Do Targeted Hiring Subsidies and Profiling Techniques Reduce Unemployment?

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ABSTRACT

Do Targeted Hiring Subsidies and Profiling Techniques Reduce Unemployment?

To reduce unemployment targeted hiring subsidies for long-term unemployed are often recommended. To explore their effect on employment and wages, we devise a model with two types of unemployed and two methods of search, a public employment service (PES) and random search. The eligibility of a new match depends on the applicant's unemployment duration and on the method of search. The hiring subsidy raises job destruction and extends contrary to *Mortensen-Pissarides* (1999, 2003) the duration of a job search, so that equilibrium unemployment increases. Like the subsidy, organizational reforms, which advance the search effectiveness of the PES, crowd out the active jobseekers and reduce overall employment as well as social welfare. Nevertheless, reforms are a visible success for the PES and its target group, as they significantly increase the service's placement rate and lower the duration of a job search via the PES.

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1 INTRODUCTION

To fight unemployment, instruments of active labour market policy (ALMP) have been in use for many years in Europe. The quest for such policy measures continues to be central to the policy debate, particularly in the large European countries with persistently high unemployment. To reduce intervention costs and deadweight losses, ALMP instruments target on special groups of unemployed, particularly on the long-term unemployed. In addition, it is often recommended, for instance in the European Employment Strategy (*European Commission* 2006) or in the German "Hartz Proposals" (*Hartz* et al. 2002), that the public employment service (PES) increases its search effectiveness. A wide variety of tools have been proposed to achieve this objective. The implicit assumption behind all PES reforms is that coordination failures hamper the labour market, which can be overcome by ALMP and a comprehensive search assistance by the PES.

To analyze the effects of targeted hiring subsidies and enhancement strategies, we use a search model of the Mortensen-Pissarides type with an endogenous separation rate which combines two methods of search, the PES and random search. Workers are employed or unemployed, and if unemployed they are either short-term, medium-term or long-term unemployed. Firms must retrain entrants and pay the training costs. If eligible, they receive a job creation assistance. Eligibility depends on the worker status and the method of search. Two subsidy regimes are compared. In the first regime, only placements by the PES are eligible, in the second also matches established by random search take part in the subsidy competition.

The paper is structured as follows. Section 2 reviews the literature. Section 3 develops the model. Section 4 presents simulation results. Section 5 concludes. The Appendices I-II contain a summary of the model equations and definitions of some indicator functions, which are used in order to evaluate the simulation outcomes.[1]

2 LITERATURE

There is a growing empirical literature estimating the short-term employment effects for the participants of ALMP programmes. Even if the net effects of hiring subsidies and profiling

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[1] Proofs can be found in Appendix III which is available upon request.

measures are positive, the effect on the aggregate rate of unemployment remains inconclusive (*Heckman* et al. 1999, *Kluve* 2006). One reason may be that there are interactions between ALMP participants and other employed or unemployed jobseekers, specifically crowding out and general equilibrium effects. Without incorporating them into a macro framework, the microeconometric treatment effects will provide poor guides to policy makers.

Matching models are a powerful instrument to analyze the macroeconomic impact of ALMP. The theoretical literature of the effect of hiring subsidies and instruments which increase the efficiency of the PES on the aggregate rate of unemployment is still in its infancy and relatively small (*Yashiv* 2006 and *Rogerson* et al. 2005). Moreover, to our knowledge, the literature has paid little attention to the effects of instruments that target on particular groups of the unemployed. Furthermore, this strand of the literature does not take into account that ALMP can influence the matching process itself (*Brown* et al. 2006, 4f). It is the objective of our paper to address these questions.

Millard and *Mortensen* (1997), *Mortensen* and *Pissarides* (1999, 2003) and *Pissarides* (2000, ch. 9) analyze the effects of non-targeted hiring subsidies on equilibrium unemployment. The labour market is characterized by a matching technology which represents the two-sided search process with its frictions arising from imperfect information, mobility costs and worker and job heterogeneities. Each new match of a jobseeker with a vacancy is eligible for a hiring subsidy. The hiring subsidy increases both the number of vacancies and the amount of job destruction. Therefore, its overall effect on equilibrium unemployment is ambiguous. *Millard* and *Mortensen* (1997) and *Mortensen* and *Pissarides* (1999, 2003) thus simulate the net effects of the subsidy and show that the hiring subsidy increases the aggregate unemployment rate.

Yashiv (2004) addresses the question whether hiring subsidies, employment subsidies, unemployment benefits and wage tax reductions under a given budget constraint have substantial effects on labour market outcomes and the business cycle when labour market frictions are present. Within a stochastic, discrete-time version of a matching model, he shows that hiring subsidies reduce unemployment while employment subsidies and wage tax cuts do not.

The positive effect of the hiring subsidy is partly induced as *Yashiv* (2006, 276) states by the setup of a model that does not allow for endogenous separations.

Brown et al. (2006) introduce self-financing hiring vouchers targeted at the long-term unemployed and unskilled workers into a dynamic model setting. Their calibrated model shows that in contrast to wage subsidies hiring vouchers can raise employment, are "approximately welfare efficient" and reduce inequality. The model setup determines that entrants or outsiders earn a wage that is negotiated between the insiders and the firm. The hiring voucher does therefore not influence the outsider wage and the initial labour costs of the firm, which pockets the entire subsidy. While the entrant's hiring rate increases with the cash inflow from the voucher, the insider's firing rate is unaffected.

Within a general equilibrium search model *Vereshchagina* (2002) evaluates the long-run implications of shifting unemployment benefits into a subsidy programme targeting the long-term unemployed. In general, the effect of the subsidy is ambiguous. On the one hand, the introduction of a subsidy reduces government's expenditure on unemployment benefits and weakens the distortionary effects of the unemployment insurance system by putting the unemployed back to work. On the other hand, the subsidy programme puts additional economic pressure on the government budget caused by the subsidy. The higher the elasticity of labour demand, the more likely it is that the subsidy programme generates a government budget surplus and positively affects welfare.

Our discrete-time model differs from the above-mentioned in the following respects. First, two competing search methods are available, the PES and random search (*Pissarides* 1979). Second, the unemployed choose between a passive and an active search strategy. Third, we distinguish between two types of unemployed namely the short-term unemployed (STU) and the medium-term and long-term unemployed (MLTU). In view of the response time which the PES needs to make a first job offer to a newly registered unemployed, STU workers can count on no search assistance from the PES.

Fourth, the matching process comprises three stages. In the last stage, the PES matches the stock of registered vacancies with the stock of registered MLTU workers. In the second stage, the stock of active MLTU workers randomly searches for a vacancy. In the first stage, the

active jobseekers among the inflow of STU workers is matched with the stock of vacancies. Thus, the model combines stock-flow with random matching.

Fifth, most of the above-mentioned models assume an exogenous job destruction rate. In these models, ALMP primarily affects job creation rather than job destruction which seems counter-factual. We build on the theoretical matching literature by adding an endogenous job destruction rate (*Mortensen* and *Pissarides* 1994, 1999, 2003).

Sixth, we investigate two instruments of active labour market policy, organizational reforms to advance the placement effectiveness of the PES and a hiring subsidy. In most of the above-mentioned papers, the subsidy is a lump-sump payment, which creates a pure quasi-rent. In *Mortensen* and *Pissarides* (1999, 2003), the subsidy offsets a fixed job creation cost of the employers. In the present paper, firms bearing stochastic training costs to reintegrate the MLTU workers into production compete for the subsidy. Training costs are compensated up to an endogenous subsidy limit. We compare two subsidy regimes. In the first, only matches with a MLTU worker arranged by the PES are eligible. In the second, also matches established through random search compete for the hiring subsidy.

The calibrated version of the model generates the following results. Both instruments of ALMP increase equilibrium unemployment. They induce both a fundamental change of the equilibrium composition of search methods, boosting passive and curbing active job search. They raise the equilibrium fraction of MLTU workers and the duration of unemployment, and reduce social welfare. Both instruments are a visible and significant success for the PES and its target group. For example, reforms to advance the search effectiveness of the PES boost the service's placement rate and reduce the average duration of a job search via the PES. Active policy approaches like the "Hartz-reforms" recommend increasing the productivity of the PES in order to overcome the coordination failures of the labour market. This popular *ceteris paribus* reasoning is not confirmed by our simulation results.

3 THE MODEL

3.1 EQUILIBRIUM UNEMPLOYMENT

A discrete-time model is devised with two methods of search, random search and organized search by a PES, and two types of unemployed. The first type is the STU workers who have lost their job at the end of the previous period. The second type has been unemployed for at least one period and is either medium-term or long-term unemployed (MLTU). Jobseekers choose between an active and a passive search strategy. Active jobseekers use both methods of search; passive jobseekers wait for a placement by the PES. The search strategy of the vacancies is not specialized; they are simultaneously advertised and registered with the PES.

Matching process. The timing of the matching process is as follows (Figure 1). In the first stage, the inflow of active STU workers meet the advertised vacancies. Active STU have the best information about current labour market conditions and apply earlier than the PES or the active MLTU. In the second stage, vacancies are matched with the active MLTU. In the last stage, the PES arranges matches between the registered vacancies and the registered unemployed. The meetings of the first and second stage are not observed by the PES, so a fraction of the PES matches do not fit. This can happen either because the selected vacancies are already occupied or the selected jobseekers have already found a job.

Training costs. Each match with a MLTU worker causes match-specific training costs $t \geq 0$, of which *ex ante* only the c.d.f. $F(t)$ with support $0 \leq t < \infty$ and the endogenous reservation costs T_i are known. Reservation costs depend on the search method $i = P, S$. T_P are the training costs up to which vacancies are ready to take on applicants placed by the PES. T_S are the reservation costs of the method of random search. A match, which is arranged through the search method i, is dissolved, if firm and worker draw training costs $t > T_i$.

Separation rate. The labour force is normalised to one. Job creation takes place at the beginning and job destruction at the end of a period. Of the $1 - u$ employed, $I \equiv \lambda G(R)(1 - u)$ lose their job at the end of a period. $\lambda G(R)$ is the unemployment incidence where λ is the probability of an idiosyncratic productivity shock, $G(x)$ with support $0 \leq \alpha \leq x \leq 1$ is the c.d.f. of the multiplicative shock x and R is the endogenous reservation productivity. yx, with

$y > 0$, is the flow output of a job. If a firm draws x and $R \leq x \leq 1$, the job is continued. If $x < R$, the match is dissolved, and the worker becomes unemployed.

[Figure 1 about here]

Transition probabilities. Of the inflow of I STU workers, S_I opt for active job search. As the STU are at least as productive as the MLTU and do not cause training costs, it does not pay for a vacancy to wait for the subsequent stages of the matching process. Therefore, $m(S_I, v)$ of the v advertised vacancies are filled by the active STU. The matching technology $m(s, v)$ of the method of random search has constant returns to scale and is strictly concave and monotone in both arguments. For a given vacancy, the probability of a match with a STU worker is $q(\theta_I) \equiv m(1/\theta_I, 1) = m(S_I, v)/v$, where $\theta_I = v/S_I$ is the stock-flow tightness of the labour market in the first stage. For convenience, we write $q_I = q(\theta_I)$ and $p_I \equiv p(\theta_I) = \theta_I q(\theta_I)$. In the steady state p_I, the transition probability of an active STU worker, is equal to the proportion of newly unemployed jobseekers who match immediately after separation (*Forslund* and *Johansson* 2007).

