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Insurable Assets, and Wealth Accumulation**

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ABSTRACT

The Dynamics of Market Insurance, Insurable Assets, and Wealth Accumulation*

We analyze dynamic interactions between market insurance, the stock of insurable assets and liquid wealth accumulation in a model with non-durable and durable consumption. The stock of the durable is exposed to risk against which households can insure. Since the model does not have a closed form solution we first provide an analytical approximation for the case in which households own abundant liquid wealth. It turns out that precautionary motives still matter because of fluctuations of the predetermined durable stock. Second we solve the model numerically. With deterministic labor income the representative agent demands a non-negligible amount of market insurance. The deductible is substantially higher than in static models because agents can time-diversify their risk. Market insurance implies welfare gains of around .6% in terms of non-durable consumption. Introducing labor income risk into the model does not necessarily increase the importance of market insurance if the borrowing constraint endogenously tightens.

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1 Introduction

Durables are a substantial part in the balance sheets of households. E.g., in the US and the UK durable consumption accounts for 13-14 % of total consumption in the time period 1959-96 (see Attanasio (1999)) where the ratio of durable consumption over non-durable consumption (excluding services) is roughly one third. In the US the current-cost net stock of private fixed assets amounts to twice the GDP in the year 2000 where alone the stock of privately-owned manufacturing durable goods accounts for 10% of GDP.¹ The stock of durables is exposed to risk. E.g., houses can be broken into or destroyed by natural catastrophes, cars can be stolen etc. The topic of this paper is to assess the importance of market insurance compared with other possible actions of households to manage this risk. Empirically, market insurance seems to be important. Non-life insurance penetration, defined as direct gross premiums over GDP, is 10% in the US in the 90s (see OECD (1998)) and the OECD average is 8%. Expenditure on motor-vehicle insurance and fire and property insurance each account for roughly a quarter of the total. However, besides buying market insurance, households can accumulate liquid wealth to self-insure or adjust the stock of durables exposed to risk. We want to find out which of these actions dominates and how they interact dynamically.

The following papers are closest to our analysis. Ehrlich and Becker (1972) analyze the interactions of market insurance, self insurance and self protection in a static model. In their work self insurance is defined as actions of agents to decrease the size of the loss and self protection as actions to decrease the probability of the loss. They find that self insurance and market insurance are substitutes whereas this is not unambiguously the case for market insurance and self protection. Eeckhoudt, Meyer and Ormiston (1997) perform a static analysis of interactions between market insurance and insurable assets. They show that the utility derived from market insurance increases as the share of the risky asset in the portfolio increases if the utility function has the property of decreasing absolute risk aversion. Eeckhoudt, Gollier and Schlesinger (1991), Proposition 2, find

¹The figures are obtained from the national income and product accounts of the Bureau of Economic Analysis, U.S. Department of Commerce.

that in a static model market insurance and precautionary savings are substitutes for a particular increase in risk of the insurable loss. Gollier (1994, 2000) investigates *dynamic* interactions between market insurance and the accumulation of liquid wealth as buffer stock. Whereas Gollier (1994) derives a closed form solution for CRRA utility and no liquidity constraints in a continuous-time model, Gollier (2000) provides a numerical solution to the more general problem. He finds that allowing for time diversification through wealth accumulation reduces the scope for market insurance substantially. However, in his model the insurable risk is exogenous.

In this paper we want to analyze simultaneously the size of the insurable asset, liquid wealth accumulation and market insurance and their interactions. I.e., we allow that the risk associated with the insurable asset can be adjusted not only by buying market insurance, but also by adjusting the size of the asset itself. We get the following set of results.

First we derive an analytical approximation for the special case of abundant liquid wealth. It turns out that precautionary motives still matter in this case because the risky durable stock is predetermined and enters the utility function. This is in contrast with standard models of non-durable consumption under uncertainty in which risk-averse households accumulate sufficient wealth dynamically so that utility costs resulting from fluctuations are small as the value function becomes approximately linear (see Krusell and Smith (1998) and their references). Our numerical results and the empirical evidence are at odds with linearity of the value function for the representative agent. If the value function was linear, the representative agent should not demand any market insurance.

Second we solve the model numerically. For the case of deterministic labor income we find that the representative agent does not accumulate substantial liquid wealth and spends a non-negligible part of his income on market insurance. The agent buys less market insurance in our model compared to the results of Gollier (2000). This is because in our model it is possible to adjust the asset at risk. Furthermore, we solve the model for stochastic labor income. Interestingly, if labor income is risky and thus the borrowing constraint becomes tighter, the accumulation of liquid wealth becomes more important whereas the opposite holds for market insurance. The latter

result is contrary to findings in static models given that we assume utility functions that have the property of decreasing absolute risk aversion. The reason is that the tighter borrowing constraint forces the agent to be relatively more patient. The smaller amount of resources needed to service the debt allows the agent to finance relatively more consumption. Moreover, labor income risk creates an endogenous incentive to accumulate buffer-stock wealth which can be used to smooth fluctuations of the durable stock as well. With a tighter borrowing constraint adverse shocks imply smaller fluctuations in terms of marginal utility in the steady state and hence less scope for market insurance. Finally, we find that the motive for market insurance is stronger if labor income shocks are permanent rather than transitory whereas the opposite holds for liquid wealth accumulation.

The rest of the paper is structured as follows. In Section 2 we present and discuss the model. In Section 3 we provide an analytic approximation of the policy functions for the special case of abundant liquid wealth. In Section 4 we present the numerical results and analyze the solution for various parameter sets. We conclude in Section 5.

2 The model

Households derive utility from a durable good v , e.g., cars, and a non-durable good c , e.g., food. For simplicity we assume v to be a homogenous, divisible good. This assumption is made for tractability given that it is more realistic to assume that, e.g., cars are a bundle of characteristics. Household's utility is separable so that $U(c, v) = u(c) + \phi w(v)$ where $U(\cdot)$ is the instantaneous utility function, $u(\cdot)$ and $w(\cdot)$ are both concave, and ϕ is the weight given to utility derived from the durable. This assumption allows us to derive interpretable analytical results because otherwise cross-derivatives would render our results ambiguous. We assume $w(v)$ to be well defined at $v = 0$ so that our model is able to generate households with no durable stock. A possible functional form is $w(v) = (v + \underline{v})^\tau$, $\tau \leq 1$, where $\underline{v} > 0$. The asymmetry in the utility function with respect to non-durable and durable consumption is justified in the sense that durable consumption is less essential than non-durable consumption such as food. Note that c and v are measured in the same units so that we do not

introduce a price in our notation.

We specify our model in discrete time so that we have to make assumptions about the timing. On the basis of the state variables financial wealth, a_t , and durable stock, v_t , households choose the controls non-durable consumption, c_t , investment into the durable, d_t , and the retention rate D_t , i.e., the percentage-of-loss deductible for the insurance of the durable good. D is defined as the proportion of the loss l which is not insured so that the payment the agent receives from the insurance is

$$\max(l - Dl, 0) = l * \max(1 - D, 0) = l * (1 - D) ,$$

where the last equality holds since $D \in [0,1]$. It is well known that optimal insurance contracts indeed contain a deductible (see, e.g., Raviv (1979)). Finally, the durable stock depreciates at rate δ and the values of labor income, \tilde{y}_t , and the loss of the durable, \tilde{l}_t , are realized.

Notice that v_t is predetermined. Households' investment d_t only materializes in $t+1$. This timing assumption is equivalent to the one made in standard growth models with capital accumulation. The assumption is clearly stylized because the adjustment speed will in general be a function of the size of the shock. It is quite realistic, however, if one considers large durable items such as cars and houses whose purchase take a considerable amount of time.

Defining $V(\cdot)$ as the value function, the maximization problem is:

$$V(a_t, v_t) = \max_{c, d, D} [u(c_t) + \phi w(v_t) + \beta E_t \{V(a_{t+1}, v_{t+1}) | \Omega_{y, l}\}] \quad (1)$$

s.t.

$$a_{t+1} = (1 + r)a_t - c_t - d_t - \mu(1 - D_t)E_t \tilde{l}_t + (1 - D_t)\tilde{l}_t + \tilde{y}_t$$

$$v_{t+1} = (1 - \delta)v_t + d_t - \tilde{l}_t$$

$$a_s \geq \underline{a} , s \geq t$$

$$v_s \geq 0 , s \geq t,$$

where β is the discount factor, r is the interest rate² and δ is the depreciation rate which we assume to be constant for simplicity, $\Omega_{y,t}$ is the support of the joint distribution of the two random variables \tilde{y}_t , income, and \tilde{l}_t , the loss associated with the durable as a result of accidents, theft etc. The relative weight of durable versus non-durable consumption depends on ϕ . The expectation operator conditional on time t is denoted by E_t . For the case of deterministic income $\tilde{y} = y$ where y is non-random.

