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Wolfgang Höchtl
Rudolf Kerschbamer
Rudi Stracke
Uwe Sunde

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Forschungsinstitut
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Institute for the Study
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Wolfgang Höchtl
University of Innsbruck

Rudolf Kerschbamer
University of Innsbruck

Rudi Stracke
University of St. Gallen

Uwe Sunde
University of St. Gallen,
CEPR and IZA

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IZA

P.O. Box 7240
53072 Bonn
Germany

Phone: +49-228-3894-0
Fax: +49-228-3894-180
E-mail: iza@iza.org

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ABSTRACT

Incentives vs. Selection in Promotion Tournaments: Can a Designer Kill Two Birds with One Stone?*

This paper studies the performance of promotion tournaments with heterogeneous participants in two dimensions: incentive provision and selection. Our theoretical analysis reveals a trade-off for the tournament designer between the two goals: While total effort is maximized if less heterogeneous participants compete against each other early in the tournament, letting more heterogeneous participants compete early increases the accuracy in selection. Experimental evidence supports our theoretical findings, indicating that the optimal design of promotion tournaments crucially depends on the objectives of the tournament designer. These findings have important implications for the optimal design of promotion tournaments in organizations.

JEL Classification: M52, J33

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Corresponding author:

Uwe Sunde
SEW-HSG
University of St. Gallen
Varnbüelstrasse 14
CH-9000 St. Gallen
Switzerland
E-mail: uwe.sunde@unisg.ch

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1 Introduction

Most employment relationships are characterized by competition among employees for promotion to a higher level position. The prospect of moving up the ladder to a better paid, more attractive position is a strong motivator for employees to exert effort on their current job. By creating incentives for workers through deferred compensation that can be linked to performance, promotion tournaments are a prominent instrument in the practice of human resource management. The central advantage of a promotion tournament is the modest informational requirement (only ordinal knowledge of workers' performance is necessary) and the fact that the employees' performance needs only to be observable and not necessarily verifiable by the employer (see, e.g., Lazear and Rosen, 1981 and Prendergast, 1999). This gives tournaments a clear advantage over many other compensation schemes and provides an explanation for their widespread use, especially in higher ranks of firms and organizations, which are typically characterized by multi-dimensional tasks, lack of cardinal information on individual performance and non verifiable information.

In addition to being a powerful incentive device, promotion tournaments can also perform a second important function: sorting heterogeneous employees by ability. The selection properties of tournaments are especially relevant in dynamic promotion tournaments with multiple stages. This has already been stressed by Rosen in his seminal paper on promotion tournaments, stating that "the inherent logic [of sequential elimination tournaments] is to determine the best contestants and to promote survival of the fittest" (Rosen 1986, p.701). Despite its practical importance, little is known about the selection properties of tournaments with heterogeneous workers in theory, even though some recent papers address this aspect.[1]

This paper addresses the central question that naturally arises once the tournament designer optimizes along two dimensions: Can promotion tournaments kill two birds with one stone? Or, put differently, is there a trade-off between incentive provision and selection performance? Answering this question is of high practical relevance, as it as it helps tournament designers to find the optimal structure of promotion tournaments given the relative weights assigned to each of the two goals. However, the two central goals of promotion tournaments – providing employees with incentives to exert effort and selection of the most able employees for higher-ranked positions – have typically been studied independently from each other in the literature. This is rather surprising, since recent theoretical work by Koch and Nafziger (2011) shows in a very general framework that it is not optimal for a company to separate the two goals, and both dimensions are certainly of central interest to designers of promotion

[1] See, e.g., Clark and Riis (2001) or Ryvkin and Ortmann (2008).

tournaments: First, heterogeneity between workers is obviously rather the rule than the exception in real world promotion tournaments.[2] Therefore, accuracy in selecting high ability employees for higher levels in the hierarchy is clearly an important issue in any organization. Secondly, a large proportion of observed wage variations within firms is associated with promotions, whereas the link between pay variation within hierarchical levels and performance is rather weak.[3] Instead, most of the literature has concentrated on studying the properties of different tournament formats for effort provision, with the typical result that heterogeneity in the workforce is detrimental for effort exertion.[4] Part of the literature has also been devoted to the investigation of how these detrimental effects of heterogeneity on incentives can be moderated, for example by handicapping of strong workers (Lazear and Rosen, 1981) or by type dependent wages (Gürtler and Kräkel, 2010).

This paper investigates the question of a trade-off between incentive provision and selection performance in two steps. In the first step, we theoretically investigate a promotion tournament with heterogeneous workers. In this tournament, workers are perfectly informed about both their own ability and the ability of their co-workers – due to, e.g., repeated interactions in the work place. The tournament designer in contrast possesses only some noisy prior knowledge about the ability of his workers. We investigate whether the designer can use this prior knowledge to structure the tournament in a way as to achieve strong incentives for effort exertion as well as a high accuracy in selecting the better workers. More specifically, we ask whether the designer should structure the tournament such that either relatively similar or relatively diverse workers compete against each other early in the tournament.

The analysis of the model suggests the existence of a fundamental trade-off between incentive provision and selection. While total effort is maximized by letting relatively similar workers compete early in the tournament, accuracy in selection is maximized by letting relatively heterogeneous workers compete early in the tournament. Intuitively, when relatively heterogeneous workers compete early, weak workers are discouraged and as a response, strong workers slack-off as well, such that total effort is reduced. On the other hand, letting diverse workers compete early is good for accuracy in selection, because the weak employee's decrease in effort due to discouragement is more pronounced than that of the strong due to slacking off. This increases the probability that weak employees drop out early, which translates in an increased probability of promoting "the fittest". As a consequence, the optimality of

[2] For instance, O'Keeffe, Viscusi and Zeckhauser (1984, p.42) note that "Most contests in this world are among unequal contestants".

[3] See Gibbs and Hendricks (2004), for example, as well as the literature cited in their paper.

[4] See Sunde (2009) for empirical evidence from heterogeneous tennis tournaments, and Bull, Schotter, and Weigelt (1987) or Harbring and Lünser (2008) for experimental evidence.

the tournament design crucially depends on the relative weights that the principal attaches to these goals.

As the second step, we present the results of experiments that were designed to test the behavioral relevance of the fundamental trade-off between incentives and selection. We find strong evidence that the tournament design matters for relative performance in the two dimensions. More specifically, we find that the tournament design has a strong effect on the accuracy of selection, and a minor effect on incentive provision as long as heterogeneity is modest. Results of an alternative treatment where the degree of heterogeneity between workers is higher delivers a more sizable effect on incentives in the direction predicted by the theory. Put together, these results support the theoretical prediction that a tournament designer faces a trade-off between incentive provision and selection if the tournament participants are heterogeneous.

This paper contributes to different strands of the tournament literature. The paper related closest to ours is the one by Tsoulouhas, Knoeber, and Agrawal (2007) who study a one-stage CEO promotion tournament with heterogenenous agents, assuming that both the quality of the promoted agent and the provision of incentives matter for the designer. While they consider the same trade-off as we do, their setting is quite different from the one we analyze: Their focus is on optimal handicapping in a contest in which outsiders and insiders compete for a CEO position, assuming that only effort provision by insiders matters on this stage of the competition, whereas the ability matters on a second stage when they work as a CEO under a revenue sharing contract. In contrast, we analyze the optimal design of a multi-stage tournament in the spirit of Rosen (1986) with heterogeneous agents, in which incentives as well as selection performance at all stages of the tournament matter for the designer. Essentially, our results show that the trade-off established by Tsoulouhas, Knoeber, and Agrawal (2007) for a competition between insiders and outsiders also holds for purely internal promotion tournaments if a company employs workers of different types. That tournament design can involve a trade-off between incentives and sorting accuracy has also been shown by Groh, Moldovanu, Sela, and Sunde (2011). However, while they analyze the case of an all-pay-auction, our analysis complements theirs by focusing on a standard Tullock contest framework. Moreover, we contribute to the relatively small literature on the optimal design of multi-stage elimination tournaments. By considering heterogeneity between participants, our analysis complements the study by Fu and Lu (2011), who determine the optimal number of stages as well as the optimal allocation of prizes across stages when the designer's goal is the maximization of aggregate effort and agents are homogeneous. Further, we extend the theoretical work on behavior of heterogeneous agents in two-stage contests by Stein and Rapoport (2004) and Harbaugh and Klumpp (2005) by considering a situation in which continuation values in early stages of the tournament are endogenously determined, and the effort choice of agents is unrestricted. Finally, our analysis complements experimental work of Orrison, Schotter, and Weigelt (2004), Altmann, Falk, and Wibral (2008), and Sheremeta (2010), who analyze different aspects of the optimal design of tournaments, but who restrict attention to the case of homogeneous players.

The remainder of this paper is structured as follows. Section 2 presents the theoretical model and its implications. We test the theoretical predictions experimentally in section 3. Section 4 provides some concluding remarks.

