Formal and Informal Risk Sharing in LDCs: Theory and Empirical Evidence

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ABSTRACT

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We develop and estimate a model of dynamic interactions between households where commitment is limited and contracts are incomplete to explain the patterns of income and consumption growth in village economies of less developed countries. Households can insure through both formal contracts and informal agreements, that is, agreements specifying voluntary transfers that need to be self-enforceable. This theoretical setting nests the case of complete markets and the case where only informal agreements are available. We derive a system of non-linear equations for income and consumption growth. A key prediction of our model is that both variables are affected by lagged consumption as a consequence of the interplay of formal and informal contracting possibilities. In a semi-parametric setting, we prove identification, derive testable restrictions and estimate the model with the use of data from Pakistan villages. Empirical results are consistent with the economic arguments. Incentive constraints due to self-enforcement bind with positive probability and formal contracts are used to reduce this probability.

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1   Introduction[1]

Limited commitment is an attractive argument to accommodate consistent empirical evidence that full-risk sharing is rejected in village economies. In this paper, we nest complete markets and limited commitment in a unified set-up. We show the empirical relevance of the joint consideration of incomplete markets and dynamic interactions using household data from villages in Pakistan.

Limited commitment refers to situations where there is no legal contract enforceability (Thomas and Worrall, 1988). Some level of insurance can nonetheless be achieved through informal agreements between agents engaged in a long-term relationship. These agreements take the form of voluntary transfers that are self-enforceable in a game theoretic manner. Yet, even if a complete market structure is beyond reach, village institutions might be able to enforce specific types of formal contracts like sharecropping or other land-leasing contracts. In this paper we propose a model of risk sharing that allows for the two types of transactions between agents – voluntary transfers and formal contracts.

A distinguishing feature of our model from the literature is the peculiar interaction between contracts and informal agreements. Indeed as informal agreements are identified with behavior at equilibrium, they not only involve voluntary transfers but also specific formal contracts. Contracts are indeed used as an enforcement tool for voluntary transfers, so that formal and informal contracts can be seen as complements. To some extent contracts are used as a collateral in informal lending-borrowing agreements. It implies that individuals receiving a large transfer – say because of negative income shocks – should concede more favorable terms on the formal contracts in exchange, raising the future prospects of the transferring-in parties. The existence of formal contracting enables to relax incentive constraints due to limited commitment and thus to improve on global efficiency.

Our paper builds on a rich literature. Starting with the seminal paper of Townsend (1994), the empirical evaluation of the complete market hypothesis in village economies has led to a better understanding of market failures and to a better identification of the characteristics of households who are most affected by these failures (Deaton, 1997, Morduch, 1999, Fafchamps, 2003, Dercon, 2004). These results were paralleled by tests of complete markets in developed economies both at the aggregate level (see Attanasio and Ríos-Rull, 2003) and the micro level (Cochrane, 1991, and Mace, 1991). Most papers report rejections of the complete market hypothesis and how agents

are partially insured is now the question of interest (Blundell, Pistaferri and Preston, 2003).

This empirical literature goes with a theoretical literature which highlights two aspects. One stream of literature emphasizes the role of asymmetric information (see Thomas and Worrall, 1990, Doepke and Townsend, 2006, and references therein). The other stream highlights the problem of contract enforcement and limited commitment in risk sharing (see Thomas and Worrall, 1988, Kocherlakota, 1996). The literature about village economies in LDCs focuses mostly on this latter aspect, the rationale being that village economies lack institutions which are able to enforce the whole set of contracts and to achieve perfect risk sharing. For instance, there are no written records for transfers such as loans, no proper institutions designed to enforce repayments and repayments can be delayed or altogether canceled. As a consequence, villagers sign informal agreements that are Pareto-improving provided that they can be self-enforced (Coate and Ravallion, 1993, Fafchamps, 1992). This requirement restricts the set of informal agreements which may not be rich enough to lead to complete risk sharing in the village. A few recent papers show the empirical relevance of such analyses and show that self-enforcing transfers play their part in sharing risk within villages (Ligon, Thomas and Worrall, 2000, 2002), within networks or extended families (Foster and Rosenzweig, 2001).

