma_t + (1-\alpha) E_t \Gamma_{t+1}\right). \qquad (4.1)
\end{aligned}
$$

where $\nu_t = E_{t-1}P_t - P_t$ is an expectational error term. Just as the current nominal wage depends on past and future wages (by (3.8)), so the current price level depends on past and future prices. This equation implies the following forward-looking short-run Phillips curve:[24]

$$
\begin{aligned}
\pi_t \quad = \quad & \left(\frac{1-\alpha}{\alpha}\right) E_t \pi_{t+1} + \frac{\gamma c}{\alpha} \qquad (4.2) \\
& - \frac{\gamma}{2\alpha}\left[\alpha u_{t-1} + \alpha u_t + (1-\alpha) E_{t-1}u_t + (1-\alpha) E_t u_{t+1}\right] + \eta_t
\end{aligned}
$$

where $\eta_t = \frac{1-\alpha}{\alpha}\nu_t + \frac{1}{2\alpha}\left(\omega_t + \omega_{t-1}\right)$.

This equation differs from the standard New Phillips curve ($\pi_t = E_t \pi_{t+1} - b\left(u_t - u^n\right) + \varepsilon_t$) in two important respects:

- Inflation depends not just on current unemployment, but also on past and future unemployment. It has been argued that since unemployment has a high degree of serial correlation, the weighted average of past, current, and future unemployment may be approximated by the current unemployment rate.[25] But this argument runs afoul of the Lucas critique: the degree to which current unemployment depends on past and future unemployment is affected by macro policy (the monetary policy equation (3.7)) and thus cannot be specified a priori.

- The coefficient on future inflation ($(1-\alpha)/\alpha$) is not unity unless $\alpha = 1/2$ which is the case only when the future is not discounted ($\alpha = \frac{1}{1+\delta}$ and $\delta = 1$). Under discounting, $\alpha > 1/2$ and thus the coefficient on future inflation is less than unity. This implies that the NAIRU does not exist, i.e. there does not exist a unique unemployment rate (at any time t) that is consistent with constant inflation.[26]

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[24] Add the term $-(1-\alpha)P_t$ to both sides of the previous equation and note that $\Gamma_t = Q_t - L_t = N_t - L_t = -u_t$.

[25] See, for example, Roberts (1995).

[26] Staggered pricing à la Calvo can also yield a coefficient on future inflation that is not unity, as shown in Bernanke, Gertler and Gilchrist (1998), Gali (2002), and others.

Of course the forward-looking Phillips curve (4.2) is not the full solution of our macroeconomic model, since this Phillips curve involves expectations of future inflation. To solve the model, these expectations must be derived from the model's underlying stochastic processes and expressed in terms of current and past macroeconomic variables.

We proceed to do so and thereby find a closed-form expression of our short-run Phillips curve. The first step is to find the equilibrium wage and price level in terms of current and past variables. It can be shown[27] that the equilibrium nominal wage is

$$W_t = (1-\lambda)c + \lambda W_{t-1} + (1-\lambda)M_t + \kappa(1-\lambda)\mu_t - (1-\lambda)L + \omega_t; \qquad (4.3)$$

where $\lambda = \frac{\frac{\phi_2}{\phi_3} - \sqrt{\left(\frac{\phi_2}{\phi_3}\right)^2 - 4\frac{\phi_1}{\phi_3}}}{2}$, $\phi_1 = \alpha\left(1-\frac{\gamma}{2}\right)$; $\phi_2 = \left(1+\frac{\gamma}{2}\right)$; $\phi_3 = (1-\alpha)\left(1-\frac{\gamma}{2}\right)$, $\kappa = \frac{\alpha(1+\lambda)}{\frac{\alpha}{1-\alpha}-\lambda} > 0$, and $0 < \lambda < 1$. The equilibrium price level is[28]

$$\begin{aligned} P_t &= (1-\lambda)c + \lambda P_{t-1} + (1-\lambda)M_t + (1-\lambda)\left(\kappa - \frac{1}{2}\right)\mu_t \qquad (4.4)\\ &\quad - \frac{1}{2}\kappa(1-\lambda)\varepsilon_t - (1-\lambda)L + \frac{1}{2}(\omega_t + \omega_{t-1}). \end{aligned}$$

Thus the inflation rate is[29]

$$\begin{aligned} \pi_t &= \lambda\pi_{t-1} + (1-\lambda)\mu_t + \frac{1}{2}(1-\lambda)(\kappa-1)\varepsilon_t \\ &\quad + \frac{1}{2}\kappa(1-\lambda)\varepsilon_{t-1} + \frac{1}{2}(\omega_t + \omega_{t-2}). \qquad (4.5) \end{aligned}$$

The price equation (4.4) also implies that equilibrium real money balances are[30]

$$\begin{aligned} M_t - P_t &= -(1-\lambda)c + \lambda(M_{t-1} - P_{t-1}) + (1-\lambda)\left(\frac{2\alpha-1}{\gamma}\right)\mu_t \\ &\quad + \frac{1}{2}\kappa(1-\lambda)\varepsilon_t + (1-\lambda)L - \frac{1}{2}(\omega_t + \omega_{t-1}). \qquad (4.6) \end{aligned}$$

Thus the equilibrium unemployment rate is[31]

$$\begin{aligned} u_t &= (1-\lambda)c + \lambda u_{t-1} - (1-\lambda)\left(\frac{2\alpha-1}{\gamma}\right)\mu_t \qquad (4.7)\\ &\quad - \frac{1}{2}\kappa(1-\lambda)\varepsilon_t + \frac{1}{2}(\omega_t + \omega_{t-1}). \end{aligned}$$

[27]See Appendix 2.1.
[28]See Appendix 2.2.
[29]See Appendix 2.3.
[30]See Appendix 2.4.
[31]See Appendix 2.5.

By the inflation equation (4.5), the unemployment equation (4.7), and the money supply equation (3.7), we obtain our closed-form short-run Phillips curve (Appendix 2.6):

$$\pi_t = d_0 + d_1 \pi_{t-1} - d_2 u_t - d_3 u_{t-1} + d_4 u_{t-2} + \varepsilon_t; \qquad (4.8)$$

where

$$d_0 = \psi c; \; d_1 = \frac{\psi \kappa}{2}; \; d_2 = \frac{\psi (1+\kappa)}{2}; \; d_3 = \frac{\psi}{2}; \; d_4 = \frac{\psi \kappa}{2}; \; \psi = \frac{1}{\frac{2\alpha - 1}{\gamma} + \frac{\kappa}{2}} \qquad (4.9)$$

$$\varepsilon_t = \frac{\left[\left(1 + \frac{\psi(1+\kappa)}{2}\right) \omega_t + \frac{3\psi}{2} \omega_{t-1} + \left(1 + \frac{\psi(1-\kappa)}{2}\right) \omega_{t-2} - \psi \kappa \omega_{t-3} \right]}{2(1 - \lambda B)}. \qquad (4.10)$$

The above error term is an infinite moving average (IMA) process in terms of ω_t, with parameters which are non-linear functions of the theoretical parameters ψ, κ, and λ.[32] Inspection of equations (4.9) shows the following relationships among the slope coefficients of (4.8):

$$d_4 = d_1; \text{ and } d_3 = d_2 - d_1. \qquad (4.11)$$

Note that the closed-form Phillips curve (4.8) looks like the traditional backward-looking Keynesian Phillips curve. Nevertheless, given our macroeconomic model, our closed-form Phillips curve (4.8) is of course equivalent to our forward-looking Phillips curve (4.2). This is noteworthy because the standard way of distinguishing the backward-looking from the forward-looking Phillips curves is in terms of lags and leads: in the backward-looking curve, current inflation depends on past inflation, whereas in the forward-looking curve it depends on expected future inflation. Our analysis suggests that this distinction is bogus. Since expectations of future inflation can be restated in terms of the current and past values of the variables, any Phillips curve with forward-looking inflation expectations can be turned into a Phillips curve in which current inflation depends on past inflation.

What, then, is the relation between the traditional backward-looking, expectations augmented Keynesian Phillips curve and our forward-looking one? In the traditional Phillips curve, the coefficients on past inflation and on unemployment are unrestricted, with one exception: since the traditional expectations-augmented Phillips curves is compatible with the NAIRU, the coefficient on past inflation was restricted to $d_1 = 1$. In our forward-looking Phillips curve, as we have seen,

[32] ψ, κ, and λ are of course functions of the more basic time-discount parameter α and the demand-sensitivity parameter γ.

this restriction does not apply.[33] Instead, the coefficients of this forward-looking Phillips curve must satisfy the restrictions (4.11) and its error term ($\hat{e}_t$) follows the IMA process given by (4.10).[34]

5. The Long-Run Phillips Curve

In the long-run steady state, $\pi_t = \pi_{t-1}$, $u_t = u_{t-1}$, and the white noises error terms ε_t, and ω_t are zero. Thus, by (4.5), the long-run inflation rate is[35]

$$\pi_t^{LR} = \mu_t^{LR}. \tag{5.1}$$

The long-run unemployment rate is (by (4.7))

$$u_t^{LR} = -\left(\frac{2\alpha - 1}{\gamma}\right)\mu_t^{LR} + c. \tag{5.2}$$

Substituting equation (5.1) into (5.2), we obtain the long-run Phillips curve:

$$\pi_t^{LR} = -\left(\frac{\gamma}{2\alpha - 1}\right)u_t^{LR} + \left(\frac{\gamma}{2\alpha - 1}\right)c. \tag{5.3}$$

Note that the sign of the slope depends critically on the value of the discounting parameter $\alpha = \frac{1}{1+\delta}$, where δ is the discount factor.

Table 1: Slope of the long-run Phillips curve

			slope		
r (%)	δ	α	$\gamma = 0.05$	$\gamma = 0.07$	$\gamma = 0.10$
1.0	0.990	0.502	−10.1	−14.1	−20.1
2.0	0.980	0.505	−5.05	−7.07	−10.1
3.0	0.971	0.507	−3.38	−4.74	−6.77
4.0	0.962	0.510	−2.55	−3.57	−5.10
5.0	0.953	0.512	−2.05	−2.87	−4.10

[33] In this respect, our forward-looking Phillips curve resembles the old-style Phillips curves prior to the "discovery" of the NAIRU. Our long-run Phillips curve is vertical only when the rate of time discount is zero.