2 Theoretical Analysis

2.1 A Model of Promotion Tournaments With Heterogeneous Workers

We follow Rosen (1986) by modeling the promotion competition in a company as a multi-stage pairwise elimination tournament. The simplest contest of this type with heterogeneous participants is the two-stage pairwise elimination contest with four workers of two different types, who compete under a Tullock contest success function with discriminatory power of 1. Hence, we consider a setting with two hierarchical levels and four workers of two possible types. The workers compete pairwise and the winning probability of each worker is proportional to the relative effort he provides. An advantage of this highly stylized and simplified model is that it remains analytically tractable. As will become clear later in the discussion of the results, however, our model provides important insights that are robust to changes in the simplifying assumptions we make.

In such a two-stage pairwise elimination tournament, there are three two-person interactions to consider: Two parallel interactions on the first stage, and one interaction on the final stage among those workers who were promoted from stage 1. Under the assumption that equal shares of the workers in the tournament are "weak" (type W) and "strong" (type S), two different settings are possible: The principal can separate strong and weak workers in the first stage, i.e., the two strong and the two weak employees compete against each other in the two parallel interactions (setting A: SSWW); alternatively, the stage 1 interactions can be mixed, i.e., each strong worker competes with a weak one on stage 1 (setting B: SWSW).

Heterogeneity between workers is modeled by assuming type-dependent linear cost of effort. Specifically, the effort cost of strong workers, $c_{\mathtt{S}}$, is assumed to be lower than the effort cost $c_{\mathtt{W}}$ of weak workers ($c_{\mathtt{S}} \leq c_{\mathtt{W}}$). It is assumed throughout that all workers are perfectly informed about both their own type and the type of their colleagues. Modeling heterogeneity in terms of effort cost is without loss of generality. The theoretical results presented below hold also if heterogeneity is modeled in terms of differing ability or valuation of the prize.[5]

To induce effort provision by workers, being promoted must have some value. In our simplified model, there are three events of promotion: First, two workers are promoted from stage 1 to stage 2, one from each pairwise interaction. Then, on stage 2, one of these two workers is promoted to the top level position. To keep the analysis simple, we assume that the promotion to the top level position entails an exogenously given wage increase P, whereas there is no immediate reward for a promotion from stage 1 to stage 2.

Due to the strictly positive value of promotion to the top level position, workers have an incentive to exert effort on each stage of the tournament because effort provision increases the probability to receive the prize P. In their effort decision, workers face a trade-off between higher costs and a higher promotion probability associated with an increase in own effort. In equilibrium, workers choose effort in such a way that the marginal cost of effort provision $c_k \in \{c_{\mathtt{W}}, c_{\mathtt{S}}\}$ equals the expected marginal monetary gain of promotion to the top level position. If two workers i and j, with $j \neq i$, compete against each other on a given stage of the tournament, and if the effort of worker $k \in \{i, j\}$ is $x_k(c_k)$, then the winning probability p_i of worker i as given by:

$$p_i = \frac{x_i(c_i)}{x_i(c_i) + x_j(c_j)}.$$

This formulation of the winning probability is standard in the literature and implies that the winning probability of a worker i is increasing in the effort he provides, and decreasing in the effort of his opponent j.

Below, we will first derive the analytical solution for both settings, A: `SSWW` and B: `SWSW`, in section 2.2. In section 2.3 we analyze the equilibrium properties of both settings in terms of selection performance and aggregate effort provided. Finally, we discuss the implications of these findings on the optimal tournament design in section 2.4 and address the role of the simplifying assumptions for our results.

2.2 Equilibrium Behavior

The solution concept for a two-stage tournament is subgame perfect Nash equilibrium. Solving the game via backwards induction, we start by analyzing all possible interactions of stage 2, and subsequently solve stage 1, taking the optimal actions in stage 2 as given. Recall that the only reward for winning

[5] Proofs are available from the authors upon request.

stage 1 is the participation in stage 2, which provides the possibility of an eventual promotion to the top position with the associated prize P. The expected equilibrium payoffs for both types and all potential stage 2 interactions thus determine the continuation values and the optimal efforts on stage 1.

2.2.1 Solution for Stage 2

With four players of two types, there are three potential stage 2 games, namely (1) SS, (2) WW, or (3) SW, each of which is considered below. Independent of the specific interaction, $x_{i2}(c_i)$ denotes the effort of agent i with effort cost c_i on stage 2. Equilibrium efforts are marked with an asterisk.

(1) SS: If two strong agents l and k of type S compete with each other on stage 2, they both face the same maximization problem. Without loss of generality, we consider the optimization by agent l, who maximizes his expected payoff $\pi_l(\texttt{SS})$ by choosing an optimal level of effort x_{l2}. Formally, the maximization problem reads

$$\max_{x_{l2}\geq 0} \pi_{\texttt{S}l}(\texttt{SS}) = \frac{x_{l2}}{x_{l2}+x_{k2}} P - c_{\texttt{S}} x_{l2}.$$

Using the first-order condition for agent l ($x_{k2}P - c_{\texttt{S}}(x_{l2}+x_{k2})^2 = 0$) and invoking symmetry ($x^*_{k2} = x^*_{l2}$) delivers $x^*_{l2} = x^*_{k2} = \frac{P}{4c_{\texttt{S}}}$. Inserting optimal actions in the objective function gives the payoff that a "strong" agent can expect if he meets another "strong" agent on stage 2.[6] This yields

$$\pi^*_{\texttt{S}}(\texttt{SS}) = \frac{P}{4}\,. \tag{1}$$

(2) WW: The symmetry argument with respect to the maximization problem also holds if two weak agents l and k of type W compete with each other on stage 2. As before, we consider the optimization by player l without loss of generality: $\max_{x_{l2}\geq 0} \pi_{\texttt{W}l}(\texttt{WW}) = \frac{x_{l2}}{x_{l2}+x_{k2}} P - c_{\texttt{W}} x_{l2}$. The same steps as before deliver equilibrium efforts $x^*_{l2} = x^*_{k2} = \frac{P}{4c_{\texttt{W}}}$. When inserting these efforts in the objective function above, we get the expected equilibrium payoff for a weak player in a stage 2 interaction WW:

$$\pi^*_{\texttt{W}}(\texttt{WW}) = \frac{P}{4}\,. \tag{2}$$

(3) SW: Finally, consider the situation where a strong agent S meets a weak agent W on stage 2. The

[6]Note that the expected payoff is the same for player l and player k, therefore the indices can be dropped.

agents solve the following optimization problems:

$$\begin{aligned}\max_{x_{S2}\geq 0} \pi_{\mathtt{S}}(\mathtt{SW}) &= \frac{x_{s2}}{x_{s2}+x_{w2}}P - c_{\mathtt{S}}x_{s2}\ ,\\ \max_{x_{W2}\geq 0} \pi_{\mathtt{W}}(\mathtt{SW}) &= \frac{x_{w2}}{x_{s2}+x_{w2}}P - c_{\mathtt{W}}x_{w2}\ .\end{aligned}$$

The combination of first-order conditions implies equilibrium efforts $x^*_{\mathtt{S}2} = \frac{c_{\mathtt{W}}}{(c_{\mathtt{S}}+c_{\mathtt{W}})^2}P$ for the strong agent and $x^*_{\mathtt{W}2} = \frac{c_{\mathtt{S}}}{(c_{\mathtt{S}}+c_{\mathtt{W}})^2}P$ for the weak agent, respectively. Inserting optimal actions in the two objective functions gives the expected payoffs for strong and weak agents in a stage 2 interaction SW:

$$\pi^*_{\mathtt{S}}(\mathtt{SW}) = \frac{c_{\mathtt{W}}^2}{(c_{\mathtt{S}}+c_{\mathtt{W}})^2}P\ , \tag{3}$$

$$\pi^*_{\mathtt{W}}(\mathtt{SW}) = \frac{c_{\mathtt{S}}^2}{(c_{\mathtt{S}}+c_{\mathtt{W}})^2}P\ . \tag{4}$$

2.2.2 Solution for Stage 1

With all potential stage 2 interactions solved, we proceed to compute the equilibrium on stage 1 for the two specifications A: SSWW (homogeneous stage 1 interactions), and B: SWSW (heterogeneous stage 1 interactions). We begin with setting A: SSWW. The effort of agent i ($i = 1, 2, 3, 4$) on stage 1 in setting n ($n = A, B$) is denoted by x^n_{i1}; as before, equilibrium efforts are marked with an asterisk.

Setting A: SSWW Two-stage tournaments with this structure have been analyzed in the previous literature.[7] In this setting, one strong and one weak worker reach stage 2 with certainty and consequently, SW is the only possible constellation on stage 2. This implies that both strong workers know that conditional on reaching stage 2, a weak worker will be the opponent, while weak workers know that they will interact with a strong one in case they get promoted to stage 2.[8] We assume without loss of generality that workers 1 and 2 are strong, while workers 3 and 4 are weak. The value of participation on stage 2 is given by (3) for workers 1 and 2, while the continuation value for workers 3 and 4 is defined by (4). Note that due to symmetry of the optimization problems, it suffices to solve the optimization problem of one strong worker (1 or 2) and one weak worker (3 or 4). Without loss of

[7] This situation corresponds to what Stein and Rapoport (2004) refer to as "semifinals" model in their paper.

