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Returns to Tenure or Seniority?

Sebastian Buhai
Miguel Portela
Coen Teulings
Aico van Vuuren

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Institute for the Study
of Labor

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Sebastian Buhai
Aarhus School of Business, University of Aarhus,
Erasmus University Rotterdam and Tinbergen Institute

Miguel Portela
NIPE-University of Minho and IZA

Coen Teulings
CPB Netherlands Bureau for Economic Policy Analysis,
University of Amsterdam, Tinbergen Institute and IZA

Aico van Vuuren
Free University Amsterdam
and Tinbergen Institute

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IZA

P.O. Box 7240
53072 Bonn
Germany

Phone: +49-228-3894-0
Fax: +49-228-3894-180
E-mail: iza@iza.org

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ABSTRACT

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This study documents two empirical regularities, using data for Denmark and Portugal. First, workers who are hired last, are the first to leave the firm (Last In, First Out; LIFO). Second, workers' wages rise with seniority (= a worker's tenure *relative* to the tenure of her colleagues). We seek to explain these regularities by developing a dynamic model of the firm with stochastic product demand and hiring cost (= irreversible specific investments). There is wage bargaining between a worker and its firm. Separations (quits or layoffs) obey the LIFO rule and bargaining is efficient (a zero surplus at the moment of separation). The LIFO rule provides a stronger bargaining position for senior workers, leading to a return to seniority in wages. Efficiency in hiring requires the workers' bargaining power to be in line with their share in the cost of specific investment. Then, the LIFO rule is a way to protect their property right on the specific investment. We consider the effects of Employment Protection Legislation and risk aversion.

JEL Classification: J31, J41, J63

Keywords: irreversible investment, efficient bargaining, seniority, LIFO, matched employer-employee data, EPL

Corresponding author:

Sebastian Buhai
Department of Economics
Aarhus School of Business
University of Aarhus
Prismet, Silkeborgvej 2
DK-8000, Aarhus C
Denmark
E-mail: sebu@asb.dk

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1 Introduction

Why does Lars earn a lower wage than Jens, while they both do exactly the same job, at the same firm, and with equal skills? And why is Pedro fired when his employer has to scale down employment and his colleague Miguel allowed to stay at the firm, while again they do the same job? Some might think that the answer to these questions is obvious: it is simply because Jens and Miguel have a longer tenure at the firm than Lars and respectively, Pedro. Nevertheless, we do not know of any paper within economics that establishes and justifies these regularities. This paper seeks to fill this gap and to provide a simple explanation for the occurrence of these phenomena. Using matched worker-firm data for Denmark and Portugal, we show that a worker who is hired last, is likely to be fired first (Last In, First Out; LIFO). Analogously, we show that there is return to *seniority* in wages. In both cases, our claims are different from saying that your *tenure* at the job affects negatively your job exit hazard or, respectively, affects positively your wage. Seniority is different from tenure in that it measures the worker's tenure *relative* to the tenure of her colleagues. Your seniority is your rank in the tenure hierarchy of the firm. Hence, we need all-encompassing matched worker-firm data to establish a worker's seniority because we need to know the tenure of all the firm's workers. Thus, when we claim that seniority affects your separation risk, we mean that on top of the negative duration dependence of the hazard rate, being a senior worker with many more junior colleagues has a further negative effect. Similarly, when we claim that there is a return to seniority in wages, we mean that on top of the return to tenure as usually measured, there is return to seniority. We offer a simple economic theory of why firms and workers would agree on applying a LIFO layoff rule and why that leads to a return to seniority in wages. A LIFO layoff rule is a way to protect the interests of incumbent insiders when hiring and training new workers. Without this protection, the incumbents would have an incentive not to train any new worker. The LIFO layoff rule provides protection against layoff for senior workers, and hence gives these workers additional power to bargain for a higher wage, leading to a return to seniority. To the extent that this return to seniority is a compensation for the worker bearing part of the cost of specific investment in the relation between the worker and the firm, the LIFO rule can be interpreted as a protection of the worker's property right on her specific human capital in the relation with the firm. We show that worker turnover is maximal and the expected job duration is minimal when the surplus and the cost of the specific investments are shared between the worker and the firm in the same proportions, which is an application of the Hosios (1990) condition. In some sense, stringent Employment Protection Legislation (EPL) acts as an artificial way to increase the specific investment in the relation, thereby reducing turnover and increasing the expected job duration. Comparing Denmark and Portugal, we see that Portugal has much more stringent EPL than Denmark, and in accordance with our theoretical predictions, a much higher expected job duration than Denmark.

Our theory is based on a dynamic model of the firm with stochastic product demand and irreversible specific investments for each newly hired worker, similar to Bentolila and Bertola (1990). Dixit (1989) considers the same model, but then for an individual worker. Labor demand follows a geometric random walk in these models. Bentolila and Bertola calculate the optimal hiring and firing points, by considering, for the current employment level, the expected discounted marginal revenue of hiring an additional worker, accounting for the expected moment when it is efficient to fire that worker, taking as given all workers currently employed by the firm and disregarding any workers that might be hired in the future. In this way, the hiring and firing of each worker can be considered separately of the hiring and firing of all other workers, transferring a firm level model into a model of an individual worker, as in Dixit (1989). This turns out to be equivalent to applying a LIFO separation rule. Whereas Bentolila and Bertola (1990) and Dixit (1989) take wages as given, we allow for wage bargaining over the surplus generated by the specific investment. Here, we apply an idea developed by Kuhn (1988) and Kuhn and Robert (1989). They start from the distinction in trade union theory between the right-to-manage model, where the union bargains for wages above the market wage and the firm reduces its labor demand in response to this higher wage (it has the right to manage) -leading to an inefficiently low employment- and the efficient bargaining model, where the union and the firm bargain simultaneously over wages and employment, so that employment remains at its efficient level. Kuhn and Robert observe that there is an alternative way for workers to extract rents from the firm, while retaining both the right-to-manage feature and efficiency in employment setting. Their idea is to bargain for a layoff order and for a wage schedule where inframarginal workers get higher wages than marginal workers. The firm cannot fire the expensive inframarginal workers without first firing the cost effective marginal workers. When this wage schedule is properly set, the firm will pick the efficient employment level. As a consequence of this setup, equally productive workers receive different wages, just based on their position in the layoff order, just like Lars and Jens in the opening sentence of this paper. Kuhn and Robert elaborate their ideas in a static framework. Here, we introduce them in the dynamic model of Bentolila and Bertola, leading to a return to seniority in wages. We take an eclectic approach, that is, we do not start from an explicit bargaining game, but from positing a log linear sharing rule of the surplus of the specific investment. However, we impose one feature that characterizes Nash bargaining, namely efficient bargaining: as long as there is a surplus, the worker and the firm will be able to agree on a distribution of that surplus that makes continuation of the relation mutually beneficial. This guarantees that there is efficiency on the firing side. However, the efficiency of hiring decisions depends on a different issue, namely the Hosios condition, which requires the surplus generated by the specific investment to be shared between the worker and the firm in the same proportions as their shares in the cost of the investment. If not, hiring is below the efficient level due to a hold up problem. We elaborate our model under the assumption that the firm must pay for the full cost of the specific investment, so that any return to seniority implies sub-efficient hiring.

Under risk neutrality, contractibility of either specific investment or wages suffices to achieve efficiency, since we can always satisfy the Hosios condition by using one to match the other. When workers are risk averse, any return to seniority is inefficient, as it assigns the worker a risky return that can better be assigned to the risk neutral firm. As an extension, we consider the effect of firing cost, accounting for its upward effect on wages.[1] By the efficient bargaining assumption and the Coase theorem, firing cost does not affect firing, but further deteriorates hiring. Finally, we consider the role of trade unions in this model. The ideas in Kuhn (1988) and Kuhn and Robert (1989) seem to suggest that the return to tenure should be higher in unionized firms, since unions are predicted to use the tenure profile as a rent extraction mechanism. This turned out to be counter-factual: unionized firms generally have a lower return to tenure, not a higher return, see for instance Teulings and Hartog (1998: 225). We observe that this fits our theory. Incumbent workers have sufficient bargaining power to extract returns to their seniority even in the absence of a formal union: their cooperation is indispensable when the firm wants to transfer the tacit knowledge to newly hired workers. The LIFO layoff rule allows for a decentralisation of the bargaining process, leading to higher wages for senior workers. Instead, the political process within a union would lead to a more egalitarian distribution of the rents among the workers, that is, to higher wages but a lower wage return to seniority.

In the empirical part, we establish a number of features of our model. We show that seniority is an important determinant of job separation. Junior workers have a larger separation probability than senior workers. This effect comes on top of the duration dependence of the hazard, that is, in addition to the fact that the separation probability declines with the elapsed tenure at the job. Second, we show that there is a wage return to seniority. Starting from the seminal papers by Altonji and Shakotko (1987) and Topel (1991), there is a large and still flourishing literature on the estimation the wage return to tenure. The problem in this literature is that within a job spell, tenure is perfectly correlated with experience. Hence, the first order term of this return can only be estimated using variation between job spells, but that introduces all kind of selectivity problems, which this literature sets out to resolve. This problem is absent in the estimation of the return to seniority, since seniority is not perfectly correlated with experience. Seniority increases for example because new workers enter the firm. From that perspective, changes in seniority are correlated with changes in firm size, since an increase in firm size requires new workers to be hired and, hence, the seniority of the incumbents to increase. Luckily however, seniority is not perfectly correlated to firm size, since then the return to seniority could not be disentangled from the firm size wage effect: seniority does also increase by more senior workers leaving the firm, for example due to retirement. In our regressions, we use within job spell variation and we include both tenure and firm size as controls. Nevertheless, we are still able to find wage returns to seniority of 1 to

[1] In Bentolila and Bertola's (1990) analysis of the effect of firing costs, wages are fixed. Accounting for the effect on wage setting turns out to be important for the conclusions.

2 % in Portugal, and returns half that range in Denmark. Including seniority reduces the coefficients for tenure and firm size by 5-30 %, suggesting that tenure and firm size served at least partly as proxies for seniority in previous regressions. The return to seniority turns out to be of the same order of magnitude for males and females, but much larger for high than for low educated workers. Our theory also implies that returns to seniority are higher in industries with a high degree of monopoly power. We make an attempt to test this prediction, but our explanatory variables proxying for monopoly power are not strong enough to find an effect.

The paper is set up as follows. Section 2 presents our theoretical framework. In Section 3, we describe the data and the relevant labour market institutions in Denmark and Portugal, and we present our estimation results. Section 4 summarizes and concludes.

2 Theoretical framework

2.1 Setup

The model of Bentolila and Bertola (1990) provides a nice starting point for our analysis. Firms face a stochastic iso-elastic demand curve for their output, in logs:

$$n_t = z_t - \eta p_t, \tag{1}$$

where $\eta > 1$ is the price elasticity of demand, n_t is log demand, p_t is its log price, and z_t is a market index capturing the exogenous evolution of demand; z_t is assumed to follow a Brownian with drift, such that $\Delta z \sim N(\mu, \sigma^2)$. Labor is the only factor of production. The production function exhibits constant returns to scale. Without loss of generality, productivity is normalized to unity, so that output is equal to employment. In the model of Bentolila and Bertola (1990), hiring and firing of workers is costly. At this stage, we focus on hiring cost, denoted by I. This cost is interpreted more broadly as the specific investment that has to be made by the firm at the start of an employment relation. It is irreversible: once made, the cost cannot be recouped by ending the employment relation. For simplicity, we assume that this investment can be made instantaneously, so that no time elapses between the start and the end of the investment process. At the outside market, workers can earn a reservation wage, which is constant over time. It is most convenient to think of this reservation wage as the return to self employment. Without loss of generality, it is normalized to unity. Hence $w^r = 0$, where w^r denotes the log reservation wage. We assume both workers and firms to be risk neutral.