The other $I - S_I \geq 0$ workers of the inflow wait for a placement through the PES. Passive STU cannot expect a job offer until the second period of the unemployment spell. The reason for this delay is the response time of the PES, which is the time the agency needs to process the worker's data and to prepare a first job offer. The German Federal Employment Office estimates an average response time of three months for 2005. We assume that the response time of the PES lasts one period plus the duration of a job search. One period is equal to a quarter of a year in the calibrated model version.

The number of STU, who either do not find a job or prefer to wait for the PES, u_S, is given by:

$$I - p_I S_I = u_S . \tag{1}$$

u_S is the inflow into the pool u of MLTU workers, of which S opt for random search. They face the same v advertised vacancies of which $q_I v$ are already occupied with active STU. $m(S, v)$ represents the measure of contacts, and $q(\theta_S) \equiv m(1/\theta_S, 1) = m(S, v)/v$ is the contact probability of a given vacancy with an active MLTU jobseeker, where $\theta_S = v/S$ is the tight-

ness of the labour market in the second stage. The contact probability for a given jobseeker is $p(\theta_S) = \theta_S q(\theta_S)$, where we write $q_S = q(\theta_S)$ and $p_S = p(\theta_S)$. Of the S active MLTU $p_+ F(T_S)S$ draw training costs $t \leq T_S$ and make a transition into employment, where $p_+ = p_S(1 - q_I)$.

The pool $u - p_+ F(T_S)S$ consists of MLTU workers who either prefer to wait for the PES or who have decided for active job search but did not find a job or even hit on a vacancy but the job was already filled or the vacancy was still available but the match would have caused too high training costs. These workers have a last chance to be placed by the PES. The probability of a suitable placement arranged by the PES is $P_+ F(T_P)$, where $P_+ = P(1 - q_I)(1 - q_S)$. P is the probability of a contact with a vacancy found by the PES and $(1 - q_I)(1 - q_S)$ is the probability that the vacancy is not yet filled by any of the active jobseekers.

The contact probability P is the output of the placement activities of the PES. For given search effort ef, $M(ef, u, v)$ is the measure of arranged contacts between the v registered vacancies and the u registered unemployed, where $ef \in [0,1]$. The production function of the PES is multiplicative separable with respect to ef, so that $M(ef, u, v) = \mu(ef)M(u, v)$. The effort function $\mu(ef)$, with $\mu(0) = 0$, is strictly increasing and exhibits diminishing returns. The matching function of the PES, $M(u, v)$, has standard properties. $Q(ef, \Theta) \equiv \mu(ef)M(1/\Theta, 1) = \mu(ef)M(u, v)/v$ is the contact probability of a given vacancy with a worker via the PES with $\Theta = v/u$ denoting the tightness between both registers, and $P(ef, \Theta) = \Theta Q(ef, \Theta)$ is the contact probability of a given worker with a registered vacancy. We write $P = P(ef, \Theta)$ and $Q = Q(ef, \Theta)$.

Considering the flows out of the pool u and taking into account the inflow u_S into u yields the steady state condition:

$$u_S = p_+ F(T_S)S + P_+ F(T_P)[u - p_+ F(T_S)S]. \tag{2}$$

LEMMA 1 [UNEMPLOYMENT]. (i) *If a newly unemployed worker opts for a placement through the PES, he will be out of work for D_P periods. In view of the response time of the PES, D_P is given by $D_P = 1 + d_P$, where $d_P = 1/P_+ F(T_P)$.*

(ii) *An unemployed who combines the passive search strategy in the first period of his unemployment spell with the active search strategy in all subsequent periods faces a duration of*

unemployment of $D_S = 1 + d_S$ periods, where $d_S = 1/[p_+ F(T_S) + [1 - p_+ F(T_S)] P_+ F(T_P)]$ is the expected duration of job search of an active MLTU worker. Obviously, $D_S < D_P$.

(iii) *A newly unemployed worker who opts for the active search strategy can expect a spell length of $D_I = p_I 0.5 + (1 - p_I) D_S$ periods, where we assume to overcome time aggregation problems that the workers who immediately after separation find a new job face a duration of unemployment equal to half a period.*

(iv) *With the share of active MLTU $\sigma_S = S/u \leq 1$ and the ratio of active jobseekers among the STU $\sigma_I = S_I/u$, the steady-state equilibrium stock of unemployed u is*

$$u = \frac{\lambda G(R)}{\lambda G(R) + \sigma_I p_I + \sigma_S / d_S + (1 - \sigma_S)/d_P}. \tag{3}$$

(v) *Let $S_I = I$, so that all newly unemployed workers actively search. Then the expected duration of a job search D of a worker who is randomly drawn from the inflow I is given by*

$$D = p_I 0.5 + (1 - p_I)[\sigma_S D_S + (1 - \sigma_S) D_P]. \tag{4}$$

Hiring subsidy. The government assigns to the PES an exogenous subsidy budget B which pays a job-specific creation assistance $\eta \geq 0$ to firms that employ a MLTU worker. η is disbursed as soon as the training costs $t \geq 0$ are observed, where due to the subsidy limit H: $\eta = \min\{t, H\}$. Matches with training costs $t - H > 0$ have to finance the balance out of the match rent.

There are two subsidy regimes $\tau \in \{0,1\}$. If only matches arranged by the PES are subsidized, $\tau = 0$. Otherwise, if all matches with a MLTU worker are eligible, $\tau = 1$. The PES sets H, the endogenous subsidy limit, such that for given B the following budget constraint is fulfilled:

$$B = P_+[u - p_+ F(T_S)S]\left[\int_0^H t dF(t) + H[F(T_P) - F(H)]\right] + \\ \tau p_+ F(T_S)S\left[\int_0^H t dF(t) + H[F(T_S) - F(H)]\right] \tag{5}$$

The first summand on the right hand side is the subsidy demand of the matches arranged by the PES, the second summand is the demand of the matches established through random search, which are entitled only in the regime $\tau = 1$. The overall subsidy demand is a continuous and strictly increasing function of the subsidy limit per eligible job H and equal to zero if $H = 0$. Thus, for all $B \geq 0$ a unique subsidy limit $H \geq 0$ exists, such that the budget constraint (5) is fulfilled.

3.2 FILLED JOBS AND EMPLOYED WORKERS

An employment contract $[w_i(t), w(x), R]$ includes the outside wage $w_i(t)$, the inside wage $w(x)$, and the reservation productivity R at which the firm will destroy the job. Wages are paid at the end of a period. The initial wage depends on the training costs and on the job-seeker type, where $i = I$ if the applicant is a STU worker, and $i = P, S$ if the applicant is placed by the PES or is an active MLTU jobseeker, respectively.

Continuation period. Let $\Pi(x)$ be the present value of a filled job with idiosyncratic productivity $x \in [\alpha, 1]$. The steady state asset price equation for $\Pi(x)$ is

$$\Pi(x) = \rho \left\{ yx - w(x) + \lambda \int_R^1 \Pi(h) dG(h) + (1-\lambda)\Pi(x) \right\} . \tag{6}$$

Flow and stock variables are discounted at the rate ρ, where $0 < \rho = 1/(1+r) < 1$ with the real interest rate $r > 0$. With probability λ the job is hit by a shock and the new productivity is h. If $R \leq h \leq 1$, the job is continued and its value is $\Pi(h)$ otherwise, if $h < R$ the job is destroyed, where R is the reservation productivity for which $\Pi(R) = 0$.

The present value of the worker $W(x)$ is

$$W(x) = \rho \left\{ w(x) + \lambda \left[\int_R^1 W(h) dG(h) + G(R)U_I \right] + (1-\lambda)W(x) \right\}. \tag{7}$$

If a shock $h \geq R$ occurs, the value of the worker is $W(h)$. If $h < R$ the job is destroyed, the worker becomes a STU jobseeker and his value is U_I.

Training period. To re-integrate the MLTU workers into the production, firms must spend training costs t.[2] The outside wage $w_P(t)$ and the initial value of the job $\Pi_P(t)$ as well as the initial value of the worker $W_P(t)$ depend on t if t exceeds the subsidy limit H. The present value before training costs t and subsidy payments $\eta = \min\{t, H\}$ of a job filled with a MLTU worker who is placed by the PES is given by

$$\Pi_P(t) = \rho \left\{ y - w_P(t) + \lambda \int_R^1 \Pi(h) dG(h) + (1-\lambda)\Pi(1) \right\}, \tag{8}$$

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[2]   The asset equations for a match with a STU worker can be found in the Appendix III, see equations (B3), (B4).

while the present value of the worker during the training period is

$$W_P(t) = \rho\left\{w_P(t) + \lambda\left[\int_R^1 W(h)dG(h) + G(R)U_I\right] + (1-\lambda)W(1)\right\}. \tag{9}$$

If no shock arrives, the match enters the continuation period with productivity $x = 1$, and job and worker take on the values $\Pi(1)$ and $W(1)$ respectively.

3.3 JOB CREATION

Market entry for vacancies is free but feasible only in the first of the three subsequent stages of the matching process. Due to the perfectly elastic supply in the first stage, the asset value of vacancies is driven to zero and the job creation condition is $0 = -k + q_I \Pi_I + (1-q_I)V_I$, where k is the flow recruiting cost, q_I is the probability of a match with a STU worker, Π_I is the initial job value, and V_I is the value of a vacancy's outside option.

Three factors are responsible for $V_I \geq 0$. First, the search strategy of a vacancy is not specialised, all vacancies are registered and advertised. Second, a vacancy that is not filled during the first stage has the option to meet a jobseeker in the second or third stage of the matching process. Third, the supply of vacancies is perfectly inelastic in the last two stages, such that in equilibrium $V_I \geq 0$, where V_I is given by

$$V_I = q_S V_S + (1-q_S)Q_+ V_P. \tag{10}$$

q_S is the probability that the vacancy will meet an active MLTU worker, and V_S is the value of the vacancy in the second stage. If the vacancy does not meet an active MLTU, then it still has the chance to contact a worker placed by the PES. The probability of a suitable match with a worker placed by the PES is $Q_+ = Q[1 - p_+ F(T_S)\sigma_S]$, where Q is the contact probability and $[1 - p_+ F(T_S)\sigma_S]$ is the probability that the worker is still available and the training costs are feasible. The probability that the PES arranges a match with a registered worker who already found a job in the preceding stage is $p_+ F(T_S)\sigma_S$, where $\sigma_S = S/u$ is the share of active MLTU workers. Finally, V_P is the value of the vacancy in the final stage.