[8] Strong agents do not care which of the two weak agents they meet on stage 2, since any agent of type W chooses the same equilibrium strategy. An analogous argument holds for weak agents.

generality, we consider workers 1 and 3 and obtain:

$$\begin{aligned}\max_{x_{11}^A \geq 0} \Pi_1^A &= \frac{x_{11}^A}{x_{11}^A + x_{21}^A} \pi_{\mathtt{S}}^*(\mathtt{SW}) - c_{\mathtt{S}} x_{11}^A \,, \\ \max_{x_{31}^A \geq 0} \Pi_3^B &= \frac{x_{31}^A}{x_{31}^A + x_{41}^A} \pi_{\mathtt{W}}^*(\mathtt{SW}) - c_{\mathtt{W}} x_{31}^A \,.\end{aligned}$$

First-order and symmetry conditions deliver the following stage 1 equilibrium efforts:

$$x_{11}^{A*} = y_{21}^{A*} = \frac{c_{\mathtt{W}}^2}{4c_{\mathtt{S}}(c_{\mathtt{S}} + c_{\mathtt{W}})^2} P \,, \tag{5}$$

$$x_{31}^{A*} = y_{41}^{A*} = \frac{c_{\mathtt{S}}^2}{4c_{\mathtt{W}}(c_{\mathtt{S}} + c_{\mathtt{W}})^2} P. \,. \tag{6}$$

Setting B: `SWSW` In setting B: `SWSW`, optimal behavior is somewhat harder to characterize because the continuation values are more complicated as there is a mutual interdependence of the workers' continuation values. In fact, to the best of our knowledge, a closed-form solution for this setting in the context of a Tullock contest success function has not been presented in the literature.[9] As will become clear below, continuation values are endogenously determined in this setting, which complicates the solution. Nevertheless, properties of this setting were previously discussed by Rosen (1986) who conjectured (without analytical proof) that certain properties of numerical simulations hold in general.

As for setting A: `SSWW`, we again define that workers 1 and 2 are strong, whereas workers 3 and 4 are weak. Further, we assume that workers 1 and 3 as well as workers 2 and 4 interact on stage 1, i.e., each strong worker meets a weak one, and vice versa. Note that the two stage 1 interactions are identical, i.e., it suffices to analyze one of the two.[10] Without loss of generality, we consider the interaction between workers 1 and 3, who face the following maximization problems, respectively:

$$\max_{x_{11}^B} \Pi_1 = \frac{x_{11}^B}{x_{11}^B + x_{31}^B} \underbrace{\left[\frac{x_{21}^B}{x_{21}^B + x_{41}^B} \pi_{\mathtt{S}}^*(\mathtt{SS}) + \frac{x_{41}^B}{x_{21}^B + x_{41}^B} \pi_{\mathtt{S}}^*(\mathtt{SW}) \right]}_{\equiv P_1(x_{21}^B, x_{41}^B)} - c_{\mathtt{S}} x_{11}^B,$$

$$\max_{x_{31}^B} \Pi_3 = \frac{x_{31}^B}{x_{11}^B + x_{31}^B} \underbrace{\left[\frac{x_{21}^B}{x_{21}^B + x_{41}^B} \pi_{\mathtt{W}}^*(\mathtt{SW}) + \frac{x_{41}^B}{x_{21}^B + x_{41}^B} \pi_{\mathtt{W}}^*(\mathtt{WW}) \right]}_{\equiv P_3(x_{21}^B, x_{41}^B)} - c_{\mathtt{W}} x_{31}^B \,.$$

Note that $P_1(x_{21}^B, x_{41}^B)$ and $P_3(x_{21}^B, x_{41}^B)$ denote the continuation values for workers 1 and 3, respectively,

[9] The only paper that deals with endogenously determined continuation values is Groh, Moldovanu, Sela, and Sunde (2011), but they consider the limit case of an all-pay auction, i.e., in the case in which the contest-success function is perfectly discriminating.

[10] The unique equilibrium in pure strategies exists regardless of the degree of heterogeneity. A proof is available upon request.

which depend on the behavior of workers 2 and 4 in the other stage 1 interaction, i.e., the continuation values are endogenously determined. The reason for this complication is that the tournament structure allows for three different stage 2 interactions, namely SS, WW, and SW, which are of different value to agents of the two types and which occur with different probabilities.

Independent of this complication, the following two first-order conditions are still necessary equilibrium conditions:

$$\begin{aligned} x_{31}^{B} P_1(x_{21}, x_{41}) - c_{\mathtt{S}}(x_{11}^{B} + x_{31}^{B})^2 &= 0\,, \\ x_{11}^{B} P_3(x_{21}, x_{41}) - c_{\mathtt{W}}(x_{11}^{B} + x_{31}^{B})^2 &= 0\,. \end{aligned}$$

If we combine these conditions, we obtain another necessary equilibrium condition which defines a relation between equilibrium actions of workers 1 and 3:

$$\frac{x_{11}^{B*}}{x_{31}^{B*}} = \frac{c_{\mathtt{W}}}{c_{\mathtt{S}}} \frac{P_1(x_{21}^{B}, x_{41}^{B})}{P_3(x_{21}^{B}, x_{41}^{B})} = \frac{c_{\mathtt{W}}}{c_{\mathtt{S}}} \frac{(c_{\mathtt{S}} + c_{\mathtt{W}})^2 x_{21}^{B} + 4c_{\mathtt{W}}^2 x_{41}^{B}}{4c_{\mathtt{S}}^2 x_{21}^{B} + (c_{\mathtt{S}} + c_{\mathtt{W}})^2 x_{41}^{B}}\,. \tag{7}$$

Recall that the two stage 1 interactions are identical. This implies that the conditions $x_{11}^{B*} = x_{21}^{B*}$ and $x_{31}^{B*} = x_{41}^{B*}$ do hold in the (unique symmetric) equilibrium. This gives the following quadratic equation in x_{11}^{B*} and x_{31}^{B*}:

$$\begin{aligned} \frac{x_{11}^{B*}}{x_{31}^{B*}} &= \frac{c_{\mathtt{W}}}{c_{\mathtt{S}}} \frac{(c_{\mathtt{S}} + c_{\mathtt{W}})^2 x_{21}^{B} + 4c_{\mathtt{W}}^2 x_{41}^{B}}{4c_{\mathtt{S}}^2 x_{21}^{B} + (c_{\mathtt{S}} + c_{\mathtt{W}})^2 x_{41}^{B}} \\ \Leftrightarrow \quad 0 &= 4c_{\mathtt{S}}^2 \left[\frac{x_{11}^{B*}}{x_{31}^{B*}}\right]^2 + \left(1 - \frac{c_{\mathtt{W}}}{c_{\mathtt{S}}}\right)(c_{\mathtt{S}} + c_{\mathtt{W}})^2 \left[\frac{x_{11}^{B*}}{x_{31}^{B*}}\right] - 4\frac{c_{\mathtt{W}}^3}{c_{\mathtt{S}}} \\ \Leftrightarrow \quad 0 &= \left[\frac{x_{11}^{B*}}{x_{31}^{B*}}\right]^2 + \frac{(c_{\mathtt{S}} - c_{\mathtt{W}})(c_{\mathtt{S}} + c_{\mathtt{W}})^2}{4c_{\mathtt{S}}^3} \left[\frac{x_{11}^{B*}}{x_{31}^{B*}}\right] - \frac{c_{\mathtt{W}}^3}{c_{\mathtt{S}}^3} \\ \Leftrightarrow \quad \frac{x_{11}^{B*}}{x_{31}^{B*}} &= F^*(c_{\mathtt{S}}, c_{\mathtt{W}}), \end{aligned}$$

where

$$F^*(c_{\mathtt{S}}, c_{\mathtt{W}}) = \frac{(c_{\mathtt{W}} - c_{\mathtt{S}})(c_{\mathtt{S}} + c_{\mathtt{W}})^2 + \sqrt{64 c_{\mathtt{W}}^3 c_{\mathtt{S}}^3 + (c_{\mathtt{S}} - c_{\mathtt{W}})^2 (c_{\mathtt{S}} + c_{\mathtt{W}})^4}}{8c_{\mathtt{S}}^3}. \tag{8}$$

The above expression allows for an analytical solution of the game. Essentially, $F^*(c_{\mathtt{S}}, c_{\mathtt{W}})$ is a measure for the additional heterogeneity between strong and weak agents on stage 1 that is due to differences in continuation values.[11] Thus, the two interdependent stage 1 interactions can be disentangled in

[11] The continuation value for strong agents is always higher than the continuation value for weak agents, as shown by the first-order conditions above.