As a benchmark, we analyze first the simple case where firms pay workers their reservation wage and where there are no specific investments required for starting an employment relation, $I = 0$. In that case, labor demand can be adjusted costlessly at each point in time. Hence, the optimal strategy is to maximize instantaneous profits Π_t:

$$\Pi_t = e^{n_t} \left(e^{p_t} - 1 \right),$$

subject to the demand curve (1). The first order condition implies

$$\begin{aligned} p_t &= \pi, \\ mr\left(z_t - n_t\right) &= \frac{1}{\eta}\left(z_t - n_t\right) - \pi, \\ n_t &= z_t - \eta\pi, \\ \pi &\equiv \ln \frac{\eta}{\eta - 1} > 0. \end{aligned} \tag{2}$$

where $mr\left(z_t - n_t\right)$ denotes the log of the marginal revenue for the firm of hiring an additional worker, conditional on the state of demand z_t and log employment n_t. The parameter π is the log of the ratio of price over wage cost, when marginal cost and marginal revenue are equal. This ratio is greater than unity due to the monopoly power of the firm at the product market. The firm's price is constant over time, while its labor demand follows a random walk. The latter implication is consistent with Gibrat's law that tends to hold for large firms, see for instance Jovanovic (1982).

Next, consider the optimal strategy with specific investments, $I > 0$. Then, labor demand cannot be adjusted costlessly. On the hiring side, an additional worker requires a specific investment, which has to be recouped from future profits. Moreover, this investment is irreversible, so that delaying hiring has an option value. On the firing side, firing per se is costless, but irreversible. If demand surges after having fired the worker, the firm is unable to benefit from that demand without incurring the cost of the specific investment again. Hence, retaining the worker has an option value, too. Bentolila and Bertola (1990) show that the optimal policy of a firm is to hire workers whenever p_t reaches a constant upper bound $p^+ > \pi$ and to fire them whenever p_t reaches a lower bound $p^- < \pi$. The hiring bound p^+ exceeds π due to the necessity for the firm to recoup the cost of specific investments and due to the option value of postponing hiring, while the firing bound p^- is below π due to the option value of postponing firing. The situation is sketched in Figure 1. below.

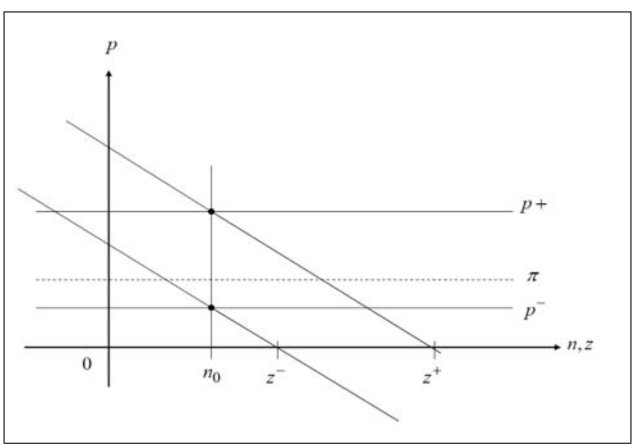

Figure 1: Firing-hiring boundaries with stochastic market index

The present employment level is denoted by n_0 and the present market index by z_0. If the market index rises above z^+, the firm hires additional workers to avoid p rising above p^+. If the market index falls below z^-, the firm fires workers to avoid p falling below p^-. Hence, p_t follows a random walk between p^- and p^+, while n_t is constant in this interval. However, when p_t drifts outside these boundaries, the firm uses n_t as an instrument to control p_t. Then, p_t is held constant, and n_t starts drifting, either up (if $p = p^+$), or down (if $p = p^-$). Bentolila and Bertola (1990) provide expressions for both boundaries.

Suppose we impose a LIFO separation rule upon this firm. We can index each worker by the log employment level of the firm at the date that the worker is hired. A worker hired at time h gets rank q, $q = n_h = z_h - \eta p^+$. Her seniority index at time t is defined as $n_t - q$. The less senior the worker, the shorter her tenure, and the lower her seniority index $n_t - q$. Hence, the most senior worker $q = 0$ has seniority index $n_t - q = n_t$, while the least senior worker at time t with $q = n_t$ has seniority index $n_t - q = 0$. The LIFO layoff rule implies that a worker hired at time h with index $n_t - n_h$, will be fired at the first moment $f > h$ that employment is back at the level n_h and $p_f = p^-$, that is, when $z_f - z_h = -\eta\left(p^+ - p^-\right)$. By construction, f is the first point in time that z_t has travelled down a distance $\eta\left(p^+ - p^-\right)$ from its initial value z_h. Whether or not new workers have been hired after time h by z_t rising above z_h for some $t, h < t < f$, and if so, how many, is immaterial to this conclusion, since these workers are indexed $n_f - n_t < n_f - n_h$, and hence, the LIFO separation rule imposes that these workers will be fired before the worker with seniority $n_f - n_h$. Bentolila and Bertola's model of firm level employment supplemented with a LIFO layoff rule corresponds one-to-one with a simple model of individual job tenures. In this model, a worker hired at time h, with $z_t = z_h$, will be fired a the first time that z_t has travelled down a distance $\eta\left(p^+ - p^-\right)$; see Buhai and Teulings (2005) for a recent elaboration of that model.

2.2 Rationale for LIFO

Why would a firm use a LIFO layoff strategy? In the simple world discussed above, where the firm pays the worker her reservation wage, there is no rationale for such a rule. Since the worker receives her reservation wage, she is indifferent between working at the firm or being laid off. Hence, there is no point in fixing an order of layoff. However, if we relax the assumption that the firm pays its workers their reservation wage and we attribute incumbent workers some bargaining power, the quasi rents of the specific investment enable these workers to capture wages above the reservation wage. In that case, a layoff order carries practical relevance, as it protects the 'rights' of senior workers (those who are hired first). Kuhn (1988) and Kuhn and Robert (1989) offer a neat further legitimation for using such a rule. Their idea is based on the classic distinction in the theory of unionized wage setting between right-to-manage or labour demand curve models, on the one hand, and efficient bargaining models, on the other hand. In the former, unions raise wages above the reservation wage. However, the firm can set employment unilaterally ("the firm has the right to manage"). From the point of view of the firm, wages are equal to marginal cost. Hence, the higher wage rate reduces the firm's labour demand. This outcome has been criticized for leaving gains from trade between the firm and the workers unexploited. Additional workers would be willing to join the firm at a wage rate between the reservation wage and the negotiated wage, and the firm would be willing to hire them at that wage. By bargaining on wages and employment simultaneously, the firm and the union can exploit these gains from trade. Kuhn and Robert's idea is that these gains from trade can also be exploited, while maintaining the right-to-manage feature that wages are negotiated between the firms and the union (or: its workers) and that the firm sets employment unilaterally. This can be done by fixing the order of layoff, and differentiating wages by the position in the layoff order. A firm can only fire senior workers after having first fired all junior workers. Senior workers earn the highest wage since they can only be fired after all juniors been fired and therefore they feel sufficiently protected to demand higher pay, any resultant job loss falling on their less senior colleagues. This is a form of price discrimination on the side of the union. First degree price discrimination results when the union has full bargaining power. Inframarginal senior workers receive part of their inframarginal productivity surplus.

Kuhn and Robert (1989) specify their theory in static framework. In that case, the layoff ordering can be based on any variable, height, IQ, experience, or what else springs to mind. Combining the model of Bentolila and Bertola (1990) with a LIFO layoff ordering provides a straightforward way to cast the ideas of Kuhn and Robert in a dynamic framework. Then, the prevalence of a LIFO ordering has a natural economic interpretation. The senior worker's future wage claims are sensitive to the firm hiring new workers, since after the specific investments have been made, these new workers are perfect substitutes for senior workers. The firm could in principle hire new workers for a low wage, and fire the senior workers instead. The lack of commitment on the side of the firm of not benefitting from this strategic option has an adverse effect on the set of feasible contracting arrangements open to the firm and its workers. Suppose that the specific investment of new workers is largely made up from acquiring the tacit knowledge of the firm's production process and the transfer of this knowledge can be blocked by senior workers, or suppose that senior workers can harass newcomers, as suggested by Lindbeck and Snower (1988). In that case, hiring new workers requires the consent of senior workers. At the same time, the firm has a commitment problem: how can it credibly promise senior workers not to use new workers as a replacement for them, after the transfer of the tacit knowledge to the new workers is completed? Due to this commitment problem, gains from trade from hiring new workers cannot be exploited. A LIFO separation rule is a solution to this commitment problem by providing senior workers protection against being laid off before newly hired workers, so that there are no disincentives to cooperate in training new workers.

2.3 Wage sharing rule

We operationalize the LIFO idea by positing a linear relation for the log wage as a function of the current state of product demand z_t and the seniority index $n_t - q$:

$$w(z_t - q) = \omega + \frac{\beta}{\eta}\left(z_t - q - \eta p^-\right) \tag{3}$$

where $0 < \beta < 1$ and where ω is the log wage at which a worker is indifferent between remaining employed at the firm and being laid off, given the LIFO layoff rule and equation (3). The parameter ω can therefore be interpreted as the reservation wage of an incumbent worker. We shall derive an expression for it below, when discussing the worker's problem. At the moment of firing the worker with seniority index $n_t - q$, it must be true that $z_t = q + \eta p^-$, by the definition of the firing bound. The factor $\eta^{-1}(z_t - \eta p^- - q)$ is equal to $mr(z_t - q)$, the log of surplus of marginal revenue above marginal cost, compare expression (2), conditional on employment being equal to the worker's seniority index $n_t - q$; that is, if there were no workers in the firm with higher seniority, or equivalently, there is nobody in the firm with a lower tenure than worker $z_t - q$. This marginal revenue is a counterfactual, in the sense that actual employment can be larger, $n_t \geq q$. We can therefore just as well refer to this "marginal" revenue as the "infra marginal" revenue, because it would be the marginal revenue only if the firm were first to fire all workers with a higher q. Equation (3) implies that senior workers receive a share β of this surplus of log inframarginal revenue above the log marginal outside option. The parameter β can be interpreted as the bargaining power of workers, though strictly speaking this interpretation lacks a foundation in a formal bargaining model.[2] The log linearity of equation (3) is just imposed for the sake of analytical convenience. Equation (3) implies efficient bargaining: as long as there is a positive surplus both parties get a share of it, so that it is rational for both to continue the employment relation. As soon as the surplus is vanished, separation will occur, which by then is the efficient outcome. Note that equation (3) depends on $z_t - q$, not on z_t and q separately. Since z_t is closely related to log firm size n_t (apart from the effect of insulating n_t from fluctuations in z_t whenever p_t is in between the hiring and firing bounds, p^+ and p^-), $z_t - q$ can be interpreted as an index of seniority relative to firm size. The return to the seniority index (or simpler, to seniority), β/η, is increasing in the bargaining power of the workers, β, and in the monopoly power of the firm on its product market, η^{-1}. Since $0 < \beta < 1$ and $\eta > 1$, the elasticity of wages with respect to the index $n_t - q, \beta/\eta$, must be between zero and unity.

2.4 The worker's problem

The value of ω can be derived using the theory of option values, see Dixit and Pindyck (1994). Let $V(z_t - q)$ be the asset value of holding a job at a firm. By Ito's lemma

[2] In the case of a single worker firm, where we could apply the theory of two player bargaining, as in Buhai and Teulings (2005), the log linear sharing rule would be almost equivalent to Nash bargaining, which would yield a linear sharing rule.