Full risk sharing and limited commitment are two polar cases though. Some institutions may help enforce some formal contracts. In particular, sharecropping and fixed rent formal contracts are commonly observed in villages of LDCs and their role in allocating risk have been repetitively emphasized (Stiglitz, 1974, Newbery, 1977). The ability of landlords to enforce contracts on land is a critical aspect and the design of such contracts is studied in many empirical studies (Shaban, 1987). Dubois (2000) provides empirical evidence using the same data that sharecropping or other land contracts enable risk sharing. Households use these instruments to smooth income as well as consumption (Morduch, 1995).

Envisioning formal contracts along with informal agreements explains why formal contracts are accompanied by informal transfers that can attenuate their effects in bad states of nature (Bell, 1988, Udry, 1994). Whether formal transfers facilitate informal agreements is a different question. Indeed the existence of formal transfers affects not only the incentives during the agreements, but also the possibilities without an agreement (see for instance Albarran and Attanasio, 2003).

Finally, the issue of formal and informal risk sharing is of wider importance than this specific application, as it is relevant for many other domains such as international debt, finance, internal labour markets or social networks.

Outline and main findings   In Section 2, we construct and analyze a theoretical setting of long-term dynamic interactions between households where several cases are nested: Complete

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markets if all risks can be insured by formal contracts; One non contingent transfer only as in Gauthier, Poitevin and Gonzalez (1997); Limited commitment and no formal contracts as in Ligon *et al.* (2002). A noticeable aspect is that we embed both income and preference shocks into the two-agent model, where one agent, who is interpreted afterwards as representing the "village" can commit while the other agent (the "household") cannot.

The market allocation (consumption, income and contracts) is obtained as an equilibrium of an infinite horizon game. Using recursive methods, we characterize the constrained Pareto efficient allocations where an allocation specifies the sequence of contracts and informal transfers that should be implemented. Because of the possibility to sign formal contracts, the value function may not be concave. We solve this issue by randomizing over contracts and transfers.

We derive optimality conditions and monotone comparative results. Any efficient equilibrium allocation is characterized by two monotonic (stochastic) mappings: one maps current consumption into contracts, and the other is an Euler-type equation where consumption growth, instead of being a martingale, is affected by lagged consumption and current income. In contrast to the limited commitment case where once an incentive compatibility constraint is binding, all the past becomes irrelevant, an attractive feature is that the process keeps a richer memory of the past.

A key contribution of the paper is the econometric and empirical implementation. It aims at showing that our theoretical setting leads to a credible empirical alternative to the complete market and limited commitment hypotheses. It achieves more than simply testing these alternatives and exhibits the conditions under which the model is semi-parametrically identified and its implications are testable.

The enforcement of contracts depends on specific institutions within each village and evaluating their degree of formality is a difficult task. This is why we work within the set-up of Deaton (1997) where we do not examine specific institutions in villages but the consequences they have on the behavior of households in terms of income and consumption only. In Section 3, we specify preferences and income processes and we derive two equations of interest, an Euler-type equation of consumption dynamics and an income equation.

The first prediction that is testable is that income depends monotonically on lagged consumption. The second prediction is that the ratio of marginal utilities is a random walk with jumps as in Ligon *et al.* (2002) although the occurrence and size of jumps are not exogenous and depend on the ratio of marginal utilities last period. It explains why lagged consumption enters the right hand side of the consumption growth equation.

In Section 4, the model is estimated by semi-parametric methods using panel data that come from a survey in Pakistan done by IFPRI at the end of the 1980s. Because of non linearities, we

estimate the model of income and consumption growth by penalized splines (Yu and Ruppert, 2002). Results are consistent with the predictions of the theoretical model. Lagged consumption affects income and this result is robust to the presence of measurement errors, of superior information that households might have on future income shocks and of effects of nutrition on productivity (Behrman, Foster and Rosenzweig, 1997). Among other results, the estimation of the consumption growth equation enables us to identify a lower bound that the probability that a self-enforcing constraint binds. This estimated bound is roughly equal to 15%.