equilibrium. The continuation values satisfy

$$\begin{aligned} P_1(x_{21}^{B*}, x_{41}^{B*}) = P_2(x_{11}^{B*}, x_{31}^{B*}) &= \frac{(c_{\mathtt{S}} + c_{\mathtt{W}})^2 F^*(c_{\mathtt{W}}, c_{\mathtt{S}}) + 4c_{\mathtt{W}}^2}{4(c_{\mathtt{S}} + c_{\mathtt{W}})^2[1 + F^*(c_{\mathtt{S}}, c_{\mathtt{W}})]} P \, , \\ P_3(x_{21}^{B*}, x_{41}^{B*}) = P_4(x_{11}^{B*}, x_{31}^{B*}) &= \frac{(c_{\mathtt{S}} + c_{\mathtt{W}})^2 + 4c_{\mathtt{S}}^2 F^*(c_{\mathtt{W}}, c_{\mathtt{S}})}{4(c_{\mathtt{S}} + c_{\mathtt{W}})^2[1 + F^*(c_{\mathtt{S}}, c_{\mathtt{W}})]} P \, , \end{aligned}$$

since, due to symmetry, $P_1(x_{21}^{B*}, x_{41}^{B*}) = P_2(x_{11}^{B*}, x_{31}^{B*})$ and $P_3(x_{21}^{B*}, x_{41}^{B*}) = P_4(x_{11}^{B*}, x_{31}^{B*})$. Given these continuation values, one can determine equilibrium efforts as

$$x_{11}^{B*} = x_{21}^{B*} = \frac{(c_{\mathtt{S}} + c_{\mathtt{W}})^2 F^*(c_{\mathtt{W}}, c_{\mathtt{S}})^2 + 4c_{\mathtt{W}}^2 F^*(c_{\mathtt{W}}, c_{\mathtt{S}})}{4c_{\mathtt{S}}(c_{\mathtt{S}} + c_{\mathtt{W}})^2[1 + F^*(c_{\mathtt{S}}, c_{\mathtt{W}})]^3} P \, , \tag{9}$$

$$x_{31}^{B*} = x_{41}^{B*} = \frac{(c_{\mathtt{S}} + c_{\mathtt{W}})^2 F^*(c_{\mathtt{W}}, c_{\mathtt{S}}) + 4c_{\mathtt{S}}^2 F^*(c_{\mathtt{W}}, c_{\mathtt{S}})^2}{4c_{\mathtt{W}}(c_{\mathtt{S}} + c_{\mathtt{W}})^2[1 + F^*(c_{\mathtt{S}}, c_{\mathtt{W}})]^3} P \, . \tag{10}$$

2.3 Properties of the Equilibrium Solutions

Based on the closed form solutions derived previously we proceed by analyzing the properties of the equilibrium. To investigate the properties in the two dimensions of interest, incentives and selection, we define performance measures to capture them.

Incentives: In the existing literature, elimination tournaments have been analyzed mostly as a means to provide participants with incentives for effort provision, following the approach taken in the seminal paper by Lazear and Rosen (1981). In this literature, it is common to use total expected equilibrium effort, i.e., the sum of equilibrium efforts on stage 1 and (expected) equilibrium efforts on stage 2, as a measure for the provision of incentives. The underlying idea of this approach is that effort provision by workers and productivity are directly linked, i.e., total output is an increasing function of total effort. For simplicity, but without loss of generality, we assume that effort translates one to one into output, such that total output equals the amount of effort provided in both stages of the tournament.[12] This implies that we do not need to distinguish between total output and total effort. For the remainder of this paper, we concentrate on total effort provided by the workers.

In setting A: `SSWW`, total effort provision in both stages amounts to

$$\mathcal{E}_{\mathtt{SSWW}} = \underbrace{\frac{c_{\mathtt{S}}^3 + c_{\mathtt{W}}^3}{2c_{\mathtt{S}}c_{\mathtt{W}}(c_{\mathtt{S}} + c_{\mathtt{W}})^2} P}_{\text{stage 1 effort}} + \underbrace{\frac{2}{(c_{\mathtt{S}} + c_{\mathtt{W}})^2}}_{\text{stage 2 effort}} P \tag{11}$$

The characterization of the corresponding measure for setting B: `SWSW` is more involved since three

[12] This assumption is without loss of generality as long as output is a weakly increasing function of effort.

different interactions can occur on stage 2, each with some probability. Therefore, $\mathcal{E}_{\texttt{SWSW}}$ represents an *expected* value, which has a variance from an ex-ante perspective. *Realized* ex-post equilibrium effort provision varies and depends on the type composition of the stage 2 interaction; it is highest in case of the interaction SS which is observed with probability $[F^*(c_{\texttt{S}}, c_{\texttt{W}})]^2/[1 + F^*(c_{\texttt{S}}, c_{\texttt{W}})]^2$, lowest in situation WW which realizes with probability $1/(1 + F^*(c_{\texttt{S}}, c_{\texttt{W}}))^2$, and intermediate in pairing SW, which occurs with probability $2F^*(c_{\texttt{S}}, c_{\texttt{W}})/[1+F^*(c_{\texttt{S}}, c_{\texttt{W}})]^2$. This is different in setting A: SSWW, where the interaction SW is always observed with certainty on stage 2.

The computation of the effort measure yields

$$\mathcal{E}_{\texttt{SWSW}} = \underbrace{\frac{(c_{\texttt{S}} + c_{\texttt{W}})^2 F^*(c_{\texttt{W}}, c_{\texttt{S}})^2 + 4c_{\texttt{W}}^2 F^*(c_{\texttt{W}}, c_{\texttt{S}})}{2c_{\texttt{S}}(c_{\texttt{S}} + c_{\texttt{W}})^2[1 + F^*(c_{\texttt{S}}, c_{\texttt{W}})]^2} P}_{\text{stage 1 effort}} + \underbrace{\frac{(c_{\texttt{S}} + c_{\texttt{W}})^2[c_{\texttt{S}} + c_{\texttt{W}} F^*(c_{\texttt{S}}, c_{\texttt{W}})^2] + 4c_{\texttt{S}}c_{\texttt{W}} F^*(c_{\texttt{S}}, c_{\texttt{W}})}{2c_{\texttt{S}}c_{\texttt{W}}(c_{\texttt{S}} + c_{\texttt{W}})^2[1 + F^*(c_{\texttt{S}}, c_{\texttt{W}})]^2} P}_{\text{stage 2 effort}}. \quad (12)$$

A comparison of $\mathcal{E}_{\texttt{SSWW}}$, and $\mathcal{E}_{\texttt{SWSW}}$ gives the following relation between the two settings in terms of total effort:

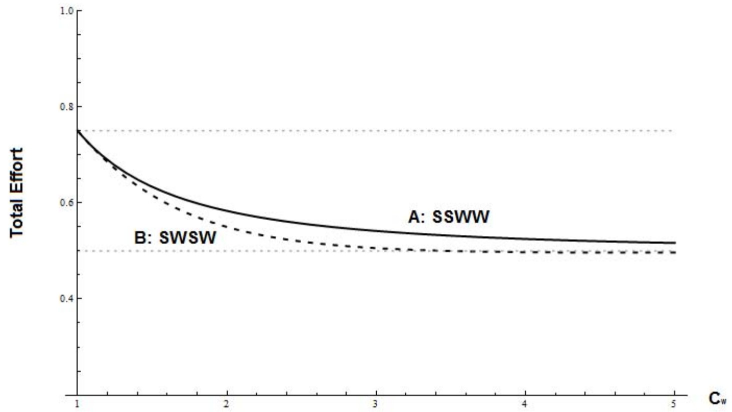
Proposition 1 (Incentive Provision)**.** *When the cost of effort is strictly higher for weak than for strong agents ($c_{\texttt{W}} > c_{\texttt{S}}$), total effort is always strictly higher in setting A: SSWW than in setting B: SWSW; formally, this implies that $\mathcal{E}_{\texttt{SSWW}} > \mathcal{E}_{\texttt{SWSW}}$ for all $c_{\texttt{W}} > c_{\texttt{W}}$.*

Proof. See Appendix. □

Proposition 1 implies that heterogeneity between workers has different effects on effort provision in setting A: SSWW than B: SWSW. We know from previous work on tournaments with heterogeneous workers that two effects are responsible for the detrimental effect of heterogeneity on effort provision: On the one hand, weak workers reduce their effort (compared to a situation with lower costs) because it is more costly for them. On the other hand, strong workers optimally react to the effort reduction of their competitor by also reducing their equilibrium effort. Intuitively, the distortion that weak workers invest less effort in the tournament is less pronounced in setting A: SSWW, where weak workers compete among themselves on stage 1 and do not have to interact with strong ones. In contrast, weak workers meet a strong worker for sure on stage 1 of setting B: SWSW, and with a high probability also on stage 2 (if they reach stage 2). Consequently, they reduce their effort provision even more. The advantage of setting A: SSWW over B: SWSW in terms of effort provision is further accentuated by the reaction of strong workers to heterogeneity.