$V(z_t - q)$ satisfies the Bellman equation

$$\rho V(z_t - q) = e^{w(z_t - q)} + \mu V'(z_t - q) + \frac{1}{2}\sigma^2 V''(z_t - q),$$

where ρ is the interest rate, such that $\rho > \mu + \frac{1}{2}\sigma^2$; $e^{w(z_t - q)}$ is the current wage. The relevant solution to this second order differential equation reads

$$\begin{aligned} V(z_t - q) &= \frac{1}{r(\beta/\eta)} \exp\left[\omega + \frac{\beta}{\eta}\left(z_t - q - \eta p^-\right)\right] + A\exp\left[\lambda^-(z_t - q)\right], \\ r(x) &\equiv \rho - \mu x - \frac{1}{2}(\sigma x)^2 \Rightarrow r(0) = \rho, r(1) = \rho - \mu - \frac{1}{2}\sigma^2 > 0 \\ \lambda^{+,-} &\equiv -\mu/\sigma^2 \pm \sqrt{\mu^2/\sigma^4 + 2\rho/\sigma^2} \Rightarrow \lambda^- < 0, \lambda^+ > 1. \end{aligned}$$

where we substitute $w(z_t - q)$ for equation(3) and where A is a constant of integration that remains to be determined. $r(x)$ is a modified discount rate, accounting for the drift and the variability of z_t; note that $r(0) = \rho$; $r(1)$ must be positive for a bounded solution for the welfare of the worker to exist. Hence, the first term is the net discounted value of expected wage payments, disregarding the worker's option to quit the firm when wages fall too far below the reservation wage. The second term, $A\exp\left[\lambda^{+,-}z_t\right]$, is the option value of separation. Only one of the roots $\lambda^{+,-}$ is relevant here. For large values of z_t, the firm is doing well and, hence, keeping the job is attractive for the foreseeable future. The option value of separation must converge to zero, which is the case for the negative root λ^-, since $\lim_{z\to\infty} A\exp\left[\lambda^- z\right] = 0$. Hence, the constant of integration for the positive root λ^+ is equal to zero. Efficient bargaining implies that it is optimal for the worker to separate when $z_t = q + \eta p^-$. Two conditions to hold for that value of z_t to be optimal: the value matching and the smooth pasting condition. The value matching condition states that the asset value of holding the job should be equal to the asset value after separation, that is, the net discounted value of the reservation wage, ρ^{-1}. The smooth pasting condition states that for small variations in z_t, the worker remains indifferent between holding the job and separation, since the worker should not regret separation after a small perturbation of z_t because separation is irreversible. This requires the first derivative of $V(z_t - q)$ with respect to z_t to be zero. Using $z_t - q = \eta p^-$ at the moment of separation, both conditions read

$$\begin{aligned} V\left(\eta p^-\right) &= \frac{1}{r(\beta/\eta)} e^{\omega} + Ae^{\lambda^- z} = \frac{1}{\rho}, \\ V'\left(\eta p^-\right) &= \frac{\beta}{\eta r(\beta/\eta)} e^{\omega} + \lambda^- Ae^{\lambda^- z} = 0, \\ \omega &= \ln r(\beta/\eta) - \ln\rho - \ln\left(1 - \frac{\beta}{\eta\lambda^-}\right) < 0, \end{aligned} \tag{4}$$

where the final equation follows from the elimination of A from the first two. ω is below the log reservation wage $w^r = 0$ since separation is an irreversible decision. If the demand for the firm's product, z_t, goes up after the separation decision, the worker is no longer able to benefit from the wage increase. Hence, workers are prepared to incur some loss before they decide to separate. The higher the worker's share β in the log surplus, the lower is ω, since expected future revenues are higher so that workers are prepared to accept greater losses before separation. Similarly, ω is declining in the drift μ, since a higher drift raises expected future revenues, and it is declining in the variability of demand σ^2, since a higher variability raises the option value of hoping for a future increase in the surplus.

2.5 The firm's problem

Given this wage setting rule, we derive the firm's optimal strategy. We use a methodology that is similar to Bentolila and Bertola (1990). We attribute to each worker her marginal revenue and her wage, taken the employment of workers hired previously as given, and then consider when it is optimal for the firm to hire and subsequently fire this worker. In this way, we can consider the decision to hire and fire the N_t-th worker ($N_t \equiv \exp n_t$) separately of the hiring and firing of workers hired before this worker, and of workers hired afterwards. Then, the model is a straightforward extension of Dixit and Pindyck (1994: 216), the only difference being that wages are constant in Dixit and Pindyck, while they vary with the state of demand z_t in this model. Let $F(n_t, z_t)$ be the asset value of the firm for the N_t-th worker. The Bellman equation for $F(n_t, z_t)$ satisfies

$$\rho F(n_t, z_t) = \exp[mr(z_t - n_t)] - \exp[w(z_t - n_t)] + \mu F_z(n_t, z_t) + \frac{1}{2}\sigma^2 F_{zz}(n_t, z_t). \quad (5)$$

The first term is the marginal revenue of that worker, see equation (2), the second term is the wage for that worker. The relevant solution to this differential equation reads

$$F(n_t, z_t) = \frac{1}{r(\eta^{-1})} \exp\left[\frac{1}{\eta}(z_t - n_t) - \pi\right] - \frac{1}{r(\beta/\eta)} \exp[w(z_t - n_t)] + B^- \exp\left[\lambda^-(z_t - n_t)\right].$$

The final term is the option value of separation, with B^- being the constant of integration. As in the case of the worker, the positive root λ^+ is irrelevant, since the option value must converge to zero for large values of z_t (since then the option to fire the worker has no value). Suppose the firm employs less than N_t workers. Then, the option value of hiring the N_t-th worker at some future date reads

$$G(n_t, z_t) = B^+ \exp\left[\lambda^+(z_t - n_t)\right],$$

where B^+ is the constant of integration. There are no current costs or revenues, hence only the option value term matters. Since this option value converges to zero for low values of z_t, here only the positive root λ^+ applies. The value matching and smooth pasting conditions read

$$\begin{aligned} F\left(z_t - \eta p^-, z_t\right) &= G\left(z_t - \eta p^-, z_t\right), \\ F_z\left(z_t - \eta p^-, z_t\right) &= G_z\left(z_t - \eta p^-, z_t\right), \\ F\left(z_t - \eta p^+, z_t\right) &= G\left(z_t - \eta p^+, z_t\right) + I, \\ F_z\left(z_t - \eta p^+, z_t\right) &= G_z\left(z_t - \eta p^+, z_t\right). \end{aligned} \tag{6}$$

The first pair refers to the firing decision, the second to the hiring decision. The first condition states that at the moment of firing, when by definition $n_t = z_t - \eta p^-$, the asset value of keeping the worker is equal to the option value of a vacancy. The second equation is the smooth pasting condition, which states that this condition also applies for slight variations of z_t, so that firm wouldn't regret a decision to fire after a slight variation in z_t. The third equation is the value matching condition for the moment of hiring, when $n_t = z_t - \eta p^+$: the asset value of hiring the worker should be equal to the cost of investment and option value of filling the vacancy at a later point in time. The final equation is the smooth pasting conditions for the moment of hiring. This system of four equations determines four unknowns, the constants of integration, B^- and B^+, and the hiring and firing boundaries, p^- and p^+. This system is non-linear, and its analysis is relegated to Appendix A, where we prove the subsequent results. A unique, economically meaningful solution to this system exists, where $B^- > 0$ and $B^+ > 0$, and where $p^+ > 0 > p^-$. The elimination of B^- from the first two equations of (6) yields

$$p^- = \ln r\left(\eta^{-1}\right) - \ln \rho + \pi - \ln\left(\frac{\eta\lambda^- - 1}{\eta\lambda^-}\right) - \ln\left(1 - \rho\frac{\lambda^+ - \lambda^-}{\lambda^-}B^+ \exp\left[\eta\lambda^+ p^-\right]\right). \tag{7}$$

The firing bound p^- does not depend on β, except for the effect of β on B^+, which is the option value of refilling the vacancy at a later moment. This is an application of the Coase theorem: under the efficient bargaining, the distribution of the surplus of the employment relation does not matter for the actual level of employment. The option value of rehiring comes in because when the firm decides to fire the N_t-th worker, it always has the option to rehire at a later moment. The larger the distance between the hiring and firing threshold, $p^+ - p^-$, the longer it will take (in expectation) before the firm will find it attractive to refill the vacancy, and hence the smaller is the option value associated with that. Keeping constant all other parameters of the model, an increase in the bargaining power of the workers β raises the distance $p^+ - p^-$

$$\frac{d\left[p^+ - p^-\right]}{d\beta} > 0.$$

The higher the workers' bargaining power β, the less volatile will be the employment, since employment is insulated from shocks in demand z_t over a larger interval of

$\eta p^- < n_t - z_t < \eta p^+$, and the larger is therefore the expected tenure of a newly hired worker. This is the consequence of a hold up problem. The larger the workers' bargaining power, the higher the hiring threshold p^+, since the firm reaps a smaller share of future surpluses created by the specific investments in new workers, so that a larger initial surplus of marginal revenue above the reservation wage is required to recoup the cost of these investments. Since $p^+ - p^-$ is larger, the option value of rehiring is lower, and hence the firing threshold p^- is lower, though this indirect effect of β on p^- is smaller than the direct effect on p^+. Hence, a higher bargaining power of workers reduces the firing threshold and postpones separation. This implication squares well with the findings in Bertrand and Mullainathan (2003), who show that when firms are insulated from takeovers, the wages of the incumbent employees are higher, suggesting a higher value of β. This goes hand in hand with lower rates of creation of new plants, which in the context of our model is similar to hiring new workers. Bertrand and Mullainathan also report a lower rate of destruction of old plants, or in the context of our model, a lower firing bound, p^-. The larger is β, the lower is the option value of future rehiring, and hence the less attractive it is to fire a worker.

2.6 Explanation of the firm size wage effect?

The firm size effect on wages has been extensively documented, see Brown and Medoff (1989). Can our model offer an explanation for the firm size wage effect? When we look at the issue from the point of view of an individual worker, the evolution of her seniority $n_t - q$ is driven by the evolution of log firm size n_t. In reality workers retire at some point in time. When more senior workers retire, a worker's seniority goes up even at constant firm size. Here, we abstract from retirement, so that firm size is the only driver of changes in seniority. At first sight, this suggests that our theory could explain the firm size wage effect. Nevertheless, this turns out not to be true. The average log wage in a firm at the firing bound p^- satisfies

$$e^{-n_t} \int_{-\infty}^{n_t} w(z_t - q) e^q dq = e^{-n_t} \int_{-\infty}^{n_t} \left[\omega + \beta/\eta \left(z_t - q + 1 - \eta p^-\right)\right] e^q dq = \omega + \beta/\eta,$$

where in the final equality we use the fact that at the firing bound, $n_t = z_t - \eta p^-$. Hence, the average log wage does not depend on firm size. The intuition is that the positive effect of the wage increase for the incumbent workforce is exactly offset by the negative effect of the below average log wage for the new hires. Thus, although this model predicts firm size to be a driver for the changes in the wages of incumbent workforce, it does not explain why wages for the firm as a whole depend on firm size. However, the average log wage does depend on the parameter β/η. Other things equal (in particular, human capital of the workforce), the model predicts the return to seniority, β/η, to be increasing in the average log wage.

2.7 Who gets hired and the welfare cost of hold up

To close the model, we have to explain who gets hired by a firm and who does not. The log wage of a worker who is just hired is higher than the wage of a worker who is at the borderline of being laid off, that is

$$w\left(z_t - \eta p^{+}, z_t\right) > w\left(z_t - \eta p^{-}, z_t\right) = \omega.$$

Since the asset value of a worker who is on the borderline of being laid off is equal to the net present value of her reservation wage, $1/\rho$, the asset value of a worker who is just hired must be higher than $1/\rho$. Hence, new jobs at the firm are rationed. A convenient way to model this rationing process is to introduce unemployment. A worker who is just laid off has two options. Either she can decide to collect her outside wage by becoming self employed, or she can decide to queue for a new job at a firm. During this waiting period she cannot produce as a self employed. For simplicity, we assume that leisure has no value.[3] New jobs at firms arrive at a rate λ per unit of the labor force and are distributed randomly among the unemployed. Hence, the asset value of unemployment, V^U, satisfies

$$\rho V^U = \frac{\lambda}{u}\left[V\left(\eta p^{+}\right) - V^U\right],$$

where u is the unemployment rate. λ/u is the arrival rate of a new job for unemployed. The lower unemployment, the higher this arrival rate, since there are less people among whom new jobs have to be distributed. $V\left(\eta p^{+}\right) - V^U$ is the asset gain of getting a job offer. The level of unemployment follows from the no-arbitrage condition between self employment and unemployment

$$u = \lambda\left[V\left(\eta p^{+}\right) - \frac{1}{\rho}\right], \tag{8}$$

where we use $V^U = 1/\rho$, the asset value of self employment[4]. The higher the asset gain of getting a job at a firm, the higher must be unemployment. Though the efficient bargaining assumption generates efficiency in the layoff decision of firms, it does not achieve efficiency on the hiring side. There are two types of inefficiency.[5] First, not all gains from trade between the worker and the firm are exploited. Firms would hire more workers if $\beta = 0$, since p^{+} is an increasing function of β. Firms'

[3] Allowing for a value of leisure would not change the predictions of model. It would make unemployment less costly per unit of time, but this effect would be exactly offset by the rise in unemployment.