[34] These conditions, however, should not be viewed as restrictions imposed on an estimated Phillips curve equation, for two related reasons. First, the IMA error term is not estimable. Second, as we argue in Section 7, the phenomenon of frictional growth cannot be captured through single-equation estimation of the inflation-unemployment tradeoff, but requires multi-equation estimation, describing how wages and price depend on the money supply and how unemployment depends on the relation between money and prices (or some other relation between real and nominal variables).

[35] Since money growth follows a random walk, the long run money growth rate varies through time (μ_t^{LR} has a time subscript) and the long-run inflation rate is time-varying as well.

In much of the literature on the New Phillips curve,[36] this parameter is set equal to a half, thereby making the New Phillips curve consistent with the NAIRU hypothesis. However the underlying reasoning - that the discount factor is close to unity - turns out to be misleading because (a) the discounting parameter α depends nonlinearly on the discount rate and (b) the slope of the long-run Phillips curve depends nonlinearly on the discounting parameter. Thus small variations in the discounting parameter may have large effects on the slope of the long-run Phillips curve, depending on the magnitude of the demand sensitivity parameter γ. There is little agreement in the literature about the appropriate value of γ. Taylor (1980b) estimates it to be between 0.05 and 0.1; Sachs finds it in the range 0.07 and 0.1; calibration of microfounded models (e.g. Huang and Liu (2002)) assigns higher values. Table 1 presents the slope of the long-run Phillips curve associated with various values of the discount rate r (where $\delta = \frac{1}{1+r}$) and the γ parameter.

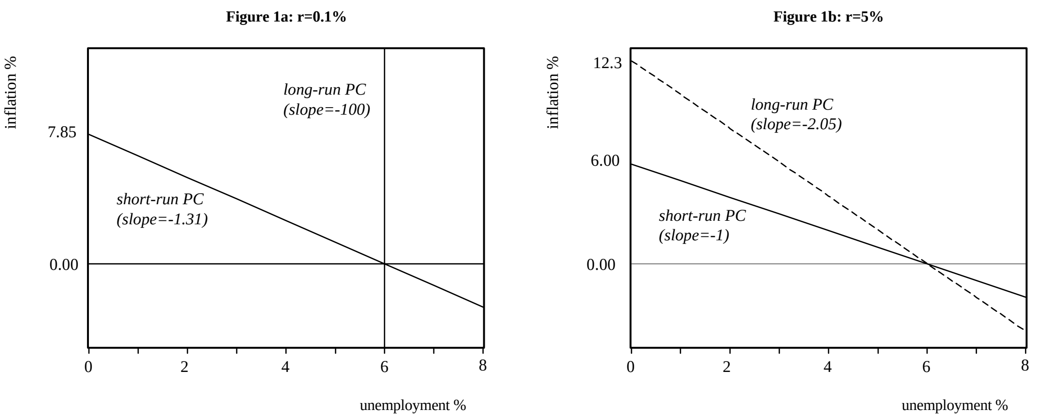
Observe that for discount rates above 2 percent and the above range of γ values, the slope of the long-run Phillips curve is quite flat. These results, however, are merely suggestive, since the theoretical model above is obviously far too simple to provide a reliable account of the long-run inflation-unemployment tradeoff under frictional growth. For that purpose it would be necessary to examine the role of other growing variables (such as capital and productivity) in conjunction with other frictions (such as unemployment inertia). The illustrative empirical model in Section 7 is a small step in this direction.

It can be shown that, for plausible parameter values, our short-run Phillips curve has a flatter slope and lower intercept than its long-run counterpart.[37] Figures 1 provide two examples of associated short- and long-run Phillips curves. Observe that although the long-run Phillips curve is nearly vertical when the discount rate is very low (at 0.1%) and much flatter when the discount rate is high (5%), the short-run Phillips curve remains quite flat in both cases.

[36] See, for example, Blanchard and Fisher (1989, p. 395). The authors however express discomfort with this: "Even under lognormality of money and the price level (actually, even under certainty) the optimal rule is not one in which the parameter is equal to a half" (p. 420).

[37] In particular, the slope of the short-run Phillips curve (4.8) is $\frac{\partial \pi_t}{\partial u_t} = d_2 = -\frac{\gamma + \gamma\kappa}{2(2\alpha - 1) + \gamma\kappa}$ whereas the slope of the long-run Phillips curve (5.3) is $\frac{\partial \pi_t^{LR}}{\partial u_t^{LR}} = -\frac{\gamma}{2\alpha - 1}$. It can be shown that if, as is plausible, the long-run slope is less than -1, the long-run Phillips curve is steeper than the short-run one. (This is a sufficient but not necessary condition, as shown in Appendix 2.8). The intercept of the short-run Phillips curve (4.8) is given by $d_0 = \left(\frac{2\gamma}{2(2\alpha - 1) + \gamma\kappa}\right) c$, which is smaller than the long-run Phillips curve (5.3) intercept: $\left(\frac{\gamma}{2\alpha - 1}\right) c$. (See Appendix 2.8).

Figures 1: The Short- and Long-Run Phillips Curves



$\gamma = 0.05, \;\; c = 0.06$

6. Theoretical Impulse Response Functions

We now examine the connection between the short- and long-run Phillips curves by deriving the impulse response functions of inflation and unemployment to a monetary shock. Specifically, consider a one-off unit shock to money growth (3.7), occurring at time $t = 0$: $\varepsilon_0 = 1$ and $\varepsilon_t = 0$ for $t > 0$. This represents a permanent change in money growth. At time $t = 0$, economic agents know the process (3.7) generating money growth, but not the realizations of the error term ε_{t+i}; $i \geq 1$.

Thus the monetary shock ε_0 is known to the wage setters at time $t = 0$, but not at time $t = -1$ (so that the expectations of wage setters at time $t = -1$ are $E_{-1}\varepsilon_0 = 0$). Since the current wage W_0 depends on the past wage W_{-1}, the current wage W_0 does not adjust fully to the shock ε_0. On this account, the shock has real effects.

Let $R(\pi_t)$ and $R(u_t)$ be the period-t responses of inflation and unemployment (respectively) to the above money growth shock, ceteris paribus. By the inflation equation (4.5), we find that the inflation responses through time are:

$$\begin{aligned} R(\pi_0) &= 1 + \frac{1}{2}\left[(1-\lambda)\kappa - (1+\lambda)\right]; \\ \underset{t \geq 1}{R(\pi_t)} &= 1 + \lambda^{t-1}\left(\frac{1+\lambda}{2}\right)\left[(1-\lambda)\kappa - \lambda\right]; \\ R(\pi_{LR}) &\equiv \lim_{t \to \infty} R(\pi_t) = 1 \text{ (long-run response).} \end{aligned} \tag{6.1}$$

By the unemployment equation (4.7), the unemployment responses through time are:

$$R(u_t)_{t \geq 0} = -\left(\frac{2\alpha - 1}{\gamma}\right) - \frac{\lambda^t(1+\lambda)}{2(1-\lambda)}[(1-\lambda)\kappa - \lambda];$$

$$R(u_{LR}) \equiv \lim_{t \to \infty} R(u_t) = -\left(\frac{2\alpha - 1}{\gamma}\right); \text{ (long-run response).} \qquad (6.2)$$

The impulse-response function for inflation always lies above the initial ($t = 0$) inflation rate, and the impulse-response function for unemployment always lies below the inital ($t = 0$) unemployment rate. It can be shown,[38] that the inflation and unemployment responses fall into two broad classes:

1. Inflation and unemployment under-shooting: If $\kappa < \frac{\lambda}{1-\lambda}$, inflation gradually rises toward its new long-run equilibrium ($\pi_t < \pi_{LR}$; and $\pi_{t+1} > \pi_t$ for $t \geq 0$); unemployment gradually falls towards its new long-run equilibrium ($|u_t| < |u_{LR}|$ for $t \geq 0$).

2. Inflation over-shooting slowly and unemployment over-shooting quickly: If $\frac{\lambda}{1-\lambda} < \kappa < \frac{1+\lambda}{1-\lambda}$,[39] then inflation rises, over-shooting its new long-run equilibrium after one period, and then gradually falls toward this equilibrium ($\pi_0 < \pi_{LR}$; $\pi_t > \pi_{LR}$; and $\pi_{t+1} < \pi_t$ for $t \geq 1$). Unemployment falls, over-shooting its new long-run equilibrium, and then gradually rises toward this equilibrium ($|u_t| > |u_{LR}|$; and $|u_{t+1}| < |u_t|$ for $t \geq 0$). The maximum impact of the monetary shock on unemployment is achieved before the maximum impact on inflation.

For most of the empirically reasonable parameter values given in Table 1, the impulse-response functions can be shown to fall into Class 2, the class that accords with the stylized facts (viz., the inflation responses to monetary shocks are delayed and gradual, the unemployment responses occur more quickly). Figures 2 depict the impulse response functions for inflation, unemployment, and the slope of the Phillips curve for the same parameter values as in Figures 1.[40] The horizontal axis measures time; the left-hand vertical axis measures the slope of the Phillips curve; and the right-hand vertical axis measures the inflation and unemployment rates.

Observe when the discount rate is very low ($r = 0.1\%$), in Fig. 2a, the long-run Phillips curve is virtually vertical, but the short-run Phillips curve at time

[38] See Appendix 2.9.

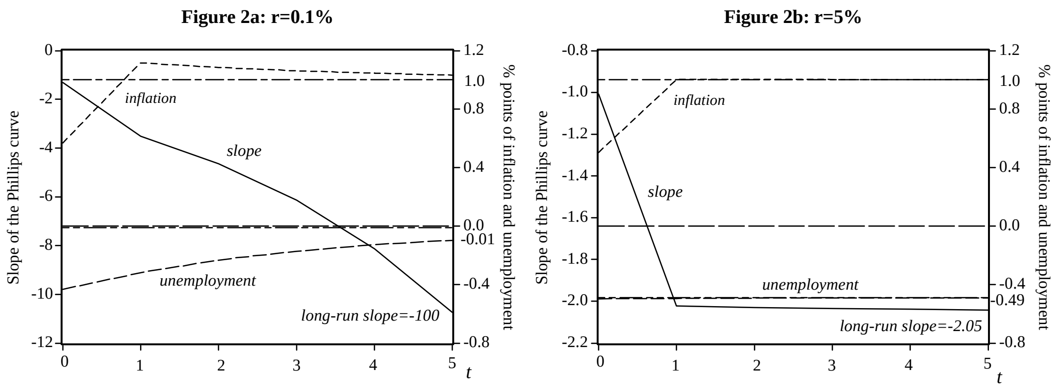
[39] It can be shown that κ cannot exceed $\frac{1+\lambda}{1-\lambda}$. (See Appendix 2.9.)