The first constraint is the budget constraint. The amount of assets tomorrow depends on the amount of assets today plus the interest, the amount of durable and non-durable consumption, and labor income. Moreover, it depends on the amount of insurance demanded in period t , $\mu(1 - D_t)E_t\tilde{l}_t$, and the money received from the risk-neutral insurance company if damage occurred, $(1 - D_t)\tilde{l}_t$. Note that there is a wedge introduced by the mark-up μ which is assumed constant for simplicity. This wedge is necessary because otherwise risk-averse households always would be fully insured by risk-neutral insurers. Note, that we take the design of insurance contracts as exogenous. E.g., Briys and Viala (1995) analyze optimal insurance design under background risk and hence endogenize insurance contracts.

The second constraint is the law of motion of the durable stock. The size of the durable stock tomorrow depends on its size today net of depreciation plus the investment today minus the random loss. Note that we abstract from adjustment costs. Adjustment costs certainly imply more realistic adjustment dynamics for the durable stock, but are not essential for the focus of the paper. Given that the model structure is already quite rich, we chose to abstract from adjustment costs as a starting point also because otherwise the numerical solution would become much more computing intensive. For an analysis of durable investment under uncertainty with adjustment costs see, e.g.,

²Note that r is exogenous and hence our model is partial equilibrium. This not crucial for the main message we want to convey, but necessary to retrieve numerical results in finite time. It implies that below we have to assume that the discount rate is larger than r so that liquid assets are finite in the steady state with incomplete markets. This would be an endogenous result if the interest rate had to equal the marginal product of capital and the returns to capital are decreasing (see Aiyagari (1994)).

Bertola et al. (2002) or Lam (1991).

The third constraint is the solvency constraint. It implies that households cannot borrow more than \underline{a} which guarantees that households are always able to repay their debt. Aiyagari (1994) derives in a model with non-durable consumption that $\underline{a} = -\frac{\min y}{r}$, where $\min y$ is the smallest y attainable on the support of the distribution. In our model this constraint remains the same because we assume that households cannot use the durable stock as collateral. This is done for clarity and simplifies the model. If it were possible for households to use the durable as a collateral instead, the solvency constraint would vary over time together with v . In this case $\underline{a}_t = -\frac{\min y}{r} - \min v_{t+1}$ where $\min v_{t+1}$ is the minimum appropriable amount of the durable in period $t + 1$. This amount negatively depends on the maximum of the support of the loss distribution, the depreciation rate and interest rate since selling the durable takes one period of time. Moreover, it depends on the specification of the appropriation technology.

The fourth constraint implies that households cannot go short in the durable. Finally, for both state variables a and v a transversality condition has to be satisfied, respectively.

Ignoring the constraints $a_s \geq \underline{a}$ and $v_s \geq 0$ in order to derive an analytical approximation, problem (1) yields the following Euler equations for the controls c, d and D , respectively:

$$u'(c_t) = \beta(1+r) E_t u'(c_{t+1}) , \quad (2)$$

$$u'(c_t) = \beta [(1-\delta)E_t u'(c_{t+1}) + \phi E_t w'(v_{t+1})] \quad (3)$$

and

$$\beta E_t \{u'(c_{t+1}) \tilde{l}_t\} = u'(c_t) \mu E_t \tilde{l}_t , \quad (4)$$

where primes denote first-order derivatives of the functions with respect to the variables in brackets.

Equations (2) and (3) can be used to solve for the intertemporal behavior of v :

$$E_t w'(v_{t+1}) = \beta(1+r) E_t w'(v_{t+2}) . \quad (5)$$

Equation (2) is standard and relates non-durable consumption intertemporally. Equation (5) is the equivalent for the durable stock.

On the left-hand side of equation (3) are the foregone benefits resulting from one unit of durable investment d , in terms of utility derived from non-durable consumption. These have to equal the benefits on the right-hand side of equation (3). The benefits are the discounted expected utility afforded by the increase of the durable stock, $\phi E_t w'(v_{t+1})$, plus the expected utility of non-durable consumption resulting from selling d units of the durable stock adjusted for depreciation in the next period.

Equation (4) is the Euler equation for insurance demand. Again, marginal benefits on the left-hand side of the equation equal marginal costs which are on the right-hand side. The marginal benefits are that households can consume more tomorrow in bad states of the world once they are insured today because they expect to have relatively more resources resulting from insurance payments. Since $E_t \{u'(c_{t+1})\tilde{l}_t\} = E_t u'(c_{t+1})E_t \tilde{l}_t + cov(u'(c_{t+1}), \tilde{l}_t)$, households are willing to buy more insurance if large realizations of \tilde{l}_t occur in states of the world in which the households' consumption is already small. In terms of equation (4), a higher $cov(u'(c_{t+1}), \tilde{l}_t)$ implies a higher $u'(c_t)$. I.e., the marginal cost on the right-hand side is relatively higher because households are willing to forego more non-durable consumption to buy more insurance.

3 Approximation of the policy functions

In general the model presented above does not have a closed-form solution. Before we perform a simulation of the model based on the numerical solution we present the results of a second-order approximation of the policy functions and laws of motion to develop some intuition. The main point

of this exercise is that in our model precautionary motives are important even if households own abundant liquid wealth. This results from the fact durables are a state variable and directly enter the utility function. Alternatively, one could assume a model structure symmetric to non-durable consumption in which utility is derived from a flow out of the durable stock. In this case utility from durable consumption does not necessarily decrease if the durable stock does. We decided that our modeling choice is more natural for risks which substantially reduce the durable stock like theft or destruction.

To approximate the solution we employ the perturbation method which is explained in detail in Schmitt-Grohé and Uribe (2001) using the Euler equations (2)~(4) and the two laws of motion for a and v . We provide the derivation of the approximation in the Appendix. Because we do allow the state variables to be at interior optima only, for the purpose of the approximation, the results simplify considerably and can be used to develop intuition on the mechanics of the model. I.e., we assume that the household's stock of liquid assets is sufficient so that liquid assets buffer income shocks and allow the durable stock to return to the steady state after one period through the appropriate durable investment if a loss occurs. The policy functions can then be approximated³ by

$$\begin{aligned}
c &= c_{ss} \\
d &= d_{ss} - (1 - \delta)(v - v_{ss}) + \frac{1}{2}\alpha((1 - \delta)(v - v_{ss}))^2 - \frac{1}{2}\gamma\sigma_l^2 \\
D &= D_{ss} \\
a &= a_{ss} + (1 + r)(a - a_{ss}) - (1 - \delta)(v - v_{ss}) - \frac{1}{2}\alpha((1 - \delta)(v - v_{ss}))^2 + \frac{1}{2}\gamma\sigma_l^2 \\
v &= v_{ss} + \frac{1}{2}\alpha((1 - \delta)(v - v_{ss}))^2 - \frac{1}{2}\gamma\sigma_l^2,
\end{aligned}$$

where

$$\alpha \equiv \frac{u'''(c_{ss})}{\beta\phi w''(v_{ss})}$$

³We derived the approximation for the more general case in which households do not have abundant liquid wealth. However, expressions become very messy so that they do not help much to develop intuition on the mechanics of the model. Hence, we focus on a special, but important case.

and

$$\gamma \equiv \frac{w'''(v_{ss})}{w''(v_{ss})}.$$

The steady state is obtained solving the model without uncertainty. Hence, no market insurance is demanded, i.e., $D_{ss} = 1$. Note that the steady state of liquid wealth a_{ss} is the one which makes the steady state non-durable consumption c_{ss} and durable investment $d_{ss} = \delta v_{ss}$ feasible. We now discuss the solution in detail.

First-order deviations In the scenario considered first-order deviations of a or v from the steady state do not affect non-durable consumption or the durable stock. Shocks occurring to the durable stock are offset after one period (net of the depreciation rate) by durable investment which is fully financed by liquid assets. Shocks occurring to liquid wealth do not result in any reaction of the controls, but only change liquid wealth by the same amount plus the return on liquid assets.