Figure 1 plots total expected effort for the two settings as a function of heterogeneity, reflected by the constant marginal cost by the weak agent, $c_{\texttt{W}}$, while the costs for the strong agent and the prize

Figure 1: Incentive Provision



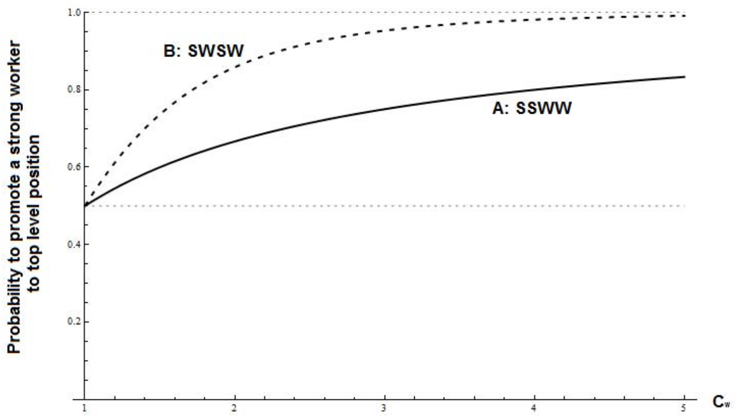
Notes: The figure plots expressions (11) and (12) with $c_{\mathtt{S}} = 1$ and $P = 1$.

for being promoted to the top level position are normalized to 1 ($c_{\mathtt{S}} = 1$ and $P = 1$). The figure illustrates that the difference between the two settings is most pronounced for intermediate degrees of heterogeneity, since total effort provision in both settings converges towards that of a tournament with strong workers only, $\mathcal{E}_{\mathtt{SSSS}} = \frac{3}{4c_{\mathtt{S}}}P$, if $c_{\mathtt{W}} \to c_{\mathtt{S}}$, while this measure approaches $\frac{P}{2c_{\mathtt{S}}}$ if $c_{\mathtt{W}} \to \infty$. However, even in situations with very low or very high degree of heterogeneity in which the difference in total effort exerted is rather small in equilibrium, the difference becomes increasingly relevant if one allows for multiple prizes, with a high prize for the winner of stage 2, and another, lower prize for the loser of the interaction on stage 2.[13] Such a prize structure is likely to be common practice rather than an exception in reality. If part of the prize P is moved to the earlier stages, total effort provision decreases in both specifications, but especially so in setting B: `SWSW`, where the stage 1 interactions are heterogeneous. Contrary, both stage 1 interactions are homogeneous in setting A: `SSWW`, such that the difference in total effort provision between the two settings increases. In this sense, the baseline specification of the theoretical investigation is likely to underestimate the difference between settings A: `SSWW` and B: `SWSW` in terms of effort provision.

Selection: There is no unambiguous way to measure the selection accuracy of a tournament. In the context of promotion tournaments, it appears natural to assume that it is in the principal's interest to promote strong (type `S`) rather than weak workers (type `W`) to the top level position. However, it is rather difficult to generally justify the relative importance of promotion on the two stages. As will become clear below, this does not constitute a problem for current application, because one of

[13]The solution presented above can be directly extended to the consideration multiple prizes. Formal proofs for the verbal arguments are available from the authors upon request.

Figure 2: Selection



Notes: The figure plots expressions (13) and (14) with $c_S = 1$.

the two settings has a higher selection performance on both stages. Therefore, we define the selection performance of a tournament as the probability that one of the two strong workers is promoted to the top level position. We refer to this selection measure as $\mathcal{S}$.[14] The respective selection measures $\mathcal{S}_{\texttt{SSWW}}$ and $\mathcal{S}_{\texttt{SWSW}}$ for settings A: SSWW and B: SWSW are given by

$$\mathcal{S}_{\texttt{SSWW}} = \frac{c_{\texttt{W}}}{c_{\texttt{S}} + c_{\texttt{W}}} \tag{13}$$

$$\mathcal{S}_{\texttt{SWSW}} = \frac{(c_{\texttt{S}} + c_{\texttt{W}})F^*(c_{\texttt{S}}, c_{\texttt{W}})^2 + 2c_{\texttt{W}}F^*(c_W)}{(c_{\texttt{S}} + c_{\texttt{W}})[1 + F^*(c_{\texttt{S}}, c_{\texttt{W}})]^2}. \tag{14}$$

In setting A: SSWW, the selection performance is necessarily determined on stage 2. The measure $\mathcal{S}_{\texttt{SSWW}}$ is therefore equal to the ratio of stage 2 efforts provided by strong and weak workers. Relative incentives for strong and weak workers also determine the selection performance in setting B: SWSW. However, in this setting selection occurs on both stages of the tournament. As with the incentive measure $\mathcal{E}$, we compare settings A: SSWW and B: SWSW in terms of their selection performance and get to the following result:

Proposition 2 (Selection performance)**.** *When the cost of effort is strictly higher for weak than for strong agents ($c_W > c_S$), the ex-ante probability that a strong agent wins the tournament is always higher in setting B: SWSW than in setting A: SSWW; formally, this implies that $\mathcal{S}_{SSWW} < \mathcal{S}_{SWSW}$ for all $c_W > c_S$.*

Proof. See Appendix. □

The finding that setting B: SWSW performs better than setting A: SSWW in terms of promoting a strong worker to the top level position is due to the differences in selection on stage 1. While there

[14]This would be a natural candidate for selection also in other applications, like procurement tournaments, or sports.

is selection on both stages of the tournament in setting B: SWSW, a weak agent reaches stage 2 with certainty in setting A: SSWW. Consequently, stage 1 does not contribute to the selection in setting A: SSWW, which implies a lower selection accuracy overall. This fact also explains why relative importance of promotions on stages 1 and 2 does not affect the results: Both, the probability to promote strong workers from stage 1 to stage 2, and the probability to promote a strong worker from stage 2 to the top level position, are higher in setting B: SWSW than in A: SSWW. Figure 2 displays the relation in terms of selection performance between settings A: SSWW and B: SWSW graphically: The Figure shows that the probability that a worker of type S is promoted to the top level position is close to 50% in both settings if strong and weak workers are almost identical ($c_{\mathtt{W}} \rightarrow c_{\mathtt{S}} = 1$). However, as heterogeneity between worker types increases, the probability that a strong worker is promoted to the top level approaches 1 rather fast in setting B: SWSW compared to setting A: SSWW, where the probability becomes high only for extreme degrees of heterogeneity.

2.4 Implications for the Optimal Design

Under the assumption that the selection performance and the incentives for effort provision are important for the tournament designing principal, one can describe the preferences of the principal by use of an objective function $Z(\mathcal{E}, \mathcal{S})$, where $Z(\mathcal{E}, \mathcal{S})$ is increasing in both arguments, in the total effort $\mathcal{E}$ and the selection performance $\mathcal{S}$. As Propositions 1 and 2 show, there is no clear solution to this problem: Whether the principal prefers a situation in which he pairs workers of equal types on stage 1 (A: SSWW) over or setting where stage 1 interactions are mixed (B: SWSW), depends on the relative weights that the principal attaches to effort provision by the agents and to selection performance. In other words, the two goals are conflicting, such that the designer has to trade-off gains in one dimension against losses in the other. The intuition for the existence of this trade-off becomes clear when distinguishing *absolute* and *relative* incentives for effort provision. Selection performance is driven by the *ratio* of the workers' efforts: The higher the incentives for strong workers to provide effort are *relative* to the incentives for weak workers, the better is the performance of a tournament in the selection dimension. This implies that a high degree of heterogeneity between workers is beneficial, because the heterogeneity discourages weak workers more than it induces strong workers to slack off, which facilitates selection. At the same time, however, heterogeneity reduces incentives for effort provision, a measure which depends on *absolute* incentives. In this dimension, the performance of a tournament is better the higher incentives for effort provision are for both strong and weak workers in *absolute* terms.

This trade-off between absolute and relative incentives for effort provision is a result that holds in general. Essentially, implementing design A: `SSWW` can be understood as an indirect way of handicapping strong workers. In this setting, being promoted to stage 2 is harder for strong workers than in setting B: `SWSW`. Therefore, it is generally the case that actions which increase the incentives for effort provision in heterogeneous settings, like handicapping à la Lazear and Rosen (1981), or wage discrimination à la Gürtler and Kräkel (2010), or seeding of worker types as in this paper, automatically reduce the selection performance of a tournament. This result holds regardless of the tournament design and the number of worker types.