Outside options. Firms know the c.d.f. of the training costs $F(t)$, the reservation costs T_i, $i = P, S$, and the subsidy limit H, which is announced by the PES. Before the training costs are revealed, the value of a vacancy which is in contact with a worker placed by the PES is

$$V_P = F(H)\Pi_P + \int_H^{T_P} [\Pi_P(t) + H - t] dF(t). \tag{11}$$

If the training costs are fully refunded, an event with probability $F(H)$, the job has the initial value Π_P. The integral expression in (11) denotes the value of the job if the training costs are not fully refunded but are below T_P. If the training costs exceed T_P, the match dissolves.

Before the training costs are known, the conditional option value of a vacancy that meets an active jobseeker among the MLTU in the second stage of the matching process is

$$V_S = F(\tau H)\Pi_S + \int_{\tau H}^{T_S} [\Pi_S(t) + \tau H - t] dF(t) + [1 - F(T_S)]Q_+ V_P. \tag{12}$$

If the training costs exceed T_S, the match is dissolved and, in view of the third stage, the vacancy takes on the expected value of its outside option $Q_+ V_P$.

Reservation costs. The hiring subsidy refunds the training costs up to the subsidy limit H. The allocation of the remaining costs is part of the contract negotiation, and the value of a job, which is filled with a worker who was placed by the PES, $\Pi_P(t)$, depends on t. As will be shown in Lemma 3, $\Pi_P(t)$ is an increasing function of t, while the net value of the job, $\Pi_P(t) + H - t$, is a contraction and fulfils the reservation property with respect to t. Hence, reservation costs T_P exist for which

$$T_P = \Pi_P(T_P) + H. \tag{13}$$

A job which meets an active MLTU worker will only be filled if its net value exceeds the value of the outside option $Q_+ V_P$, such that $\Pi_S(t) + \tau H - t \geq Q_+ V_P$. Thus, the reservation training costs T_S of the method of random search are given by:

$$T_S = \Pi_S(T_S) + \tau H - Q_+ V_P. \tag{14}$$

3.4   JOBSEEKERS AND WAGES

Search strategy. Given the unemployment benefit b, STU workers opt for the search strategy which maximises their value U_I

$$U_I = \max \{\rho(b + U), \ -c_I + p_I W_I + (1 - p_I)\rho(b + U)\}. \tag{15}$$

The Bellman equation (15) represents the choice set of the STU workers. A worker who prefers search via the PES receives at the end of the period the unemployment benefit b and takes on the value U. In case the worker opts for random search, he would have to bear search costs $c_I > 0$. With probability p_I, he will locate a vacancy, and his value is W_I. With probability $1 - p_I$, his search fails.

In order to determine U, note that at the beginning of the subsequent period the worker is still unemployed and has to decide whether to wait for a placement by the PES or to search actively. In the first case, his value is U_P, in the second, it is U_S. The worker will choose the strategy that maximises his present value so that $U = \max \{U_P, U_S\}$. We will focus on equilibria in which the number of active MLTU jobseekers, S, is an interior solution to the model, while all newly unemployed workers prefer random search, such that $0 < S < u$ and $S_I = I$, respectively. Thus, the MLTU workers are indifferent in the equilibrium between a placement via the PES or random search, so that $U = U_P = U_S$.

The value of a MLTU worker, U_P, who is waiting for a placement by the PES is given by

$$U_P = P_+ \left[F(H)W_P + \int_H^{T_P} W_P(t) dF(t) \right] + \left[1 - P_+ F(T_P) \right] \rho(b + U). \qquad (16)$$

If the vacancy to which the worker is matched is not yet filled, the probability for this event is P_+, his value is W_P provided that the hiring subsidy offsets the training costs, an event with probability $F(H)$. Otherwise, if the training costs are not fully refunded, the expected value of the worker is given by the integral expression in (16). If the training costs exceed T_P, firm and applicant separate, and the present value of the worker is $\rho(b + U)$ as in the other two cases where the vacancy is already filled, or the worker is not offered a vacancy by the PES. These events, where the job search fails for one or the other reason, have the composite probability $1 - P_+ F(T_P)$.

If the MLTU worker decides for random search, he will incur search costs $c_S > 0$. Considering the probability of a suitable match p_+, his value U_S is

$$U_S = -c_S + p_+ \left[F(\tau H)W_S + \int_{\tau H}^{T_S} W_S(t) dF(t) \right] + \left[1 - p_+ F(T_S) \right] U_P \qquad (17)$$

If the job search fails, either because the worker is confronted with a vacancy already filled or because he incurs training costs that exceed T_S or because he does not meet a vacancy, a composite event with the probability $1 - p_+ F(T_S)$, his value is equal to U_P, because placement via the PES concludes the search process.

Wage negotiations. Each match generates a monopoly rent that is distributed between firm and worker through the wage. The distribution rules are obtained according to the generalised Nash solution to a bargaining problem, with $\beta \in (0,1)$ denoting the bargaining strength of the jobseeker.

If a vacancy meets a STU worker, the outside wage w_I is derived from the sharing rule

$$W_I - U_I = \frac{\beta}{1 - \beta}(\Pi_I - V_I). \tag{18}$$

If the vacancy meets a MLTU worker, the sharing rule depends on whether the PES refunds the training costs. For wage negotiations with an active MLTU, the sharing rules are

$$W_S(t) - U_S = \begin{cases} \dfrac{\beta}{1 - \beta}[\Pi_S - Q_+ V_P], & \text{for } 0 \leq t \leq \tau H \\[2mm] \dfrac{\beta}{1 - \beta}[(\Pi_S(t) + \tau H - t) - Q_+ V_P], & \text{for } \tau H \leq t \leq T_S \end{cases}, \tag{19}$$

The next equations show the sharing rules of a match with a MLTU worker arranged by the PES:

$$W_P(t) - U_P = \begin{cases} \dfrac{\beta}{1 - \beta}\Pi_P, & \text{for } 0 \leq t \leq H \\[2mm] \dfrac{\beta}{1 - \beta}[\Pi_P(t) + H - t], & \text{for } H \leq t \leq T_P \end{cases}. \tag{20}$$

Taking into account the idiosyncratic shock $x \in [R,1]$, the value of a STU worker U_I and the fact that the asset price of a vacancy at the first stage of the search process is equal to zero, the sharing rule implemented by negotiations with an insider is

$$W(x) - U_I = \frac{\beta}{1 - \beta}\Pi(x). \tag{21}$$

Bargained wages. To determine the bargained wages we need to know the reservation wages of the different worker types which depend on the expected returns from random search, where J_I and J_S denote the expected returns of the STU and the MLTU jobseekers, respectively. Taking account of the expected search costs c_I / p_I and c_S / p_+, and considering the fact that jobseekers can avoid active job search by waiting for a PES placement, we get $J_I = \max \left\{ 0, (W_I - U_I) - c_I / p_I \right\}$ and $J_S = \max \left\{ 0, \int_0^{T_S} [W_S(t) - U_S] dF(t) - c_S / p_+ \right\}$.

Considering the asset equations (6)–(9), (16) and the sharing rules (18)–(21), we obtain

LEMMA 2 [BARGAINED WAGES]. (i) *The difference between the reservation wages of a MLTU worker who waits for a placement by the PES and an active STU worker depends on the differential rent $U_P - U_I$, which is determined by*

$$U_P - U_I = \frac{\beta}{1-\beta} \frac{P_+ V_P}{[1 - P_+ F(T_P)]} - \frac{p_I}{1-p_I} J_I. \tag{22}$$

(ii) *The reservation wage of the insiders, rU_I, is given by*

$$rU_I = b + \frac{\beta}{1-\beta} \frac{P_+ V_P}{[1 - P_+ F(T_P)]} + \frac{p_I}{1-p_I} rJ_I, \tag{23}$$

(iii) *The bargained inside wage at $x \in [R,1]$ is*

$$w(x) = rU_I + \beta(yx - rU_I). \tag{24}$$

(iv) *The outside wages of the STU workers and the MLTU workers if the PES refunds the training costs are*

$$w_i = \begin{cases} w(1) - \beta V_I \rho^{-1}, & i = I \\ w(1) + (1-\beta)(U_P - U_I)\rho^{-1}, & t \leq H \text{ and } i = P \\ w(1) + (1-\beta)(U_S - U_I)\rho^{-1} - \beta Q_+ V_P \rho^{-1}, & t \leq \tau H \text{ and } i = S \end{cases}, \tag{25}$$

where $w(1)$ is the inside wage for $x = 1$, and $\rho^{-1} = 1 + r$.
(v) *If the training costs exceed H or τH, the outside wages are*

$$w_i(t) = \begin{cases} w_P - \beta(t - H)\rho^{-1}, & H \leq t \leq T_P \text{ and } i = P \\ w_S - \beta(t - \tau H)\rho^{-1}, & \tau H \leq t \leq T_S \text{ and } i = S \end{cases}, \tag{26}$$

where the outside wages w_P and w_S are given by (25).

Scar of unemployment. Newly employed workers suffer from a scar of unemployment. The outside options, which the employers can claim during the wage negotiations with a jobseeker, and, in the case of a MLTU worker, the training costs are the reasons for the wage penalty which a jobseeker has to accept. Take for example the sharing rule (18) which shows, that the value of the outside option V_I reduces the share of the current match rent a STU worker can appropriate in the wage negotiation. While V_I refers to the beginning of the period in the discrete-time model, wages are paid at the end. Therefore, V_I is discounted in (25) to the end of the period.

3.5   VALUE OF THE FILLED JOBS AND JOB DESTRUCTION

With the wage equations from Lemma 2, the asset equations from Section 3.2, and the condition of the reservation productivity, $\Pi(R) = 0$, we can now derive the value of the filled jobs.

LEMMA 3 [FILLED JOBS]. (i) *The continuation value of a filled job producing with the idiosyncratic productivity $x \in [R,1]$ is*

$$\Pi(x) = (1-\beta)y\frac{x-R}{\lambda + r}.$$

(27)

(ii) *The initial asset value of a job filled by a STU worker or a MLTU worker if the training costs are refunded by the PES is given by*

$$\Pi_i = \begin{cases} \Pi(1) + \beta V_I, & i = I \\ \Pi(1) - (1-\beta)(U_P - U_I), & t \leq H \text{ and } i = P \\ \Pi(1) - (1-\beta)(U_S - U_I) + \beta Q_+ V_P, & t \leq \tau H \text{ and } i = S \end{cases}$$

(28)

(iii) *A job filled by a MLTU worker whose training costs exceed H or τH has the asset price*

$$\Pi_i(t) = \begin{cases} \Pi_P + \beta(t - H), & H \leq t \leq T_P \text{ and } i = P \\ \Pi_S + \beta(t - \tau H), & \tau H \leq t \leq T_S \text{ and } i = S \end{cases}$$

(29)

With the above equations for the filled jobs, we can derive the reservation costs T_P and T_S, where we use the fact that in the equilibrium the MLTU workers are indifferent between the two methods of search, so that $U_S = U_P$.