[4] We assume: $u < 1$. If $u > 1$, the outside option of self employment would become irrelevant, and the reservation wage ω and job arrival rate λ would become endogenous. ω would rise till so many workers are fired, and so few workers are hired till λ is such that the no arbitrage condition holds for $u = 1$.

[5] Throughout the paper, we do not pay attention to a third type, the inefficiency caused by the monopoly power of the firm vis-a-vis consumers.

perception of the marginal cost of hiring a worker in net present value terms exceeds the social cost by the same amount as the asset gain for an unemployed of getting a job offer, $V(\eta p^{+}) - 1/\rho$. This gives rise to a Harberger triangle. Next to this Harberger triangle, there are the cost of rationing that dissipate workers' surplus. The no-arbitrage condition (8) implies that the workers as a group spoil their whole share in the quasi rents in wasteful unemployment.

The inefficiency problem arises by a violation of the Hosios (1990) condition. Workers are able to capture a share of the quasi rents of the specific investments, while they do not bear a corresponding share of the burden of the investment cost. Hence, firms restrain new hiring to push up the net present value of all rents of a new hire above the cost I till their share in the rent suffices to recoup the cost. The inefficiency is due to the non-verifiability of specific investment and the inability of workers to commit on not using their bargaining power after the specific investment has been made. If wages were contractible, workers could commit on not demanding any return to seniority, so that the firm bears the full cost and gets the full revenues of the specific investment, thereby satisfying the Hosios condition. Alternatively, if specific investments were verifiable, the inefficiency would be resolved by shifting some share of the burden of investment to the worker, such that workers bear an equal share of cost of the specific investment as they get from its revenues, again satisfying the Hosios condition. For the latter case, note that the asset value of a worker at the moment of hiring, $V(\eta p^{+})$, is independent of the moment of hiring. Hence, although at a particular point in time senior workers get more quasi rents than juniors, at the moment of hiring each worker has the same net present value of expected rents, independent of her rank q, that is, independent of the level of employment at the moment she is hired. Seniors getting higher rents than juniors at a particular point in time reflects the fact that they are able to realize the upside of the risky returns on their share in the specific investment I. Hence, the LIFO separation rule can be interpreted as a protection of their property right on their share in the quasi rents, against the temptation of the firm to fire the expensive senior workers, thereby depriving them from the upside of their risky returns. A LIFO separation rule is then a device for implementing an efficient contract. This argument implies that as long as we do not know what share of the cost of specific investment is born by workers, empirical evidence supporting the relevance of equation (3) for wage setting is inconclusive on the issue of whether or not employment is below its inefficient level.

Finally, note that when workers have to pay the full cost of the specific investment, the hold up problem is precisely the reverse. Then, the non-verifiability of workers investment and the inability of firms to commit on not using their bargaining power $1 - \beta$ leads to inefficiency. Workers are only willing to enter the firm when the net present value of quasi rents of their investment are so high that their share in this present value suffices to cover the cost of investment. There is a simple statistic enabling the observer to establish which side is overcompensated in the ex post bargaining over the surplus of the specific investment: when workers queue for jobs, so that there is unemployment, firms are held up, as in the basic model; when firms chase after workers, so that there are vacancies, workers are held up.

2.8 Extensions

Some extensions to this model are worth discussing. First, relaxing the assumption of risk neutrality on the side of the worker introduces a trade off. As discussed in the previous section, verifiability of the specific investment I is sufficient to implement first best in the standard case with risk neutral workers. With risk averse workers, this conclusion no longer applies. First best requires that workers get paid their reservation wage all the time, and hence that firms bear the full cost of investment. The inability of workers to commit on not using their bargaining power makes first best unattainable in that case. The case of risk averse workers and risk neutral firms is particularly relevant because one can expect capital markets to be much more complete for firms than for workers. It is much easier for the firm to diversify firm specific risks on the capital market than for its workers.

A second extension is the introduction of firing cost, imposed either by law or by trade unions. We think of a firing cost as a wealth transfer W from the firm to the worker at the moment of firing. By the assumption of efficient wage bargaining, this wealth transfer has an impact on the wage bargaining process. The value matching condition of the worker for the moment of firing reads, compare equation (4)

$$V\left(\eta p^{-}\right)=\frac{1}{\rho}+W\Rightarrow\omega=\ln r\left(\beta/\eta\right)-\ln\rho-\ln\left(1-\frac{\beta}{\eta\lambda^{-}}\right)+\ln\left(1+\rho W\right)<0. \quad (9)$$

Firing cost raise value of the outside option of the worker by the wealth transfer W. This raises ω. Hence, there are two counteracting effects on the firing bound p^{+}: the direct effect of firing cost makes layoffs less attractive to the firm, while the indirect effect via higher wages makes layoffs more attractive, since workers are more costly due to the higher level of ω. The first order condition for optimal firing now reads, compare equation (6)

$$F_z\left(z_t-\eta p^{-},z_t\right)=G\left(z_t-\eta p^{-},z_t\right)+W,$$

where we use the value of ω from equation (9) to account for the effect of firing cost on wages. Some calculation, see Appendix A, shows that the value for p^{-} remains the same as in equation (7). The direct and the indirect effect cancel therefore exactly, except for the indirect effect via B^{+}, the option value of rehiring. Again, this is an implication of the efficient bargaining assumption and the Coase theorem. On the hiring side, firing cost has two effects with the same sign: first, it raises wages via its effect on ω and, second, there is the prospect of having to pay firing cost in case of future layoff. To the extent that workers have excessive bargaining power, in the sense that the net present value of their share in the surplus exceeds the share in the cost of specific investment, which shows up as workers queueing for a job and hence as unemployment, this increase comes to the detriment of efficiency. The paradox here is that firing costs aggravate the unemployment problem that they were meant to resolve. Since the hiring threshold p^+ goes up due to the introduction of firing cost, the expected value of future rehiring is lower, so the firing threshold is lower. Hence, firing cost raises the distance $p^+ - p^-$ and therefore the expected tenure of newly hired workers.

Now that we have discussed these extensions, risk aversion and firing cost, it makes sense to consider the nature of workers' bargaining power. Most economists associate this power with the trade unions.[6] Only unions provide workers bargaining power. Without unions, firms are supposed to have complete bargaining power. A notable exception is the article by Lindbeck and Snower (1990), who point out that the insiders' ability to harass new hires gives them bargaining power vis-a-vis their employer. Without the insiders' consent, firms are effectively unable to introduce new hires. The interesting aspect of Kuhn (1988) and Kuhn and Robert (1989) is that their rank related compensation scheme allows a decentralization of the bargaining process. As soon as the layoff order has been set, each worker can negotiate for herself. When a marginal worker negotiates a wage increase raising her wage above marginal cost, she endangers her own employment, not that of the inframarginal workers.[7] Hence, a LIFO scheme enables workers to exploit their individual bargaining power without workers having to solve their collective action problem. When workers are united in a trade union, more elaborate strategies are available, that yield a higher expected payoff, in particular when workers are risk averse. By trading a higher transfer W, in case of layoff, in exchange for a lower seniority premium, such that the firm's asset value $F(n_t, z_t)$ remains the same, the expected utility for new hires can be improved by shifting the firm specific risk z_t to the firm. The transfer W allows this insurance to be extended even beyond the time spell covered by the employment relation with the firm, compare the discussion on the efficient bargaining model of the union. Moreover, the political decision making process within the union, where senior and junior workers have to compromise on the distribution of the rents, is likely to generate support for an egalitarian outcome, as implied by the median voter model.[8] All these arguments suggest that the LIFO model is probably a more appropriate description of a non-union than of a unionized environment. These arguments can also explain, why tenure profiles seem to be flatter in unionized firms, in contrast to what Kuhn (1988) and Kuhn and Robert (1989) seem to predict, see e.g. Teulings and Hartog (1998: 225).

[6] This problem has been suggested to us by Kevin Murphy.

[7] The reverse is not necessarily true. An inframarginal worker can bargain a wage above her productivity, if workers with lower seniority capture less than their full productivity. In that case, the firm has an incentive not to fire the inframarginal worker because it first has to fire the marginal worker.

[8] Equation (3) implies a Pareto distribution of wages within the firm, which is heavily skewed to the right.

3 Empirical framework

The model discussed in the previous section has three testable implications:

1. Gibrat's law: log firm size n_t follows a random walk. This should hold better when the distance between the hiring and firing boundaries is small. We use various standard procedures to test for Gibrat's law for log firm sizes.

2. The Last-in-First-Out separation rule: the workers hired last, leave the firm first. Note that we apply an efficient bargaining model. Hence, the observational distinction between quits and layoffs is arbitrary, compare McLaughlin (1991). As long as there is a positive surplus of the worker's marginal revenue to the firm above the worker's reservation wage, the worker and the firm will strike a deal. As soon as this surplus has vanished, it is in their mutual interest to separate. Whether the separation is initiated by the worker or by the firm is irrelevant. Hence, the model predicts the LIFO separation rule to apply to separations as a whole, not just to layoffs separately. We use duration analysis to test this for this implication. Furthermore, we test one cross country implication. Our theoretical model predicts that EPL raises the distance between the hiring and the firing threshold, and hence the expected completed job tenure. Since Portugal has a much stricter EPL, one would expect tenure to be higher there than in Denmark.

3. The dependency of wages on seniority: wages depend on a worker's seniority in the firm, relative to his colleagues, see equation (3). We use wage regressions, both in levels and in first differences, to test this implication. Moreover, the higher the monopoly power of the firm, η^{-1}, the higher should be the return to seniority. We use variation in the estimated return to seniority between industries to test this implication.

The challenging aspect of this paper is testing the second and third implication. For that purpose, we need longitudinal matched worker firm data. Only by knowing the tenure distribution of the entire workforce of the firm, at all times, we can calculate the seniority of a worker. Though using this type of data has become more fashionable in recent years, they are still not widely available. We have been able to get access to such data on Denmark and Portugal. The two countries are a nice combination since their level of employment protection differs widely. We give a description of both data sets and the relevant institutions from these countries in the next subsection. Subsequently, we discuss the test of the three implications of our model, each in a separate subsection.

3.1 Data and relevant labour market institutions

For Denmark, we use the *Integrated Database for labor Market Research* (IDA) for 1980-2001, from the Danish Bureau of Statistics, which has been used previously e.g.

in Mortensen (2003). IDA tracks every single individual between 15 and 74 years old. The labor market status of each person is recorded once a year, at November 30. The dataset contains a plant identifier, which allows the construction of the total workforce of a plant, and hence of the firm as a whole. There is information on earnings, occupation, education, and age, and on the plant's location, firm size, and industry. Industry is defined as the industry employing the largest share of the firm's workforce. Firm size is defined as the number of individuals holding primary jobs in that firm and earning a positive wage. The tenure of workers hired since 1980 can be calculated straightforwardly from the IDA. For workers hired between 1964 and 1980, the tenure can be calculated from a second dataset on the contribution histories to a mandatory pension program, the *ATP*. The tenure in job spells started before 1964 is left censored (less than 3% of the observations). We calculate potential experience as age-schooling-6.

For Portugal, we use the *Quadros de Pessoal* for 1991-2000 provided by the Ministry of Employment, which has been used before e.g. in Cabral and Mata (2003). It is based on a compulsory survey of firms, establishments and all their workers; the compulsory participation enhances the quality of the data. The information available is similar to that for Denmark except that workers' tenure is directly reported and firm size is measured by sales when available, otherwise by employment; the industry of the firm is that industry with the highest share of sales or, when the allocation by sales is not possible, the industry with the highest employment share. We use all full-time employees in their main job, aged between 16 and 66, and working for a firm located in Portugal's mainland. The hourly gross earnings were computed as the monthly base-wage plus seniority-indexed components plus other regularly paid components, divided by normal hours of work per month.