Second-order deviations The second-order deviations of liquid wealth from its steady state do not matter. Instead the second-order deviations of the durable stock do matter for a , d , and v where the absolute value of the size of the effect is the same. Because concavity and precautionary motives imply $u''(c_{ss}) < 0$, $w''(v_{ss}) < 0$ and $u'''(c_{ss}) > 0$, $w'''(v_{ss}) > 0$, it follows that $\alpha < 0$. Hence, the second-order effect decreases durable investment so that the durable stock falls and liquid wealth rises. The denominator of α is a measure of concavity of the utility function for the durable stock which is adjusted for the fact that durable investment today affects utility derived from the durable stock tomorrow. Intuitively, the second-order effect is smaller, the more concave the utility function $w(v_{ss})$, the smaller β and the larger ϕ . In this case the marginal utility derived from the durable stock would increase relatively more if durable investment falls. Instead, the more important the precautionary motive for non-durable consumption, i.e., the larger $u'''(c_{ss})$, the more important is the second-order effect. The link between the durable investment and non-durable consumption results from the budget constraint, i.e., the fact that durable investment potentially “crowds out” non-durable consumption.

Variiances The variance of the durable shock turns out to be important whereas the variance of income does not affect the solution. The asymmetric effect of the variances is resulting from the model's structure that v is predetermined whereas c is not. Since v enters the utility function, the fluctuations of v directly result in variation of utility whereas this is not the case for fluctuations of a . In our scenario there are no effects of deviations of a from its steady state level on non-durable consumption c .

The determinants of the effect of the variance of the loss, γ , depend on the prudence with respect to the durable stock $-w'''(v_{ss})/w''(v_{ss})$ which is defined according to Kimball (1990). Since households derive utility from the durable stock, they invest more into durables if durables are more exposed to risk. Thus, the durable stock rises and liquid wealth falls. This is in contrast to investment behavior for risky assets from which households do not directly derive utility but only indirectly as more assets afford more units of non-durable consumption.

As we pointed out in the introduction, empirically durable consumption is a non-negligible fraction of total consumption, and hence considering the second moment of the distribution of the durable stock might be important although the second moment of the wealth distribution is negligible.⁴ Although there are no costs resulting from volatility of liquid assets in terms of utility in the case of abundant liquid wealth, there remain costs resulting from the volatility of the durable stock.

Having developed some intuition on the mechanics of the model we now present numerical simulations which will tell us whether households indeed accumulate abundant wealth or whether market insurance plays an important role in our model.

⁴In a model with only non-durable consumption Krusell and Smith (1998) have argued that it is sufficient to only consider the first moment of the wealth distribution.

4 Numerical simulations

The numerical solutions are interesting because ex-ante it is not obvious how the accumulation of liquid wealth, adjustment of the durable stock and market insurance interact dynamically. E.g., if households have a lot of liquid assets there is less need to buy market insurance. However, since the durable investment positively depends on liquid assets and durables are exposed to risk also the exposure of households increases. This in turn can feed back into the accumulation of liquid assets and/or the demand for market insurance.

The benchmark parameters used in the numerical simulations are summarized in Table 1. We choose contemporaneous utility functions $u(c) = \frac{c^{1-\sigma_1}}{1-\sigma_1}$ and $w(v) = \frac{(v+\underline{v})^{1-\sigma_2}}{1-\sigma_2}$.⁵

Table 1: Benchmark Parameters

$\sigma_1 = 2$	$E \ln \tilde{y} = 8.8$
$\sigma_2 = 2$	$s_{yt} = .1 * E\tilde{y}$
$\underline{v} = 1$	$g_y = 0$
$\phi = 1$	$s_{yp} = .08$
$r = .03$	$\lambda = .2$
$\beta = .95$	$E\tilde{l}_t = -\lambda(.8v_{t+1})$
$\delta = .2$	$\mu = 1.3$

For the benchmark case we assume that the representative household is equally elastic or risk-averse for utility derived from the non-durable and durable good, i.e., $\sigma_1 = \sigma_2$. Note, that the parametrization of the utility function as CRRA does not allow us to disentangle risk aversion from the elasticity of intertemporal substitution. A risk aversion of 2 is within the interval of values commonly used in the literature. We set $\underline{v} = 1$ so that $v = 0$ is feasible. The parameter \underline{v}

⁵Hence, relative risk aversion is constant for non-durable consumption c . It is increasing for the durable stock because $\underline{v} > 0$: $\partial[-w''(v)v/w'(v)]/\partial v = \sigma_2 [\underline{v}/(v+\underline{v})^2]$. However, we choose \underline{v} to be very small compared with average income and v is always substantially larger than 0 so that for the interpretation of the numerical solution we can treat relative risk aversion as constant for both the durable stock and non-durable consumption.

determines the relative importance of durable compared to non-durable consumption. The smaller \underline{v} , the higher the marginal utility of the durable stock at $v = 0$ and thus the higher the durable investment. The other preference parameter capturing the relative importance of durable versus non-durable consumption –in this case in a linear way, however– is ϕ . In our benchmark case $\phi = 1$, i.e., the only asymmetry between durables and non-durables in the consumer preferences is resulting from $\underline{v} > 0$. The value of $\beta = .95$ is chosen as in Aiyagari (1994) and $r = .03$ is chosen to satisfy $\beta < \frac{1}{1+r}$. As is well known, we need to assume impatience for the steady state of a to be finite in incomplete markets. The depreciation rate $\delta = .2$ is consistent with micro evidence on cars provided by Alessie, Devereux and Weber (1997). The loading factor (mark-up) for insurance premiums μ is 1.3 as reported by Gollier (2000). We assume that the event whether a loss occurs is binomially distributed where we set the probability of a loss to $\lambda = .18$ which is consistent with values reported by the car insurance industry in the US or UK. Once a loss occurs 80% of the durable stock is lost. This has the realistic implication that exposure increases with the size of the durable stock. Moreover, we assume that a substantial fraction of the durable is lost in order to give market insurance a chance to be important in our numerical simulations.

We first solve the model with households obtaining deterministic labor income.⁶ We then consider the case of transitory and permanent income shocks, subsequently. For transitory income we assume a log-normal distribution of income where the mean of log-income for the representative agent is $E \ln \tilde{y} = 8.8$. This corresponds to the mean of log-quarterly-output of the US in 1995 as reported by King and Rebelo (1999), Figure 1, which we then transform to annual income and normalize to 1 in the simulations below.⁷ The standard deviation of transitory income, s_{yt} , is assumed to be 10 percent of average income. Once we consider permanent income shocks we assume that the average growth rate of income, g_y , is 0 and the standard deviation of permanent income,

⁶If the agent does not earn any labor income, numerical solutions which are not reported show that the agent only holds a positive amount of the durable asset in rather special cases. This is not plausible for the representative agent so that we focus on the case where households have positive income, be it deterministic or stochastic.

⁷The absolute value only matters relative to the value assigned to \underline{v} .

s_{yp} , is 8 percent. The standard deviations are in line with evidence reported by Carroll (1992) for the variation of the transitory and permanent component of income in the US. We assume $g_y = 0$ to disentangle the effect of permanent income risk and income growth. Below we discuss the results for the case $g_y = .03$ as well.

4.1 Deterministic labor income

We first discuss the results for the benchmark case before we vary some parameters of interest to further understand the numerical solution. The main finding is that with deterministic labor income our model is only able to match the empirically observed insurance expenditure if agents are substantially risk averse. Moreover, market insurance turns out to be more important than the accumulation of liquid wealth.

The numerical results reported are obtained by simulating the economy for 1000 periods and calculating summary statistics for the last 900 periods. This is done because we do not want initial conditions to matter. Mean labor income is normalized to 1 to facilitate comparison. The borrowing constraint is set to 6.5 times labor income. This is done to decrease the grid size of the state space and affects the result for the resources that agents need to service their debt. Dynamically, agents turn out to be close to the borrowing limit in finite time because we assume them to be impatient.