In the last section, we discuss the economic relevance of our empirical results and the agenda of further research to overcome limitations of this approach. We conclude by discussing some implications for policy conduct. Proofs are in appendices.

2 A Theoretical Model of Risk Sharing with Formal and Informal Contracts

We consider a two-agent framework and assume full commitment by one agent and none by the other.[2] We first define preferences, income processes, the contractual environment as well as the timing of the game. We then study payoffs and feasible allocations. We analyze the properties of the optimal allocations and we characterize the dynamics. The extension to account for the presence of observed heterogeneity makes the transition with the next econometric section.

2.1 The model

Consider an economy with two agents and dates $t = 1, ..., \infty$. Let s_t denote a random state of the world at date t, belonging to a finite set $\mathcal{S}$, and whose distribution is identically and independently distributed (i.i.d.). In state s which occurs with probability p_s, agent i receives exogenous resources $\omega^i(s)$. The extension to a general Markov process is undertaken at the end of the section.

The utility function of agent 1 is written at date $t \geq 1$ as $u_t(.) = \eta_t u(.)$, where $(\eta_t)_{t=1}^{\infty}$ is a random process. In the empirical part, η_t represents the new information affecting preferences that is revealed at the beginning of a period and that is never observed by the econometrician. In the theoretical part, its presence ensures enough variability in the allocation, implying in particular that the problem is strictly concave and the allocation is uniquely defined with probability 1. Setting $\eta_0 = 1$, we denote $\chi_t = \frac{\eta_t}{\eta_{t-1}}$ and we follow the literature by assuming that χ_t is i.i.d. on an interval of $\mathbb{R}^+$, with mean 1 and positive density.

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[2] We analyze the two-sided limited commitment case in a companion paper (Dubois, Jullien and Magnac, 2002).

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The second agent has non-stochastic preferences since what matters are ratios of marginal utilities between the two agents. Her preferences are described by a Von-Neumann Morgenstern utility function $v(.)$. We assume that u and v are concave, twice continuously differentiable, $u(0) = v(0) = 0$ and $u'(0) = v'(0) = +\infty$, which ensures that consumption levels are positive. Both agents share the same discount rate ρ. The ex-ante expected intertemporal utility of agents 1 and 2 are respectively $E\left[\sum_{t=1}^{\infty} \rho^{t-1} u_t(c_t^1)\right]$ and $E\left[\sum_{t=1}^{\infty} \rho^{t-1} v(c_t^2)\right]$.

Because of convexity issues, we allow for some randomization beyond the fundamentals, by assuming that there is a public random variable, ε_t, which is i.i.d. and continuous on a compact support.

The precise timing of realization of the various events within period t, is the following.

- At the beginning of period t, the shocks χ_t and ε_t are publicly realized. A formal contract is signed.

- At the end of period t, the income shock s_t is publicly realized. The formal contract is enforced. Then parties may complement it by voluntary transfers and consumptions take place.

Once signed, formal contracts are valid for period t and binding for both agents. They are restricted and incomplete as follows:

1. Contracts are short term, specifying how resources will be shared within the period only.

2. Contracts can be contingent only on the realization of one of E exclusive events: $e \in \mathcal{E}$, where $\mathcal{E}$ is a partition of the set of states of nature $\mathcal{S}$. An event $e \in \mathcal{E}$ is a subset of $\mathcal{S}$, and occurs with probability $p_e = \sum_{s \in e} p_s$.

3. Contracts entail real costs, while voluntary transfers involve no distortions.[3]

A contract is represented by a vector $(\tau(1), ..., \tau(E))$ of net transfers received by agent 1. Transfers belong to some compact set $\times_e [\underline{\tau}(e), \bar{\tau}(e)]$, where $\underline{\tau}(e) < 0 \leq \bar{\tau}(e)$. Transferring τ in event e through a contract entails a cost $\varphi(e, \tau)$ for agent 2. As a consequence, total resources in state $s \in e$ are $\omega^1(s) + \omega^2(s) - \varphi(e, \tau(e))$, positive in all states. We assume that $\varphi(e, .)$ is continuous, quasi-convex, with a minimum at $\tau = 0$ and $\varphi(e, 0) = 0$. To avoid corner solutions we also assume that $\varphi(e, \underline{\tau}(e))$ is large enough so that $\tau(e)$ remains interior. If $w^1(s) + \tau(e) < 0$ in some state, we adopt the convention that $u(w^1(s) + \tau(e)) = 0$, and only $-w^1(s)$ is transferred.