Table 1: Descriptive Statistics Denmark and Portugal

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Pooled sample												
DK 1980-2001	22364083	2771627	6870869	301015	164.22	5.41	20.25	12.05	37.79	0.35	0.63	4.68
	.	.	.	.	(59.94)	(5.58)	(12.19)	(3.05)	(11.95)	.	(0.72)	(2.44)
PT 1991-2000	11420191	3211990	4268149	330270	3.68	8.43	20.26	6.81	36.73	0.40	0.87	4.04
	.	.	.	.	(2.52)	(8.61)	(11.39)	(3.65)	(11.18)	.	(0.85)	(2.14)
Cross-section year 2000												
DK 2000	1087922	1087922	1087922	79188	23.53	5.26	21.10	12.69	38.98	0.36	0.61	4.86
	.	.	.	.	(9.02)	(5.78)	(11.68)	(2.74)	(11.75)	.	(0.73)	(2.48)
PT 2000	1403686	1403686	1403686	184957	4.06	7.93	20.47	7.59	37.14	0.42	0.75	3.64
	.	.	.	.	(2.75)	(8.54)	(11.42)	(3.83)	(11.08)	.	(0.81)	(2.18)

Columns' labels: (1) Observations, (2) Workers, (3) Spells, (4) Firms, (5) Average Real Hourly Wage (base year=2000, measure unit: Euro), (6) Average Tenure, (7) Average Potential Experience, (8) Average Education, (9) Average Age, (10) Proportion of Women, (11) Average Relative Log Rank, (12) Average Log Firm Size. (Standard deviations in paratheses.)

For both countries, we use data for all private sector jobs, except agriculture, fishing and mining. We eliminate outliers by deleting all wage observations lower than the legal minimum wage for each year and drop the top 1% of the wage distribution. Summary statistics for both countries are presented in Table 1, both for the pooled data and for 2000 separately[9]. There are several obvious differences between the two countries. The mean level of education is more than 5 years higher in Denmark than in Portugal, while the mean tenure is almost 3 years longer in Portugal than in Denmark. The number of firms is far higher in Portugal than in Denmark, and the average firm size in Portugal is only 30% of that in Denmark. Finally, Danes earn on average almost six times more than Portuguese.

Denmark and Portugal are both members of the European Union, both small open economies, and both with (in EU terms) low unemployment rates over recent years, see Nickell et al (2005). The two countries differ substantially with respect to their labour market flexibility and social safety net. In Denmark, wage bargaining is de facto done at the firm level over the observation period, although there are some collective industry-level bargaining agreements for minimum wages, see Kenworthy (2001). The Danish private labor market is characterized by very low EPL compared to most OECD countries, but similar to the United States, the United Kingdom and Australia, see OECD Employment Outlook (2004). The EPL applicable to privately owned firms is limited to basic provisions such as white-collar workers being given an advance notice and a minimum of EU enacted rules relating to mass layoffs. General rules and procedures for dismissal are absent, see also Albaek et al (1999). Unemployment benefits for wage earners are high and can be obtained for a long period, being generous compared to most other countries. In Portugal, wage negotiations start at the national level, defining a national minimum standard and setting guidelines for collective bargaining at a lower level. Massive collective agreements dominate the labor market as a result of extension mechanisms. However, individual firms are able to pay higher wages than those bargained at the aggregate level, see Cardoso and Portugal (2005). The EPL in Portugal is the other extreme compared to Denmark: according to the OECD Employment Outlook (2004), Portugal has the about highest overall summary index of all countries. The notice period for layoff is 60 days, and the severance pay for individual dismissals is 1 month per year of service, with a minimum indemnity of 3 months. The minimum duration of any contract is 6 months, although the law defines some exceptions. Following an unjustified dismissal the worker can be reinstated or compensated. The maximum duration of benefits in Portugal varies from 10 months for people aged under 25 and 30 months for those aged above 45. The replacement rate is 65 % of the previous wage, but the benefit cannot be below the minimum wage or above 3 times the minimum wage.

[9]Summary statistics for each separate industry (see Appendix B for our broad industry classification) are available upon request.

3.2 Testing Gibrat's law

Our theoretical model predicts log employment size of firm j, n_{jt}, to follow a random walk, apart from the dampening effect of the hiring and firing boundary on short run fluctuations in n_{jt}. This regularity is known as Gibrat's law. Though slight deviations from Gibrat's law do not affect the main economic implications of the theoretical model, it is useful to have at least some idea how close this assumption is in the data. There is a massive literature on testing Gibrat's law, see e.g. De Wit (2005) or Sutton (1997). Here, we use two tests.

The first approach is laid out in Abowd and Card (1989) and Topel and Ward (1992) for log wages; we adapt this methodology for log firmsizes. First, we estimate

$$\Delta n_{jt} = \delta_0 + \delta_1 Z_{jt} + \varepsilon_{jt} \tag{10}$$

where Δ is the first difference operator and where Z_{jt} is vector of controls: age category of the firm, time effects and industry indicators. Second, we construct the autocovariance matrix of the residuals ε_{jt} of this regression. If n_{jt} follows a random walk, ε_{jt} should be uncorrelated across time t.

The resulting covariograms for (10) are reported in Table 2, both for the whole sample of firms and for the subsample of larger firms (at least 20 employees each year over the sample period of that firm). The evidence from Table 2. suggests that the process characterizing the residuals in firm size changes is very close to a random walk, except for the case of Denmark when including the small firms (column 2). In Portugal the residual auto-covariogram support the specification of an autoregressive process very close to a unit root, although when using only the subsample of larger firms we find evidence of mild positive serial correlation in first differences.

The second approach follows the literature on testing for unit roots in panel data. Breitung-Meyer (1994) and Bond et al. (2005) show that for micro panels with large cross-sectional and small time dimension, OLS in levels is consistent and typically more efficient than more complex GMM and ML estimators. Consider a simple dynamic AR(1) panel data model, where for expositional brevity we do not include any covariates (in the estimation we use specifications where we control for age category of the firm, industry and time effects):

$$n_{jt} = \beta n_{j,t-1} + u_{jt}, \tag{11}$$

where $u_{jt} \equiv (1-\beta)\gamma_j + v_{jt}$ and the initial firm size $n_{j1} = \alpha_0 + \alpha_1\gamma_j + \varepsilon_j\gamma_j$, with ν_{jt} and γ_j error terms such that $\text{E}(\gamma_j)$ $=\text{E}(v_{jt}) = 0$ and $\text{E}(v_{jt}v_{js}) = 0$ for $t \neq s$. Mean stationarity in (11) requires $\beta < 1$, $\alpha_0 = 0$ and $\alpha_1 = 1$. In addition, covariance stationarity requires homoskedasticity over time of v_{jt}, i.e. $\text{Var}(v_{jt}) = \sigma^2_{v_j}$ and $\text{Var}(\varepsilon_j) = \sigma^2_{v_j}/(1-\beta^2)$. Bond et al (2005) show that under the null of $\beta = 1$ the OLS estimator of β in (11) is consistent. We refer to this estimator of β as the

Table 2: 1st Gibrat's Law Test: Residual Autocovariances

	Denmark		Portugal	
Lag	(1)	(2)	(1)	(2)
0	0.1587	0.0424	0.1162	0.0255
	(0.0005)	(0.0112)	(0.0005)	(0.0007)
1	-0.0030	-0.00003	0.0002	-0.0001
	(0.0002)	(0.0005)	(0.0002)	(0.0003)
2	-0.0094	-0.0008	-0.0024	0.0012
	(0.0002)	(0.0003)	(0.0002)	(0.0004)
3	-0.0020	-0.0002	-0.0013	0.0006
	(0.0002)	(0.0002)	(0.0002)	(0.0003)
4	-0.0016	-0.00004	-0.0008	0.0006
	(0.0002)	(0.0002)	(0.0003)	(0.0002)
5	-0.0008	-0.0006	-0.0010	0.0001
	(0.0002)	(0.0005)	(0.0003)	(0.0003)
6	-0.0008	-0.0002	-0.0013	0.0002
	(0.0002)	(0.0005)	(0.0004)	(0.0004)
N obs generating reg	1505926	79425	878919	66369

Specification (1) uses all the firms; specification (2) uses all firms that have at least 20 employees in each year of their life spans. All generating regressions use the first differenced log firm size as dependent variable and control for age of the firm, time and industry effects. (Robust standard errors in parentheses)

OLS estimator. Under the alternative $\beta < 1$, the OLS estimator is biased upwards; this is more the case when $\text{Var}(\gamma_j)/\text{Var}(v_{jt})$ is large. In cases where this difference in the variances is high, one could use the transformed statistic in Breitung and Meyer (1994), which estimates β from a transformed version of (11):

$$n_{jt} - n_{j1} = \beta(n_{j,t-1} - n_{j1}) + \varepsilon_{jt} \tag{12}$$

where $\varepsilon_{jt} = v_{jt} - (1-\beta)\left(n_{j1} - \gamma_j\right)$. The OLS estimator of (12) above is consistent again under the null and again upwards biased under the alternative $\beta < 1$, but this time the asymptotic bias does not depend on $\text{Var}(\gamma_j)/\text{Var}(v_{jt})$ when the process is mean stationary. The t tests based on these two estimators should be considered jointly when testing for the unit root and that tests based on other estimators that are consistent under both the null and under certain alternatives are found to have less power. For the purpose of our exercise, estimating β by least squares both in (11) and in (12) would provide a good indication whether the process is close to a random walk.

Table 3: 2nd Gibrat's Law Test: Unit Root Type Regressions

	Denmark				Portugal			
	all firms		large firms		all firms		large firms	
Coef	OLS1	BM1	OLS2	BM2	OLS1	BM1	OLS2	BM2
β	.9361	.9208	.9755	.9806	.9594	.9537	.9791	1.043
	(.0003)	(.0006)	(.0012)	(.0030)	(.0004)	(.0009)	(.0011)	(.0030)
N obs	1505926		79425		878934		66340	
R^2	0.87	0.70	0.95	0.82	0.91	0.66	0.96	0.84
MSE	0.42	0.43	0.21	0.21	0.36	0.36	0.17	0.17

The dependent variable is logfirmsize in OLS columns and (logfirmsize-initial logfirmsize) in BM columns. Columns indexed 1 correspond to estimates using the sample of all firms, while columns 2 correspond to the sample of firms with at least 20 employees in each year of their life spans. Both regressions control for age of the firm, time and industry effects. (Robust standard errors in parentheses).

The results of the regressions in (11) and (12) are shown in Table 3, for the methodologies of both Bond et.al. (2005) and Breitung and Meyer (1994), and both for the sample using all firms and the sample with only the large firms. The results are very similar to those in Table 2. Looking at the estimates for Denmark, the conclusions from the first Gibrat test are confirmed. While the coefficient on the first lag is somewhat lower than unity when using the sample of all firms, it approaches the unit root once we look at the subsample of large firms. The value for the MSE, a good estimate for the parameter σ of the theoretical model, is very similar for both countries; it is quite large (0.40) for the whole sample, but it is about half of that (0.20) for the subsample of larger firms, suggesting a lot of variation due to small firms. This confirms results from previous literature that Gibrat's law is more accurate for large firms, e.g. Jovanovic (1982). Similar conclusions hold for Portugal, where in the subsample of large firms we seem to be closer to the unit root. We conclude that the Gibrat law holds for large firms in both countries, while there is some mean reversion for small firms, in particular in Denmark.

3.3 Testing the LIFO separation rule

Next, we turn to the second prediction of our model, the LIFO separation rule. Let the function $j(i,t)$ denote the firm j in which worker i is employed in period t. We drop the arguments of this function whenever the identification of the individual and the period of observation are clear. The seniority level q_{ijt} is defined as the log of number of workers employed at firm $j\,(i,t)$ at time t at least as long as or longer than worker i; hence, this number includes worker i herself. Hence, q_{ijt} is equal to n_{jt} at the moment t when worker i is hired (assuming that i is the only one hired at time t). Furthermore, for the most senior worker $q_{ijt} = 0$ because there is only one worker who is employed at the firm as least as long as herself. Then, the seniority index r_{ijt} is defined as the log of the ratio of the number of people employed at least as long as worker i to the size of firm j at time t, in logs

$$r_{ijt} \equiv n_{jt} - q_{ijt}. \tag{13}$$

The seniority index r_{ijt} is a reasonable proxy for the variable $z_t - q$, since z_t is equal to n_t, up to a constant, ηp, and except for the insulation of n_t from shocks in z_t when $p^- < p_t < p^+$, recall the setup of our theoretical model. Were the LIFO separation rule to apply literally, the seniority index r_{ijt} would be the only determinant of separation. However, there are two reasons why this is not likely to be the case. First, the workforce of the firm is not completely homogeneous, so that a firm may wish to diminish its workforce in one skill class but not necessarily for other skill classes employed within that firm. This may disrupt a strict application of the LIFO separation rule. Second, workers separate not only due to shocks of the demand for the firm's product, but also due to worker specific shocks, e.g. when a worker's partner gets a new job in another city, that causes the worker to quit from his or her current job. A particularly important worker specific factor that does not fit in the LIFO model is retirement. Hence, our ambition is more limited than what would follow from a strict interpretation of the LIFO separation rule. We just want to show that r_{ijt} has a strong impact on the job separation rate.