[40] The value of c has no effect on the slope of the Phillips curve.

t = 0 is very °at, and it takes a very long time for unemployment, in°ation, and the Phillips curve slope to reach their long-run values.

By contrast, when the discount rate is higher (r = 5%), the long-run Phillips curve is quite °at, and it takes a short time for unemployment, in°ation, and the slope to reach their long-run values.

Figures 2: Impulse Response Functions



The shock is a 1 % point increase in the money growth rate at t=0

° = 0:05

It is easy to show that this pattern holds for the full range of discount rates: The lower the discount rate (for a given value of the demand-sensitivity parameter °):

- the steeper is the long-run Phillips curve and
- the longer it takes for the slope of the Phillips curve to converge to its long-run value.

Thus, observationally, it may make little di®erence whether the long-run Phillips curve is °at - so that an increase in money growth permanently reduces unemployment - or near-vertical - so that the e®ect is not permanent, but very prolonged. In other words, it may be di±cult, if not impossible, to distinguish in practice between a world in which there is quick convergence to a °at long-run Phillips curve and one in which there is slow convergence to a steep one. In both cases, monetary shocks have long-lasting e®ects on unemployment.

The underlying theme of our analysis has been that (a) in the presence of staggered wage contracts and time discounting, current prices depend more heavily on past prices than on future prices, (b) this asymmetry gives rise to inflation inertia, and (c) this inflation inertia, interacting with money growth, leads to downward-sloping inflation-unemployment tradeoff. Given the impulse response functions above, we are now able to give a formal characterization of inflation inertia and provide a rigorous foundation for this argument.

Inflation inertia arises in our model when the inflation response to a permanent money growth shock is delayed, i.e. inflation responds only partially in the short run, taking time to reach its long-run equilibrium value. In particular, we measure inflation inertia as the sum of the differences through time between (i) the actual change in inflation in response to the permanent money growth shock ($\Delta(\pi_t)$) and (ii) the inflation change that would have occurred if inflation had responded instantaneously ($\Delta(\pi_{LR})$): $\rho = \sum_{t=0}^{\infty} (\Delta(\pi_t) - \Delta(\pi_{LR}))$. By the impulse response functions (6.1), it is easy to show that

$$\rho = \sum_{t=0}^{\infty} (\Delta(\pi_t) - \Delta(\pi_{LR})) = -\frac{2\alpha - 1}{\gamma} \tag{6.3}$$

Observe that inflation inertia turns out to be the inverse of the slope of the long-run Phillips curve! The greater is the rate of time discount (the greater is the discounting parameter α), the more heavily do current prices depend on past prices rather than future prices. As result, by (6.3), there is more inflation inertia. On this account, the actual price level lags further behind the growing money supply, so that real money balances increase, leading to a fall in long-run unemployment along the long-run Phillips curve.

In this context it is also easy to show that we can avoid the counterfactual implication of disinflationary booms, analogously to Mankiw and Reis (2001).[41] In the context of the Calvo model of random nominal adjustment, Mankiw and Reis avoid disinflationary booms by assuming that only a fraction of agents receives updated information in each period. The analogue in the Taylor model of fixed, staggered adjustment is to assume that all agents receive information about monetary shocks with a one-period lag. It is trivial to see that if monetary shocks are announced one period in advance and if agents' information about these shocks is received one period in arrears, then the resulting model generates precisely the same results as the model above. More generally, our model avoids the implication

[41] To see the problem of disinflationary boom in our analysis, suppose that monetary shocks are announced one period in advance. Thus the money supply process is given by (3.7) and agents at time t have information on the money supply up to time $t+1$. Then a disinflationary boom occurs if a drop in money growth between period t and $t+1$ leads to a fall in unemployment.

of disinflationary booms whenever the lead time for monetary announcements is not greater than the lag time in agents' information updates.

7. Empirical Analysis

To evaluate the inflation-unemployment tradeoff analyzed above, we estimate a dynamic structural model with the following building blocks, matching those of our theoretical model: (i) an unemployment equation (the counterpart of the unemployment equation (3.6)), (ii) a wage setting equation (the counterpart of the wage equation (4.3)), and (iii) a price setting equation (the counterpart of the price equation (4.4)).[42]

We solve these three equations as a system and derive the implied inflation-unemployment tradeoff. This empirical exercise merely aims to illustrate how an estimated Phillips curve can be derived from equations describing the interplay between money growth and nominal frictions. The exercise is no more than a preliminary first step towards a full-blown empirical investigation,[43] which lies well beyond the scope of this paper.

Our empirical analysis is based on multi-equation estimation, since the phenomenon of frictional growth cannot be captured through the usual procedure of estimating a single-equation Phillips curve. When we estimate a traditional or New Phillips curve as a single equation, we are unable to assess how the effects of money growth work their way through the wage-price adjustment process and thereby affect unemployment. Money growth does not enter a single-equation Phillips curve at all; it is substituted out when the impulse-response function of inflation is substituted into the impulse-response function for unemployment to derive the Phillips curve. On this account, we estimate a system in which the wage and price equations portray nominal sluggishness (so that changes in money growth lead to changes in real money balances), and the unemployment equation indicates how the changes in real money balances affect the unemployment rate.

[42] It is important to note that although our wage and price equations are specified solely in terms of current and past variables, they can nevertheless be interpreted as the outcome of decisions by forward-looking agents. As we have seen, forward-looking wage and price equations can be restated in terms of current and past variables, since agents' expectations of the future depend on their information about current and past variables and the underlying stochastic processes.

[43] Such an analysis would, for example, contain a wider range of explanatory variables (e.g. dividing the labor force into skilled and unskilled workers, distinguishing between productivity in different sectors of the economy, etc.), a larger number of equations (e.g. the unemployment rate could be derived from labor demand and labor supply equations, the capital stock could be endogenized, etc.), and so on.

7.1. Data and Estimation

We use US annual time series data, obtained from the OECD and Datastream, covering the period 1966-2000. The definitions of the variables are given in Table 2.

Table 2: Definitions of variables

M_t :	money supply (M3)	f_t :	financial wealth $\left(\frac{\text{SP500}}{\text{labor productivity}}\right)$
P_t :	price level	o_t :	real oil price
W_t :	nominal wages	z_t :	working age population
u_t :	unemployment rate	τ_t :	indirect taxes as a % of GDP
θ_t :	real labor productivity	b_t :	real social security benefits
m_t :	real money balances$(M_t - P_t)$	c_t :	real social security contributions
k_t :	real capital stock	η_t :	real foreign demand (exports-imports)

All variables are in logs except for u_t; foreign demand, η_t; and the tax rate, τ_t.
The variables $m_t; c_t; b_t$; and η_t have been normalized by working age population.
The financial wealth variable f_t is defined as in Phelps and Zoega (2001).

The price setting, wage setting, and unemployment rate equations of our model were initially estimated individually using the autoregressive distributed lag (ARDL) approach to cointegration analysis developed by Pesaran and Shin (1995), Pesaran (1997), and Pesaran et al. (1996). These papers argue that the traditional ARDL approach justified when regressors are I(0), can also be valid with I(1) regressors. An important implication of this methodology is that, since an ARDL equation can always be reparameterized in an error correction format, the long-run solution of the ARDL can be interpreted as the cointegrating vector of the variables involved.

The dynamic specification of each equation was determined by the optimal lag-length algorithm of the Akaike and Schwarz information criteria. The selected estimated equations are dynamically stable (i.e., the roots of their autoregressive polynomia lie outside the unit circle), and pass the standard diagnostic tests (for no serial correlation, linearity, normality, homoskedasticity, and constancy of the parameters of interest) at conventional significance levels.[44] In order to take into account potential endogeneity and cross equation correlation, we then estimated the equations as a system using three stages least squares (3SLS). These results are presented in Table 3.[45] The model tracks the data very well.[46]

[44] See Tables A2-A4 in Appendix 3.
[45] Constants and trends are omitted for brevity.
[46] The actual and fitted values of the estimated system are pictured in Appendix 4.

Table 3: US model, 3SLS, 1966-2000.								
Dependend variable: u_t			Dependend variable: P_t			Dependend variable: W_t		
	coef.	std. e.		coef.	std. e.		coef.	std. e.
u_{t-1}	0.43	(0.12)	P_{t-1}	1.19	(0.13)	W_{t-1}	0.24	(0.10)
u_{t-2}	-0.30	(0.11)	P_{t-2}	-0.54	(0.08)	ΔW_{t-2}	0.48	(0.10)
m_t	-0.12	(0.03)	W_{t-1}	0.34	(0.10)	P_t	0.68	(0.09)
η_t	-0.16	(0.05)	M_t	0.01	$(*)$	M_t	0.09	$(*)$
Δk_t	-0.01	(0.002)	u_t	-0.72	(0.16)	u_t	-0.41	(0.17)
o_{t-1}	0.01	(0.003)	θ_t	-0.30	(0.06)	θ_t	0.32	(0.09)
f_t	-0.01	(0.005)	o_t	0.02	(0.004)	b_t	0.05	(0.02)
c_t	0.04	(0.02)	o_{t-1}	0.01	(0.004)			
			o_{t-2}	-0.01	(0.003)			
			τ_t	0.02	(0.006)			

$(*)$ coefficient is restricted so that there is no money illusion.
Δ denotes the difference operator.

In the unemployment equation, product demand-side influences are captured through real money balances and financial wealth[47] (affecting domestic demand), as well as net foreign demand. Product supply-side influences are captured through the oil price, capital accumulation, and social security contributions. Observe that the sum of the lagged dependent variable coefficients is small and positive, implying a low degree of unemployment persistence. Since the US unemployment rate is trendless, the explanatory variables in the unemployment equation need to be specified as non-trended series as well. On this account, real money balances, social security contributions and benefits, and foreign demand are normalized by working age population, whereas financial wealth is deflated by productivity.

The price and wage equations are quite standard.[48] Prices depend on wages and the money supply, and wages depend on prices and the money supply. Productivity has a positive effect on nominal wages and a negative effect on prices. The unemployment moderates the mark-up of prices on wages, and of wages on prices. The lag structure of our price and wage equations is consistent with our theoretical model. The restriction of no money illusion is imposed on the price

[47] See Phelps (1999), Fitoussi et al. (2000), and Phelps and Zoega (2001).