Benchmark The results for the benchmark case are summarized in Table 2. The behavior of the model is very simple. Liquid assets are at the borrowing limit and increase only when insurance payments occur. Hence, the mean is only slightly above the borrowing limit which we set to -6.494 , i.e., 6.5 times the labor income. When the shock occurs and 80% of the durable stock is lost, the durable stock falls and so does non-durable consumption to finance durable investment. Note that non-durable consumption falls because the agent holds no buffer stock. This is different to the standard model with labor income uncertainty and non-durable consumption in which it is optimal for agents to hold a buffer stock of liquid assets to smooth out fluctuations in income. It results from the fact that in the benchmark case labor income is deterministic and only the durable is exposed

to risk. As we will see below agents accumulate a buffer stock if risk aversion is high. As can be seen in Table 2, the durable stock amounts to 14% of mean labor income. The representative agent spends 5% on durable investment compared with 75% spent on non-durable consumption. Hence, the durable stock in our simulation roughly matches the size of privately-owned manufacturing durable goods in the US which amount to 10% of GDP (see the Introduction). The retention rate D remains nearly constant when a shock occurs. This is because the durable stock quickly returns to its initial value. The retention rate is .65 which is substantially higher than the retention rate in static models (see Drèze (1981)). A retention rate of .65 is slightly higher than in the dynamic analysis of liquid wealth accumulation and market insurance of Gollier (2000) for similar parameter values. He finds that the average retention rate of the representative agent is about .6. If we do not allow for net borrowing, $\underline{a}=0$, as in Gollier (2000), D increases in our model and is substantially larger than .6 (see below and Table 2C). This is intuitive because in our model agents have one more degree of freedom. They are able to adjust the asset at risk, the durable stock. This decreases the motive for market insurance *ceteris paribus*.

The decomposition of expenditure shows that the representative agent spends a fifth of his income to service debt and 1% on market insurance. This is substantially less than the observed non-life insurance expenditure of 8% in OECD countries. The model does better if we consider the expenditure on motor-vehicle and property insurance which is a quarter of the total expenditure, respectively. The latter measure matches better the type of insurable asset we model. Note that agents spend 11% of their exposure on market insurance whereas they do not accumulate any liquid wealth. The result that agents spend slightly more than their labor income (expenditures are slightly larger than 1) stems from the optimal behavior in periods in which the shock occurs. In this period the agent spends not only his labor income but also part of the payment received from the insurance company.

To get further insights on the solution of the model we now vary some of the model's parameters.

Different risk aversion for non-durable consumption and the durable stock The left panel of Table 2A displays the results when we lower σ_1 so that $\sigma_1 = .6$. For CRRA utility σ_1 denotes risk aversion as well as the inverse of the intertemporal elasticity of substitution for non-durable consumption. Hence, we let risk aversion decrease and the intertemporal elasticity of substitution of non-durable consumption increase. If σ_1 is smaller, the marginal utility of non-durable consumption, $c^{-\sigma_1}$, increases relative to the marginal utility derived from the durable stock. Hence, the agent substitutes durable investment with non-durable consumption. For the parameter values chosen the representative agent holds such a small amount of the durable asset that the exposure is negligible. Consequently insurance expenditure is 0, the retention rate is 1 and there is no need to hold a buffer stock of liquid assets. Liquid assets are at the borrowing limit. The labor income which remains after servicing the debt is spent on non-durable consumption.

Results which are not reported show that if we let $\sigma_1 = 2$ and $\sigma_2 = .6$ instead, the agent holds a larger durable stock. This is because the marginal utility derived from the durable increases ceteris paribus. However, this stock is relatively less insured because the durable stock is also more substitutable intertemporally and the agent is less risk averse with respect to the durable stock.

High risk aversion The right panel of Table 2A displays the results when we let risk aversion increase for both the durable stock and non-durable consumption, i.e., $\sigma_1 = \sigma_2 = 10$. The main difference is that the representative agent now holds a buffer stock of liquid assets which amounts to the size of the durable stock at risk. The agent fully insures and invests substantially more into the durable stock. Although ceteris paribus marginal utility decreases for both the durable and non-durable as σ_1 and σ_2 increase, the agent shifts his consumption towards the durable. Both the higher risk aversion and the lower intertemporal elasticity of substitution explain this behavior. Since the agent only loses 80% of the durable stock, a higher level of the durable stock guarantees a higher amount of the durable should a loss occur.

Note that a representative agent with high risk aversion spends 5.6% of his income on insurance which is slightly more than the average OECD insurance expenditure on property and motor

vehicle insurance. The ratio of durable investment over total expenditure is 11% and in line with empirical evidence for the US and UK. However, the fact that the agent chooses a retention rate of 0 is counter to the empirical observation that insurance contracts often do contain a non-zero deductible. Finally, it is interesting to mention that non-durable consumption *increases* in the period in which the shock occurs whereas it decreases in the benchmark case. This is because the buffer stock of liquid assets allows the agent to smooth utility in periods in which the durable stock is low by extra non-durable consumption. This is particularly beneficial because compared with the benchmark the value function is relatively more concave.

Impatience The left panel of Table 2B displays the results if we decrease the discount factor from .95 to .9. In this case the representative agent spends less on insurance compared to the benchmark case. The retention rate rises to .75. This is because impatience increases the disutility of foregoing one unit of consumption today because of insurance expenditure. The durable stock decreases slightly and the agent consumes more of the non-durable in “good” times, but has to consume less when a shock occurs because he is at the borrowing limit and less insured. The standard deviation of non-durable consumption increases compared with the benchmark case. The same holds if we compare the respective coefficients of variation which is not surprising since the mean is almost the same.

Loss probability The middle panel in Table 2B displays the results if we increase the loss probability λ from .18 to .4 whereas the right panel displays simulation results for $\lambda = .05$. The higher λ , the higher the retention rate chosen by the agent. This is because one unit of insurance becomes relatively more expensive and insurance is not actuarially fair. However, insurance expenditure increases if λ does because the retention rate does not increase enough to offset the increase in premium cost. Hence, less of the labor income remains for the representative agent to consume the non-durable and durable good if λ is high. The mean of the durable stock is not only smaller because shocks occur at a higher frequency, but also because the level of the durable in “good”

times is smaller.

Tighter borrowing constraint Table 2C displays the results if we do not allow net borrowing, i.e., $\underline{a}=0$. One might expect that a tighter borrowing constraint does increase the motive for market insurance. The opposite is the case, however, in the steady state. The retention rate is 1, i.e., the agent buys no market insurance at all. The intuition is the following. We assumed the agent to be impatient so that liquid assets will be close to the borrowing limit in finite time. Once the softer borrowing limit is binding the agent has to spend more of his income on servicing the debt compared to the case with a tighter borrowing constraint. This crowds out non-durable consumption and investment into the durable. Thus, marginal utility is higher for both, the durable stock and non-durable consumption, in the case when the soft borrowing constraint becomes binding and the value function is relatively more concave. Hence, a loss of part of the durable stock hurts relatively more so that the agent increases market insurance. Note that, of course, the agent is better off with a softer borrowing constraint because he can consume more on the transition path until the borrowing constraint becomes binding.

We have illustrated the numerical solution of the model with deterministic labor income. Interestingly, the solution for agents with a fraction κ of mean labor income has simply to be scaled down by κ .⁸ This is because exposure is endogenous and the agent can adjust the exposure associated with the durable to a fraction κ . Hence, aggregation is simple in our modeling framework. Results which are not reported confirm, however, that as soon as agents cannot adjust their exposure proportionally, low income households spend relatively more on market insurance expenditure, as is intuitive.

Our results are related to the static analysis of Eeckhoudt et al. (1997), Theorem 4, which states that if the utility function features decreasing absolute risk aversion –which is the case in our simulations– either investment in the risky asset is increasing in wealth or the retention rate

⁸Note that we also scale down the borrowing constraint by proportion κ . This is true endogenously if the borrowing constraint equals the solvency constraint.

is increasing in wealth or both.⁹ In our simulations we find that on the transition path durable investment and the retention rate indeed increase for higher levels of liquid wealth.