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[3]This assumption is necessary to obtain an interior solution with one-sided commitment, it can be dispensed with in the case of bilateral limited commitment (see Dubois *et al.* , 2002).

When no contract is feasible ($\mathcal{E} = \varnothing$), we obtain the model of informal risk sharing as in Thomas and Worrall (1988). In the polar case when $\mathcal{E} = \mathcal{S}$ and the cost of transfers is zero ($\varphi(e, \tau) \equiv 0$), our formulation yields full risk sharing. Moreover, our setting nests the model of Gauthier, Poitevin and Gonzalez (1997) since, among other things, they consider one ex-ante transfer only ($\mathcal{E} = \{\mathcal{S}\}$ i.e. $card(\mathcal{E}) = 1$).

Denote $H_t = [(s_{r-1}, \chi_r, \varepsilon_r)_r]_{r=1}^t$ the history of shocks up to the beginning of period t. An allocation is a random profile of formal transfers τ_t and consumptions c_t^i that are measurable with respect to the relevant information, i.e. $\tau_t = \tau(H_t, e_t)$ and $c_t^i = c^i(H_t, s_t)$, and that satisfy the resource constraint, for all H_t and s_t :

$$c_t^1 + c_t^2 = \omega_t, \tag{1}$$

where $\omega_t = \omega^1(s_t) + \omega^2(s_t) - \varphi(e_t, \tau_t)$ denotes aggregate resources available at date t.

2.2 Intertemporal Payoffs and Feasible Allocations

In the benchmark case of complete markets when optimal insurance is achieved, consumption at date t only depends on the realization of aggregate resources that are maximal at $\tau(e_t) = 0$. According to Borch rule, the ratio of marginal utilities is the same in all states of nature so that consumption dynamics is given by

$$\eta_{t+1} \frac{u'(c_{t+1}^1)}{v'(c_{t+1}^2)} = \eta_t \frac{u'(c_t^1)}{v'(c_t^2)}.$$

Assume now that there is no financial market. By assumption, agent 2 can fully commit to a future behavior while agent 1 cannot commit at all and at any point in time maximizes his payoffs. Any equilibrium allocation must be self-supporting in a game theoretic manner.

Because of the presence of the random parameter η_t, our model is not truly a repeated game. Yet, it is stationary in the sense that the sub-game starting at date t is identical to the game starting at date 1 provided that the utilities are re-normalized. To see that, define the expected utility at the beginning of date t, normalized by η_t for agent 1, as:

$$U_t = E\left[u\left(c_t^1\right) + \sum_{r=1}^\infty \rho^r \frac{\eta_{t+r}}{\eta_t} u(c_{t+r}^1) \mid H_t\right], \quad V_t = E\left[\sum_{r=0}^\infty \rho^r v(c_{t+r}^2) \mid H_t\right]. \tag{2}$$

Notice that $U_t = \frac{1}{\eta_t} E\left[\sum_{r=0}^\infty \rho^r \eta_{t+r} u(c_{t+r}^1) \mid H_t\right]$ and using $\eta_{t+1} = \chi_{t+1}\eta_t$:

$$U_t = E\left[u\left(c_t^1\right) + \rho \chi_{t+1} U_{t+1} \mid H_t\right], \quad V_t = E\left[v\left(c_t^2\right) + \rho V_{t+1} \mid H_t\right].$$

Now consider the subgame starting at date t with H_t known. Given that $\frac{\eta_{t+r}}{\eta_t} = \Pi_{l=1}^r \chi_{t+l}$, the distribution of $\left\{\frac{\eta_{t+r}}{\eta_t}, \varepsilon_{t+r}, s_{t+r}\right\}_{r \geq 1}$ is independent of the period t; and the subgame starting at

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date $t > 1$ is identical to the initial game. The sets of equilibria of the two games coincide and using normalized utility U_t, the tools of repeated game theory apply.