We model the transition process by a mixed proportional hazard rates model with discrete time periods. This implies that the conditional probability of leaving the firm (i.e. the hazard rate) after T_{ijt} years of tenure can be written as:

$$\theta(r_{ijt}, Z_{ijt}, T_{ijt}, v_i) = \frac{\exp\left(\beta r_{ijt} + \gamma Z_{ijt} + \psi_{T_{ijt}} + v_i\right)}{1 + \exp\left(\beta r_{ijt} + \gamma Z_{ijt} + \psi_{T_{ijt}} + v_i\right)} \tag{14}$$

where Z_{ijt} is a vector of observed characteristics of the individual and the job, and where v_i represents the unobserved worker heterogeneity. We include a full set of dummies ψ_T for every tenure category, which is equivalent to a fully flexible specification of the baseline hazard. Identification of the parameter β of the seniority index r_{ijt} separate of the parameters of the baseline hazard ψ_T requires variation in r_{ijt} that is independent of the tenure T_{ijt}. Such independent variation is available since the seniority index also depends on the hiring and firing of other workers and hence is a non-deterministic function of tenure. A LIFO separation rule implies that β should be negative. For our estimation method we use a two mass-point distribution for the unobserved heterogeneity. We use up to 10 spells of an individual, which helps to estimate the unobserved heterogeneity distribution. The main reason for using a discrete time model is practical. For Denmark, the worker is observed only once per year. Hence, we cannot observe the exact moment at which the worker enters or leaves the firm.[10] In addition, short spells are underrepresented since a worker has to stay at least till the next period of observation. With the data at hand, we cannot correct for these problems in a continuous time model and even though we do not claim that a discrete time analysis solves all these problems, this is the simplest accurate representation of the data.

As noted before, older workers may leave the firm for retirement. This process is likely to run counter to the LIFO separation rule, since retired workers tend to have long tenures. Therefore, we exclude workers above the age of 55 from the analysis. Spells started before the age of 55 and finished afterwards are therefore right censored. Women are also more likely to leave the firm for non-participation. Hence, we separate our results for men and women. We delete spells that are left censored since we cannot compute the seniority of an individual for the periods before she enters our observation sample. Since this seniority affects the probability that the individual survives till the start of the sample period, we cannot easily correct for left censoring. Deleting the left-censored spells implies that we have a maximum of 22 years of tenure in Denmark and 10 for Portugal. The vector Z_{ijt} includes education, potential experience and indicators for region, industry and occupation.

Table 4 lists the main results. We find a negative and significant impact of seniority for both women and men, with small differences between these categories, in both Denmark and Portugal, in accordance with the LIFO separation rule. Furthermore, education and experience have a negative impact on the job separation

[10] For Portugal, tenures is reported in months. We use this information in the estimation. For the rest, the modelling is identical to that for Denmark.

hazard. Though the actual coefficients are not reported here, we also find negative duration dependence and evidence of unobserved heterogeneity, in both countries.[11] Apparently, seniority does not pick up all the variation in separation rates over the course of a job spell. There are two explanations for this phenomenon. First, as noted before, our seniority index might not exactly correspond to the actual layoff ordering, since the firm's workforce is likely to be heterogeneous with separate LIFO ordering to apply to subsets of the workforce. This is equivalent to measurement error in our seniority index r_{ijt}, leading to an attenuation bias in the estimate of β and unobserved variation in the seniority index being picked up by correlated variables, i.e. the tenure dummies ψ_T. Second, not all separations are driven by the fluctuations in the demand for the firm's product, and hence, the log seniority index. In particular, some separations are driven by the worker and the firm learning about the quality of the match. This learning process leads to hump shape hazard, with many separations early on and a quickly declining hazard for higher elapsed tenures, see Jovanovic (1979).

The analysis in Section 2.8 shows that firing cost increases the expected duration of job spells. Since EPL is more stringent in Portugal than in Denmark, one would expect longer job durations in Portugal. In Figure 2 below, we present the estimated cumulative distribution of completed tenures for job spell of a male with 12 years of education which started at the age of 25 and $r_{ijt} = 0$ along the whole spell, for both Denmark and Portugal. Indeed, the expected tenure is clearly higher in Portugal than in Denmark. This conclusion generalizes to any other type of worker.

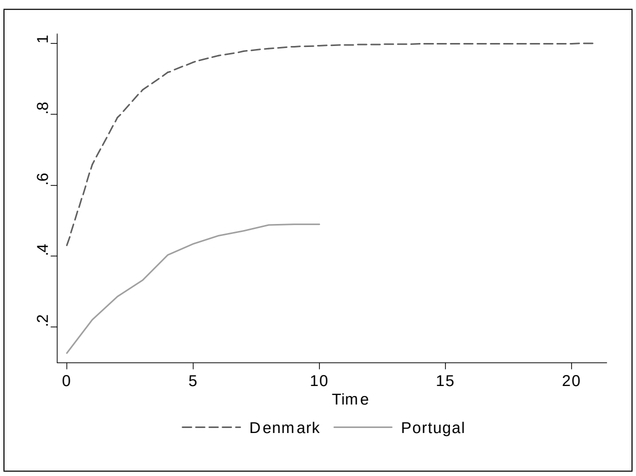

Figure 2: CDF expected completed tenure Denmark and Portugal

3.4 Testing dependency of wages on seniority

The third and main prediction of the model is the dependency of wages on seniority. This can be tested by extending the standard specification of the log earnings equa-

[11] The full estimation results are available upon request.

Table 4: Main results LIFO test

	Denmark		Portugal	
	Males	Females	Males	Females
Logrank	-0.0577	-0.0357	-0.0549	-0.0669
	(0.0019)	(0.0025)	(0.0054)	(0.0065)
Education	-0.1169	-0.1267	-0.1204	-0.1446
	(0.0003)	(0.0005)	(0.0009)	(0.0012)
Experience	-0.0771	-0.0732	-0.0490	-0.0656
	(0.0001)	(0.0001)	(0.0003)	(0.0004)
N obs	10788368	5990891	2118405	1488687

The estimation also controls for occupation, region and industry indicators. (Standard errors in parantheses)

tion with the seniority index, r_{ijt}, as defined in (13) above. Consider the following specification of log wages w_{ijt}

$$w_{ijt} = \alpha + \chi X_{ijt} + \gamma T_{ijt} + \delta r_{ijt} + \zeta n_{jt} + \varepsilon_{ijt}, \tag{15}$$

where X_{ijt} is experience. We omit higher order terms in experience and tenure and other controls (including time effects) from equation (15) for the sake of convenience, but include them in the estimation. The unobservable term can be decomposed into four orthogonal components, a match, a firm, a worker, and an idiosyncratic effect[12]

$$\varepsilon_{ijt} = \varphi_{ij} + \psi_j + \mu_i + \nu_{ijt}. \tag{16}$$

The idiosyncratic effect ν_{ijt} can also include measurement error. There are all kinds of reasons for ϕ_{ij}, ψ_j, and μ_i to be correlated to T_{ijt}, see Topel (1991) or Altonji and Williams (2005): good worker-firm relationships tend to survive as the worker and the firm learn about the quality of their match and bad matches are broken up, leading to a positive correlation between $\varphi_{ij} + \psi_j + \mu_i$ and T_{ijt}. Search theories imply that workers sample new jobs from a job offer distribution. The longer this selection process is going on, the higher the expected value of $\varphi_{ij} + \psi_j$ since bad jobs do not survive, leading to a positive correlation between $\varphi_{ij} + \psi_j$ and T_{ijt}. There are two obvious solutions to this problem, either within-job first differencing (FD) or adding fixed effects for every job spell (FE). First differencing yields

$$\Delta w_{ijt} = \chi + \gamma + \delta \Delta r_{ijt} + \zeta \Delta n_{jt} + \Delta \nu_{ijt}. \tag{17}$$

Adding fixed effects per job spell is equivalent to estimating (15) by taking deviations from the mean over time, within a job spell:

$$\widetilde{w}_{ijt} = (\chi + \gamma) \widetilde{T}_{ijt} + \delta \widetilde{r}_{ijt} + \zeta \widetilde{n}_{jt} + \widetilde{\nu}_{ijt}, \tag{18}$$

[12]This formulation is similar to Topel (1991: 150), except that we add a firm effect and that we delete the subscript t from the match effect ϕ_{ij}, as Topel does in his application.

where the upper tilde denotes deviations from the mean per job spell, e.g. $\widetilde{w}_{ijt} = w_{ijt} - \overline{w}_{ijt}$, with $\overline{w}_{ijt}$ the mean over time of w_{ijt}. We exclude $\widetilde{X}_{ijt}$ from (18) because it is perfectly collinear with $\widetilde{T}_{ijt}$. In both specifications above, it is immediately clear that the first order effects of tenure and experience are not separately identified. However, this problem does not affect the estimation of δ, since r_{ijt} is not perfectly correlated to T_{ijt}. The choice between the FE and FD estimators above depends on the error structure of v_{ijt}. The closer is v_{ijt} to a unit root, the more efficient is the FD method; the closer v_{ijt} is to being serially uncorrelated, the more efficient estimation method is the FE estimator. Previous empirical studies have typically found a high degree of autocorrelation in v_{ijt}, even close to a unit root, see for instance Abowd and Card (1979) and Topel and Ward (1992). From that perspective, equation (17) is likely to be most efficient. However, this equation assumes that the effect of r_{ijt} and n_{jt} on w_{ijt} is immediate. Any lagged impact will not be captured after first differencing. From that perspective, equation (18) is preferred, since there lagged effects of r_{ijt} and n_{jt} will be captured. Hence, one would expect higher estimates for δ and ζ from using equation (18) than from (17).[13] In the strict version of our model, where separation is completely governed by the LIFO separation rule, r_{ijt} and n_{jt} are perfectly correlated within a job spell, since more senior workers will never leave the firm before worker i, so that the only variation in r_{ijt} comes from variation in n_{jt}. The same argument applies to $\widetilde{r}_{ijt}$ and $\widetilde{n}_{jt}$. Hence, δ and ζ are not separately identified in that world neither in equation (17) nor in (18). Happily, LIFO does not apply in a strict sense. The most compelling reason for a violation of the LIFO separation rule is workers' retirement, but also other individual specific shocks discussed earlier in this section. These separations allow separate identification of δ and ζ with FE and FD estimators.

First, we check the characteristics of the dynamic process of v_{ijt}. Table 5 reports the variance-covariance of Δv_{ijt}, analogous to what we did for log firm sizes in Table 2. For both countries, the covariance of ε_{ijt} with its first lag is substantial, the covariance with higher lags is negligible. Hence, the process is well approximated by an MA(1) process, made up of a mixture permanent and transitory shocks. Abowd and Card (1979) and Topel and Ward (1992) find similar results for the United States. The standard deviation of the permanent shocks can be calculated as 0.12 for Denmark and 0.10 for Portugal.[14] These numbers are of the same order of magnitude as found for the United States.

[13] We report robust standard errors, so that correlation between the residuals over time does not affect their validity.

[14] Let q_{ijt} and u_{ijt} be the transitory and permanent shock respectively. Then:

$$\Delta v_{ijt} = u_{ijt} + q_{ijt} - q_{ij,t-1}.$$

Hence: $\text{Var}(\Delta v_{ijt}) = \text{Var}(u_{ijt}) + 2\text{Var}(q_{ijt})$ and $\text{Cov}(\Delta v_{ijt}, \Delta v_{ij,t-1}) = -\text{Var}(q_{ijt})$, so that: $\text{Var}(u_{ijt}) = \text{Var}(\Delta v_{ijt}) + 2\text{Cov}(\Delta v_{ijt}, \Delta v_{ij,t-1})$.