We find that in our benchmark model the welfare gains derived from the existence of market insurance are similar to those found by Gollier (2000). In his model insurance increases welfare by about .7% in terms of non-durable consumption. To assess the welfare implications within our model, we proceed as in Gollier (2000). We compare certainty-equivalent non-durable consumption c^* for the case in which agents can insure with the case in which they cannot. We evaluate the value function at the sample mean of the liquid assets, \bar{a} , and the durable stock, \bar{v} , of the simulations. From (1) it follows that in the steady state under certainty

$$V(\bar{a}, \bar{v}) = \frac{(c^*)^{1-\sigma_1}}{1-\sigma_1} + \phi \frac{(\bar{v} - \underline{v})^{1-\sigma_2}}{1-\sigma_2} + \beta V(\bar{a}, \bar{v})$$

and we get

$$c^* = \left\{ (1 - \sigma_1) \left[(1 - \beta)V(\bar{a}, \bar{v}) - \phi \frac{(\bar{v} - \underline{v})^{1-\sigma_2}}{1 - \sigma_2} \right] \right\}^{\frac{1}{1-\sigma_1}}.$$

For the representative agent the welfare gains are .6% in terms of non-durable consumption. It is not surprising that the welfare gains are close to those in Gollier (2000) given that the retention rate is of the same order of magnitude.

4.2 Stochastic labor income

Unless we assume that risk aversion is high, a representative-agent economy cannot explain why market insurance is as important in OECD countries as the empirical evidence shows. In our benchmark model with deterministic income and commonly assumed levels of risk aversion the representative agent spends only 1% of his income on insurance and the retention rate is substantially higher than observed empirically. The fact that commonly assumed levels of risk aversion do not result in realistic retention rates is also found in static models (see Drèze (1981)), but becomes more important quantitatively in our dynamic model. This is intuitive because dynamically agents can time-diversify their risk by accumulating wealth and adjusting the asset at risk.

⁹Note that $1 - D$ is denoted with θ in their notation.

Obviously, assuming deterministic labor income is quite unrealistic. Hence, a natural extension is to make labor income stochastic in order to investigate whether market insurance becomes relatively more important. E.g., Gollier (2001) derives conditions which need to be imposed on the utility function so that labor income risk has a tempering effect for the decision how much other risks to bear in a standard non-durable consumption model. The interesting question is whether such an effect is present in our framework and how large this effect is quantitatively. We consider two cases: stochastic transitory and stochastic permanent income.

4.2.1 Transitory income shocks

We report results for the representative agent in Table 3. The standard deviation of labor income is set to 10% of mean income which is in line with empirical evidence provided by Carroll (1992). At the same time we assume a tighter borrowing constraint, i.e., agents cannot borrow at all so that $a \geq 0$. This captures that the solvency constraint will become relatively more binding endogenously if labor income is stochastic since it depends on the minimum of the support of the income distribution (see above). We are able to disentangle the effect of the tighter borrowing constraint from the direct effect of labor income risk using the results of the benchmark case (Table 2) and the results for the tighter borrowing constraint (Table 2C). Comparing the results with the latter identifies the direct effect of labor income risk whereas comparison with the benchmark case gives us the composite effect.

Comparing the left panel of Table 3 with Table 2C we observe that labor income risk increases the motive for market insurance keeping the borrowing constraint constant. The retention rate falls from 1 to .83. Moreover, transitory labor income risk creates an endogenous incentive to accumulate buffer-stock wealth which can be used to smooth fluctuations of the durable stock as well. The agent holds 1.3 times his exposure related to the durable stock as liquid wealth. Note that with labor income risk fluctuations in non-durable consumption increase. The standard deviation doubles (and so does the coefficient of variation which is not reported). Instead, average consumption and the durable stock remain roughly the same. On the one hand the agent spends

some of his income on insurance, but on the other hand he earns interest on the buffer stock wealth.

Comparing the results to the benchmark case (Table 2) we observe that the direct effect of labor income risk on market insurance is outweighed by its indirect effect on the borrowing constraint. The retention rate of .83 is higher than .65 in the benchmark case. Thus, whereas statically one would expect a tighter borrowing constraint and more labor income risk to increase the utility derived from insurance because the agent becomes more vulnerable to fluctuations in the durable stock, this is not necessarily the case in a dynamic framework.

4.2.2 Permanent income shocks

Introducing permanent income does not increase the state space because we can exploit the homogeneity of the value function. This is well known (see, e.g., Haliassos and Michaelides (2002)) and a simple proof is provided in the appendix. With permanent income the following condition has to be satisfied for liquid wealth to be finite in the steady state (see, e.g., Deaton (1991) or Haliassos and Michaelides (2002)):

$$\frac{r - \rho}{\sigma} + \frac{\sigma (s_{yp})^2}{2} < g_y,$$

where ρ is the discount rate, $\sigma = \sigma_1 = \sigma_2$ and the other parameters are defined as in Table 1 above. Again $a \geq 0$. In order to disentangle the effect of uncertainty about permanent income and income growth we introduce permanent income shocks and set $g_y = 0$.

The results of the simulation are displayed in the right panel of Table 3. Compared to uncertain transitory income the accumulation of liquid wealth becomes less important whereas the motive for market insurance is stronger. The retention rate is .68, i.e., about the same as in the benchmark case. Permanent income shocks make the agent more vulnerable with respect to insurable risk because the shocks cannot be mitigated by accumulating liquid wealth. With permanent income shocks, consumption fully adjusts so that there is less need to accumulate a buffer stock. *Ceteris paribus* one would expect non-durable consumption to be more variable with permanent income shocks. However, permanent income uncertainty induces substantially more market insurance of the

durable stock so that the coefficient of variation of non-durable consumption (which is not reported) is smaller than in the case with transitory labor income shocks. Given the higher expenditure on market insurance and the smaller amount of positive assets, the mean of consumption of the non-durable and durable falls slightly compared with the case of transitory labor income shocks.

Hence, neither transitory nor permanent income uncertainty as such do increase the benefit of market insurance as long as the borrowing constraint tightens endogenously. We also calculated the numerical solution for the case in which $g_y = .03$. Since income growth makes the agent relatively more impatient, one would expect effects similar to the case of high impatience displayed in the left panel of Table 2B. Indeed, results which are not reported confirm that the motive for market insurance decreases substantially.

5 Conclusion and further research

We analyze the interplay of accumulation of liquid wealth, the size of the insurable asset and market insurance in a dynamic model. In our model the durable stock and in some simulations also labor income are uncertain. For the case of deterministic labor income we find that market insurance matters for the representative agent. This is inconsistent with approximate linearity of the value function. Unless we assume high risk aversion, insurance expenditure is smaller and the retention rate larger than observed empirically. Introducing labor income risk into the model is per se no remedy for this finding. This is because the borrowing constraint endogenously tightens. For a given borrowing constraint, we find that labor income risk strengthens the motive for market insurance and this relatively more if labor income shocks are permanent. For a realistic parametrization of the income processes, however, this effect is quantitatively too small to generate insurance expenditure which is in line with empirical evidence.

To increase the importance of market insurance the following modifications of the model seem promising. The assumption of perfect divisibility of the durable good is clearly a big simplification. Alternatively, one could consider the case that v cannot fall below a positive value $v^{\min} > 0$ unless it

is 0. This assumption is more realistic because, e.g., one has to invest a substantial amount to buy any reasonable house or car. With this indivisibility, it is optimal for some relatively poor agents to hold a durable stock which is larger than the optimal amount chosen under perfect divisibility. As mentioned above this increases the importance of market insurance since the durable asset cannot be adjusted downward as desired. Adding adjustment costs to the model instead will not necessarily deliver this result because the durable stock of some household will be larger than desired and smaller than desired for others. The aggregate implication is unclear. Another more promising alternative would be to assume a finite horizon. This is likely to increase insurance expenditure as time diversification becomes less perfect especially for older agents. Gollier (2000) provides some first results consistent with this conjecture. He finds that the retention rate is smaller if the time horizon is relatively shorter. Finally, we have assumed that the insurable risk is i.i.d. The results for permanent or transitory background risk suggest that permanent or highly autocorrelated shocks for the insurable risk will increase the scope for market insurance *ceteris paribus*. However, such shocks seem more realistic in the context of health than in the context of durables such as motor vehicles or property.

Another interpretation of our results is that institutional constraints force households to insure far more than would be optimal given that they can partly time diversify their risk. Casual observations suggest that this might indeed be the case because a substantial part of insurance expenditure is obligatory in OECD countries.

Our model shows that if households derive utility directly from the durable stock, utility costs of fluctuations might not be small even if risk-averse households accumulate substantial wealth dynamically. This is in contrast with standard models of non-durable consumption under uncertainty (see Krusell and Smith (1998) and their references). Furthermore, instead of assuming heterogeneity in preferences as in Krusell and Smith (1998), incorporating durables into an otherwise standard macro-model has the potential to generate substantial wealth heterogeneity because households do not fully insure the durable stock.