LEMMA 4 [RESERVATION COSTS AND OUTSIDE OPTIONS]. (i) *The reservation costs T_P which are applied to the MLTU workers who are placed by the PES follow from (29) together with $\Pi_P(T_P) + H - T_P = 0$:*

$$T_P = \frac{\Pi_P}{1-\beta} + H . \tag{30}$$

From the asset pricing equations (28)-(29), and $T_S = \Pi_S(T_S) + \tau H - Q_+ V_P$ we can derive the reservation costs for applicants who are active MLTU:

$$T_S = T_P - (1-\tau)H - Q_+ V_P . \tag{31}$$

(ii) From the above reservation costs and the asset equations of the outside options (11) and (12) we get the values of the outside options:

$$V_S = (1-\beta) \left[F(\tau H)(T_S - \tau H) + \int_{\tau H}^{T_S} (T_S - t) dF(t) \right] + [1 - F(T_S)] Q_+ V_P \tag{32}$$

$$V_P = (1-\beta) \left[F(H)(T_P - H) + \int_{H}^{T_P} (T_P - t) dF(t) \right]. \tag{33}$$

There are two reasons for the fact that $T_P - T_S > 0$ as equation (31) points out, first the hiring subsidy and second the sequential matching process. However, H has a direct impact only in the subsidy regime $\tau = 0$, where only PES placements are supported. If all matches of MLTU workers are subsidized, then $\tau = 1$, and the subsidy has no direct impact on the reservation cost differential $T_P - T_S = Q_+ V_P$.

The job destruction condition can be derived given the wage equation (24) by evaluating the asset equation for a filled job (6) at the reservation threshold $x = R$. We obtain

$$R = \frac{rU_I}{y} - \frac{\lambda}{\lambda + r} \int_R^1 (h - R) dG(h). \tag{34}$$

Through the reservation wage of the insiders, rU_I, the odds in favour of finding a job through random search immediately after a separation and the annuity of the capital gain from active job search positively influence the reservation productivity R.

Random search. Taking account of the sharing rules (18) and (19), the asset equation (28) for the initial value of a job filled with a STU worker, Π_I, and the equation (12) for the option value V_S, we can rewrite the gains from active job search for the STU and MLTU workers as in Lemma 5.

LEMMA 5 [RANDOM SEARCH]. (i) *The expected gain from active job search by the STU workers is given by:*

$$J_I = \max\left\{0, \frac{\beta}{1-\beta}\left[\Pi(1)-(1-\beta)V_I\right]-\frac{c_I}{p_I}\right\}.$$

(ii) *For the expected gain from active job search by the MLTU workers we get:*

$$J_S = \max\left\{0, \frac{\beta}{1-\beta}\left[V_S - Q_+V_P\right]-\frac{c_S}{p_+}\right\}.$$

Equilibrium. The equilibrium of the discrete-time search model consists of solutions $[\Theta, \theta_I, \theta_S, R, T_P, T_S, H]$ to the model equations (A1) – (A7) in Appendix I. The comparative static effects of the subsidy budget, the subsidy regime and the productivity enhancing reforms of the PES are presented below through a series of numerical experiments with the policy variables $[B, \tau, ef]$. To evaluate the results, we introduce the following steady-state welfare criterion.

3.6 WELFARE

The social planner takes the budget constraint (5), the distribution functions of the productivity shocks and the training costs as well as the matching functions of the two methods of search as given. The maximization problem of the planner has two state variables (u, Y) and seven control variables $[\Theta, \theta_I, \theta_S, R, T_P, T_S, H]$, where Y is the average steady-state per-capita output of a job, which is available at the end of a period:

$$0 = yp_I S_I + y\,p_+ F(T_S)S + y\left[u - p_+F(T_S)S\right]P_+ F(T_P) + \lambda(1-u)\int_R^1 yxdG(x) - \lambda Y. \tag{35}$$

During the initial period of a match, $p_I S_I$ active STU workers and $p_+ F(T_S)S$ active MLTU workers find a job and produce the output y. The unemployment pool $u - p_+ F(T_S)S$ consists of passive and active MLTU workers whose job search has failed. The fraction $P_+ F(T_P)$ of the pool is successfully placed by the PES and also produces the output y in the initial period of the match. $\lambda(1-u)$ of the continuing jobs are hit by a shock. The integral expression in equation (35) is the average output of the continuing jobs. In the steady-state the

output of the start-ups and the jobs in the continuation periods must be equal to the average per-capita output which is lost in the case a shock arrives, λY.

To evaluate the simulation results, we use the following steady-state version of the planner's discrete-time objective function subject to the budget constraint (5):

$$
\begin{aligned}
\Omega = {} & Y + b_l u - k\Theta u - c_I S_I - c_S S - ef\, c_U u - ef\, c_V \Theta u - p_+ F(T_S) S \int_0^{T_S} t\, dF(t) - \\
& P_+ F(T_P)\big[u - p_+ F(T_S) S\big] \int_0^{T_P} t\, dF(t)
\end{aligned}
\tag{36}
$$

where Y is the steady-state output (35) and $b_l u$ is the utility of leisure of the unemployed. The next terms denote the recruiting costs of the v vacancies, where $v = \Theta u$, the search costs of the S_I active STU, the S active MLTU and the PES, respectively; while the last two terms refer to the average training costs of, respectively, the MLTU workers who found a job through random search or via the PES. We take account of the production costs of the placement service in a simple way, by weighting the unit costs c_U and c_V with the effort $ef \in [0,1]$ the PES puts into job search. Increasing the effort will proportionally increase the effective unit costs.

4 SIMULATION

4.1 DATA, BASELINE PARAMETERS AND INDICATORS

Data. The calibration targets are taken from the European employment data. In particular the unemployment rate u, the average unemployment duration D and the fraction of MLTU workers of the year 2006, *MLTU*, are used to calibrate the model. There exist no consistent time series of the fraction of active jobseekers and the performance of the European PESs which we could use for calibration. However, for the wage penalty associated with unemployment and the instantaneous transition probability of the newly unemployed STU workers we can consult the descriptive statistics of *Garcia-Perez* and *Sanz* (2006, Table 3) and the estimation results of *Forslund* and *Johansson* (2007, Table 3), respectively. Although it is not necessary for our purpose to generate an exact mirror image of the EU employment statistics, the numerical simulation output of our model fits quite well with the data.

For comparison, the unemployment rate, the fraction of MLTU workers and the average duration of an unemployment spell of the years 2001–2006 for Europe, the USA and the OECD are reported in Table 1. In 2006, our target values, the European unemployment rate and the average spell length, were 8.2 % and 15.7 months, respectively, while 77.1 % of the unemployed had an ongoing spell length of more than 3 months and belonged to the type of MLTU workers.

Table 1: Data 2001 - 2006

		2001	2002	2003	2004	2005	2006
u (%)	Europe	8.5	8.9	9.0	9.0	8.9	8.2
	USA	4.7	5.8	6.0	5.5	5.1	4.6
	OECD	6.4	6.9	7.1	6.9	6.7	6.1
$MLTU$ (%)	Europe	77.4	77.4	77.7	78.3	77.9	77.1
	USA	25.8	34.7	38.5	37.7	34.6	32.4
	OECD	60.2	62.2	63.1	63.6	62.9	61.9
D (months)	Europe	16.0	15.6	16.1	15.7	15.7	15.7
	USA	3.0	3.8	4.4	4.5	4.3	3.9
	OECD	9.8	9.6	9.9	10.0	9.9	9.6

Source: OECD (2008), Eurostat (2008).

For the wage penalty, we follow *Garcia-Perez* and *Sanz* (2006, Table 3) who find a penalty of $Pen = 14.3$ % with the German data from the eight waves (1994–2001) of the European Community Household Panel (ECHP). The authors calculate the ratio of the wages at the time of the interview and prior to unemployment for men between 20 and 60 years of age and distinguish between stayers, voluntary movers, and involuntary movers. The benchmark of 14.3 % is the wage penalty of the German involuntary movers.

Although we choose the model period to be a quarter it is useful to consult *Forslund* and *Johansson* (2007, Table 3) for an estimation of the transition probability of the newly unemployed workers. The authors evaluate a data set of the Swedish PES with records of all contacts between jobseekers and the PES during the period 1991–2002 and estimate an instantaneous (weekly) transition probability of 21 %.

Baseline parameters. We use the baseline parameters of Table 2. The marginal product of a job at full productivity is set to $y = 100$; the UI benefits, $b = 60$, match the continental European replacement payments which vary between 40 % (Italy) and 90 % (Denmark) of the last net-income (OECD 2004).

Table 2: Baseline parameters

Parameters			
Output y	100	Elasticity w.r.t. vacancies PES Φ	0.30
Unemployment benefit b	60	Elasticity w.r.t. search effort of PES κ	0.90
Utility of leisure b_l	60	Total factor productivity RM d	0.20
Recruiting costs of a vacancy k	38	Total factor productivity PES a	0.20
Search costs STU c_I	28.2	Real interest rate r	0.01
Search costs MLTU c_S	28.2	Probability of a shock λ	0.10
Search costs PES c_U	38	Lower support of G α	0.532
Search costs PES c_V	38	Mean training costs $1/\delta$	30
Bargaining power β	0.50	Duration of job search of a STU j	1.5
Elasticity w.r.t. vacancies SM ϕ	0.30		
Policy instruments			
Hiring Subsidy H	0	Effort of PES ef	0.60

Data on search and recruiting costs are not available and we set these parameters to obtain the target values. The recruiting costs of a vacancy are $k = 38$; the search costs of the active STU and the active MLTU workers are $c_I = c_S = 28.2$. In the policy-free equilibrium, all newly unemployed STU workers choose the active search strategy, such that $S_I = I$, and earn the capital gain from active job search $J_I = 55.6$ (see Figure 2). With increasing search costs c_I, J_I decreases monotonically and approaches zero for $c_I = 51.725$. Recall, that at $J_I = 0$ STU workers are indifferent between the active and the passive search strategy with the consequence that the hiring subsidy would induce an increasing fraction of passive jobseekers among the STU workers. With search costs of $c_S = 28.2$, the fraction of active jobseekers among the MLTU workers in the policy-free environment is equal to $\sigma_S = 50.9$ % (see Figure 2). If c_S would fall *ceteris paribus* by 5 % to $c_S = 26.79$, the equilibrium fraction of active MLTU workers would rise by 41.3 % to $\sigma_S = 71.8$ %. Roughly one half of the European unemployed are LTU workers and, presumably, do not actively search for a job, therefore the equilibrium fraction of 50.9 % active MLTU workers seems to be not too low.

For the effort of the public placement service, we set $ef = 0.60$. To calculate the aggregate per capita welfare we additionally assume that the PES has to bear effective search costs of $ef\, c_U = 22.8$ per registered unemployed and the same unit costs per registered vacancy. The utility of leisure of an unemployed worker is assumed to be $b_l = 60$.