3 Experimental Evidence

3.1 Implementation

The experiment implements the exact same tournament structure that was analyzed theoretically in the previous section. Four subjects competed in a two-stage elimination tournament for a unique prize P of 240 Taler (the currency of the experiment), which the winner of the stage 2 interaction received; 200 Taler equal 1.00 Euro. The prize had the same value for all participants. Following the theoretical model, we introduce heterogenenity between agents through different costs of effort provision: *strong* agents had a cost of effort equal to $c_{\mathtt{S}} = 1.00$ Taler in all treatments, while *weak* agents had costs of $c_{\mathtt{W}} = 1.50$ Taler. Effort provision was implemented in terms of investments in a lottery: Participants were told that they could buy a discrete number of balls in the stage 1 interaction.[15] The balls purchased by the subjects as well as those purchased by their respective opponents in the stage 1 interaction were then said to be placed in the same urn, of which one ball was randomly drawn. The agent who purchased this ball proceeded to the stage 2 interaction. This setting closely reflects the theoretical counterpart of the specification of the success probability in terms of a Tullock contest success function with discriminatory power 1 that was used in the theoretical analysis. The two stage 1 interactions are independent from each other, i.e., subjects know that there are two separate urns on stage 1. When agents made their decision on stage 1, they did not know whom they would encounter on stage 2; they only knew the types of those players who competed in the other stage 1 interaction. In stage 2, the two agents who won the stage 1 interaction met. Again, both players could buy a certain number of balls, which were then placed in a third urn; the player whose ball was randomly drawn received the prize of 240 Taler. The two players who did not proceed to stage 2 saw a waiting screen until the decision round was completed. Note that players had to buy and pay for a certain number of balls *before* they

[15] With the given parameterization, there is a unique equilibrium in pure strategies and the discrete grid has no consequences for the equilibrium strategies.

knew whether or not they won the prize in a given game. To avoid limited liability problems, each participant received an endowment of 240 Taler which he could use to buy a certain number of balls on stage 1; if he reached stage 2, he could use whatever remaines of his endowment to buy balls on stage 2; the part of the endowment which a participant did not use to buy balls was added to the payoffs for that round. Since the endowment was as high as the prize which could be won, agents were not budget-constrained at any time.

The experimental variation is reflected in two different main treatments: A: `SSWW` and B: `SWSW`, with weak players in treatments A: `SSWW` and B: `SWSW` exhibiting *low heterogeneity* in terms of $c_W = 1.5$. In additional robustness treatments (*high heterogeneity*), we increase the degree of heterogeneity by setting $c_W = 2.5$ to test the predictions about the effect of increased heterogeneity on both performace measures. Moreover, as Figures 1 and 2 nicely show, the theoretically predicted difference between settings A: `SSWW` and B: `SWSW` is much higher if $c_W = 2.5$, in particular with respect to incentive provision. The information participants received about their own cost parameter, the cost parameters of their co-players, and the structure of the tournament (A: `SSWW` versus B: `SWSW`) constitutes the only difference across treatments; the wording was identical in all cases.[16] We adopted a between-subject design, such that each participant encountered only one of the four treatments. Each participant played the same tournament 30 times. Note that the endowment could only be used in a given decision round, it could not be transferred. Therefore, the strategic interaction was the same in each of the 30 decision rounds. Random matching in each round ensured that the same participants did not interact repeatedly. After each game, participants were informed about their own decision and about their own payoff. This allows for an investigation whether players learn when completing the task repeatedly. To avoid income effects, however, the participants were told that only four decision rounds (out of 30) would be chosen randomly and paid out at the end of the experiment.

The procedures in an experimental session were as follows for all treatments: First, the participants received some general information about the experimental session. Then, instructions for part one of the experimental session, the two-stage tournament with four players as described above, were distributed. After each participant confirmed that he had understood the instructions on the computer screen, participants were informed about their individual cost parameter. Subsequently, participants had to answer a set of control questions correctly to ensure that they had fully understood the instructions as well as the implication of their personal cost parameter. Only then did the first decision round start. Only at the end of a session were participants informed about their overall payoff in the experiment. We ran a total of eight computerized sessions with 20 participants each using the software z-Tree

[16]Detailed information as well as the instructions for tournament participants are available upon request.

(Fischbacher 2007). In six sessions, we implemented a low degree of heterogeneity between agents, and an equal number of sessions was conducted for settings A: SSWW and B: SWSW, i.e. three sessions for each setting. In two additional sessions, we increased the degree of heterogeneity between strong and weak agents as previously mentioned to test the robustness of our findings. All 160 participants were students from the University of Innsbruck, which were recruited with ORSEE (Greiner 2004). Each session lasted approximately 1.5 hours, and participants earned between 10-20 Euro (approximately 15 Euro on average).

3.2 Results

This section presents experimental evidence to test whether the decisions of the subjects participating in the experiment are consistent with the predictions of Propositions 1 and 2.

Table 1 presents the theoretical predictions for the parameterization of the experiment and compares them to the actual behavior of the subjects in the experiment. The upper part of the table presents results for outcomes in terms of aggregate measures of effort provision. The numbers represent the average of aggregate effort provision in terms of the measure $\mathcal{E}$ across the different sessions of the respective treatment and allow for testing the properties of the tournament regarding incentives as predicted in Proposition 1. Columns (1) and (2) present the results for the baseline parameterization with low heterogeneity ($c_W = 1.50$) for both settings A: SSWW and B: SWSW. The theoretical predictions reflect the relatively better incentive properties of setting A: SSWW in terms of a predicted total effort of 152, as compared to 146.32 in setting B: SWSW. However, it should be noted that the difference is relatively modest (with predicted effort being a mere 4% higher in setting A: SSWW). Compared to this theoretical benchmark, we find substantial overprovision of effort by the subjects in the experiment, with average total effort levels of above 300 in both settings. The substantial overprovision of more than 100% compared to the theoretical prediction is consistent with similar findings in the literature.[17] Also in contrast to the theoretical prediction, the experimental results reveal a higher effort exertion in setting B: SWSW (with 322 compared to 304 in setting A: SSWW), which is potentially due to the high degree of noise which is common in experimental test of tournament theory.[18] It fits into this picture that the difference is not statistically significant (the p-value of a Mann-Whitney u-test of the null that $\mathcal{E}_{\texttt{SSWW}} = \mathcal{E}_{\texttt{SWSW}}$ is 0.51).[19]

[17]See, for example, Sheremeta (2010) or Sheremeta (2011) who finds a high degree of overprovision in tournaments with homogeneous agents when using a similar experimental design. According to the study by Bull, Schotter, and Weigelt (1987), overprovision is also common in tournaments with heterogenous agents.

[18]See Sheremeta (2011) for a potential explanation.

[19]An investigation of the effort provision of the individual subjects of the different types on the two stages of the tournament reveals that these results are mainly driven by the substantial overprovision of effort of players of type W on

Table 1: Aggregate Incentive Provision and Selection

		low het.		high het.	
		A: SSWW (1)	B: SWSW (2)	A: SSWW (3)	B: SWSW (4)
Total Effort ($\mathcal{E}$)	*Data*	**303.751**	**322.171**	**275.440**	**247.480**
	Theory	152	146.32	133	123.72
Selection ($\mathcal{S}$)	*Data*	**0.524**	**0.706**	**0.620**	**0.853**
	Theory	0.604	0.744	0.710	0.921
Observations		450	450	150	150

Note: Aggregate incentives in terms of the sum of total effort exerted by all players on all stages (in experimental currency, Taler). Selection refers to the expected probability of a strong type winning the tournament, conditional on the effort exertion of all players and the Tullock CSF. See text for details.

The lower part of Table 1 presents the respective results for the selection properties as predicted in Proposition 2 in terms of the selection measure $\mathcal{S}$. According to the theory, a player of type S is expected to succeed on stage 2 with probability 0.604 in setting A: SSWW, compared to a probability of 0.744 in setting B: SWSW. To evaluate this hypothesis empirically, we use the realized outcomes in terms of the winner of the tournament being of type S as the respective measure.[20] The results indicate that the theoretical prediction is in line with the data, with the empirical probabilities of 0.524 in setting A: SSWW and 0.706 in setting B: SWSW. This difference in the winning probability in the two settings is statistically significant (the p-value of a Mann-Whitney u-test test is 0.049). Summing up, we find a strong and significant difference between settings A: SSWW and B: SWSW in terms of selection performance, but no significant difference in terms of incentive provision.

To test whether this finding might be due to the modest difference in the theoretically predicted effort across the two settings for this low level of heterogeneity (see Figure 1), we ran additional sessions where the trade-off between the two tournament designs was more pronounced due to a higher level of heterogeneity between the two types. In this high heterogeneity scenario, we set $c_W = 2.50$. The

stage 1 in the setting B: SWSW. The respective results are presented in Table B1 in the Appendix. They also show that overprovision is generally higher for players of type W than of type S, in particular on stage 1 of the tournament. A more detailed investigation of the reasons for this pattern is beyond the scope of this paper.