In our model fluctuations in non-durable consumption are an optimal response to fluctuations

in the durable stock. The results are thus in the spirit of the real-business-cycle literature where consumption volatility arises, e.g., because of technology shocks. It would be interesting to explore the business-cycle implications of our model further in future research.

Appendix

A Numerical solution

We solve the program numerically using standard discrete state-space methods (see, e.g., Burnside (1999)). We first compute the value function for every state of a and v conditional on the state of the random variables. I.e., for the model without labor income risk we compute the value function for the case when a loss occurs and for the state when no loss occurs. In the case with labor income risk we have a grid of five states, i.e., the mean and ± 2 standard deviations of labor income. The conditional value functions are then used to calculate the unconditional value function. We also need to calculate the conditional value function over the grid of retention rates because the amount of insurance influences the amount of the loss, the shape of the conditional value function and hence also the unconditional value function. We choose a grid size of 21 points for the retention rate and the state variables. This results in $21 \times 21 \times 21 = 9261$ grid points for the case of deterministic income and $9261 \times 5 = 46305$ grid points for the case of stochastic labor income. The fineness of the grid was chosen to obtain solutions in finite time. To account for optimal choices not on the grid we interpolate the value function once per iteration using cubic splines which make the grid three times finer. Cubic splines preserve the prudence feature of the utility function, i.e., a non-zero third derivative. For the simulations of the results we increase the fineness of the grid to $64 \times 21 = 1344$ points per state variable using cubic splines.

B Solution for the analytic approximation

Consistent with the notation used in Schmitt-Grohé and Uribe (2001) we define the matrix F as

$$F \equiv \begin{bmatrix} u'(c_t) - \beta(1+r) E_t u'(c_{t+1}) \\ u'(c_t) - \beta[(1-\delta)E_t u'(c_{t+1}) + \phi E_t w'(v_{t+1})] \\ \beta E_t \{u'(c_{t+1}) \tilde{l}_{t+1}\} - u'(c_t) \mu_t E_t \tilde{l}_t \\ a_{t+1} - (1+r)a_t + c_t + d_t + \mu(1-D_t)E_t \tilde{l}_t - (1-D_{t-1})\tilde{l}_t - \tilde{y}_t \\ v_{t+1} - (1-\delta)v_t - d_t + \tilde{l}_t \end{bmatrix},$$

where $F = 0$. We define the controls as $\zeta = (c, d, D)'$ and the state variables as $x = (a, v)'$. The shocks can be rewritten as

$$\tilde{l} = m_l + \sigma_l \varepsilon_l$$

and

$$\tilde{y} = m_y + \sigma_y \varepsilon_y,$$

where $\varepsilon_i \sim N(0, 1)$, $i = l, y$ and $m_y = E\tilde{y}$, $m_l = E\tilde{l}$. The shocks are assumed to be i.i.d.

We know that the solution will take the form $\zeta_t = g(x'_t, \sigma')$ and $x_{t+1} = h(x'_t, \sigma') + \eta \varepsilon_t$, where $\varepsilon_t = (\varepsilon_{lt}, \varepsilon_{yt})'$. The 2x2 matrix η and $\sigma = (\sigma_y, \sigma_l)'$ are known. In our model

$$\eta = \begin{bmatrix} (1-D)\sigma_l & \sigma_y \\ -\sigma_l & 0 \end{bmatrix}.$$

Note that \tilde{y}_t and \tilde{l}_t are i.i.d. distributed shocks. To perform a second-order approximation, first and second derivatives of the functions $g(\cdot)$ and $h(\cdot)$ need to be determined. As explained in more detail in Schmitt-Grohé and Uribe (2001) this is done by taking first and second derivatives of F with respect to x and exploiting the fact that these derivatives are 0.

We find that

$$F_x = \begin{bmatrix} -\beta(1+r)E_t u''(c_{t+1}) [g_a^c h_a^a + g_v^c h_a^v] + u''(c_t) g_a^c \\ -\beta(1-\delta)E_t u''(c_{t+1}) [g_a^c h_a^a + g_v^c h_a^v] + u''(c_t) g_a^c - \beta \phi E_t w''(v_{t+1}) h_a^v \\ \frac{\partial F(3,1)}{\partial c_{t+1}} [g_a^c h_a^a + g_v^c h_a^v] - \mu E_t \tilde{l}_t u''(c_t) g_a^c + \frac{\partial F(3,1)}{\partial D_t} g_a^D \\ g_a^c + g_a^d - \mu E_t \tilde{l}_t g_a^D + h_a^a - (1+r) \\ -g_a^d + h_a^v \\ -\beta(1+r)E_t u''(c_{t+1}) [g_a^c h_v^a + g_v^c h_v^v] + u''(c_t) g_v^c \\ -\beta(1-\delta)E_t u''(c_{t+1}) [g_a^c h_v^a + g_v^c h_v^v] + u''(c_t) g_v^c - \beta \phi E_t w''(v_{t+1}) h_v^v \\ \frac{\partial F(3,1)}{\partial D_{t+1}} [g_a^c h_v^a + g_v^c h_v^v] - \mu E_t \tilde{l}_t u''(c_t) g_v^c + \frac{\partial F(3,1)}{\partial D_t} g_v^D \\ g_v^c + g_v^d - \mu E_t \tilde{l}_t g_v^D + h_v^a \\ -g_v^d + h_v^v - (1-\delta) \end{bmatrix},$$

where derivatives of F with respect to a are in rows 1-5 and derivatives with respect to v are in rows 6-10. The notation g_a^c denotes the derivative of non-durable consumption c with respect to a and $F(3,1)$ is the element in the third row and first column of F . Expressions for $\frac{\partial F(3,1)}{\partial c_{t+1}}$ and $\frac{\partial F(3,1)}{\partial D_t}$ can be derived using the fact that $Eab = EaEb + cov(a, b)$, but are lengthy so that we use shorthand notation. F_x is a system of 10 equations in 10 unknowns g_j^i, h_j^k with $i = c, d, D; j = a, v$ and $k = a, v$.

It turns out that one solution of the system of equations is

$$\begin{aligned} g_a^c &= g_v^c = g_a^d = g_a^D = g_v^D = h_a^v = h_v^v = 0, \\ h_a^a &= 1+r, \\ g_v^d &= -(1-\delta), \\ h_v^a &= 1-\delta. \end{aligned}$$

This is the case of abundant liquid wealth when $D = D_{ss}$ and $c = c_{ss}$. The solution for the general case is messy and does not add to the intuition.

We calculate F_{xx} which gives us 20 equations to determine 20 second-order derivatives. Given the results for the case of abundant liquid wealth for the first derivatives, we get that

$$F_{xa} = \left[\begin{array}{c} -\beta(1+r)E_t u''(c_{t+1})g_{aa}^c (h_a^a)^2 + u''(c_t)g_{aa}^c = 0 \\ -\beta(1-\delta)E_t u''(c_{t+1})g_{aa}^c (h_a^a)^2 + u''(c_t)g_{aa}^c - \beta\phi E_t w''(v_{t+1})h_{aa}^v = 0 \\ \frac{\partial F(3,1)}{\partial c_{t+1}} g_{aa}^c (h_a^a)^2 - \mu E_t \tilde{l}_t u''(c_t)g_{aa}^c + \frac{\partial F(3,1)}{\partial D_t} g_{aa}^D = 0 \\ g_{aa}^c + g_{aa}^d - \mu E_t \tilde{l}_t g_{aa}^D + h_{aa}^a = 0 \\ -g_{aa}^d + h_{aa}^v = 0 \\ -\beta(1+r)E_t u''(c_{t+1})g_{aa}^c h_a^a h_v^a + u''(c_t)g_{va}^c = 0 \\ -\beta(1-\delta)E_t u''(c_{t+1})g_{aa}^c h_a^a h_v^a + u''(c_t)g_{va}^c - \beta\phi E_t w''(v_{t+1})h_{va}^v = 0 \\ \frac{\partial F(3,1)}{\partial c_{t+1}} g_{aa}^c h_a^a h_v^a - \mu E_t \tilde{l}_t u''(c_t)g_{va}^c + \frac{\partial F(3,1)}{\partial D_t} g_{va}^D = 0 \\ g_{va}^c + g_{va}^d - \mu E_t \tilde{l}_t g_{va}^D + h_{va}^a = 0 \\ -g_{va}^d + h_{va}^v = 0 \end{array} \right]$$