Table 5: Residual Autocovariances for Within-Job LogWage Innovations

Lag	Denmark	Portugal
0	0.0231	0.0273
	(0.00002)	(0.00007)
1	-0.0043	-0.0082
	(0.00001)	(0.00006)
2	-0.0006	-0.0008
	(8.7e-06)	(0.00003)
3	-0.0003	-0.0004
	(9.0e-06)	(0.00003)
4	-0.0003	9.2e-06
	(9.5e-06)	(0.00003)
5	-0.00008	-0.00008
	(0.00001)	(0.00004)
6	-0.0001	-0.0006
	(0.00001)	(0.00005)
N obs generating reg	14907897	5758655

The generating regressions are the FD wage regressions with logrank includes, see the FD2 columns in the next table. (Robust standard errors in parentheses)

Table 6: FE and FD Wage Regressions for the Entire Private Sector in Denmark and Portugal

	Denmark				Portugal			
	FD1	FD2	FE1	FE2	FD1	FD2	FE1	FE2
logrank		.003***		.008***		.016***		.022***
		(.0003)		(.0003)		(.0005)		(.0005)
logfsize	.013***	.011***	.026***	.021***	.025***	.015***	.040***	.028***
	(.0002)	(.0003)	(.0003)	(.0003)	(.0004)	(.0005)	(.0004)	(.0004)
tenure+exper	.047***	.045***	.010***	.007***	.068***	.065***	.059***	.055***
	(.0003)	(.0003)	(.0001)	(.0002)	(.0005)	(.0005)	(.0003)	(.0003)
$tenure^2$	.191***	.199***	-.052***	-.036***	-.086***	-.069***	-.083***	-.067***
	(.002)	(.002)	(.002)	(.002)	(.003)	(.003)	(.002)	(.002)
$tenure^3$	-.101***	-.105***	.014***	.008***	.027***	.021***	.024***	.019***
	(.001)	(.001)	(.099)	(9.88e-07)	(.001)	(.001)	(.0007)	(.0007)
$tenure^4$	.002***	.002***	-.0009***	-.0002	-.003***	-.002***	-.003***	-.002***
	(.0002)	(.0002)	(.0002)	(.0002)	(.0002)	(.0002)	(.00009)	(.00009)
$exper^2$	-.224***	-.223***	.099***	.100***	-.204***	-.204***	-.149***	-.147***
	(.002)	(.002)	(.0006)	(.0006)	(.004)	(.004)	(.002)	(.002)
$exper^3$	.039***	.039***	-.039***	-.039***	.043***	.043***	.030***	.029***
	(.0007)	(.0007)	(.0002)	(.0002)	(.001)	(.001)	(.0007)	(.0007)
$exper^4$	-.003***	-.003***	.004***	.004***	-.003***	-.003***	-.002***	-.002***
	(.00007)	(.00007)	(.00003)	(00003)	(.0001)	(.0001)	(.00007)	(.00007)
N obs	14907897		22364083		5758655		10743244	
Workers	2116307		277162		1752000		3092329	
Spells	3745050		6870869		1965560		4053649	
Firms	221106		301015		206361		322502	

The dependent variable is the (Δ) time-detrended log real hourly wage for the (FD) FE columns; the covariates have Δ in front for FD columns. Columns 1 report results for the same regressions as corresponding columns 2, but without logrank included as covariate. The higher order polynomials in tenure and experience are divided by the corresponding powers of 10. All regressions also control for region, industry and occupation indicators. Significance levels: $*$: 10% $**$: 5% $***$: 1%. (Robust standard errors in parentheses).

This evidence suggests that in terms of efficiency of the estimation method we might prefer FD, while in terms of allowing for a lagged effect of Δr_{ijt} on Δw_{ijt} we might prefer FE. Hence, we report both the FD and FE estimator. Our regressions control for up to quartic terms in tenure and experience, log firmsize and industry, occupation, and region dummies. In Table 6 we report the results[15]. We present the estimation results for two specifications, one excluding log seniority r_{ijt} and another including it. We can draw the following conclusions. First, all coefficients for log seniority are positive and statistically significant. Second, the coefficients are larger for FE than for FD, as was expected because FE allows for a lagged effect of r_{ijt} on w_{ijt}, while FD does not. Third, comparing the estimation results with and without seniority, including seniority reduces the estimates for the first order effect of tenure + experience and for log firmsize by 5-30 %. The coefficients for the higher order effects of tenure and experience are hardly affected by including seniority. The effect of log firmsize and tenure on wages is at least partly a proxy for the effect of seniority. Of the three variables, tenure, firm size, and seniority, we can expect seniority to be measured with the greatest amount of measurement error. Apart from straightforward reporting errors, the main source of measurement error in tenure is who exactly is the relevant employer. Some job changes might either be classified as between firms, justifying the tenure clock being set back to zero, or as within the firm, which does not affect the tenure clock. However, this source of measurement error only affects changes at the borderline of the definition of a firm. This is likely to be only a small fraction of the firm's workforce. However, misclassification of the tenure of even a single worker can affect the measurement of the seniority of all other workers of the firm. In general, any measurement error in tenure or firm size automatically feeds into seniority, while on top of that, seniority is also affected by measurement errors because separate seniority statistics are likely to apply for subgroups of the workforce. Both the upward effect on the coefficients for tenure and log firm size and downward effect on the coefficient for seniority of the measurement error in seniority can therefore be expected to be substantial. Finally, the effect of seniority is twice as high in Portugal as in Denmark.[16] It is tempting to link this difference between Denmark and Portugal to the differences in EPL in both countries, but our theoretical model does not allow for a link between the bargaining power β and the firing cost W.

[15] Results for the same analysis performed for each of the broad industry categories defined in Appendix B are qualitatively identical (with some heterogeneity in the magnitude of the estimated seniority coefficients, among the various industries) to the results at the national level in Table 6. They are available upon request from the authors.

[16] A corrolary of that is that also the return to tenure is very low in Denmark, compared to most other countries, and certainly so when compared to Portugal. The fact that the common linear effect of tenure and experience is further reduced when accounting for seniority rank suggests however that seniority rank, keeping in mind the proportions, is a much more important factor in wage determination than tenure (whose still significant impact, is in our interpretation due to measurement error in seniority).

3.4.1 Returns to seniority within gender and education subgroups

We repeat the analysis separately for males and females, and for low- and high-educated workers. The results are reported in Table 7. The results for male and female categories do not differ much. The only apparent exception is for Denmark, when using the FE estimator, where the estimated coefficient for males is twice as high as for females, though they are the same when using the FD estimator. At the same time, and linked to the previous observation, the estimates by gender categories really do not differ much from the estimate when using the whole samples, for either country. Our interpretation is therefore that seniority positions within gender categories are not more relevant for wage determination than the seniority position within the firm as a whole and hence splitting by gender is not likely to attenuate the measurement error in seniority index. The estimation results for education groups show that the effect of seniority is much larger for higher educated workers than for low educated workers. The impact of seniority on wages is lower for the low-educated workers, compared to corresponding estimates from Table 6, and the FE estimate is even significantly negative for Denmark, though small in absolute value. The impact of seniority on wages within the high educated group is much larger, both in Denmark and in Portugal. These results are consistent with the fact that high educated workers have steeper wage-tenure profiles than their low-educated peers. At the same time, they give support to the fact that the relevant seniority hierarchy within the firm is already more realistically captured when accounting for education levels.

Table 7: FE and FD Regressions by Gender and Education Rank Groups

	Denmark				Portugal			
	Gender Categories							
	Females		Males		Females		Males	
	FD	FE	FD	FE	FD	FE	FD	FE
logrank	.005***	.005***	.005***	.010***	.015***	.019***	.014***	.019***
	(0.0004)	(0.0005)	(0.0004)	(0.0004)	(0.0007)	(0.0006)	(0.0007)	(0.0006)
logfsize	.002***	.014***	.014***	.025***	.015***	.028***	.019***	.031***
	(0.0005)	(0.0005)	(0.0004)	(0.0004)	(0.0007)	(0.0006)	(0.0006)	(0.0006)
ten+exp	.032***	.009***	.052***	.007***	.053***	.042***	.080***	.073***
	(0.0004)	(0.0002)	(0.0004)	(0.0002)	(0.0007)	(0.0005)	(0.0007)	(0.0005)
N obs	5049388	7745676	9858509	14618407	2300767	4353808	3457888	6389436
	Education Categories							
	HighEduc		LowEduc		HighEduc		LowEduc	
	FD	FE	FD	FE	FD	FE	FD	FE
logrank	.010***	.020***	.002***	-.002***	.029***	.032***	.013***	.016***
	(.0003)	(.0004)	(.0004)	(.0004)	(.002)	(.002)	(.0005)	(.0005)
logfsize	.007***	.016***	.014***	.024***	.026***	.026***	.016***	.031***
	(.0004)	(.0004)	(.0005)	(.0005)	(.002)	(.002)	(.0005)	(.0004)
(ten+exp)	.040***	.006***	.031***	.006***	.116***	.099***	.056***	.049***
	(.0004)	(.0002)	(.0007)	(.0002)	(.002)	(.001)	(.0005)	(.0003)
N obs	9567345	14054988	5268672	8309095	259793	536920	5492034	10206324

The dependent variable is the (Δ) time-detrended log real hourly wage for the (FD) FE columns; the covariates have Δ in front for FD columns. Logrank has been computed separately for each category. "LowEduc" stands for category of people with at most 12 years of education. All regressions include also up to 4th order polynomials in tenure and experience and indicators for region, occupation and industry. Significance levels: $*: 10\%$ $**: 5\%$ $***: 1\%$. (Robust standard errors in parentheses).

3.4.2 Returns to seniority and firm monopoly power

Our theoretical model also predicts that the return to seniority, β/η, is partly driven by the degree of monopoly power, η^{-1}. We test this hypothesis by analyzing whether the variation in the return to seniority across industries can be explained by the degree of monopoly power in each industry. We take the log of the number of firms in each industry as proxy for the degree of monopoly power. We regress the estimated coefficient for seniority for each industry on this log number of firms and a constant term, using both simple OLS and Weighted Least Squares (WLS) specifications. We use two measures as "number of firms" in an industry: the sum of all firms that were at any time active in that industry during the sample period, and respectively, the median number of firms over the sample period. We use two industries classifications, a broad classification with 12 industries for Denmark and 11 for Portugal, see Appendix B, and a more refined classification where we use all 2-digit Standard Industry Classification (SIC) industry sub-categories available, increasing the number of observations in the regressions to 40 for Denmark and 49 for Portugal. For our prediction to be verified, we expect negative estimates of the coefficients of log number of firms.

The estimation results for the regressions of returns to seniority on the log number of firms by industry are presented in Table 8. Most of the estimated coefficients of interest are not significantly different from 0 (though most slightly negative in magnitude), both when using the WLS and the OLS methods and regardless of using as dependent variables the FD or the FE coefficients previously estimated in this paper, and as independent variables the sum or the median of the number firms in an industry. There are very few cases where the results are statistically significant: when using the broad industry categories for Portugal we get significant coefficients of the expected sign with the FE method, but significant coefficients of the opposite sign in Denmark; when using the OLS for 2-digit industries in Portugal we get significantly positive coefficients for the FD method and again significantly positive when using the WLS for the FD, sum of firms, and FE, median of firms. In conclusion, we regard this test as inconclusive. The explanatory variables used as proxy for the monopoly power of an industry are not strong enough to isolate the effect of the degree of monopoly power on the return to seniority.