[48] In order for all variables in our price and wage equations to be integrated of the same order, the equations need to be reparameterized before estimation. For instance, consider the price equation in Table 2: $P_t = a_0 + a_1 P_{t-1} + a_2 P_{t-2} + a_3 W_{t-1} + (1 - a_1 - a_2 - a_3) M_t + \beta' x_t$; where β' is a row vector of parameters, and x_t is a column vector of the real variables. The above can be reparameterized as $(P_t - M_t) = a_0 + a_1 (P_{t-1} - M_{t-1}) + a_2 (P_{t-2} - M_{t-2}) + a_3 (W_{t-1} - M_{t-1}) - (a_1 + a_2 + a_3) \Delta M_t - a_2 \Delta M_{t-1} + \beta' x_t$: These two equations are statistically equivalent. We estimate our price equation using the latter equation, and present the Table 2 results in the format of the former equation. The analogous procedure is applied to the wage equation.

and wage equations, so that each equation is homogeneous of degree zero in all nominal variables. Specifically, we restrict the coefficient of money in each of our nominal equations to be equal to one minus the coefficients of all nominal variables on the right-hand side of that equation.[49] These restrictions could not be rejected at conventional significance levels.

7.2. Empirical Impulse-Response Functions

In this empirical context, we examine the influence of a money growth shock on inflation and unemployment through time. Specifically, suppose that the economy is initially in a steady state, with the money supply growing at the constant rate μ. Then, at time $t = 0$, the money growth rate increases by a fixed amount to μ'. This shock is unanticipated and may be interpretted as a single realization of the stochastic process generating the money supply.[50] We derive the inflation and unemployment responses to this shock for time $t \geq 0$.[51]

Figure 3 presents the impulse response functions (IRFs) that correspond to a 10% permanent increase in the growth rate of money supply. The inflation IRF has all the desirable properties,[52] namely, the influence of the monetary shock on inflation is delayed and gradual, and in the long run inflation is equal to money growth. The unemployment IRF also exhibits plausible behavior: the unemployment effect of the monetary shock is also delayed and gradual, but this effect occurs sooner than the inflation effect (e.g. the maximum unemployment effect occurs well before that on inflation.) Also observe that the inflation and unemployment responses take a long time to converge to their long-run values.

The only strikingly unconventional property of the unemployment IRF is that the unemployment effect does not die down to zero; rather, a 10 percent increase in money growth leads to a 2.73 percent fall in long-run unemployment.[53] Thus, the slope of the long-run Phillips curve is -3.66 $\left(= \frac{10}{-2.73}\right)$.

[49] For example, the price equation in Table 2 (first equation in the previous footnote) is clearly homogeneous of degree zero in M_t, P_t, P_{t-1}, P_{t-2}, and W_{t-1}. The analogous restriction is imposed on the wage equation.

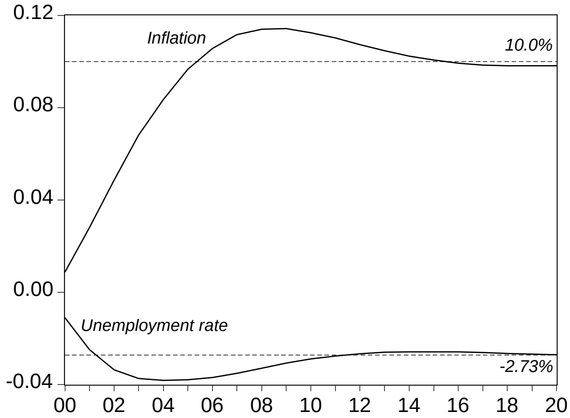
[50] See Appendix 1a. Since the shock is a realization of the actual money growth process, this exercise does not run afoul of the Lucas critique.

[51] We assume that the future values of the exogenous variables are unaffected by the monetary shock (which is obvious, for otherwise these variables would not be exogenous). Thus, given the linearity of our model, the simulation is unaffected by these future variables.

[52] See Mankiw (2001), for instance.

[53] Also observe that the unemployment IRF overshoots substantially: the maximum effect on unemployment is nearly 4 percent.

Figure 3: Impulse-response functions to
a 10% permanent increase in money growth



7.3. Montecarlo Simulations

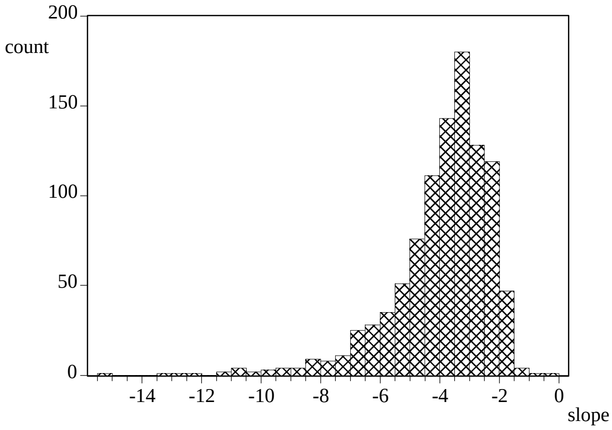
To have confidence that our long-run Phillips curve is indeed not vertical, we need to examine whether our point estimate of the slope (-3.66) is significantly different from infinity. For this purpose, we perform the following Monte Carlo experiment, consisting of 1000 replications. In each replication (i), a vector of error terms $\varepsilon_t^{(i)} = \left(\varepsilon_{u,t}^{(i)}, \varepsilon_{P,t}^{(i)}, \varepsilon_{W,t}^{(i)}\right)'$, $t = 1, 2, \ldots, T$ (of the unemployment rate, price, and nominal wage equations, respectively) is drawn from the normal distribution,[54] $N(0; \Sigma)$. The vector $\varepsilon_t^{(i)}$ is then added to the vectors of estimated equations to generate a new vector of endogenous variables $y_t^{(i)} = \left(u_t^{(i)}; P_t^{(i)}; W_t^{(i)}\right)$. Next, the equations of the model are estimated using the new vector of endogenous variables $y_t^{(i)}$, and the set of exogenous variables. Finally, the simulation exercise of the previous section is conducted on the newly estimated system to derive a new estimate of the slope of the long-run Phillips curve. In this way, each replication (i) yields a new value for the slope: $S^{(i)}; i = 1, 2, \ldots, 1000$.

Figure 4 presents the histogram of the 1000 simulated values of the long-run Phillips curve slope. This shows clearly that the estimated slope of the long-run Phillips curve is indeed significantly downward-sloping and reasonably flat, rather than vertical.[55]

[54] We used the normal distribution because the assumption of normality is valid in the estimated system of equations. ($\varepsilon_t \sim N(0; \Sigma)$, where Σ is the variance-covariance matrix of the estimated model.)

[55] Appendix 5 provides further evidence in support of this result.

Figure 4:
Slope of the long-run Phillips curve
(1000 observations)



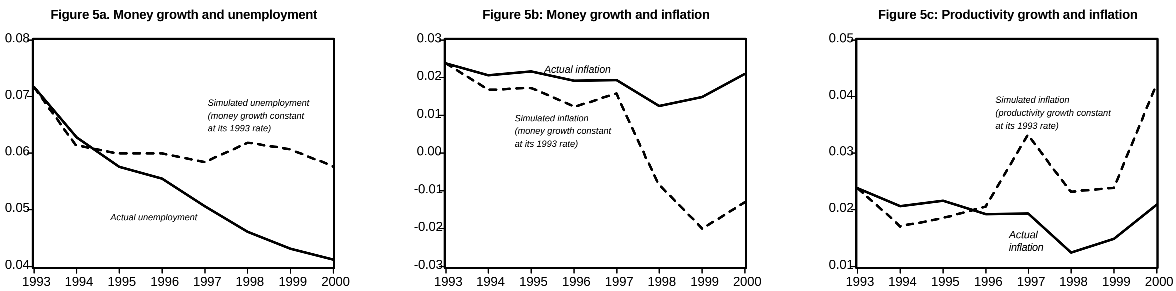
Mean=-3.93, Median =-3.57
Maximum=-0.13, Minimum =-15.22

8. Conclusions

This paper has proposed an alternative to the currently dominant New Phillips curve. Our analysis focuses on the interaction between nominal frictions and money growth. While the choice between our analysis and the New Phillips curve is an empirical issue, three of our results suggest that our analysis is more closely in accord with the established empirical regularities. First, our analysis can explain how money growth shocks have a delayed and gradual effect on inflation, so that there is inflation persistence. Second, it shows that monetary shocks usually have a quicker effect on unemployment and the time path of this effect tends to be hump-shaped. Third, movements in inflation and unemployment in our analysis do not have the knife-edge property.

Inevitably, our analysis suggests a reevaluation of the role monetary policy in the macroeconomic system. It shows that since the effects of monetary policy on inflation and unemployment generally take a long time to work themselves out, we cannot expect close correlations between current money growth (on the one hand) and current inflation and unemployment (on the other), even though monetary policy may have a major influence on these variables over time. Significantly, our analysis indicates that monetary policy can have long-term effects on unemployment. Whether these effects are permanent (along a downward-sloping long-run Phillips curve) or very prolonged (slow adjustment to a near-vertical long-run Phillips curve), may make little observational difference. Indeed, our theoretical model indicates that, in response to variations in the real interest rate, steeper long-run Phillips curves are associated with slower adjustment.

These considerations can have far-reaching implications for our understanding of monetary policy e®ectiveness. To illustrate brie°y, consider the puzzling U.S. macroeconomic developments of the 1990s, when the unemployment rate declined (after 1992) and in°ation remained subdued even though the rate of money growth surged. Although our empirical model is merely illustrative of our approach and should not be viewed as a serious tool for evaluating monetary policy, it nevertheless points to a simple story consistent with the facts. Figure 5a depicts the time path of the actual unemployment rate against the one the unemployment rate would have followed, in our model, had money growth remained constant at its 1993 rate. The di®erence between these two time paths represents the unemployment e®ect that is attributable to money growth, as an accounting exercise.[56] Figure 5b illustrates the actual in°ation rate against the simulated in°ation rate under money growth ¯xed at its 1993 rate, so that the di®erence represents the in°ation e®ect attributable to money growth. Finally, Figure 5c depicts the actual in°ation rate against the simulated in°ation rate under productivity growth ¯xed at its 1993 rate, so that the di®erence represents the in°ation e®ect attributable to productivity growth.