We follow *Mortensen* and *Pissarides* (MP 1999, 2003) and make the following additional assumptions. The bargaining power of the workers is $\beta = 0.50$; the matching functions of the PES and the method of random search (RS) are of the Cobb Douglas type (*Petrongolo* and

Pissarides 2001). For a given vacancy the probability for a contact arranged by the PES is $Q(ef, \Theta) = a * ef^{\kappa} * (1/\Theta)^{1-\Phi}$ and for a contact via RS it is $q(\theta) = d * (1/\theta)^{1-\phi}$. For the vacancy elasticities we assume $\phi = \Phi = 0.30$.[3] The effort of the PES exhibits diminishing returns since the effort elasticity is $\kappa = 0.90$. From this it follows that the production function of the PES has increasing returns to scale. The values of the total factor productivities are $a = d = 0.20$.

The real interest rate r is 1 % per quarter; the probability of a productivity shock λ is 10 %; the shock distribution $G(x)$ is uniform on $[\alpha, 1]$, with the lower support $\alpha = 0.532$; training costs are exponentially distributed with mean $1/\delta = 30$; the mean duration of unemployment of the STU workers who immediately after separation find a new job is equal to 1.5 months.

Indicators. In order to evaluate the simulation output we use the indicators in Table 3. The defining equations for the fraction of MLTU workers, *MLTU*, the average wage penalty, *Pen*, and the placement rate of the PES, *PES*, are reported in Appendix II.

Table 3: Indicators

$u*100$	Unemployment rate in per cent, see Lemma 1;
$\lambda G *100$	Unemployment incidence in per cent;
J_I	Gain from active job search by the STU, see Lemma 5;
$\sigma_S = S/u *100$	Fraction of active jobseekers among the MLTU, see Lemma 1;
MLTU	Fraction of the MLTU;
D, D_I, d_S, d_P (months)	Unemployment duration, see Lemma 1;
Pen	Average wage penalty an applicant must accept;
PES	Placement rate of the PES;
Ω	Social welfare, see equation (36).

Target achievement. Table 4 represents the values of the calibration targets in the first column and the calibration point for a policy-free environment in the second column. We have chosen the calibration point such that the unemployment rate and the average duration of unemployment, which are both influenced by the European ALMP, are somewhat lower than the target values.

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[3] Thus, the Hosios condition is not fulfilled (*Hosios* 1990).

Table 4: Matching the targets

Target	EU Employment Data	Model
Unemployment rate u	8.2 %	8.0 %
Average duration of unemployment D	15.7 (months)	14.8 (months)
Incidence of MLTU workers $MLTU$	77.1 %	83.7 %
Wage penalty Pen	14.3 %[1]	16.6 %
Transition rate STU workers p_I	21 % (week)[2]	34.3 % (quarter)

1: *Garcia-Perez* and *Sanz* (2006, Table 3); 2: *Forslund* and *Johansson* (2007, Table 3)

We find $MLTU = 83.7$ %, which is high relative to the data ($MLTU = 77.1$ %). One reason for the difference is the discrete time structure of the model. A second more important reason are the low vacancy elasticities. Low vacancy elasticities are in line with the recent empirical research (*Sunde* 2007, *Broersma* and *van Ours* 1999) but have, of course, the effect of increasing the fraction of MLTU workers. For example, with $\phi = \Phi = 0.3813$ the fraction of MLTU workers would match *ceteris paribus* the target value.

Also the wage penalty seems to be high compared with the finding of *Garcia-Perez* and *Sanz* (2006). But again, the empirical values are influenced by the European ALMP, which reduces the average penalty. Second, we relate the average initial wage to the average insider wage, which is presumably too high as a reference wage since it excludes negative selection effects which are important for the empirical measures of the wage penalty. *Garcia-Perez* and *Sanz* (2006) divide for example the wages of the respondents of the ECHP after their unemployment spell by the wages the respondents earned prior to the spell. But the average wage earned by the respondents prior to unemployment is presumably lower than the average insider wage (*Jacobsen* et al. 1993).

4.2 HIRING SUBSIDY

The results of the experiments with the subsidy budget B are reported in Figure 2. The horizontal axes of the diagrams represent the budget B, the vertical axes measure the outcome variables. The solid curves illustrate the results for the regime $\tau = 0$, where only PES placements receive subsidy payments, and the dashed curves show the corresponding results for the subsidy regime $\tau = 1$. The endogenous subsidy limit per eligible job H is depicted on the vertical axis of the first diagram. The second diagram portrays the unemployment curves and the third depicts the overall welfare induced by B in the respective subsidy regime.

MP model the hiring subsidy as a lump-sum payment that offsets the fixed job creation costs of the employers. Creation is stimulated and the labour market tightness as well as the opportunity costs of the filled jobs rise. Subsidizing job creation has therefore two opposing effects in the MP model with random search: (i) a decrease in unemployment duration and (ii) an increase of job destruction and unemployment incidence. The net effect of the subsidy is ambiguous (*Pissarides* 2000). In the numerical experiments, which the authors conduct in MP (1999, 2003), the second effect outweighs the first, so that the subsidy decreases employment.

Contrary to MP, in the present model jobseekers have two matching technologies available, random search and the PES. Thus, they can choose between an active and a passive search strategy via the PES. The hiring subsidy reduces the private gain from active job search, J_I, as the diagram in the right lower corner of Figure 2 shows and creates strong incentives to switch from random to organised search. These institutional and technological differences have consequences in particular for the above simulation result (i) by MP.

Result 1. *In the policy regime $\tau = 0$, where only PES placements are subsidised, the hiring subsidy amplifies job destruction and extends contrary to the MP model the aggregate duration of unemployment, such that the steady state unemployment increases.*

For convenience Table 5 summarizes the effects of the hiring subsidy on the outcome variables. Figure 2 shows consistent with the MP model that the unemployment effect of the subsidy budget B is positive. In regime $\tau = 0$, u increases from 8.0 % ($B = 0$) to 10.2 % ($B = 25$) which can be explained as follows.

Table 5: Effects of the Hiring Subsidy

	$\lambda G(R)$	D	$MLTU$	p_I	d_P	d_S	σ_S	u
MP (1999, 2003)	↑	↓						↑
Subsidy-Regime: $\tau = 0$	↑	↑	↑	↑	↑	↓	↓	↑
$\tau = 1$	↑	↓	↓	↓	↓	↑	↑	↑

First, the subsidy offsets the training costs of the firms which fill their vacancies with job-seekers placed by the PES up to the subsidy limit H. The subsidy therefore lowers the expected hiring costs and job creation is stimulated. Tightness in the two labour market seg-

ments with random search rise together with the opportunity costs of the filled jobs. Hence, firms and workers separate faster and job destruction increases from 2.8 % to 3.0 %.

Second, in the regime $\tau = 0$ firms and MLTU workers can only pocket the subsidy if they are matched by the PES. Therefore the subsidy increases the opportunity costs of a start-up in the first and second stage of the matching process, so that the fraction of active job-seekers among the MLTU, σ_S, falls, while the fraction of MLTU workers increases.

Third, the hiring subsidy reduces the duration of a job search of an active MLTU worker d_S, while the duration of a passive job search d_P increases from 28.3 to 29.4 months. These effects are attributable to the growing number of passive jobseekers who are concentrated in the third stage of the matching process. As a result the tightness between the registers of the PES declines, the response time of the PES given its search effort *ef* rises and the probability of a suitable match arranged by the PES falls. Nevertheless, the increasing response time is favourable for the matches arranged by the PES, as the strictly increasing and strictly convex subsidy curve in the first diagram of Figure 2 indicates. It is not surprising that in the regime $\tau = 0$ the duration of the unemployment spell of the active jobseekers among the MLTU workers falls because, on the one hand, the supply of vacancies is stimulated and, on the other hand, the number of active job-seekers falls.

Aggregating the various duration effects according to equation (4), we find that contrary to the MP model due to the crowding-out of the active jobseekers and the increasing duration of a job search via the PES the overall duration of unemployment *D* increases from 14.8 to 17.6 months.

Result 2. In the regime $\tau = 1$, where all matches with MLTU workers are subsidized, the duration effect of the hiring subsidy is negative, while unemployment is modestly rising. Nevertheless, the subsidy reduces welfare, though to a lesser degree than in the subsidy regime $\tau = 0$.

Indeed, the subsidy raises the fraction of active job-seekers among the MLTU workers in the regime $\tau = 1$ from 50.9 % to 57.3 %. But the symmetrical labour market policy nevertheless lowers overall employment, though to a much smaller degree than in the regime $\tau = 0$. The policy outcome can be explained as follows.

First, the creation effect is much stronger in the subsidy regime $\tau = 1$ than in the regime $\tau = 0$, as the comparison of the two H-curves in the first diagram of Figure 2 shows. The subsidy limit per eligible job H is for given B more than halved with the transition from regime $\tau = 0$ to $\tau = 1$. Hence, job destruction is slightly accelerated and unemployment incidence reinforced, so much that the labour market tightness during the first stage of the search process and the transition rate of the STU workers decline. Start-ups under $\tau = 1$ can pocket the subsidy not only in the third but also in the second stage of the matching process. As a result, the number and the fraction of active MLTU workers strongly increase, so much that also the labour market tightness during the second stage of the matching process declines and the transition rate of the active MLTU workers decreases too.

Second, although the transition rates of the active jobseekers who use random search decrease the outflows from both search market segments into employment increase with B, while both outflows are decreasing in the regime $\tau = 0$.

Third, the relatively strong creation effect and the weak unemployment effect of B lead to an increasing tightness between the PES registers of vacancies and unemployed and thus to a growing transition rate for the PES placements.

Finally, contrary to the regime $\tau = 0$, B reduces the aggregate duration of unemployment from 14.8 to 14.0 months. This duration effect is due to the increasing fraction of active jobseekers among the MLTU workers and the declining duration of a job search via the PES. The fraction of MLTU workers is lower than in regime $\tau = 0$ and slightly decreasing in B. But the symmetric labour market policy leads to a growing concentration of MLTU workers in the second stage of the search process and thus to an increasing duration of random search, d_S.

To sum up, four factors are responsible for the fact that the unemployment rate in the regime $\tau = 1$ rises more slowly if B increases than in the regime $\tau = 0$: the stronger creation effect of B, the increasing fraction of active jobseekers among the MLTU workers compared to the crowding-out of active job search in the regime $\tau = 0$, the increasing transition rate of the workers placed by the PES and the falling duration of unemployment.

Result 3. In the MP model, where all start-ups are eligible, STU workers who find a job would obtain a wage advantage compared to the average insider wage. In the present model, successful STU workers suffer from a wage penalty, IPen, which is strictly increasing in B. In

contrast, the average wage penalty of an entrant, Pen, is strictly decreasing in both subsidy regimes.

The scar of unemployment *Pen*, which an entrant must accept, is the weighted average of the penalties of the STU workers, *IPen*, the active MLTU workers, *SPen*, and the workers, who are placed by the PES, *PPen*, see Appendix II. All wage penalties are reported in Table 6 for the subsidy regimes $\tau = 0$ and $\tau = 1$.