Specifically, there are minimal and maximal expected utility levels of each agent that can be supported in equilibrium. Since agent 2 can commit to transfer all her wealth, the minimal level is $\underline{V} = 0$. On the contrary agent 1 refuses to enter into any agreement leading to a utility level U_t below the autarky level:

$$\underline{U} = \frac{1}{1 - \rho} \sum_s p_s u(\omega^1(s)).$$

Since agent 2 can commit not to contract at all with agent 1, $\underline{U}$ is indeed the minimal utility that he can obtain.

In order to prevent agent 1 from reneging on the agreement, it is optimal to coordinate in such a way that if he deviates, the equilibrium that follows is the worst for him. In other words one should apply an optimal penal code as defined by Abreu (1988): An allocation can be supported in equilibrium provided that, at any point in time, agent 1 prefers to abide to the informal agreement rather than to renege and be punished by receiving his minimal equilibrium expected utility. Thus at date t, agent 1 should be willing to sign the formal contract,

$$U_t \geq \underline{U}. \tag{3}$$

Once s_t is realized, he must also prefer to make the informal transfer rather than to enforce the formal contract only and continue in autarky next period:

$$u(c_t^1) + \rho E \left[\chi_{t+1} U_{t+1} \mid H_t, s_t \right] \geq u(\omega_t^1 + \tau_t) + \rho \underline{U}. \tag{4}$$

In conclusion, the set $\mathcal{F}$ of allocations that verify conditions (1), (3) and (4) at all dates and all histories is the set of feasible allocations.

2.3 Value Functions and Bellman Equation

Our goal is to derive the set of Pareto optimal allocations. An allocation is optimal if agent 1 receives the largest expected utility given agent 2's expected utility. This set, or frontier, can be evaluated at various stages. It will be useful to consider two stages: before any shock is revealed, the *ex-ante definition,* or after the revelation of the shocks (χ and ε) at the beginning of the period, *the interim definition.*

Definition 1 *At the interim stage, when $H_1 = (\chi_1, \varepsilon_1)$ is known, the Pareto frontier is the set of points $(P(V), V)$ characterized by the interim value function:*

$$P(V) = \max_{\left(c_t^1, c_t^2, \tau_t\right)_{t=1}^{\infty} \in \mathcal{F}} U_1 \quad s.t. \quad V_1 \geq V.$$

where U_1 and V_1 are date 1 normalized utilities defined in equation (2).

Definition 2 *At the ex-ante stage, the Pareto frontier is the set of feasible ex-ante utilities $(Q(V), V)$ characterized by the ex-ante value function:*

$$Q(V) = \max_{\left(c_t^1, c_t^2, \tau_t\right)_{t=1}^{\infty} \in \mathcal{F}} E\left[\chi_1 U_1\right] \quad s.t. \quad E\left[V_1\right] \geq V.$$

where the expectation operator refers to the joint distribution of (χ_1, ε_1).

Using the argument of Thomas and Worrall (1988), the functions $Q(V)$ and $P(V)$ are decreasing and continuous on the interval $[\underline{V}, \bar{V}]$. Moreover, the two frontiers are linked by the relation $Q(E[V_1]) = E[\chi_1 P(V_1)]$. Denote $F(. \mid \chi_1)$ the marginal distribution of date 1 utility V_1 conditional on the realization of the random shock χ_1 only. With the use of the random component ε_1, it is possible to implement any distribution on support $[\underline{V}, \bar{V}]$, so that $Q(V)$ is the solution of

$$Q(V) = \max_{\{F(\cdot|\chi_1)\}} E\left[\int_{\underline{V}}^{\bar{V}} \chi_1 P(V_1) dF(V_1 \mid \chi_1)\right] \quad s.t. \ E\left[\int_{\underline{V}}^{\bar{V}} V_1 dF(V_1 \mid \chi_1)\right] \geq V, \qquad (5)$$

where the expectation operator refers to the distribution of χ_1. Since this program is linear, $Q(V)$ is concave.