Figures 8: Accounting for In°ation and Unemployment

Although these ¯gures are merely suggestive - even in our illustrative model, in°ation and unemployment are explained by a lot more than just money growth and productivity growth - they make three simple points: First, the surge of money growth over the second half of the 1990s can account for about two thirds

[56] The money growth rate was less than 2 percent per annum in 1993, rose steadily to over 8 percent in 1998, before declining beneath 6 percent in 2000. Increased productivity growth is also associated with reduced unemployment in our model, but the in°uence is much weaker than that of money growth in our model.

of the decline in unemployment over this period (Fig. 5a). Second, the money growth surge was of course associated with a rise in in°ation (Fig. 5b). But, third, this in°ationary in°uence was substantially undone by the fall in in°ation associated with the increase in productivity growth over the period (Fig. 5c). This is of course a highly selective, impressionistic account of what happened, but it highlights some signi¯cant features of our analysis. In particular, since it can take a long time for the long-run in°ation e®ect of a monetary growth shock to manifest itself, a surge in money growth need not be accompanied promptly by a surge in in°ation. There is no evidence that in°ation rises inde¯nitely when unemployment is low. Finally, monetary policy can have a long-term in°uence on unemployment and, over a period of half a decade or more, it is hard to tell whether this in°uence is permanent or prolonged, since the unemployment trajectory re°ects the cumulative in°uence of lengthy impulse-response functions from an ongoing stream of monetary shocks. In any case, monetary policy may play a more important and durable role in the real economy, and with respect to unemployment in particular, than the mainstream theories allow for.

Our analysis is of course just a ¯rst step towards a thorough reevaluation of the in°ation-unemployment tradeo® in terms of frictional growth. Much remains to be done, both in exploring the microfoundations of time-contingent price adjustment and in building reliable empirical models of how monetary shocks a®ect real economic activity.

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APPENDICES

Appendix 1a: Time-Series Properties of the Money Supply

The following table presents the results of unit root tests on the US money supply. The results indicate that we cannot reject the hypothesis that the growth rate of money supply follows an $I(1)$ process at the 5% size of the test.

Table A1: Unit root tests, US money supply, 1966-2000

	Dickey-Fuller	Phillips-Perron	5% critical value
M_t	$ADF_{(c;t)} = -0.77$	$PP_{(c;t)} = -0.35$	-3.54
ΔM_t	$ADF_{(c)} = -2.80$	$PP_{(c)} = -2.72$	-2.95
$\Delta^2 M_t$	$ADF = -7.40$	$PP = -7.55$	-1.95

$ADF_{(c;t)}$; and $PP_{(c;t)}$ denote the unit root tests with constant and trend.

The lag truncation for Bartlett kernel in the PP tests is three.

The order of augmentation in the ADF tests is one.

Appendix 1b: Alternative Specification of the Money Supply Process

Suppose that money growth μ_t follows a stationary autoregressive process and the monetary authority pursues the following mixed strategy: with probability ρ it follows

$$\mu_t = g + \psi_1 \mu_{t-1} + \varepsilon_t; \tag{8.1}$$

and with probability $(1 - \rho)$ it follows

$$\mu_t = g + \psi_2 \mu_{t-1} + \varepsilon_t; \tag{8.2}$$

where ε_t is white noise, $0 < \psi_1; \psi_2 < 1$, and $\psi_1 < \psi_2$.

Thus the money supply rule is

$$\mu_t = g + \beta \mu_{t-1} + \varepsilon_t; \tag{8.3}$$

where $\beta = \rho\psi_1 + (1 - \rho)\psi_2$.

Consequently the equilibrium nominal wage is given by[57]

$$
\begin{aligned}
W_t = {} & (1-\lambda_1)c + \lambda_1 W_{t-1} + (1-\lambda_1)M_t - (1-\lambda_1)L \\
& + \sigma(1-\lambda_1)\mu_t + \left(\frac{1-\lambda_1}{1-\beta}\right)(\kappa-\sigma)g + \omega_t ,
\end{aligned}
\tag{8.4}
$$

where[58]

$$
\sigma = \frac{\beta}{1-\beta} - \frac{\alpha\beta(\lambda_2-1)}{(\lambda_2-\beta)} - \frac{\beta^2(\lambda_2-1)}{(1-\beta)(\lambda_2-\beta)} > 0. \tag{8.5}
$$

The price equation is

$$
\begin{aligned}
P_t = {} & (1-\lambda_1)c + \lambda_1 P_{t-1} + (1-\lambda_1)M_t - (1-\lambda_1)L \\
& + \left(\sigma - \frac{1}{2}\right)(1-\lambda_1)\mu_t + \left(\frac{1-\lambda_1}{1-\beta}\right)(\kappa-\sigma)g \\
& - \frac{1}{2}\sigma(1-\lambda_1)\varepsilon_t + \frac{1}{2}(\omega_t + \omega_{t-1})
\end{aligned}
\tag{8.6}
$$

The long-run solution of the first difference of above equation gives the long-run inflation rate:

$$
\pi_t^{LR} = \mu_t^{LR} = \frac{g}{1-\beta}. \tag{8.7}
$$

The real money balances equation is given by

$$
\begin{aligned}
M_t - P_t = {} & -(1-\lambda_1)c + \lambda_1(M_{t-1} - P_{t-1}) + (1-\lambda_1)L \\
& + \left[\frac{1}{2}(1+\lambda_1) - \sigma(1-\lambda_1)\right]\mu_t + \left(\frac{1-\lambda_1}{1-\beta}\right)(\sigma-\kappa)g \\
& + \frac{1}{2}\sigma(1-\lambda_1)\varepsilon_t - \frac{1}{2}(\omega_t + \omega_{t-1}).
\end{aligned}
\tag{8.8}
$$

The unemployment rate equation is

$$
\begin{aligned}
u_t = {} & (1-\lambda_1)c + \lambda_1 u_{t-1} - \left[\frac{1}{2}(1+\lambda_1) - \sigma(1-\lambda_1)\right]\mu_t \\
& - \left(\frac{1-\lambda_1}{1-\delta}\right)(\sigma-\kappa)g - \frac{1}{2}\sigma(1-\lambda_1)\varepsilon_t + \frac{1}{2}(\omega_t + \omega_{t-1}).
\end{aligned}
\tag{8.9}
$$

[57] The algebraic steps in the derivation of W_t are given in Appendix 2.

[58] $\kappa, \lambda_1, \lambda_2$ are given in Appendix 2.

The long-run unemployment rate is

$$
\begin{aligned}
u_t^{LR} &= c - \left[\frac{1}{2}\left(\frac{1+\lambda_1}{1-\lambda_1}\right) - \sigma\right]\mu_t - \left(\frac{\sigma-\kappa}{1-\beta}\right)g \\
&= c - \pi_t^{LR}\left(\frac{2\alpha-1}{\gamma}\right). \qquad (8.10)
\end{aligned}
$$

where the long-run inflation rate is $\pi_t^{LR} = g/(1-\beta)$. Changes in the policy parameters ρ, ψ_1, and ψ_2 move the economy along this long-run Phillips curve by changing the parameter β.

Appendix 2: Theoretical Model and Results

The equations of our model may be summarized as follows:

$$N_t = Q_t^S; \qquad (8.11)$$

$$L_t = L; \qquad (8.12)$$

$$u_t = L - N_t; \qquad (8.13)$$

$$Q_t^D = M_t - P_t; \qquad (8.14)$$

$$\Delta M_t \equiv \mu_t = \mu_{t-1} + \varepsilon_t; \qquad (8.15)$$

$$Q_t^S = Q_t^D = Q_t; \qquad (8.16)$$

$$P_t = \frac{1}{2}\left(W_t + W_{t-1}\right); \qquad (8.17)$$

$$\Gamma_t = Q_t - L; \qquad (8.18)$$

$$W_t = \alpha W_{t-1} + (1-\alpha)E_t W_{t+1} + \gamma\left[c + \alpha\Gamma_t + (1-\alpha)E_t\Gamma_{t+1}\right] + \omega_t; \qquad (8.19)$$

2.1: Wage Equation

Substitute (8.18) into (8.19) and use (8.14), (8.16), and (8.17) to get:

$$
\begin{aligned}
W_t = {} & \alpha W_{t-1} + (1-\alpha)E_t W_{t+1} + \gamma\alpha\left[M_t - \frac{1}{2}\left(W_t + W_{t-1}\right)\right] \\
& + \gamma(1-\alpha)\left[E_t M_{t+1} - \frac{1}{2}\left(E_t W_{t+1} + W_t\right)\right] + \gamma c - \gamma L + \omega_t.
\end{aligned}
\qquad (8.20)
$$

Apply the expectations operator E_t on the above equation, recall that $E_t(\omega_t) = 0$; collect terms together, so that

$$\psi_1 E_t W_{t-1} - \psi_2 E_t W_t + \psi_3 E_t W_{t+1} = -\gamma\left[\alpha E_t M_t + (1-\alpha) E_t M_{t+1}\right] - \gamma c + \gamma L; \tag{8.21}$$

where

$$\psi_1 = \alpha\left(1 - \frac{\gamma}{2}\right); \ \psi_2 = \left(1 + \frac{\gamma}{2}\right); \ \psi_3 = (1-\alpha)\left(1 - \frac{\gamma}{2}\right). \tag{8.22}$$

To obtain the rational expectations solution of the above eq. (8.21), we proceed as follows. Use the backward shift operator B[59] to rewrite (8.21); then multiply both sides of the resulting equation by B; divide both sides by ψ_3, and use $E_t W_t$ as a common factor on the L.H.S.:

$$\left(1 - \frac{\psi_2}{\psi_3}B + \frac{\psi_1}{\psi_3}B^2\right) E_t W_t = \frac{-B\left(E_t A_t\right) + \gamma L - \gamma c}{\psi_3}; \tag{8.23}$$

where

$$E_t A_t = \gamma\left[\alpha E_t M_t + (1-\alpha) E_t M_{t+1}\right]. \tag{8.24}$$

The B polynomial in (8.23) can de expressed as

$$\left(1 - \frac{\psi_2}{\psi_3}B + \frac{\psi_1}{\psi_3}B^2\right) = (1 - \lambda_1 B)(1 - \lambda_2 B); \tag{8.25}$$

where $\lambda_{1,2}$ are the roots of the equation

$$\lambda^2 - \frac{\psi_2}{\psi_3}\lambda + \frac{\psi_1}{\psi_3} = 0;$$

i.e.

$$\lambda_{1,2} = \frac{\frac{\psi_2}{\psi_3} \pm \sqrt{\left(\frac{\psi_2}{\psi_3}\right)^2 - 4\left(\frac{\psi_1}{\psi_3}\right)}}{2}; \text{ so} \tag{8.26}$$

$$\lambda_1 + \lambda_2 = \frac{\psi_2}{\psi_3}; \text{ and } \lambda_1 \lambda_2 = \frac{\psi_1}{\psi_3} \Rightarrow \lambda_2 = \frac{\alpha}{\lambda_1 (1-\alpha)}.$$

[59] Note that B^1 shifts the variable backward, where B^{-1} shifts the variable forward, i.e.

$$B\left[E_t W_t\right] = E_t W_{t-1}; \text{ and } B^{-1}\left[E_t W_t\right] = E_t W_{t+1};$$

where E_t is in all cases the conditional expectation as of period t.