Table 6: Wage Penalties (%)

B	IPen		SPen		PPen		Pen	
	$\tau = 0$	$\tau = 1$	$\tau = 0$	$\tau = 1$	$\tau = 0$	$\tau = 1$	$\tau = 0$	$\tau = 1$
0	8.5	8.5	25.7	25.7	15.9	15.9	16.6	16.6
5	8.6	8.6	26.4	23.4	12.0	13.7	15.1	15.3
10	8.8	8.6	27.0	21.2	8.5	11.6	13.4	14.0
15	8.9	8.7	27.5	19.1	5.2	9.6	11.7	12.7
20	9.0	8.8	28.0	17.0	2.3	7.6	10.1	11.5
25	9.1	8.9	28.3	14.9	-0.5	5.7	8.4	10.3

In the MP model, in which all start-ups receive a lump-sum subsidy payment, three factors are responsible for the wage advantage of the entrants; first, the idiosyncratic productivity component of a new match which takes on its maximal value with $x = 1$; second, the allocation of the hiring subsidy which is subject of the wage bargaining; and third, the fact that there is a single matching method in the MP model only. The value of the outside option of a vacancy is, therefore, independent from the hiring subsidy equal to zero given the free entry and the free disposal assumption of the MP model.

The positive impact of the hiring subsidy on the wage penalty of the STU workers in the present model can be explained as follows. First, start-ups which are filled with a STU worker do not receive subsidies. Second, recall the sharing rule (18) which shows the dependence of the surplus of a match with a STU worker on the employer's outside option V_I. The outside option is a consequence of the competing matching methods of random and organised job search. Now, $IPen > 0$ follows from $V_I > 0$, and the increase of *IPen* is induced by the fact that V_I is strictly increasing in H.

The wage penalty of the MLTU workers is dependent on the hiring subsidy and the policy regime. The active MLTU workers experience the penalty *SPen*. In the regime $\tau = 0$, *SPen*, which in this case reflects the uncompensated training costs, is strictly increasing, while in the regime $\tau = 1$ the penalty falls. The difference between the penalties of the two types of

MLTU workers, *SPen* and *PPen*, is attributable to the value of the outside option which the employers are able to claim during the wage negotiation with a MLTU worker who randomly searched for the job.

Why is *PPen*, the penalty experienced by the PES placements, lower in the regime $\tau = 0$ than in the regime $\tau = 1$? *PPen* can even turn into a wage advantage if the budget *B* is sufficiently high, as for example in the case $B = 25$, where $PPen = -0.5$. After all, in the regime $\tau = 1$ the creation effect of *B* is stronger, the equilibrium unemployment is lower and is increasing more slowly than in the regime $\tau = 0$. In addition, the tightness between the registers of the vacancies and the unemployed and thus the transition rate for workers, who are placed by the PES, is increasing in *B*. *Ceteris paribus* these effects should be sufficient for a lower *PPen* in the regime $\tau = 1$. However, in $\tau = 1$ only somewhat more than half of the budget *B* is allocated to the PES placements, while in $\tau = 0$ these placements enjoy all of *B*. The second effect outweighs the first, so that *PPen* is higher in the regime $\tau = 1$.

Finally, the decline of the average wage penalty, *Pen*, which is stronger in the regime $\tau = 0$ than in $\tau = 1$, can be attributed to two factors. First the diminishing search incentives, which reduce the fraction of active jobseekers and increase the number of MLTU workers, and second to the falling wage penalty of the passive workers who wait for a placement by the PES.

4.3   PES PERFORMANCE AND PROFILING

PESs are built on the conviction that the unfettered labour market fails. Equilibrium unemployment is too high, the duration of an unemployment spell is too long, the quality of the matches is too low, the life span of a job is too short or the private replacement payments are too volatile or nonexistent at all, these are some of the judgements about labour market failures. Labour law, labour courts, unions and further central help agencies beside the PES shall prevent the market failures or support and compensate the victims. Result 4 reports performance measures for the PES. Result 5 illustrates the impact of organizational reforms which serve to enhance the placement effectivity of the PES.

The performance measures *VPES*, *EPES* and *PES*, see Table 7 and Figure 2, are taken from OECD (1992). *VPES* gauges the placements of the PES as a percentage of the notified vacancies, while *EPES* is the number of PES placements as a percentage of the filled jobs. *PES*, see

Appendix II, measures the PES placements as a percentage of all hirings and is used by governments and empirical research, in order to quantify the achievements of the PES (OECD 2006). Finally, one could argue that the PES is a safety net for jobseekers. Therefore we present the indicator *SPES* which measures the number of active MLTU workers who failed to find a job by random search and are placed by the PES as a percentage of all active MLTU.

Result 4. *With the exception of SPES the performance measures are all increasing in the subsidy budget B in the regime $\tau = 0$. Due to the mobilization of the private job search of the MLTU workers, the placement volume EPES is nearly constant, while VPES and PES are strictly decreasing and SPES is strictly increasing in the regime $\tau = 1$.*

Table 7: PES Performance (%)

B	VPES		EPES		SPES		PES	
	$\tau = 0$	$\tau = 1$	$\tau = 0$	$\tau = 1$	$\tau = 0$	$\tau = 1$	$\tau = 0$	$\tau = 1$
0	9.0	9.0	0.81	0.81	8.1	8.1	34.4	34.4
5	9.5	8.9	0.87	0.81	8.0	8.2	36.7	33.9
10	10.1	8.7	0.93	0.82	7.9	8.2	38.7	33.5
15	10.5	8.6	0.98	0.82	7.8	8.2	40.5	33.1
20	10.9	8.5	1.03	0.82	7.7	8.2	42.1	32.7
25	11.3	8.4	1.07	0.82	7.6	8.3	43.6	32.3

That *PES*, the number of PES placements as a percentage of all hirings, is strictly increasing in regime $\tau = 0$, is due to the subsidy induced weakening of the search incentives and the following reallocation of the jobseekers from random to organised job search.

Advancing the PES search effort seems to be an effective policy goal, since both the contact rate of the jobseekers as well as the vacancies are strictly increasing functions of *ef*. Client opinion polls confirm that workers and firms welcome reforms, which make the PES more effective, and express their satisfaction with the achievements of the reorganized agencies (BA Geschäftsberichte 2006, 2007). IT improvement, development of the human resources for placement, counselling, profiling, screening and training measures are, therefore, much-recommended policy strategies to increase the effectivity of the PES (OECD 2006). To investigate the macroeconomic effects of such productivity enhancement strategies we conduct a numerical experiment with the search effort *ef* of the PES and calculate the comparative static impact on the outcome variables of the model. Figure 3 represents the results.

[Figure 3 about here]

Result 5. The more effort the PES puts into the job search, the higher the equilibrium un-employment is. Equilibrium unemployment increases from 6.3 % to 14.1 % when the search effort of the PES rises from $ef = 0.55$ to $ef = 1.00$.

Nevertheless, IT improvement, profiling and activation strategies are a great success for the PES and its target group. The placement rate PES increases from 24.4 % to 74.6 %, see Table 8 and Figure 3, while d_P, the duration of a job search of a MLTU worker via the PES, decreases from 29.0 to 19.1 months, and d_S, the duration of an active job search of a MLTU worker, falls from 10.1 to 5.5 months.

Table 8: PES Performance (%)

ef	VPES	EPES	SPES	PES
0.55	6.2	0.57	8.1	24.4
0.60	9.0	0.81	8.1	34.4
0.70	14.5	1.35	8.3	52.3
0.80	19.0	1.83	8.6	63.8
0.90	22.5	2.22	8.7	70.6
1.00	25.5	2.55	8.5	74.6

Contrary to the conventional opinion (OECD 2006) our model therefore suggests the hypothesis, that the mounting European unemployment during the past decades can be attributed to the interferences of the PESs into the European job markets. The establishment and growth of the European PESs fundamentally changed the weight of random and organised job search.

The reasons for the rising unemployment are threefold. First, the more effective PES raises the opportunity costs of the occupied jobs and hence the incidence of unemployment, as is shown in the diagram in the upper right corner of Figure 3. Second, the opportunity costs of the active search strategy increase too, so that the equilibrium composition of search strategies shifts from active to passive job search. The fraction of active MLTU workers σ_S for example drops from 83.7 % to 3.1 % while on the other hand the fraction of MLTU workers increases from 78.1 % to 88.6 %. Third, the aggregate duration of unemployment *D*, which is given by equation (4), is hump-shaped. *D* increases from 11.2 to 17.1 months where it attains a maximum at an effort level of $ef = 0.70$. For higher effort levels, the duration decreases again until it reaches a boundary value at 15.1 months. The hump-shaped curvature of the duration curve, see Figure 3, is the result of the following factors. On the one hand the durations of a job search for all types of search strategies decline and reduce *ceteris paribus* the overall duration *D*. The fact that d_P falls from 29.0 to 19.1 months is obviously due to the organizational reforms, which increase the search effort of the PES. The decline of D_I from 8.8 to 5.9 months and of d_S from 10.1 to 5.5 months result from the crowding-out of the active job-

seekers. This improves the chances of the remaining workers who stick to their strategy. On the other hand, the fraction of active MLTU workers, σ_S, is not only strictly decreasing in *ef*, what overcompensates at first the impact of the sinking strategy-specific durations on *D*. But the σ_S-curve is strictly convex, such that the reallocation effect of the PES effort diminishes with an increasing *ef*.

That the per capita welfare Ω is decreasing, see Figure 3, is a consequence first of the decreasing employment, which reduces the per capita production *Y*. Second, private search costs are reduced, but on the other hand public search costs are increasing as the number of passive jobseekers rises and the effort of the PES is intensified.

To sum up, organizational reforms which intensify the search effort of the PES increase the job destruction rate and sharply decrease the fraction of active MLTU workers, so that the policy will lead to a higher rate of unemployment while the unobservable social welfare declines. On the other hand, the reforms are a visible success for the PES, all four performance measures will increase and the length of a passive job search of the MLTU workers, who rely on the search assistance of the PES, will strongly decline.

5 SUMMARY

The innovation of our model is that apart from the endogenous job destruction rate the equilibrium rate of unemployment reflects, first, the composition of the two matching methods, random search and organised search by the PES, second, the duration of a job search with the active and the passive search strategy, and third, the search behaviour of the public placement service. The type-specific durations of unemployment depend on the effort the PES puts into the placement activities, the subsidy regime and the distribution of the match-specific training costs, which a firm employing a medium-term or long-term unemployed (MLTU) worker must spend to train the entrant. The subsidy regime determines whether only PES placements are eligible or all matches with a MLTU worker are entitled to compete for the payments from the subsidy budget.

The effects of the hiring subsidy depend on the target group. For short-term unemployed (STU), who are indirectly affected, the hiring subsidy is always counterproductive and re-

duces the incentives to search actively for a job. For eligible MLTU workers, the subsidy has a stimulating effect on job creation. If both active and passive MLTU are supported, the subsidy can even reduce the fraction of MLTU workers. Nevertheless, the subsidy increases unemployment. The higher unemployment is attributable to the rising job destruction, the declining fraction of active jobseekers, and, contrary to the Mortensen-Pissarides model, the increasing aggregate duration of a job search.