[20] The results are virtually identical when we compute the predicted probability of the tournament winner being of type S from the experimental data, by using the effort of all players in a given tournament and imputing the corresponding probability using the linear Tullock contest success function that determined the winning probabilities in the experimental implementation. Details are available upon request.

respective results are presented in columns (3) and (4) of Table 1. In this setting, the theoretical predictions about total effort are 133 in setting A: `SSWW` compared to 123.72 in setting B: `SWSW`, i.e., predicted effort is 7.5% higher in setting A: `SSWW`. Overprovision is still substantial in the experimental data compared to the theoretical predictions, but the experimental results are consistent with the prediction of higher effort in setting A: `SSWW`, with 275 compared to 247 in setting B: `SWSW`. Also in terms of selection properties, the difference between the two settings is more pronounced with higher heterogeneity. The probability of a player of type `S` succeeding on stage 2 is 0.62 in setting A: `SSWW` and 0.853 in setting B: `SWSW`. As in the scenario with low heterogeneity, the actual probabilities observed in the experiment closely correspond to this predicted pattern, with 0.71 compared to 0.92 in settings A: `SSWW` and B: `SWSW`, respectively. The scenario with high heterogeneity therefore delivers results that are fully consistent with the predicted trade-off faced by the tournament designer.

In order to investigate to what extent these results are affected by behavioral dynamics across experimental rounds, Table 2 presents the results disaggregated by the different rounds. A commonly found pattern in the literature is that subjects in experiments adapt their behavior as the decisions are repeated, and often converge towards the theoretical benchmark (see, e.g., Bull et al., 1987, Orrison et al., 2004, or Sheremeta, 2010b). The results presented in Table 2 do not support pattern. Rather, throughout all rounds of the experiment, we find results that are qualitatively similar to the aggregate results. We do find a successive reduction in the extent of effort overprovision over the course of the experiment, but this reduction does not affect the qualitative results regarding the trade-off between incentive provision and selection in the different tournament designs.

4 Conclusion

This paper has investigated the properties of different designs of promotion tournaments with heterogeneous participants regarding their incentives for effort provision and their selection performance. The theoretical analysis has revealed a trade-off for the realistic case of a designer who is interested in both the incentives for effort provision and the selection performance of a promotion tournament. We find that the incentive provision is maximized in a design in which workers of *similar* strength compete with each other in early stages of the tournament, while the probability of a strong worker succeeding in being promoted to the highest level is maximized in the opposite case where workers of *different* types interact early in the tournament. The trade-off between those two goals is more fundamental, however, and not restricted to the particular design parameter analyzed in this paper; a tournament designer will face the same problem when optimizing the prize structure, or deciding about handicapping in a given

Table 2: Aggregate Incentive Provision and Selection: Different Rounds

	Rounds	low het. A: SSWW (1)	low het. B: SWSW (2)	high het. A: SSWW (3)	high het. B: SWSW (4)
Total Effort ($\mathcal{E}$)	1-10	**356.40**	**374.80**	**290.26**	**274.42**
	11-20	**293.50**	**302.56**	**280.40**	**240.14**
	21-30	**261.35**	**289.15**	**255.66**	**227.88**
Selection ($\mathcal{S}$)	1-10	**0.47**	**0.77**	**0.58**	**0.88**
	11-20	**0.54**	**0.67**	**0.58**	**0.84**
	21-30	**0.56**	**0.69**	**0.70**	**0.84**
Observations		150	150	50	50

Note: Aggregate incentives in terms of the sum of total effort exerted by all players on all stages (in experimental currency, Taler). Selection refers to the expected probability of a strong type winning the tournament, conditional on the effort exertion of all players and the Tullock CSF. See text for details.

tournament format. A test of these predictions from the theoretical model using experimental methods has delivered evidence that the behavior under different tournament designs is indeed consistent with the existence of such a trade-off.

The results of this paper have important implications, since they suggest that the different measures that have been suggested in the literature to overcome the detrimental effects of heterogeneity on incentives, such as handicapping of strong participants, will lead to worse selection properties of the tournament. Also, the results suggest that heterogeneity is not always associated with worse performance of tournaments, as is suggested by a superficial review of the literature. This paper does not answer the question, however, whether the trade-off between selection performance and incentives does also exist across different tournament formats. Therefore, it would be an interesting topic for future research to analyze whether improvements in both dimensions are possible if a designer moves from a pairwise elimination to other tournament formats like a one-stage tournament or a round robin tournament.

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Appendix

A Proofs

Lemma 1. *Assume without loss of generality $c_W \geq c_S = 1$ and define $f(c_W) = \frac{5c_W^3+2c_W^2+c_W}{c_W^2+2c_W+5}$. Then, the relation $F^*(c_S, c_W) > f(c_W)$ does hold for all $c_W > 1$, where $F^*(c_S, c_W)$ is defined in (8). Further, if $c_W = 1$, it holds that $F^*(c_S, c_W) = f(c_W)$.*

Proof. From equation (7), we know that $\frac{x_{11}^{B*}}{x_{31}^{B*}} = \frac{c_W}{c_S}\frac{P_1(x_{21}^B, x_{41}^B)}{P_3(x_{21}^B, y_{41}^B)}$. Further, equation (8) tells us that $\frac{x_{11}^{B*}}{x_{31}^{B*}} = F^*(c_S, c_W)$. Consequently, using the assumption that $c_W \geq c_S = 1$, it must hold that

$$F^*(c_S, c_W) = c_W \frac{P_1(x_{21}^B, x_{41}^B)}{P_3(x_{21}^B, y_{41}^B)} = \frac{4c_W^3 + c_W(1+c_W)^2 \times \frac{x_{21}^B}{x_{41}^B}}{(1+c_W)^2 + 4 \times \frac{x_{21}^B}{x_{41}^B}}.$$

Note that

$$\frac{\partial F^*(c_S, c_W)}{\partial \frac{x_{21}^B}{x_{41}^B}} = \frac{(1+c_W)^4 - 16c_W^2}{[(1+c_W)^2 + 4 \times \frac{x_{21}^B}{x_{41}^B}]^2} > 0$$

if $c_W > 1$. Further, recall that player 4 has both higher cost ($c_W > 1$) and a lower continuation value ($P_2 > P_4$), such that $x_{21}^B > x_{41}^B$ does hold. Therefore, assuming $x_{21}^B = x_{41}^B$ underestimates $F^*(c_S, c_W)$. Since

$$f(c_W) = \frac{5c_W^3 + 2c_W^2 + c_W}{c_W^2 + 2c_W + 5}$$

is the expression we derive from $F^*(c_S, c_W)$ under this assumption, we have proven $F^*(c_S, c_W) > f(c_W)$. If we assume $c_W = 1$, all players are perfectly symmetric, such that $x_{21}^B = x_{41}^B$ does hold. Consequently, the relation $F^*(c_S, c_W) = f(c_W)$ does hold for $c_W = 1$. □

Lemma 2. *We assume $c_W \geq c_S = 1$ without loss of generality and define $f_{\text{low}}(c_W) = 2c_W - 1$. Then, the relation $f(c_W) > f_{\text{low}}(c_W)$ does hold for all $c_W > 1$. Further, if $c_W = 1$, it holds that $f(c_W) = f_{\text{low}}(c_W)$.*

Proof. We start with the relation that we want to prove, namely:

$$\begin{aligned} f(c_W) &> f_{\text{low}}(c_W) \\ \Leftrightarrow 5c_W^3 + 2c_W^2 + c_W &> (2c_W - 1)(c_W^2 + 2c_W + 5) \\ \Leftrightarrow 3c_W^3 - c_W^2 - 7c_W + 5 &> 0 \end{aligned}$$

We now have to prove that $\phi(c_W) \equiv 3c_W^3 - c_W^2 - 7c_W + 5 > 0$ does always hold for $c_W > 1$. Note that $\phi(c_W)$ has a local minimum at $c_W = 1$, and a local maximum at $c_W = -7/9$. Further, $\phi(1) = 0$. Therefore, it must be that $\phi(c_W) > 0$ for all $c_W > 1$. □

Proof of Proposition 1:
To prove the relation $\mathcal{E}_{\texttt{SSWW}} > \mathcal{E}_{\texttt{SWSW}}$ for all $c_{\texttt{W}} > c_{\texttt{S}}$, we assume without loss of generality that $c_{\texttt{W}} > c_{\texttt{S}} = 1$. In the proof, we will proceed in two steps. First, we derive a necessary and sufficient condition in terms of the function $F^*(c_{\texttt{S}}, c_{\texttt{W}})$ for the relation $\mathcal{E}_{\texttt{SSWW}} > \mathcal{E}_{\texttt{SWSW}}$ to hold. Second, we proof that the equilibrium function $F^*(c_{\texttt{S}}, c_{\texttt{W}})$ which was derived in (8) indeed satisfies this condition. We start with the relation which we want to prove:

$$\begin{aligned} \mathcal{E}_{\texttt{SSWW}} &> \mathcal{E}_{\texttt{SWSW}} \\ \Leftrightarrow \frac{c_{\texttt{W}}^3 + 2c_{\texttt{W}}(1+c_{\texttt{W}}) + 1}{2c_{\texttt{W}}(1+c_{\texttt{W}})^2} &> \frac{(1+c_{\texttt{W}})^2[1+[1+F^*(c_{\texttt{S}}, c_{\texttt{W}})]c_{\texttt{W}}F^*(c_{\texttt{S}}, c_{\texttt{W}})] + 4c_{\texttt{W}}[c_{\texttt{W}}^2 + (1+c_{\texttt{W}})F^*(c_{\texttt{S}}, c_{\texttt{W}})]}{2c_{\texttt{W}}(1+c_{\texttt{W}})^2[1+F^*(c_{\texttt{S}}, c_{\texttt{W}})]^2} \end{aligned}$$