As pointed above, the problem of maximizing U_t conditional on history H_t and $V_t = V$ is the same as the problem of maximizing U_1 conditional on H_1 and $V_1 = V$. Therefore $U_t = P(V_t)$ with probability 1. As previously,

$$E(\chi_t U_t \mid H_{t-1}, \ s_{t-1}) = Q\left(E(V_t \mid H_{t-1}, \ s_{t-1})\right). \qquad (6)$$

Therefore, the solution of the interim program is such that:

$$U_t = E\left[u\left(c_t^1\right) + \rho Q\left(E(V_{t+1} \mid H_t, \ s_t)\right) \mid H_t\right] \qquad (7)$$

It follows from $U_t = P(V_t)$ that the value function $P(.)$ can be written as the solution to the Bellman equation:

$$P(V) = \max_{\left(c^1(s), c^2(s), V(s)\right)_s, (\tau(e))_e} \sum_{s \in \mathcal{S}} p_s \left[u\left(c^1(s)\right) + \rho Q\left(V(s)\right)\right] \qquad (8)$$

$$s.t \quad \sum_{s \in \mathcal{S}} p_s \left[v\left(c^2(s)\right) + \rho V(s)\right] \ \geq \ V \qquad (9)$$

$$u\left(c^1(s)\right) + \rho Q\left(V(s)\right) \ \geq \ u\left(\omega^1(s) + \tau(e)\right) + \rho \underline{U} \qquad \forall e \in \mathcal{E}, \forall s \in e \qquad (10)$$

$$c^1(s) + c^2(s) \ = \ \omega^1(s) + \omega^2(s) - \varphi(e, \tau(e)) \qquad \forall e \in \mathcal{E}, \forall s \in e \quad (11)$$

$$0 \ = \ \underline{V} \leq V(s) \leq \bar{V} \qquad \forall s \in \mathcal{S} \qquad (12)$$

where equation (3) is implicitly translated by constraint (12).

The solution of (8) provides a mapping that associates to utility V_t a formal transfer τ_t, consumption levels c_t^i and continuation expected utility at the end of period t contingent on the realization of the shock s_t. Moreover, the solution of program (5) gives the distribution of period $t+1$ utility V_{t+1} as a function of the expected utility promised at the end of period t and the shock χ_{t+1}.

2.4 Concavity and Differentiability

Firstly, we need to address concavity and differentiability issues. Since Q is concave, the program defining P is concave for a fixed contract $\{\tau(e)\}_e$. However, when the contract is optimally designed, $P(.)$ may not be concave nor differentiable. Let $\hat{P}(.)$ be the concave envelop of $P(.)$:

$$\hat{P}(V) = \max_F \int_{\underline{V}}^{\bar{V}} P(w)dF(w) \ \ s.t. \ \int_{\underline{V}}^{\bar{V}} wdF(w) \geq V.$$

where F is a distribution on $[\underline{V}, \bar{V}]$. $\hat{P}(V)$ is a concave and decreasing function, and the argument of Benveniste and Scheinkman (1979) can be adapted along the line of Koeppl (2006). We show in Lemma 8 (Appendix A) that $\hat{P}(V)$ is differentiable, because it is not possible that the incentive constraint binds in all states. The solution F of $\hat{P}(E[V_t \mid H_{t-1}, \ s_{t-1}, \eta_t])$ defined above, is the distribution of V_t conditional on $(H_{t-1}, \ s_{t-1}, \eta_t)$, i.e. after realization of the shock χ_t but before randomization. It is immediate that $P(V_t) = \hat{P}(V_t)$ with probability 1 at an optimal allocation. Moreover,

$$\hat{P}'(E[V_t \mid H_{t-1}, \ s_{t-1}, \eta_t]) = P'(V_t) \tag{13}$$

whenever $V_t < \bar{V}$ (notice that $0 = \underline{V} < V_t$).