It can be shown that one root lies inside the unit circle and the other outside the unit circle. In particular, we can show that when $0 < \gamma < 2$ then $0 < \lambda_1 < 1$ and $\lambda_2 > 1$.

We can rewrite (8.23) using (8.25) as

$$(1-\lambda_1 B)\, E_t W_t = \frac{\gamma\,(c-L)}{\phi_3\,(\lambda_2-1)} - \frac{B\,(E_t A_t)}{\phi_3\,(1-\lambda_2 B)}. \tag{8.27}$$

Since $|\lambda_2| > 1$, a useful way to express the geometric polynomial $1/(1-\lambda_2 B)$ is as follows:[60]

$$\frac{1}{1-\lambda_2 B} = \frac{-(\lambda_2 B)^{-1}}{1-(\lambda_2 B)^{-1}}.$$

Substitute the above into (8.27) to get:

$$(1-\lambda_1 B)\, E_t W_t = \frac{\gamma\,(c-L)}{\phi_3\,(\lambda_2-1)} + \frac{E_t A_t}{\lambda_2 \phi_3 \left(1-\lambda_2^{-1} B^{-1}\right)} \tag{8.28}$$

$$= (1-\lambda_1 B)\, E_t W_t = \frac{\gamma\,(c-L)}{\phi_3\,(\lambda_2-1)} + \frac{1}{\lambda_2 \phi_3} \sum_{j=0}^{\infty} \left(\frac{1}{\lambda_2}\right)^j E_t A_{t+j};$$

or, using (8.24) and (8.15),

$$(1-\lambda_1 B)\, E_t W_t = \frac{\gamma\,(c-L)}{\phi_3\,(\lambda_2-1)} + \frac{\gamma}{\lambda_2 \phi_3} \sum_{j=0}^{\infty} \left(\frac{1}{\lambda_2}\right)^j \left(E_t M_{t+1+j} - \alpha E_t \pi_{t+1+j}\right). \tag{8.29}$$

Further algebraic manipulation leads to

$$(1-\lambda_1 B)\, E_t W_t = \frac{\gamma\,(c-L)}{\phi_3\,(\lambda_2-1)} + \frac{\gamma}{\lambda_2 \phi_3} \left[\frac{\lambda_2 M_t}{\lambda_2 - 1} - \frac{\alpha \lambda_2 \pi_t}{\lambda_2 - 1} + \frac{\lambda_2^2 \pi_t}{(\lambda_2-1)^2}\right]$$

$$= (1-\lambda_1)\,[c + M_t + \eta\, \pi_t - L];$$

where[61]

$$\eta = \frac{\lambda_2}{\lambda_2 - 1} - \alpha = \frac{\alpha\,(1+\lambda_1)\,(1-\alpha)}{\alpha - \lambda_1\,(1-\alpha)}. \tag{8.30}$$

[60] See Sargent (1987).

[61] Note that

$$(\lambda_2 - 1)\,(1-\lambda_1) = \frac{\gamma}{\phi_3};$$

so

$$\frac{\gamma}{\phi_3\,(\lambda_2-1)} = (1-\lambda_1).$$

(It can be shown that $\kappa > 0$.) So we have

$$(1-\lambda_1 B)E_t W_t = (1-\lambda_1)c + (1-\lambda_1)M_t + \kappa(1-\lambda_1)\mu_t - (1-\lambda_1)L.$$

A comparison of the above eq. with (8.19) indicates that the rational expectations reduced-form stochastic difference equation for the wage is[62]

$$W_t = (1-\lambda_1)c + \lambda_1 W_{t-1} + (1-\lambda_1)M_t + \kappa(1-\lambda_1)\mu_t - (1-\lambda_1)L + \omega_t. \quad (8.31)$$

Note that the above is the wage equation given in the text. (In the text the stable root λ_1 is denoted by λ for simplicity.)

2.2: Price Equation

To derive the equation for the price dynamics rewrite the price equation (8.17) as follows:

$$(1-\lambda_1 B)P_t = \frac{1}{2}(1-\lambda_1 B)W_t + \frac{1}{2}(1-\lambda_1 B)W_{t-1},$$

and substitute into it the wage equation (8.31). In the resulting equation, substitute the following expressions (implied by the money supply process (8.15)):

$$M_{t-1} = M_t - \mu_t, \text{ and } \mu_{t-1} = \mu_t - \varepsilon_t.$$

Next, collect terms together to get the price equation given in the text:[63]

$$(1-\lambda_1 B)P_t = (1-\lambda_1)c + (1-\lambda_1)M_t + (1-\lambda_1)\left(\kappa - \frac{1}{2}\right)\mu_t - \frac{1}{2}\kappa(1-\lambda_1)\varepsilon_t - (1-\lambda_1)L + \frac{1}{2}(\omega_t + \omega_{t-1}). \quad (8.32)$$

2.3: Inflation Rate Equation

[62] For the solution of linear difference equations under rational expectations see also Blanchard and Kahn (1980), and Sargent (1987).

[63] Note that $\kappa > \frac{1}{2}$ if $\frac{2\alpha+1}{2\alpha-1} > \lambda_2$.

Let the inflation rate be $\pi_t \equiv \Delta P_t$, and take the first difference of the price dynamics eq. (8.32) to obtain the inflation dynamics equation:

$$\begin{aligned}(1-\lambda_1 B)\pi_t &= (1-\lambda_1)\mu_t + \frac{1}{2}(1-\lambda_1)(\kappa-1)\varepsilon_t \\ &\quad + \frac{1}{2}\kappa(1-\lambda_1)\varepsilon_{t-1} + \frac{1}{2}(\omega_t+\omega_{t-2}). \qquad (8.33)\end{aligned}$$

2.4: Real Money Balances

To obtain the real money balances equation we do the following. Add and subtract on the R.H.S. of the price equation (8.32) the term $\lambda_1 M_{t-1}$, and then rearrange terms so that

$$\begin{aligned}(1-\lambda_1 B)(M_t-P_t) &= \left[\frac{1}{2}(1+\lambda_1) - \kappa(1-\lambda_1)\right]\mu_t + \frac{1}{2}\kappa(1-\lambda_1)\varepsilon_t \\ &\quad + (1-\lambda_1)L - \frac{1}{2}(\omega_t+\omega_{t-1}) - (1-\lambda_1)c. \qquad (8.34)\end{aligned}$$

Note that

$$\left[\frac{1}{2}(1+\lambda_1) - \kappa(1-\lambda_1)\right] = (1-\lambda_1)\left(\frac{2\alpha-1}{\gamma}\right). \qquad (8.35)$$

Thus we obtain the real money balances equation given in the text:

$$\begin{aligned}(1-\lambda_1 B)(M_t-P_t) &= -(1-\lambda_1)c + (1-\lambda_1)\left(\frac{2\alpha-1}{\gamma}\right)\mu_t \qquad (8.36) \\ &\quad + \frac{1}{2}\kappa(1-\lambda_1)\varepsilon_t + (1-\lambda_1)L - \frac{1}{2}(\omega_t+\omega_{t-1}).\end{aligned}$$

2.5: Output, Employment, and Unemployment

Rewrite the aggregate demand equation (8.14) as

$$(1-\lambda_1 B)Q_t = (1-\lambda_1 B)(M_t-P_t).$$

To obtain the dynamics for aggregate demand, substitute into the above equation the real money balances equation (8.36):

$$(1-\lambda_1 B)\,Q_t = (1-\lambda_1)\left(\frac{2\alpha-1}{\gamma}\right)\mu_t - (1-\lambda_1)\,c + \frac{1}{2}\kappa\,(1-\lambda_1)\,\varepsilon_t + (1-\lambda_1)\,L - \Delta_t. \tag{8.37}$$

Multiplying both sides of the production function (8.11) by $(1-\lambda_1 B)$, we obtain

$$(1-\lambda_1 B)\,N_t = (1-\lambda_1 B)\,Q_t.$$

Substituting (8.37) into the above, we derive the employment dynamics equation:

$$(1-\lambda_1 B)\,N_t = (1-\lambda_1)\left(\frac{2\alpha-1}{\gamma}\right)\mu_t - (1-\lambda_1)\,c + \frac{1}{2}\kappa\,(1-\lambda_1)\,\varepsilon_t + (1-\lambda_1)\,L - \Delta_t. \tag{8.38}$$

The labour supply (8.12) equation may be expressed as

$$(1-\lambda_1 B)\,L = (1-\lambda_1)\,L. \tag{8.39}$$

By the unemployment rate (8.13), the dynamic process for unemployment is the difference between the labor force (8.39) and employment (8.38). Thus we obtain the unemployment rate equation given in the text:

$$(1-\lambda_1 B)\,u_t = (1-\lambda_1)\,c - (1-\lambda_1)\left(\frac{2\alpha-1}{\gamma}\right)\mu_t - \frac{1}{2}\kappa\,(1-\lambda_1)\,\varepsilon_t + \frac{1}{2}(\omega_t+\omega_{t-1}). \tag{8.40}$$