Certainly, the PES can advance its placement success by improving the effort of its placement agencies through IT innovations, development of the human resources for placement, counselling, profiling, screening and training measures. But effectivity enhancement programmes will heighten the opportunity costs, first of the filled jobs and second of active job search. Job destruction will rise and the fraction of active jobseekers will sharply decline so that the organizational reforms will finally lead to higher equilibrium unemployment.

The economic policy consequences of the model are clear: the effects of a hiring subsidy and PES reforms depend on the target group. For the short-term unemployed not only indirect effects of the hiring subsidy but also of the job placement activities of the PES are counterproductive. On the other hand, these instruments have a stimulating effect on job creation for target groups who in the equilibrium without policy have no incentive to actively search for a job. But policy makers have to take into account that the two instruments of active labour market policy reduce aggregate employment.

Our model does not confirm the maintained welfare effects of a centralized labour market agency. Contrary to the prevailing opinion, the simulations suggest the hypotheses that increasing the effectiveness of the PES through counselling, profiling, screening and training measures will induce a change of the weight of the active and the passive search strategies, so that equilibrium unemployment increases and social welfare declines.

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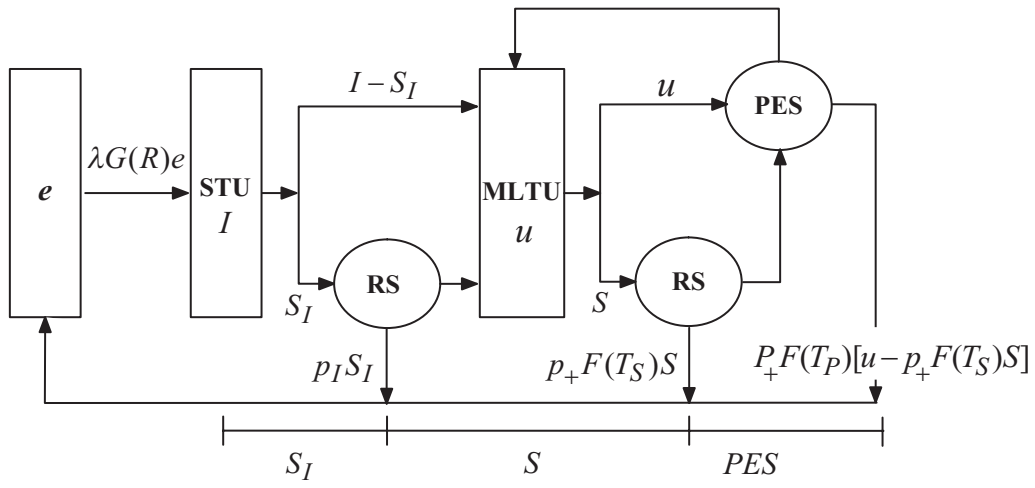
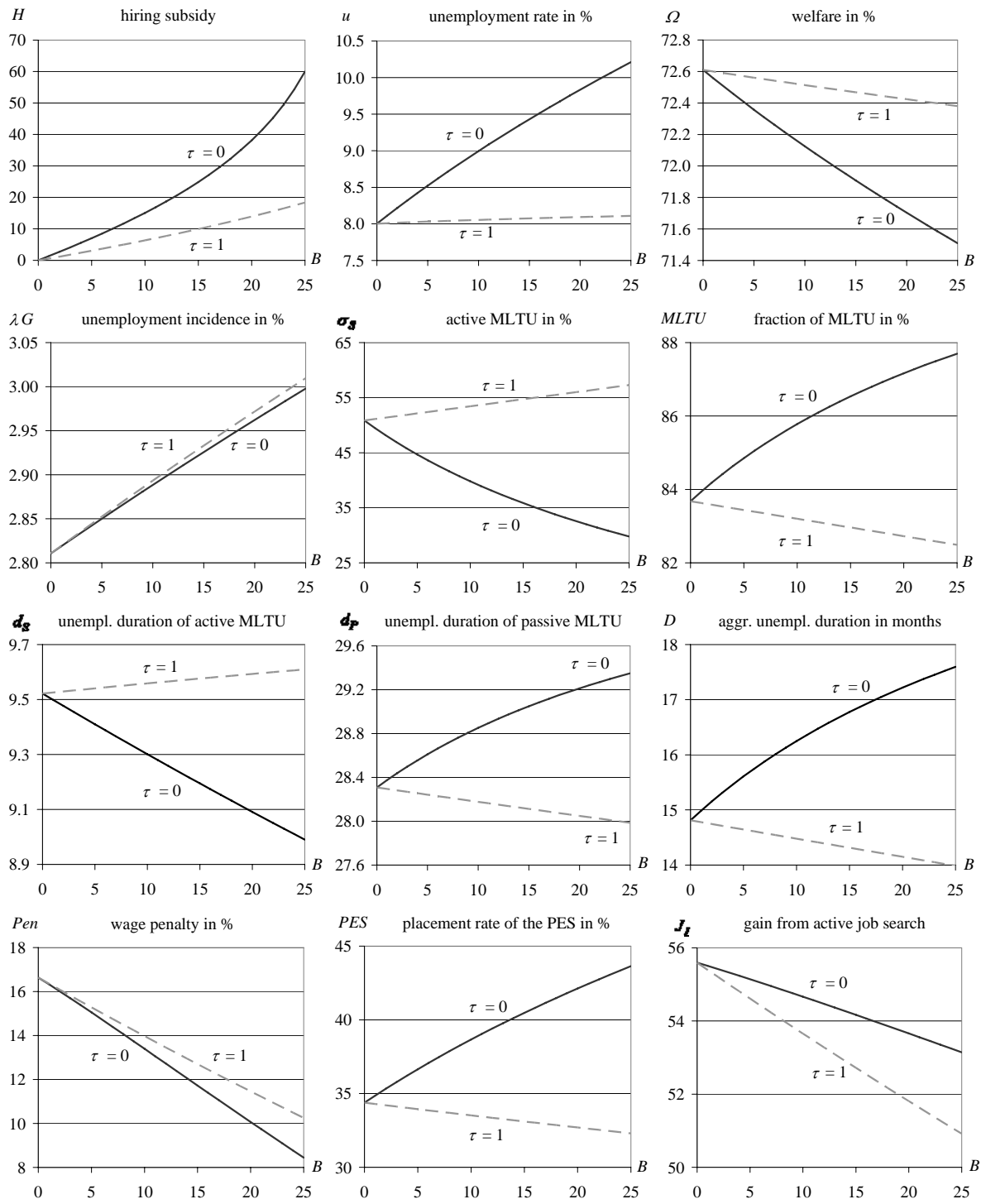Figure 1: Sequential search process, $P_+ = P(1-q_I)(1-q_S)$ and $p_+ = p_S(1-q_I)$
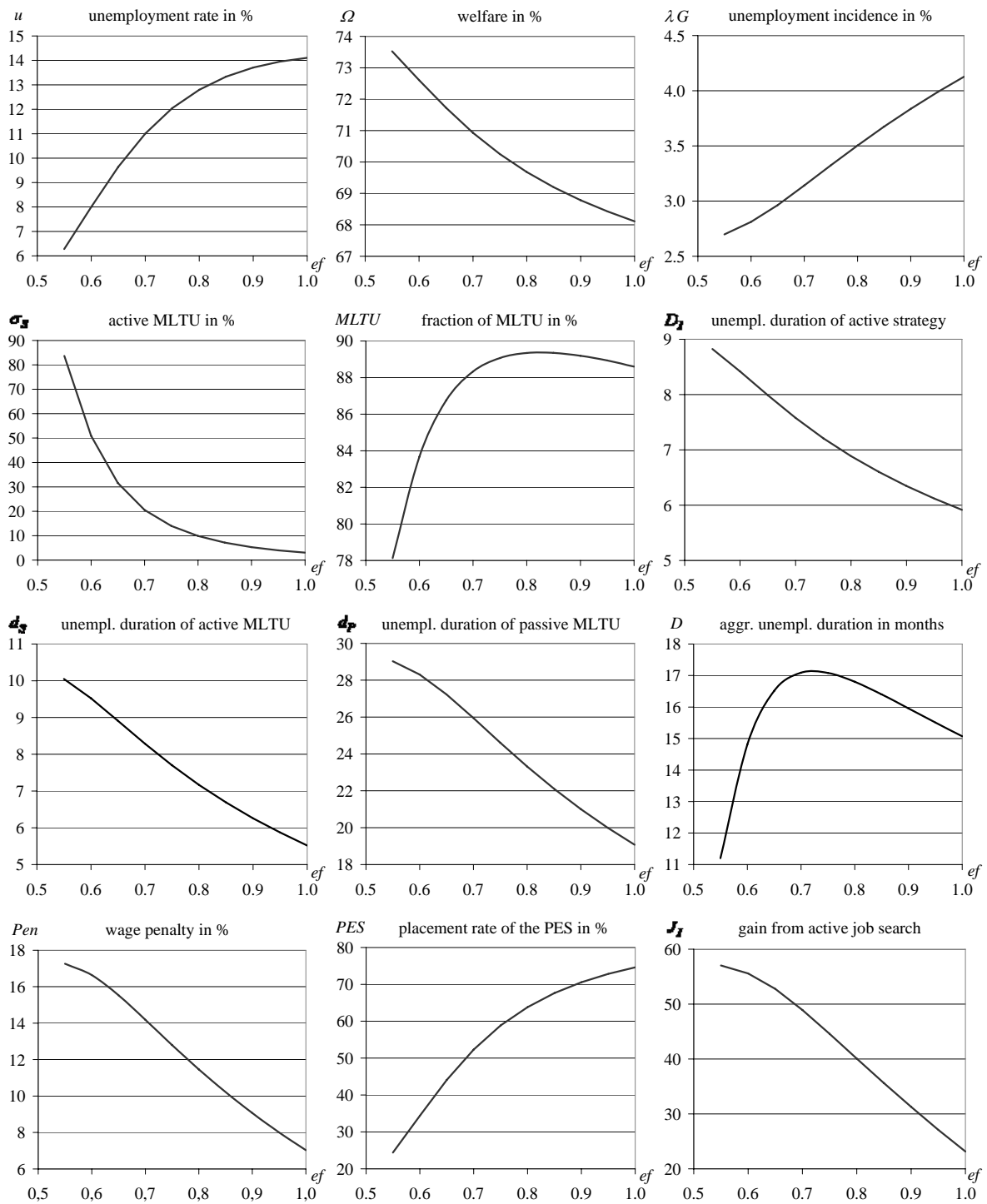
Figure 2: Hiring Subsidy Budget *B*

Figure 3: Search Effectivity of the PES *ef*

APPENDIX I: MODEL EQUATIONS

The model has seven endogenous variables $\left[\Theta, \theta_I, \theta_S, R, T_P, T_S, H\right]$ and three policy parameters $\left[B, \tau, ef\right]$. Table 9 shows the model equations in implicit form, where we make use of the following abbreviations: $P_+ = P(1-q_I)(1-q_S)$, $p_+ = p_S(1-q_I)$, $Q_+ = Q\left[1 - p_+ F(T_S)S/u\right]$, and $J^i(\cdot) = J^i(\Theta, \theta_I, \theta_S, R, T_P, T_S, H; B, ef, \tau)$, $i = 1,\dots,7$.