for the derivatives with respect to aa in rows 1-5 and derivatives with respect to va in rows 6-10 and

$$F_{xv} = \left[\begin{array}{c} -\beta(1+r)E_t u''(c_{t+1})g_{av}^c (h_v^a)^2 + u''(c_t)g_{vv}^c \\ -\beta(1-\delta)E_t u''(c_{t+1})g_{av}^c (h_v^a)^2 + u'''(c_t)(g_v^d)^2 + u''(c_t)g_{vv}^c - \beta\phi E_t w''(v_{t+1})h_{vv}^v \\ \frac{\partial F(3,1)}{\partial c_{t+1}} g_{av}^c (h_v^a)^2 - \mu E_t \tilde{l}_t u''(c_t)g_{vv}^c + \frac{\partial F(3,1)}{\partial D_t} g_{vv}^D \\ g_{vv}^c + g_{vv}^d - \mu E_t \tilde{l}_t g_{vv}^D + h_{vv}^a \\ -g_{vv}^d + h_{vv}^v \\ -\beta(1+r)E_t u''(c_{t+1})g_{av}^c h_v^a h_a^a + u''(c_t)g_{av}^c \\ -\beta(1-\delta)E_t u''(c_{t+1})g_{av}^c h_v^a h_a^a + u''(c_t)g_{av}^c - \beta\phi E_t w''(v_{t+1})h_{av}^v \\ \frac{\partial F(3,1)}{\partial c_{t+1}} g_{av}^c h_v^a h_a^a - \mu E_t \tilde{l}_t u''(c_t)g_{av}^c + \frac{\partial F(3,1)}{\partial D_t} g_{av}^D \\ g_{av}^c + g_{av}^d - \mu E_t \tilde{l}_t g_{av}^D + h_{av}^a \\ -g_{av}^d + h_{av}^v \end{array} \right]$$

for the derivatives with respect to vv in rows 1-5 and derivatives with respect to av in rows 6-10. Substituting in the results for the first derivatives derived above, it follows from $F_{xa} = F_{xv} = 0$ that

$$g_{vv}^d = h_{vv}^v = -h_{vv}^a = (1-\delta)^2 \frac{u'''(c_{ss})}{\beta\phi w''(v_{ss})},$$

where the other second-order derivatives are found to be 0.

For F_{σ_i} , $i = l, y$, we get using the results for the first derivatives g_j^i, h_j^k with $i = c, d, D$; $j = a, v$ and $k = a, v$,

$$F_{\sigma_l} = \begin{bmatrix} -\beta(1+r)E_t u''(c_{t+1})g_{\sigma_l}^c + u''(c_t)g_{\sigma_l}^c \\ -\beta(1-\delta)E_t u''(c_{t+1})g_{\sigma_l}^c + u''(c_t)g_{\sigma_l}^c - \beta\phi E_t w''(v_{t+1}) [h_{\sigma_l}^v + E_t \varepsilon_l] \\ \frac{\partial F(3,1)}{\partial c_{t+1}} g_{\sigma_l}^c - \mu E_t \tilde{l}_t u''(c_t)g_{\sigma_l}^c + \frac{\partial F(3,1)}{\partial D_t} g_{\sigma_l}^D \\ g_{\sigma_l}^c + g_{\sigma_l}^d - \mu E_t \tilde{l}_t g_{\sigma_l}^D + h_{\sigma_l}^a - (1-D_t)E_t \varepsilon_l \\ -g_{\sigma_l}^d + h_{\sigma_l}^v + E_t \varepsilon_l \end{bmatrix}$$

and

$$F_{\sigma_y} = \begin{bmatrix} -\beta(1+r)E_t u''(c_{t+1})g_{\sigma_y}^c + u''(c_t)g_{\sigma_y}^c \\ -\beta(1-\delta)E_t u''(c_{t+1})g_{\sigma_y}^c + u''(c_t)g_{\sigma_y}^c - \beta\phi E_t w''(v_{t+1})h_{\sigma_y}^v \\ \frac{\partial F(3,1)}{\partial c_{t+1}} g_{\sigma_y}^c - \mu E_t \tilde{l}_t u''(c_t)g_{\sigma_y}^c + \frac{\partial F(3,1)}{\partial D_t} g_{\sigma_y}^D \\ g_{\sigma_y}^c + g_{\sigma_y}^d - \mu E_t \tilde{l}_t g_{\sigma_y}^D + h_{\sigma_y}^a - E_t \varepsilon_y \\ -g_{\sigma_y}^d + h_{\sigma_y}^v \end{bmatrix}.$$

Since $E_t \varepsilon_l = E_t \varepsilon_y = 0$, F_{σ_l} and F_{σ_y} are linear and homogenous in $g_{\sigma_l}^j$ and $h_{\sigma_l}^k$, for the unique solution of $F_{\sigma_l} = F_{\sigma_y} = 0$ it follows that $h_{\sigma_l}^k = 0$ and $g_{\sigma_l}^j = 0$ with $j = c, d, D$; $i = l, y$ and $k = a, v$.

For the second-order derivatives we get

$$F_{\sigma_l \sigma_l} = \begin{bmatrix} -\beta(1+r)E_t u''(c_{t+1})g_{\sigma_l \sigma_l}^c + u''(c_t)g_{\sigma_l \sigma_l}^c \\ u''(c_t)g_{\sigma_l \sigma_l}^c - \beta [\phi E_t w'''(v_{t+1})E_t (\varepsilon_l^2) + (1-\delta)E_t u''(c_{t+1})g_{\sigma_l \sigma_l}^c + \phi E_t w''(v_{t+1})h_{\sigma_l \sigma_l}^v] \\ \frac{\partial F(3,1)}{\partial c_{t+1}} g_{\sigma_l \sigma_l}^c - \mu E_t \tilde{l}_t u''(c_t)g_{\sigma_l \sigma_l}^c + \frac{\partial F(3,1)}{\partial D_t} g_{\sigma_l \sigma_l}^D \\ g_{\sigma_l \sigma_l}^c + g_{\sigma_l \sigma_l}^d - \mu E_t \tilde{l}_t g_{\sigma_l \sigma_l}^D + h_{\sigma_l \sigma_l}^a \\ -g_{\sigma_l \sigma_l}^d + h_{\sigma_l \sigma_l}^v \end{bmatrix}$$

and

$$F_{\sigma_y \sigma_y} = \begin{bmatrix} -\beta(1+r)E_t u''(c_{t+1})g_{\sigma_y \sigma_y}^c + u''(c_t)g_{\sigma_y \sigma_y}^c \\ u''(c_t)g_{\sigma_y \sigma_y}^c - \beta \left[(1-\delta)E_t u''(c_{t+1})g_{\sigma_y \sigma_y}^c + \phi E_t w''(v_{t+1})h_{\sigma_y \sigma_y}^v \right] \\ \frac{\partial F(3,1)}{\partial c_{t+1}} g_{\sigma_y \sigma_y}^c - \mu E_t \tilde{l}_t u''(c_t)g_{\sigma_y \sigma_y}^c + \frac{\partial F(3,1)}{\partial D_t} g_{\sigma_y \sigma_y}^D \\ g_{\sigma_y \sigma_y}^c + g_{\sigma_y \sigma_y}^d - \mu E_t \tilde{l}_t g_{\sigma_y \sigma_y}^D + h_{\sigma_y \sigma_y}^a \\ -g_{\sigma_y \sigma_y}^d + h_{\sigma_y \sigma_y}^v \end{bmatrix} .$$

$F_{\sigma_y \sigma_y}$ is linear and homogenous in $g_{\sigma_y \sigma_y}^j$ and $h_{\sigma_y \sigma_y}^k$, $g_{\sigma_y \sigma_y}^j = h_{\sigma_y \sigma_y}^k = 0$. The same holds for $F_{\sigma_y \sigma_l}$ because $E_t \varepsilon_l \varepsilon_y = 0$; and for $F_{x \sigma_i}$, $i = l, y$. Instead given that $E_t \varepsilon_l^2 = 1$, $F_{\sigma_l \sigma_l} = 0$ implies

$$h_{\sigma_l \sigma_l}^a = -h_{\sigma_l \sigma_l}^v = -g_{\sigma_l \sigma_l}^d = \frac{w'''(v_{ss})}{w''(v_{ss})} ,$$

where the other second-order derivatives are found to be 0.