2.6: Short-Run Phillips Curve

Rewrite the unemployment eq. (8.40) and inflation eq. (8.33) as

$$(1-\lambda_1 B)\,u_t = (1-\lambda_1)\,c - \beta_1\mu_t - \beta_2\varepsilon_t + \frac{1}{2}(\omega_t+\omega_{t-1}); \tag{8.41}$$

$$(1-\lambda_1 B)\,\pi_t = \delta_1\mu_t + \delta_2\varepsilon_t + \beta_2\varepsilon_{t-1} + \frac{1}{2}(\omega_t+\omega_{t-2}); \tag{8.42}$$

where

$$\beta_1 = (1-\lambda_1)\left(\frac{2\alpha - 1}{\gamma}\right); \quad \beta_2 = \frac{1}{2}\kappa(1-\lambda_1);$$
$$\delta_1 = 1-\lambda_1; \quad \delta_2 = \frac{1}{2}(1-\lambda_1)(\kappa - 1):$$

Now substitute the money supply eq. (8.15): $(1-B)\mu_t = \varepsilon_t$ into (8.41) and (8.42) to get

$$(1-\lambda_1 B)u_t = (1-\lambda_1)c - \beta_1\mu_t - \beta_2(1-B)\mu_t + \frac{1}{2}(\omega_t + \omega_{t-1}); \tag{8.43}$$

$$(1-\lambda_1 B)\pi_t = \delta_1\mu_t + \delta_2(1-B)\mu_t + \beta_2\left(B - B^2\right)\mu_t + \frac{1}{2}(\omega_t + \omega_{t-2}): \tag{8.44}$$

Express the (8.43) in terms of μ_t:

$$\mu_t = \frac{(1-\lambda_1 B)u_t - (1-\lambda_1)c - \frac{1}{2}(\omega_t + \omega_{t-1})}{\beta(B)}; \tag{8.45}$$

where $\beta(B) = [-(\beta_1 + \beta_2) + \beta_2 B]$:

Substitution of (8.45) into (8.44) leads to the short-run Phillips curve

$$(1-\lambda_1 B)\beta(B)\pi_t = (1-\lambda_1 B)\delta(B)u_t - \delta(B)(1-\lambda_1)c + \frac{\beta(B)(\omega_t + \omega_{t-2}) - \delta(B)(\omega_t + \omega_{t-1})}{2}; \text{ or}$$

$$\beta(B)\pi_t = \delta(B)u_t - \delta_1 c + \frac{\beta(B)(\omega_t + \omega_{t-2}) - \delta(B)(\omega_t + \omega_{t-1})}{2(1-\lambda_1 B)};$$

where $\delta(B) = [(\delta_1 + \delta_2) + (\beta_2 - \delta_2)B - \beta_2 B^2]$:

After some algebraic manipulation, the above short-run Phillips curve can be written as

$$\pi_t = \frac{1}{\beta_1 + \beta_2}\left[(1-\lambda_1)c + \beta_2\pi_{t-1} - (\delta_1 + \delta_2)u_t - (\beta_2 - \delta_2)u_{t-1} + \beta_2 u_{t-2}\right] + \xi_t;$$

where

$$\xi_t = \frac{\delta(B)(\omega_t + \omega_{t-1}) - \beta(B)(\omega_t + \omega_{t-2})}{2(\beta_1 + \beta_2)(1-\lambda_1 B)}:$$

Through some algebraic manipulation we get:

$$
\begin{aligned}
\pi_t &= \frac{1}{\beta_1+\beta_2}\left[(1-\lambda_1)c + \beta_2\pi_{t-1} - (\delta_1+\delta_2)u_t - (\beta_2-\delta_2)u_{t-1} + \beta_2 u_{t-2}\right] + \phi_t \\
&= \left(\frac{1-\lambda_1}{\beta_1+\beta_2}\right)\left[c + \frac{1}{2}\kappa\pi_{t-1} - \frac{1}{2}(1+\kappa)u_t - \frac{1}{2}u_{t-1} + \frac{1}{2}\kappa u_{t-2}\right] + \phi_t \\
&= \psi\left[c + \frac{1}{2}\kappa\pi_{t-1} - \frac{1}{2}(1+\kappa)u_t - \frac{1}{2}u_{t-1} + \frac{1}{2}\kappa u_{t-2}\right] + \phi_t; \qquad (8.46)
\end{aligned}
$$

where[64] $\psi = \frac{1-\lambda_1}{\beta_1+\beta_2}$. In addition, the error term can be written as

$$\phi_t = \frac{\left[\left(1+\frac{\psi(1+\kappa)}{2}\right)\omega_t + \frac{3\psi}{2}\omega_{t-1} + \left(1+\frac{\psi(1-\kappa)}{2}\right)\omega_{t-2} - \psi\kappa\,\omega_{t-3}\right]}{2(1-\lambda_1 B)}. \qquad (8.47)$$

Note that the above error term is an infinite moving average (IMA) process in terms of ω_t, with parameters which are non-linear functions of the theoretical parameters ψ, κ, and λ_1.[65]

Express equation (8.46) as

$$\pi_t = d_0 + d_1\pi_{t-1} - d_2u_t - d_3u_{t-1} + d_4u_{t-2} + \phi_t; \qquad (8.48)$$

where

$$d_0 = \psi c;\; d_1 = \frac{\psi\kappa}{2};\; d_2 = \frac{\psi(1+\kappa)}{2};\; d_3 = \frac{\psi}{2};\; d_4 = \frac{\psi\kappa}{2}.$$

Thus we have the following relationships among the d's:

$$d_4 = d_1;\ \text{and}\ d_3 = d_2 - d_1. \qquad (8.49)$$

2.7: Long-Run Unemployment, Inflation, and the Phillips Curve

To get the long-run solution of the unemployment equation (8.40) we set the backshift operator equal to unity ($B = 1$), and set equal to zero all the error terms (ε's, ω's). This gives us the following long-run:

$$u_t^{LR} = -\left(\frac{2\alpha-1}{\gamma}\right)\mu_t^{LR} + c. \qquad (8.50)$$

[64]Note that $\frac{\beta_1+\beta_2}{1-\lambda_1} = \frac{2\alpha-1}{\gamma} + \frac{\kappa}{2}$.

[65]Recall that ψ, κ, and λ_1 are non-linear functions of the theoretical parameters α and γ of the wage contract equation.

Similarly, the long-run solution of the inflation equation (8.33) is given by

$$\pi_t^{LR} = \mu_t^{LR}. \tag{8.51}$$

To get the long-run Phillips curve we need to substitute (8.51) into (8.50):

$$\pi_t^{LR} = -\left(\frac{\gamma}{2\alpha - 1}\right) u_t^{LR} + \left(\frac{\gamma}{2\alpha - 1}\right) c. \tag{8.52}$$

2.8: Short-Run vs Long-Run Phillips Curve

The slope of the short-run Phillips curve (8.46) is

$$\frac{\partial \pi_t}{\partial u_t} = -\frac{\delta_1 + \delta_2}{\square_1 + \square_2} = -\frac{\gamma + \gamma\kappa}{2(2\alpha - 1) + \gamma\kappa}, \tag{8.53}$$

whereas the slope of the long-run Phillips curve (5.3) is

$$\frac{\partial \pi_t^{LR}}{\partial u_t^{LR}} = -\frac{\gamma}{2\alpha - 1}. \tag{8.54}$$

It can be shown that if the (absolute value of the) long-run slope is greater than unity then

$$\left|\frac{\partial \pi_t^{LR}}{\partial u_t^{LR}}\right| > \left|\frac{\partial \pi_t}{\partial u_t}\right|,$$

i.e. the long-run PC is steeper than the short run PC.[66]

[66] This can be shown as follows:

$$\begin{aligned}
\left|\frac{\partial \pi_t^{LR}}{\partial u_t^{LR}}\right| &> \left|\frac{\partial \pi_t}{\partial u_t}\right| \Rightarrow \\
\frac{\gamma}{2\alpha - 1} &> \frac{\gamma + \gamma\kappa}{2(2\alpha - 1) + \gamma\kappa} \Rightarrow \\
\gamma\left(2(2\alpha - 1) + \gamma\kappa\right) &> (2\alpha - 1)(\gamma + \gamma\kappa) \Rightarrow \\
\gamma\left((2\alpha - 1) + \gamma\kappa\right) &> (2\alpha - 1)\gamma\kappa \Rightarrow \\
\frac{\gamma}{2\alpha - 1} &> \frac{\gamma\kappa}{(2\alpha - 1) + \gamma\kappa}.
\end{aligned}$$

Since the smallest value that α is assumed to take is one half, it follows that the maximum value of right-hand side of the above inequality is unity. Therefore, we can say that a sufficient (but not necessary) condition for $\left|\frac{\partial \pi_t^{LR}}{\partial u_t^{LR}}\right| > \left|\frac{\partial \pi_t}{\partial u_t}\right|$ is that $\left|\frac{\partial \pi_t^{LR}}{\partial u_t^{LR}}\right| > 1.$

The intercept of the short-run Phillips curve (8.46) is

$$\left(\frac{1-\lambda_1}{\lambda_1+\lambda_2}\right)c=\left(\frac{2\gamma}{2\,(2\alpha-1)+\gamma\kappa}\right)c>0; \tag{8.55}$$

and the intercept of the long-run Phillips curve (8.52) is

$$\left(\frac{\gamma}{2\alpha-1}\right)c>0. \tag{8.56}$$

Since both γ and κ are positive, it is not difficult to see that the intercept of the long-run PC is greater than the intercept of the short-run PC:

$$\left(\frac{\gamma}{2\alpha-1}\right)c>\left(\frac{2\gamma}{2\,(2\alpha-1)+\gamma\kappa}\right)c.$$

2.9: Impulse Response Functions

We assume a one-off unit shock in the money growth process (8.15) which occurs at time $t=0$: $\varepsilon_0=1;\ \varepsilon_t=0$ for $t\neq0$.