Multiplying both sides by $2c_{\texttt{W}}(1+c_{\texttt{W}})^2[1+F^*(c_{\texttt{S}}, c_{\texttt{W}})]^2$ and rearranging gives

$$F^*(c_{\texttt{S}}, c_{\texttt{W}})^2 + \frac{c_{\texttt{W}}^3 - 2c_{\texttt{W}}^2 - c_{\texttt{W}} + 2}{c_{\texttt{W}}+1} F^*(c_{\texttt{S}}, c_{\texttt{W}}) - \frac{3c_{\texttt{W}}^3 - c_{\texttt{W}}^2}{c_{\texttt{W}}+1} > 0$$

Solving for $F^*(c_{\texttt{S}}, c_{\texttt{W}})$ gives us two conditions:

$$F^*(c_{\texttt{S}}, c_{\texttt{W}}) < \frac{-c_{\texttt{W}}^3 + 2c_{\texttt{W}}^2 + c_{\texttt{W}} - 2 - R(c_{\texttt{W}})}{2c_{\texttt{W}}+2} \quad \vee \quad F^*(c_{\texttt{S}}, c_{\texttt{W}}) > Z(c_{\texttt{W}}) \equiv \frac{-c_{\texttt{W}}^3 + 2c_{\texttt{W}}^2 + c_{\texttt{W}} - 2 + R(c_{\texttt{W}})}{2c_{\texttt{W}}+2},$$

where

$$R(c_{\texttt{W}}) = \sqrt{c_{\texttt{W}}^6 - 4c_{\texttt{W}}^5 + 14c_{\texttt{W}}^4 + 16c_{\texttt{W}}^3 - 11c_{\texttt{W}}^2 - 4c_{\texttt{W}} + 4}.$$

We do only have to consider the second relation, since the first one is below one for some values of $c_{\texttt{W}}$, while $F^*(c_{\texttt{S}}, c_{\texttt{W}}) \geq 1$ for all $c_{\texttt{W}} \geq 1$.[21] This completes the first part of the proof. We now have to prove that

$$F^*(c_{\texttt{S}}, c_{\texttt{W}}) > Z(c_{\texttt{W}}) \equiv \frac{-c_{\texttt{W}}^3 + 2c_{\texttt{W}}^2 + c_{\texttt{W}} - 2 + R(c_{\texttt{W}})}{2c_{\texttt{W}}+2} \tag{A1}$$

for all $c_{\texttt{W}} > 1$. From Lemmata 1 and 2 we know that $F^*(c_{\texttt{S}}, c_{\texttt{W}}) > f_{\text{low}}(c_{\texttt{W}})$. Consequently, a sufficient condition for (A1) is given by $f_{\text{low}}(c_{\texttt{W}}) > Z(c_{\texttt{W}})$. Rearranging this condition gives

$$c_{\texttt{W}}^3 + 2c_{\texttt{W}}^2 + c_{\texttt{W}} > R(c_{\texttt{W}}).$$

Squaring both sides leaves us with[22]

$$\begin{aligned} 2c_{\texttt{W}}^5 - 2c_{\texttt{W}}^4 - 3c_{\texttt{W}}^3 + 3c_{\texttt{W}}^2 + c_{\texttt{W}} - 1 &> 0 \\ \Leftrightarrow 2(c_{\texttt{W}}-1)^2(c_{\texttt{W}}+1)(c_{\texttt{W}} - \frac{1}{\sqrt{2}})(c_{\texttt{W}} + \frac{1}{\sqrt{2}}) &> 0. \end{aligned}$$

This relation is always satisfied if $c_{\texttt{W}} > 1$, which completes the proof.

Proof of Proposition 2:
In this proof, we first derive a necessary and sufficient condition which assures that the relation $\mathcal{S}_{\texttt{SSWW}} < \mathcal{S}_{\texttt{SWSW}}$ does hold in terms of the function $F(c_{\texttt{W}})$. Then, we prove that the equilibrium function $F^*(c_{\texttt{S}}, c_{\texttt{W}})$ satisfies this condition.
(1) As previously, we assume that $c_{\texttt{W}} > c_{\texttt{S}} = 1$ does hold without loss of generality. Consequently, we can use the expressions in equations (13) and (14) in what follows. We start with the relation which

[21] Note that $F^*(1) = 1$; also, we know from Lemma 1 that $\frac{\partial F^*(c_{\texttt{S}}, c_{\texttt{W}})}{\partial c_{\texttt{W}}} > 0$. Therefore, $F^*(c_{\texttt{S}}, c_{\texttt{W}}) \geq 1$ for all $c_{\texttt{W}} \geq 1$.
[22] Note that squaring is without loss of generality here, since we are only interested in solutions for $c_{\texttt{W}} > 1$.

we want to prove:

$$\begin{aligned}
\mathcal{S}_{\texttt{SWSW}} &> \mathcal{S}_{\texttt{SSWW}} \\
\Leftrightarrow (1+c_{\texttt{W}})F^*(c_{\texttt{S}},c_{\texttt{W}})^2 + 2c_{\texttt{W}}F^*(c_{\texttt{S}},c_{\texttt{W}}) &> c_{\texttt{W}}F^*(c_{\texttt{S}},c_{\texttt{W}})^2 + 2c_{\texttt{W}}F^*(c_{\texttt{S}},c_{\texttt{W}}) + c_{\texttt{W}} \\
\Leftrightarrow F^*(c_{\texttt{S}},c_{\texttt{W}})^2 &> c_{\texttt{W}} \\
\Leftrightarrow F^*(c_{\texttt{S}},c_{\texttt{W}}) < -\sqrt{c_{\texttt{W}}} &\vee \quad F^*(c_{\texttt{S}},c_{\texttt{W}}) > \sqrt{c_{\texttt{W}}}
\end{aligned}$$

Note that it is sufficient to show that $F^*(c_{\texttt{S}},c_{\texttt{W}}) > c_{\texttt{W}}$, since $c_{\texttt{W}} > \sqrt{c_{\texttt{W}}}$ for $c_{\texttt{W}} > 1$.
(2) From Lemma 1, we know that $F^*(c_{\texttt{S}},c_{\texttt{W}}) > f(c_{\texttt{W}})$. We will now proof that $f(c_{\texttt{W}}) > c_{\texttt{W}}$ for $c_{\texttt{W}} > 1$ to complete the proof. $f(c_{\texttt{W}}) > c_{\texttt{W}}$ implies that

$$\frac{5c_{\texttt{W}}^3 + 2c_{\texttt{W}}^2 + c_{\texttt{W}}}{c_{\texttt{W}}^2 + 2c_{\texttt{W}} + 5} > c_{\texttt{W}}$$

does hold. Rearranging gives

$$\begin{aligned}
5c_{\texttt{W}}^3 + 2c_{\texttt{W}}^2 + c_{\texttt{W}} &> c_{\texttt{W}}^3 + 2c_{\texttt{W}}^2 + 5c_{\texttt{W}} \\
\Leftrightarrow c_{\texttt{W}}(c_{\texttt{W}}^2 - 1) &> 0 \\
\Leftrightarrow c_{\texttt{W}} > 1 &\vee \quad -1 < c_{\texttt{W}} < 0
\end{aligned}$$

This proves the claim $\mathcal{S}_{\texttt{SWSW}} > \mathcal{S}_{\texttt{SSWW}}$ for all $c_{\texttt{W}} > 1$.

B Additional Tables

Table B1: Individual Effort Provision in Stage 1 and Stage 2

		low heterogeneity				high heterogeneity			
		A: `SSWW`		B: `SWSW`		A: `SSWW`		B: `SWSW`	
		`S`	`W`	`S`	`W`	`S`	`W`	`S`	`W`
Effort Stage 1	*Data*	**45.07**	**28.96**	**51.39**	**32.06**	**71.86**	**10.77**	**34.92**	**10.80**
	Theory	22	6	14	6	30	2	7	1
Effort Stage 2	*Data*	**83.64**	**72.05**	**89.71**	**55.65**	**69.09**	**41.09**	**92.30**	**38.39**
	Theory	58	38	59	39	49	20	56	21

Note: Individual effort provision (in experimental currency, Taler) for strong types `S` with effort cost $c_S = 1.00$ and weak types `W` with effort cost $c_W = 1.50$.