C Homogeneity of the value function

In this appendix we prove that the value function is homogenous of degree $1 - \sigma$. We assume that $\sigma_1 = \sigma_2$ so that

$$V(a, v) - \beta EV(a, v) = \frac{c^{1-\sigma}}{1-\sigma} + \phi \frac{(v - \underline{v})^{1-\sigma}}{1-\sigma} .$$

Hence,

$$F(a, v) \equiv V(a, v) - \beta EV(a, v)$$

is homogenous of degree $1 - \sigma$ because we assume that \underline{v} is defined in relative terms to permanent income. It remains to be shown that

$$F(a, v) \text{ homogenous of degree } \kappa \implies V(a, v) \text{ homogenous of degree } \kappa .$$

We prove this by contradiction. Assume that $V(a, v)$ is not homogenous of degree κ . It then follows that

$$V(\kappa a, \kappa v) = \kappa V(a, v) + \eta ,$$

where η is a constant which makes the equality hold. Assuming that the expected value is well defined we get

$$\begin{aligned} F(\kappa a, \kappa v) &= V(\kappa a, \kappa v) - \beta EV(\kappa a, \kappa v) \\ &= \kappa V(a, v) + \eta - \beta E(\kappa V(a, v) + \eta) \\ &= \kappa (V(a, v) - \beta EV(a, v)) + (1 - \beta)\eta \\ &= \kappa F(a, v), \text{ iff } \eta = 0 \text{ for } \beta \neq 1 . \end{aligned}$$

For $\beta < 1$, $\eta = 0$ for $F(a, v)$ to be homogenous of degree κ . If $\eta = 0$, then also $V(a, v)$ is homogenous of degree κ . QED

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Table 2: Simulation statistics for the benchmark case with deterministic income

Summary Statistics

<i>Variable</i>	<i>Nobs</i>	<i>Mean</i>	<i>Stddev</i>
<i>liquid assets: a</i>	900	-6.486	0.018
<i>durable stock: v</i>	900	0.142	0.052
<i>non-durable consumption: c</i>	900	0.756	0.023
<i>durable investment: d</i>	900	0.054	0.047
<i>retention rate: D</i>	900	0.649	0.002

Decomposition of Expenditure

<i>non-durable consumption</i>	0.756
<i>durable investment</i>	0.054
<i>insurance expenditure</i>	0.009
<i>insurance pay</i>	-0.007
<i>debt interest payments</i>	0.195
	1.006
<i>savings</i>	-0.006
<i>income</i>	<u>1.000</u>
<i>borrowing limit</i>	-6.494
<i>liquid wealth/exposure</i>	0.000
<i>insurance/exposure</i>	0.228

Table 2A: Risk aversion

Low risk aversion for non-durable consumption

High risk aversion for non-durable consumption and the durable stock

Summary Statistics

Summary Statistics

<i>Variable</i>	<i>Nobs</i>	<i>Mean</i>	<i>Stddev</i>
<i>liquid assets: a</i>	900	-6.494	0.001
<i>durable stock: v</i>	900	0.001	0.000
<i>non-durable consumption: c</i>	900	0.805	0.001
<i>durable investment: d</i>	900	0.000	0.000
<i>retention rate: D</i>	900	1.000	0.000

<i>Nobs</i>	<i>Mean</i>	<i>Stddev</i>
900	-6.168	0.117
900	0.292	0.109
900	0.692	0.060
900	0.111	0.096
900	0.000	0.000

Decomposition of Expenditure

Decomposition of Expenditure

<i>non-durable consumption</i>	0.805
<i>durable investment</i>	0.000
<i>insurance expenditure</i>	0.000
<i>insurance pay</i>	0.000
<i>debt interest payments</i>	0.195
	1.000
<i>savings</i>	0.000
<i>income</i>	<u>1.000</u>

0.692
0.111
0.056
-0.043
0.185
1.000
<u>0.000</u>
<u>1.000</u>

<i>borrowing limit</i>	-6.494
<i>liquid wealth/exposure</i>	-
<i>insurance/exposure</i>	-

-6.494
1.026
0.650

Table 2B: Impatience and loss probability

	High impatience			High loss probability			Low loss probability		
	Summary Statistics			Summary Statistics			Summary Statistics		
<i>Variable</i>	<i>Nobs</i>	<i>Mean</i>	<i>Stddev</i>	<i>Nobs</i>	<i>Mean</i>	<i>Stddev</i>	<i>Nobs</i>	<i>Mean</i>	<i>Stddev</i>
<i>liquid assets: a</i>	900	-6.488	0.013	900	-6.482	0.013	900	-6.491	0.014
<i>durable stock: v</i>	900	0.137	0.052	900	0.099	0.056	900	0.158	0.027
<i>non-durable consumption: c</i>	900	0.754	0.035	900	0.745	0.019	900	0.767	0.010
<i>durable investment: d</i>	900	0.053	0.051	900	0.064	0.042	900	0.037	0.025
<i>retention rate: D</i>	900	0.752	0.007	900	0.774	0.016	900	0.493	0.006
	Decomposition of Expenditure			Decomposition of Expenditure			Decomposition of Expenditure		
<i>non-durable consumption</i>		0.754			0.745			0.767	
<i>durable investment</i>		0.053			0.064			0.037	
<i>insurance expenditure</i>		0.007			0.008			0.004	
<i>insurance pay</i>		-0.005			-0.006			-0.003	
<i>debt interest payments</i>		0.195			0.194			0.195	
		1.003			1.006			1.001	
<i>savings</i>		-0.003			-0.006			-0.001	
<i>income</i>		<u>1.000</u>			<u>1.000</u>			<u>1.000</u>	
<i>borrowing limit</i>		6.494			6.494			6.494	
<i>liquid wealth/exposure</i>		0.000			0.000			0.000	
<i>insurance/exposure</i>		0.161			0.147			0.033	

Table 2C: Tighter borrowing constraint

Summary Statistics

<i>Variable</i>	<i>Nobs</i>	<i>Mean</i>	<i>Stddev</i>
<i>liquid assets: a</i>	900	0.002	0.002
<i>durable stock: v</i>	900	0.177	0.067
<i>non-durable consumption: c</i>	900	0.931	0.048
<i>durable investment: d</i>	900	0.069	0.049
<i>retention rate: D</i>	900	1.000	0.000

Decomposition of Expenditure

<i>non-durable consumption</i>	0.931
<i>durable investment</i>	0.069
<i>insurance expenditure</i>	0.000
<i>insurance pay</i>	0.000
<i>debt interest payments</i>	0.000
	1.000
<i>savings</i>	0.000
<i>income</i>	<u>1.000</u>
<i>borrowing limit</i>	0.000
<i>liquid wealth/exposure</i>	0.014
<i>insurance/exposure</i>	0.000

Table 3: Simulation statistics for uncertain labor income

	Uncertain transitory income			Uncertain permanent income		
	Summary Statistics			Summary Statistics		
<i>Variable</i>	<i>Nobs</i>	<i>Mean</i>	<i>Stddev</i>	<i>Nobs</i>	<i>Mean</i>	<i>Stddev</i>
<i>liquid assets: a</i>	900	0.223	0.113	900	0.027	0.020
<i>durable stock: v</i>	900	0.173	0.065	900	0.163	0.057
<i>non-durable consumption: c</i>	900	0.944	0.104	900	0.940	0.030
<i>durable investment: d</i>	900	0.064	0.049	900	0.058	0.053
<i>retention rate: D</i>	900	0.825	0.093	900	0.680	0.016
	Decomposition of Expenditure			Decomposition of Expenditure		
<i>non-durable consumption</i>		0.944			0.940	
<i>durable investment</i>		0.064			0.058	
<i>insurance expenditure</i>		0.006			0.009	
<i>insurance pay</i>		-0.004			-0.007	
<i>debt interest payments</i>		-0.007			-0.001	
		1.003			0.999	
<i>savings</i>		-0.003			0.001	
<i>income</i>		<u>1.000</u>			<u>1.000</u>	
<i>borrowing limit</i>		0.000			0.000	
<i>liquid wealth/exposure</i>		1.286			0.134	
<i>insurance/exposure</i>		0.045			0.083	

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