Tab. 9: Model equations in implicit form

from	*equation*	No.
(28)	$J^1(\cdot) \equiv \Pi(1) - \Pi_P - (1-\beta)(U_P - U_I) = 0$	(A1)
(31)	$J^2(\cdot) \equiv T_P - T_S - (1-\tau)H - Q_+ V_P = 0$	(A2)
(34)	$J^3(\cdot) \equiv R - \dfrac{rU_I}{y} + \dfrac{\lambda}{\lambda + r}\int_R^1 (h - R)dG(h) = 0$	(A3)
(JC)	$J^4(\cdot) \equiv -k + q_I \Pi(1) + \left[1 - (1-\beta)q_I\right]V_I$	(A4)
Lem. 5	$J^5(\cdot) \equiv \dfrac{\beta}{1-\beta}\left[V_S - Q_+ V_P\right] - \dfrac{c_S}{p_+} = 0$	(A5)
(5)	$J^6(\cdot) \equiv P_+\left(u - p_+ F(T_S)S\right)\left[\int_0^H t\,dF(t) + H\left[F(T_P) - F(H)\right]\right] + \tau\, p_+ F(T_S)S\left[\int_0^H t\,dF(t) + H\left[F(T_S) - F(H)\right]\right]$	(A6)
	$J^7(\cdot) \equiv \theta_I - \dfrac{\Theta u}{\lambda(1 - G(R))} = 0$	(A7)

APPENDIX II: INDICATORS

We measure the faction of MLTU workers by dividing the number of unemployed with an ongoing spell length strictly larger than a quarter by the sum of the unemployed u and the newly unemployed STU worker who immediately after separation find a new job:

$$MLTU = 1 - \frac{I}{p_I S_I + u}.$$

For the average wage penalty an unemployed worker must accept, *Pen*, we use the weighted mean of the initial wages of the STU workers as well as the active and passive MLTU workers and divide by the average inside wage:

$$Pen = 1 - \frac{p_I w_I + (1 - p_I)\left[(1-\sigma_S)\overline{w}_P + \sigma_S \overline{w}_S\right]}{\overline{w}},$$

where w_I is the initial wage of the STU workers, and $\overline{w}_P$, $\overline{w}_S$ and $\overline{w}$ are, respectively, the average initial wages of the MLTU workers who found a job through the PES or via random search and the average inside wage, for which (see Lemma 2):

$$\overline{w}_i = \begin{cases} \dfrac{F(H)w_P + \int_H^{T_P}\left[w_P - \beta(t-H)\rho^{-1}\right]dF(t)}{F(T_P)}, & i = P \\[4ex] \dfrac{F(\tau H)w_S + \int_{\tau H}^{T_S}\left[w_S - \beta(t-\tau H)\rho^{-1}\right]dF(t)}{F(T_S)}, & i = S \end{cases}$$

and

$$\overline{w} = (1-\beta)rU_I + \beta y\,\frac{\int_R^1 x\,dG(x)}{1-G(R)}.$$

The placement rate of the PES is measured as the number of matches arranged by the PES as a percentage of the total number of matches per period:

$$PES = \frac{P_+ F(T_P)\left[u - p_+ F(T_S)S\right]}{P_+ F(T_P)\left[u - p_+ F(T_S)S\right] + p_+ F(T_S)S + p_I S_I}.$$

APPENDIX III: PROOFS

Proof of Lemma 1. Ad (i): Since the PES arranges contacts with probability P, $1+1/P$ periods elapse in the steady state between the first day of the unemployment spell and the first job contact. $(1-q_I)(1-q_S)$ is the probability that the vacancy found is neither filled with an active STU nor with an active MLTU. So the average duration of data processing and search for an unfilled vacancy amounts to $1+1/P_+$ periods. If vacancy and applicant meet, they observe the training costs $t \geq 0$. If $t \leq T_P$, the applicant is accepted, an event with probability $F(T_P)$. Hence the average length of time required for a successful job search via the PES is equal to D_P. Ad (ii): The reasoning is similar as with (i). Ad (iii): A STU worker, who opts for the active search strategy, meets a vacancy with probability p_I. The duration of job search is equal to half a period. With probability $1-p_I$ the search fails and the worker becomes active MLTU, with a duration of unemployment equal to d_S. Therefore the duration of unemployment of an active jobseeker is: $D_I = p_I 0.5 + (1-p_I)D_S$. Ad (iv): Inserting equation (1) into equation (2), using $I = \lambda G(R)(1-u)$ and taking into account $S_I = \sigma_I u$, $S = \sigma_S u$ and $P_+ F(T_P) = 1/d_P$ we get

$$\lambda G(R)(1-u) - p_I \sigma_I u = \frac{u}{d_P} + p_+ F(T_S)[1 - P_+ F(T_P)]\sigma_S u$$
$$= \frac{u}{d_P} + \left[\frac{1}{d_S} - \frac{1}{d_P}\right]\sigma_S u$$,

from which the equilibrium unemployment (3) follows. Ad (v): p_I is the probability of an immediate transition with a duration of unemployment of a half period. With probability $1-p_I$ the job search fails and the worker becomes MLTU. Being MLTU the worker will randomly search with probability σ_S and face a spell length of d_S periods. With probability $1-\sigma_S$ the worker decides to wait for a placement via the PES and his expected unemployment duration is equal to d_P. The expected duration of unemployment conditional on becoming MLTU is therefore determined by $1 + (1-\sigma_S)d_P + \sigma_S d_S = (1-\sigma_S)D_P + \sigma_S D_S$.

Proof of Lemma 2. Ad (i) Write the sharing rule (18) as $W_I = U_I + \frac{\beta}{1-\beta}(\Pi_I - V_I)$ and use the resulting equation to substitute W_I out of (15). In view of $U = U_P$ it follows

$U_I = -c_I + p_I U_I + p_I \frac{\beta}{1-\beta}(\Pi_I - V_I) + (1 - p_I)\rho(b + U_P)$. Next rearrange terms to get

$rU_I = U_P - U_I + b + \frac{p_I}{1-p_I} J_I \rho^{-1}$. The equation of the reservation wage rU_I includes the case where S_I is an interior solution to the model and $J_I = 0$. Next we derive the reservation wage of the MLTU workers who are placed by the PES. Rearrange the asset equation (16):

$$\rho^{-1} U_P [1 - P_+ F(T_P)] = \rho^{-1} P_+ \left[F(H)(W_P - U_P) + \int_H^{T_P} [W_P(t) - U_P] dF(t) \right] +$$

$[1 - P_+ F(T_P)](b + U_P)$. Divide by $[1 - P_+ F(T_P)]$ and rearrange terms to get the reservation wage: $rU_P = b + \frac{\beta}{(1-\beta)} \frac{P_+}{[1 - P_+ F(T_P)]} V_P \rho^{-1}$. Now the equations (22) and (23) of the lemma follow from the above equations of the reservation wages rU_I and rU_P. Ad (ii) Write the sharing rule (21) as

$$(1 - \beta)U_I = (1 - \beta)W(x) - \beta\Pi(x). \tag{B1}$$

Substitute $\Pi(x)$ and $W(x)$ with the asset equations (6) and (7) out of (B1) and the inside-wage (24) follows. Ad (iii) $i = I$: Write the sharing rule (18) as

$$(1 - \beta)U_I - \beta V_I = (1 - \beta)W_I - \beta\Pi_I. \tag{B2}$$

Now for $i = I$ the outside-wage (25) follows from (B2), the asset pricing equations

$$\Pi_I = \rho \left\{ y - w_I + \lambda \int_R^1 \Pi(h) dG(h) + (1 - \lambda)\Pi(1) \right\} \tag{B3}$$

$$W_I = \rho \left\{ w_I + \lambda \left[\int_R^1 W(h) dG(h) + G(R)U_I \right] + (1 - \lambda)W(1) \right\} \tag{B4}$$

and the equation for the inside wage (24) together with (B1). Ad (iii) and (iv) $i = P$: Write the sharing rule (20) as $(1 - \beta)U_P - \beta(t - H) = (1 - \beta)W_P(t) - \beta\Pi_P(t)$. Substitute the values of the filled job and the employed worker with (8) and (9) out of the above equation, and take into account (B1) and the equation for the inside wage (24) to get the wage equation $w_P(t) = w(1) + (1 - \beta)(U_P - U_I)\rho^{-1} - \beta(t - H)\rho^{-1}$. The wage (25) for $i = P$ follows, if one takes into account that the last term on the RHS of the above wage equation is equal to zero if $t \le H$. The proof for $i = S$ is similar, where we use asset pricing equations for $\Pi_S(t)$ and $W_S(t)$, which correspond to (8) and (9), (B1) and the sharing rule (19), which we can rewrite as $(1 - \beta)U_S - \beta(t - \iota H) - \beta Q_+ V_P = (1 - \beta)W_S(t) - \beta\Pi_S(t)$.

Proof of Lemma 3. Ad (i): Equation (6) and $\Pi(R) = 0$ imply $0 = yR - w(R) +$ $\lambda \int_R^1 \Pi(h) dG(h)$ and $(\lambda + r)\Pi(x) = yx - w(x) + \lambda \int_R^1 \Pi(h) dG(h)$. From these two equations together with the wage equation (24) the statement follows. Ad (ii) and (iii): For $i = I$ insert the wage equation (25) into the asset equation (B3) and take account of equation (6) to derive the asset equation (28). For $i = P$ the asset equations (28) and (29) follow from substituting the wage equation (25) and (24) into (8) and (9) respectively. Then rearrange terms and take account of the asset equation (6). For $i = S$ we can derive (28) and (29) from the corresponding asset equation for Π_S and (8) by taking into account the wage equations (25) and (24).

Proof of Lemma 4. Ad (i): For $i = P$ write the asset equation (29) as $\Pi_P(T_P) + H - T_P = \Pi_P - (1 - \beta)(T_P - H)$, and take account of the reservation cost condition, $\Pi_P(T_P) + H - T_P = 0$. For $i = S$ equations (29) and (28) imply $\Pi_S(T_S) = \Pi_P + \beta[T_S - \tau H + Q_+ V_P]$. Taking account of the definition of the reservation cost it follows that $\Pi_S(T_S) = \Pi_P /(1 - \beta)$, hence $\Pi_P /(1 - \beta) = \Pi_P + \beta(T_S - \tau H + Q_+ V_P)$. From the last equation we get $\Pi_P /(1 - \beta) = T_S - \tau H + Q_+ V_P$, so that in view of (30) the proposition follows.