2.9a: Inflation Rate

The impulse response function of the inflation eq. (8.33) is given by

$$R\left(\pi_0\right)=1+\frac{1}{2}\left[(1-\lambda_1)\kappa-(1+\lambda_1)\right]<1 \text{ if } \kappa<\underbrace{\frac{1+\lambda_1}{1-\lambda_1}}_{\text{critical value } b_2};$$

$$\underset{t\geq1}{R\left(\pi_t\right)}=1+\lambda_1^{t-1}\left(\frac{1+\lambda_1}{2}\right)\left[(1-\lambda_1)\kappa-\lambda_1\right]<1 \text{ if } \kappa<\underbrace{\frac{\lambda_1}{1-\lambda_1}}_{\text{critical value } b_1};$$

$$R\left(\pi_{LR}\right)\equiv\lim_{t\to\infty}R\left(\pi_t\right)=1;\ \text{(long-run response)}. \tag{8.57}$$

Observe that, since $\lambda_1<1$; we have that

$$\left|R\left(\pi\right)_{t+1}-1\right|<\left|R\left(\pi\right)_t-1\right|;\ t\geq1;$$

i.e., period 1 onwards, inflation gradually approaches its new long-run value.[67]

We should note that, since $\frac{1}{2} < \alpha < 1$; we cannot have that κ is greater than b_2: That is, inflation cannot overshoot at the period that the shock is initiated $(t = 0)$.[68]

2.9b: Unemployment Rate

The impulse response function of the unemployment eq. (8.40) is given by[69]

$$\underset{t \geq 0}{R}(u_t) = -\left(\frac{2\alpha - 1}{\gamma}\right) - \frac{\lambda_1^t (1+\lambda_1)}{2(1-\lambda_1)}\left[(1-\lambda_1)\kappa - \lambda_1\right];$$

$$R(u_{LR}) \equiv \lim_{t \to \infty} R(u_t) = -\left(\frac{2\alpha - 1}{\gamma}\right); \text{ (long-run response).} \qquad (8.58)$$

Closer inspection of the above equations reveals the following pattern for unemployment responses:

$$\text{if } \kappa > \underbrace{\frac{\lambda_1}{1-\lambda_1}}_{\text{critical value } b_1} \text{ then } \underbrace{R(u_t) < -\left(\frac{2\alpha-1}{\gamma}\right)}_{\text{overshooting}}; \text{ for } t \geq 0:$$

The following table summarizes how inflation and unemployment respond to the above unit shock initiated at period $t = 0$:

---

[67] The effect of time on the inflation responses is given by

$$\frac{\partial R(\pi)_t}{\partial t} = \lambda_1^{t-1}\left(\frac{1+\lambda_1}{2}\right)\left[(1-\lambda_1)\kappa - \lambda_1\right]\ln \lambda_1; \ t \geq 1:$$

So when $\kappa < \frac{\lambda_1}{1-\lambda_1}$, $[(1-\lambda_1)\kappa - \lambda_1] < 0$; then the above derivative is positive, since $\ln \lambda_1 < 0$:

[68] Below we show that κ is always less than b_2 :

$$\kappa < \frac{1+\lambda_1}{1-\lambda_1} \Rightarrow \frac{\alpha(1+\lambda_1)(1-\alpha)}{\alpha - \lambda_1(1-\alpha)} < \frac{1+\lambda_1}{1-\lambda_1} \Rightarrow \frac{\alpha(1-\alpha)}{\alpha - \lambda_1(1-\alpha)} < \frac{1}{1-\lambda_1} \Rightarrow$$

$$\alpha(1-\alpha) - \lambda_1\alpha(1-\alpha) < \alpha - \lambda_1(1-\alpha) \Rightarrow \lambda_1(1-\alpha) - \lambda_1\alpha(1-\alpha) < \alpha - \alpha(1-\alpha)$$

$$\Rightarrow \lambda_1(1-\alpha)^2 < \alpha^2 \Rightarrow \lambda_1 < \left(\frac{\alpha}{1-\alpha}\right)^2:$$

The latter inequality is valid since $\frac{1}{2} < \alpha < 1$: Thus κ is always smaller than b_2:

[69] Also, note that the effect of time on the unemployment responses is given by

$$\frac{\partial R(u_t)}{\partial t} = -\frac{\lambda_1^t(1+\lambda_1)}{2(1-\lambda_1)}\left[(1-\lambda_1)\kappa - \lambda_1\right]\ln \lambda_1; \ t \geq 1:$$

Inflation - Unemployment Responses			
Class I	$\kappa < b_1$:		$\underbrace{\pi_t < \pi_{LR}}_{\text{undershooting}}; \text{ for } t \geq 0$
			$\underbrace{\lvert u_t \rvert < \lvert u_{LR} \rvert}_{\text{undershooting}}; \text{ for } t \geq 0$
Class II	$b_1 < \kappa < b_2$:	$\pi_0 < \pi_{LR};$	$\underbrace{\pi_t > \pi_{LR}}_{\text{overshooting}}; \text{ for } t \geq 1$
			$\underbrace{\lvert u_t \rvert < \lvert u_{LR} \rvert}_{\text{overshooting}}; \text{ for } t \geq 0$

Appendix 3: OLS Estimates of the Unemployment, Price, and Wage Equations

Table A2: Unemployment equation, OLS, 1966-2000.

Dependent variable: u_t	coefficient	s.e.	Misspecification tests*	
u_{t-1}	0.45	(0.14)	$SC[\chi^2(1)]$	1.51 $[0.22]$
u_{t-2}	-0.31	(0.13)	$LIN[\chi^2(1)]$	1.77 $[0.18]$
m_t	-0.12	(0.04)	$NOR[\chi^2(1)]$	0.84 $[0.66]$
π_t	-0.14	(0.06)	$ARCH[\chi^2(1)]$	0.11 $[0.74]$
Δk_t	-0.01	(0.002)	$HET[\chi^2(16)]$	13.9 $[0.61]$
o_{t-1}	0.01	(0.003)	CUSUM	X
f_t	-0.01	(0.005)	CUSUMSQ	X
c_t	0.04	(0.02)		

+ LL=137.77, AIC=-7.36, SC=-6.96

* Probabilities in square brackets
X Structural stability cannot be rejected at the 5% size of the test
+ Log likelihood (LL), Akaike (AIC) and Schwarz (SC) criteria

Table A3: Price equation, OLS, 1966-2000.				
Dependent variable: P_t				
	coefficient	s.e.	Misspecification tests*	
P_{t-1}	0.91	(0.20)	SC[$F(1,23)$]	7.76 [0.01]
P_{t-2}	−0.37	(0.13)	LIN[$\chi^2(1)$]	2.78 [0.10]
W_{t-1}	0.32	(0.11)	NOR[$\chi^2(2)$]	0.01 [0.99]
M_t	0.05	(0.03)	ARCH[$\chi^2(1)$]	0.00 [0.99]
u_t	−0.65	(0.18)	HET[$\chi^2(22)$]	30.0 [0.12]
μ_t	−0.53	(0.14)	CUSUM	X
o_t	0.017	(0.005)	CUSUMSQ	X
o_{t-1}	0.015	(0.006)		
o_{t-2}	−0.006	(0.004)		
τ_t	0.001	(0.007)		

+ LL=141.63, AIC=-7.41, SC=-6.87

++ [$F(1,23)$] = 4.21 [0.05]

* Probabilities in square brackets

X Structural stability cannot be rejected at the 5% size of the test

+ Log likelihood (LL), Akaike (AIC) and Schwarz (SC) criteria

++ Wald test for long-run no money illusion

Table A4: Wage equation, OLS, 1966-2000.

Dependent variable: W_t

	coefficient	s. e.	Misspecification tests*	
W_{t-1}	0.19	(0.11)	$SC[\chi^2(1)]$	$3.04\,[0.08]$
ΔW_{t-2}	0.47	(0.12)	$LIN[\chi^2(1)]$	$1.10\,[0.29]$
P_t	0.73	(0.12)	$NOR[\chi^2(2)]$	$1.76\,[0.42]$
M_t	0.08	(0.03)	$ARCH[\chi^2(1)]$	$0.06\,[0.80]$
u_t	-0.41	(0.21)	$HET[\chi^2(14)]$	$15.1\,[0.37]$
μ_t	0.35	(0.10)	CUSUM	X
b_t	0.05	(0.02)	CUSUMSQ	X

$^{+}$ LL=127.54, AIC=-6.83, SC=-6.48

$^{++}$ $[F(1;27)] = 0.07\,[0.80]$

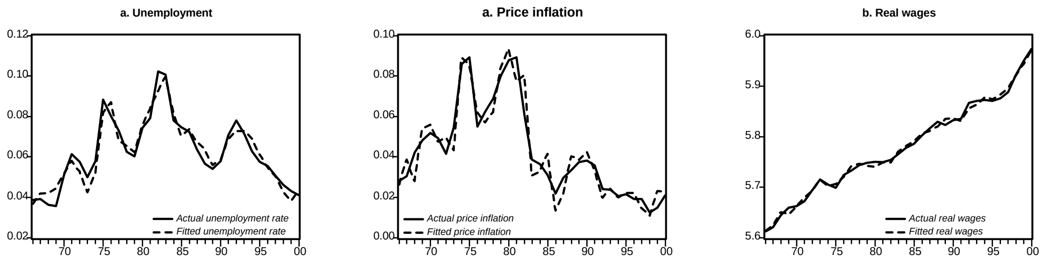
* Probabilities in square brackets

X Structural stability cannot be rejected at the 5% size of the test

$^{+}$ Log likelihood (LL), Akaike (AIC) and Schwarz (SC) criteria

$^{++}$ Wald test for long-run no money illusion

Appendix 4: Actual and Fitted Values of the Estimated System



Appendix 5: Further Evidence on Whether the Long-Run Phillips Curve is Vertical

In the following table we present the percentage count of slopes within specific class intervals. For example, the probability that the long-run Phillips curve slope lies in the interval $(-6; -1.5)$ is 89%.

Table A5: probability that the PC slope is within a specific interval			
Slope interval	$(-\infty; -6)$	$(-6; -1.5)$	$(-1.5; \infty)$
Probability	10.4 %	89.0 %	0.6 %

We also grouped the values of the generated series $S^{(i)}$; $i = 1; 2; ...; 1000$; into class intervals of 0.5 units. Using as a cut-off point a 10% count, there is no class interval below [-4.5,-4.0) or above [-2.5,-2.0) that contains at least 10% of the values of slope series S. These class intervals and their respective probabilities are given in the table below.

Table A6: Monte Carlo simulations, 1000 replications class intervals with a count above 10%					
Slope interval	$[-4.5; -4.0)$	$[-4.0; -3.5)$	$[-3.5; -3.0)$	$[-3.0; -2.5)$	$[-2.5; -2.0)$
Probability	11.1 %	14.3 %	18.0 %	12.8 %	11.9 %

IZA Discussion Papers

No.	Author(s)	Title	Area	Date
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622	J. Hassler J. V. Rodriguez Mora	Should UI Benefits Really Fall Over Time?	3	11/02
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