Table 8: Monopoly Power Test

	Denmark				Portugal			
	Sum		Median		Sum		Median	
	FD	FE	FD	FE	FD	FE	FD	FE
OLS, Broad Industry Categories								
log Nfirms	-0.00009	0.0058**	0.0003	0.0060**	0.0046	-0.0069*	0.0046	-0.0068*
	(0.0014)	(0.0024)	(0.0014)	(0.0022)	(0.0028)	(0.0032)	(0.0027)	(0.0030)
constant	0.0107	-0.0435*	0.0072	-0.0378*	-0.0275	0.0892**	-0.0234	0.0817**
	(0.0135)	(0.0227)	(0.0111)	(0.0178)	(0.0267)	(0.0303)	(0.0230)	(0.0262)
WLS, Broad Industry Categories								
log Nfirms	-0.0008	0.0053**	-0.0004	0.0002	-0.0004	-0.0057	-0.0002	0.0032
	(0.0014)	(0.0024)	(0.0014)	(0.0013)	(0.0031)	(0.0035)	(0.0030)	(0.0029)
constant	0.0174	-0.0395	0.0128	0.0075	0.0229	0.0778**	0.0212	-0.0102
	(0.0134)	(0.0227)	(0.0117)	(0.0109)	(0.0302)	(0.0340)	(0.0269)	(0.0251)
OLS, 2-Digit Industry Categories								
log Nfirms	-0.0010	-0.0010	-0.0010	-0.0007	0.0136**	-0.0003	0.0112**	-0.0011
	(0.0011)	(0.0016)	(0.0012)	(0.0016)	(0.0058)	(0.0030)	(0.0056)	(0.0028)
constant	0.0213**	0.0224*	0.0198**	0.0193*	-0.0852*	0.0314	-0.0559	0.0358*
	(0.0093)	(0.0125)	(0.0080)	(0.0108)	(0.0440)	(0.0228)	(0.0367)	(0.0187)
WLS, 2-Digit Industry Categories								
log Nfirms	-0.0012	0.0013	-0.0013	-0.0010	0.0094*	-0.0004	0.0075	0.0091*
	(0.0013)	(0.0015)	(0.0013)	(0.0013)	(0.0048)	(0.0025)	(0.0046)	(0.0047)
constant	0.0227**	-0.0022	0.0213**	0.0194**	-0.0515	0.0306	-0.0294	-0.0421
	(0.0110)	(0.0125)	(0.0085)	(0.0091)	(0.0365)	(0.0195)	(0.0307)	(0.0315)

The dependent variable is the estimated coefficient of seniority rank from FD and respectively FE regressions, by industry. "Nfirms" is measured as the sum of all firms (corresponding to columns labeled "Sum") or as the median of firms (corresponding to columns "Median"), over the sample period, respectively. There are 12 'broad industries' in Denmark and 11 in Portugal, while the number of 2-digit SIC sub-categories is 40 in Denmark and 49 in Portugal. Significance levels: $*: 10\%$ $**: 5\%$ $***: 1\%$. (Standard errors in parentheses).

4 Summary and conclusions

What have we established beyond reasonable doubt in this paper? We have shown that for Denmark and Portugal part of what has been known as the wage return to tenure is in fact a return to seniority, that is, the position of the worker in the tenure hierarchy of her firm. This implies that standard explanations of the return to tenure, like Jovanovic's learning or the classic search models, and subsequent versions of these models, cannot provide the full story, if only because these explanations focus solely on the features of the worker herself (in case of learning, her ability; in case of search, her job offer history), while the return to seniority links the fate of the worker to that of the firm as a whole. A return to seniority implies that a worker is to some extent shareholder in her own firm. Hence, it makes the link between labor economics and finance.

Our theoretical model provides a special interpretation of the return to seniority, as being due to a hold up problem, where firms pay the full cost of the specific investment, while workers capture part of the return. This setup leads to inefficiently low hiring. All these conclusions are conditional on the assumption that the firm bears the full cost of specific investments, an assumption that has not been tested empirically in this paper. How to do that remains an open question. An indirect answer can be obtained by analysing who is queueing for whom: when workers queue for jobs, so that there is unemployment, firms are held up by their incumbent workforce; when it is the other way around, and there are vacancies, workers are held up by their employer. As long as workers are risk neutral and either investment or wages are contractible, efficient hiring can be obtained by using the sharing rule of the costs for the one, to mirror the sharing rule for the other, thereby satisfying the Hosios (1990) condition. When workers are risk averse, efficiency can only be obtained when both investment and wages are contractible, such that the costs of investment are fully attributed to the firm and there is no seniority profile. Any other allocation assigns part of the risky return to the risk averse player. In that sense, our estimation results point to incompleteness in the insurance market. Nevertheless, our analysis does not imply that LIFO layoff rules are bad per se. They can offer a useful protection to the property rights of incumbent workers on their share of the specific investment, thereby helping the firm to solve a commitment problem. Without a resolution of this commitment problem, incumbents would have all reasons not to cooperate in the transfer of tacit knowledge to newly hired workers.

We have established the existence of a return to seniority for Denmark and Portugal. Whether such a return exists in other countries, in particular in the United States, remains an open question. We bet it does; the large return to tenure in the United States as compared to Denmark and Portugal strongly suggests so. One might argue that returns to seniority are largely driven by legal institutions, and that these institutions are entirely different and more market oriented in the United States. We think however that the economic mechanisms for having a LIFO layoff rule exist everywhere, and that the legal institutions might very well just be a formalisation of rules of conduct and implicit contracts that would have emerged anyway.

The return to seniority is twice as high in Portugal than in Denmark. It is tempting to relate this difference to the much more extensive Employment Protection Legislation (EPL) in the Portugal. Nevertheless, this does not follow from our theoretical model. Compared to, for example, Bentolila and Bertola (1990), our analysis has the advantage that it allows for the effect of EPL on wages, but this does not imply a higher return to seniority. What would be an interesting extension of our analysis is to allow for the fact that empirically EPL goes up with tenure. Till sofar, including this feature in theoretical models was cumbersome from an analytical point of view, but in the framework presented here this is likely to be doable. With an eye on the missing market for elderly workers in many European countries, this seems to be a worthwhile extension. We leave this for future research.

Our model suggests that hold up problems reduce turnover, and thereby specific investment (because turnover requires new specific investment to be made). This conclusion is contingent on the way specific investment is modelled here, namely as a fixed amount to be invested in one-shot at the beginning of the job. When the amount of investment can vary both in size and in timing, this conclusion might change. Then, a longer expected job duration might invoke more specific investment, which in turn would lengthen the expected job duration since the productivity at the job is raised relative to the productivity at the outside market. In such a world, a firm responds along two margins of adjustment, when the demand for its product goes up. First, it will hire additional workers, and second, it will expand the specific investment in its incumbent workforce. This model would provide further legitimation for a LIFO rule, not as legal constraint, but as an efficient economic institution. Again, we postpone this for future research.

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A Derivation

Equation (6) can be written as

$$\begin{aligned} 0 &= \left(\mu^- - \beta\right) R - \psi + \left(\mu^- - \beta\right) C^- - \left(\mu^- - \beta\right) C^+, \\ 0 &= \left(\mu^- - \beta\right) R - \psi\beta + \left(\mu^- - \beta\right) \mu^- C^- - \left(\mu^- - \beta\right) \mu^+ C^+, \\ \left(\mu^- - \beta\right) I &= \left(\mu^- - \beta\right) ER - \psi E_\beta + \left(\mu^- - \beta\right) E^- C^- - \left(\mu^- - \beta\right) E^+ C^+, \\ 0 &= \left(\mu^- - \beta\right) ER - \psi\beta E_\beta + \left(\mu^- - \beta\right) \mu^- E^- C^- - \left(\mu^- - \beta\right) \mu^+ E^+ C^+, \end{aligned} \tag{19}$$

where

$$\begin{aligned} &\eta\lambda^- \equiv \mu^- < 0, \eta\lambda^+ \equiv \mu^+ > 1, \psi \equiv \frac{\mu^-}{\rho} < 0, \Delta \equiv p^+ - p^- \\ &C^- \equiv B^- \exp\left[\mu^- p^-\right], C^+ \equiv B^+ \exp\left[\mu^+ p^-\right], R \equiv r\left(\eta^{-1}\right)^{-1} \exp\left[p^- - \pi\right] > 0, \\ &E \equiv \exp\left[\Delta\right], E_\beta \equiv \exp\left[\beta\Delta\right], E^- \equiv \exp\left[\mu^-\Delta\right], E^+ \equiv \exp\left[\mu^+\Delta\right]. \end{aligned}$$

Elimination of C^- yields:

$$\begin{aligned} 0 &= \left(\mu^- - 1\right) R - \psi - \left(\mu^- - \mu^+\right) C^+, \\ \left(\mu^- - \beta\right) I &= \left(\mu^- - \beta\right) \left(E - E^-\right) R - \psi \left(E_\beta - E^-\right) - \left(\mu^- - \beta\right) \left(E^+ - E^-\right) C^+, \\ 0 &= \left(\mu^- - \beta\right) \left(E - \mu^- E^-\right) R - \psi \left(\beta E_\beta - \mu^- E^-\right) - \left(\mu^- - \beta\right) \left(\mu^+ E^+ - \mu^- E^-\right) C^+. \end{aligned} \tag{20}$$

Rewriting the first equation yields equation (7). The system of equations (20) can be rewritten as a system of linear equation in R, C^+, and $(\mu^- - \beta) I$

$$\begin{bmatrix} R \\ C^+ \\ (\mu^- - \beta) I \end{bmatrix} = \psi \begin{bmatrix} \mu^- - 1 & -(\mu^- - \mu^+) & 0 \\ (\mu^- - \beta) D_{0-} & -(\mu^- - \beta) D_{+-} & -1 \\ (\mu^- - \beta) D_{10\mu-} & -(\mu^- - \beta) D_{\mu+\mu-} & 0 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ D_{\beta-} \\ D_{\beta\beta\mu-} \end{bmatrix} \tag{21}$$

$$\begin{aligned} &D_{0-} \equiv E - E^-, D_{+-} \equiv E^+ - E^-, D_{\beta-} \equiv E_\beta - E^-, \\ &D_{10\mu-} \equiv E - \mu^- E^-, D_{\mu+\mu-} \equiv \mu^+ E^+ - \mu^- E^-, D_{\beta\beta\mu-} \equiv \beta E_\beta - \mu^- E^-. \end{aligned}$$

Since $R > 0$, the first equation of the solution to equation (21) implies that

$$\left(\mu^+ - \mu^-\right) D_{10\mu-} - \left(1 - \mu^-\right) D_{\mu+\mu-} = -\mu^+ \left(E^+ - E\right) + \mu^+\mu^- \left(E^+ - E^-\right) - \mu^- \left(E - E^-\right) < 0$$

Hence, Δ and C^+ should be positive for a solution to exist. The third equation of this system reads

$$I = \frac{\psi}{\mu^- - \beta} \left[\frac{(\mu^- - \beta)(D_{0-} D_{\mu+\mu-} - D_{+-} D_{10\mu-})}{(\mu^+ - \mu^-) D_{10\mu-} - (1 - \mu^-) D_{\mu+\mu-}} + \frac{(\mu^+ - \mu^-) D_{0-} - (1 - \mu^-) D_{+-}}{(\mu^+ - \mu^-) D_{10\mu-} - (1 - \mu^-) D_{\mu+\mu-}} D_{\beta\beta\mu-} - D_{\beta-} \right] \tag{22}$$

which is an implicit equation in Δ. Since

$$\begin{aligned} I(0, \beta) &= 0, I(\infty, \beta) = \infty, \\ I_\Delta(\Delta, \beta) &> 0, I_\beta(\Delta, \beta) < 0, \end{aligned}$$

a unique positive solution for Δ exist for every $I > 0$, and $d\Delta/d\beta$ is positive.

With firing cost, the first equation of (19) reads

$$-\left(\mu^{-}-\beta\right)W=\left(\mu^{-}-\beta\right)R-\psi+\left(\mu^{-}-\beta\right)C^{-}-\left(\mu^{-}-\beta\right)C^{+}.$$

Elimination of C^{-} yields

$$-\mu^{-}W=\left(\mu^{-}-1\right)R-\psi-\left(\mu^{-}-\mu^{+}\right)C^{+}.$$

This can be written as

$$\begin{bmatrix} R \\ C^{+} \\ \left(\mu^{-}-\beta\right)I \end{bmatrix}=\psi\begin{bmatrix} \mu^{-}-1 & -\left(\mu^{-}-\mu^{+}\right) & 0 \\ \left(\mu^{-}-\beta\right)D_{0-} & -\left(\mu^{-}-\beta\right)D_{+-} & -1 \\ \left(\mu^{-}-\beta\right)D_{10\mu-} & -\left(\mu^{-}-\beta\right)D_{\mu+\mu-} & 0 \end{bmatrix}^{-1}\begin{bmatrix} 1-\rho W \\ D_{\beta-} \\ D_{\beta\beta\mu-} \end{bmatrix}$$

All conclusions in the text follow from this equation.

B Broad industry categories

1. Manufacturing
2. Electricity, gas and water supply
3. Construction
4. Wholesale and retail trade; repairs
5. Hotels and restaurants
6. Transport, post and communications
7. Financial intermediation
8. Real estate, renting and business activities
9. Public administration and defense; compulsory social security
10. Education
11. Health and social work
12. Other community, social and personal service activities

Note: For Portugal we miss category 9 (no firms are privately owned in that sector).