Denote $\hat{V}(\chi_1) = E[V_1 \mid \chi_1]$. The solutions of equation (5) can be written as

$$Q(V) = \max_{\hat{V}(.) \in [\underline{V}, \bar{V}]} E\left[\chi_1 \hat{P}(\hat{V}(\chi_1))\right] \ \ s.t. \ E\left[\hat{V}(\chi_1)\right] \geq V,$$

which gives the choice $E[V_1 \mid \chi_1]$ and more generally the choice of $E[V_t \mid H_{t-1}, \ s_{t-1}, \eta_t]$ as a function of $V = E[V_t \mid H_{t-1}, \ s_{t-1}]$. We show in Lemma 9 (Appendix A) that the function $Q(.)$ is strictly concave due to the random effect χ_1. As for $\hat{P}$, the function Q is differentiable.

Given differentiability, optimality conditions imply that at $\hat{V}(\chi) < \bar{V}$ solution of the program defining Q, the slopes of the value functions are linked by $Q'(V) = \chi \hat{P}'(\hat{V}(\chi))$. Combined with (13), we thus conclude that

$$Q'(E[V_t \mid H_{t-1}, \ s_{t-1}]) = \chi_t P'(V_t) \text{ when } V_t < \overline{V}. \tag{14}$$

2.5 Characterization of the Dynamics

To describe the dynamics of the system, we don't need to describe the whole frontier P but only those points that can occur in equilibrium. From what is developed above this is included in the

set of utility levels V such that $P(V) = \hat{P}(V)$. At these points, we can rely on duality theory despite the fact that $P(.)$ is not concave.

Lemma 3 *There exists a random variable* $\mu_t = -\hat{P}'(V_t)$, *measurable with respect to history* H_t, *such that with probability* 1 :

$$V_t \in \arg \max_{V \in [\underline{V}, \bar{V}]} \{P(V) + \mu_t V\}. \tag{15}$$

Proof. We have $V_t \in \arg \max_V \left\{ \hat{P}(V) - \hat{P}'(V_t)V \right\}$ since $\hat{P}(.)$ is concave. Using $\hat{P}(.) \geq P(.)$, we obtain that when $P(V_t) = \hat{P}(V_t)$ then $V_t \in \arg \max_V \left\{ P(V) - \hat{P}'(V_t)V \right\}$. The result then follows from $\Pr \left\{ P(V_t) = \hat{P}(V_t) \right\} = 1$ ∎

Characterizing the set of optimal utility V_t amounts to characterizing the solutions of program (15). They correspond to pairs $\{V, P'(V)\}$ where $P'(V) = \hat{P}'(V) = -\mu$. In what follows, we describe the dynamics by using the Pareto weight μ_t instead of V_t.

The characterization is obtained in two stages. In a first stage we fix the contract $\{\tau(e)\}$ and μ, and we characterize the solution V of program (15), along with the solution to the program (8) defining $P(V)$. This determines date t promised consumptions and continuation utilities for a given contract $\{\tau(e)\}$ and μ. A simple arbitrage condition shows that

$$\frac{u'(c^1(s))}{v'(c^2(s))} = -Q'(V(s)), \tag{16}$$

which corresponds to a risk sharing rule determining the allocation in state s, where the trade-off is between consumption and continuation utilities.

In all states where the agent 1 interim incentive constraint (10) is not binding, we have $-Q'(V(s)) = \mu$, which equalizes the ratios of marginal utilities across states, as required by Borch rule. Nevertheless, when agent 1's income is large, maintaining the ratio of marginal utilities at μ violates incentive compatibility. The interim utility of agent 1 must be increased at the level that would be obtained by reneging on the promise and enforcing contract T. In this case the allocation depends on interim resources, and the outside option, but not on the weight μ.

In a second stage we consider the choice of T for a given weight μ. Although a characterization is out of reach, a super-modularity argument shows that it is a monotonic function of the Pareto weight. As we move along the frontier $\hat{P}(V)$ toward higher absolute slopes μ (and thus higher V), the contract becomes uniformly less favorable to agent 1.

The next proposition summarizes the results:

Proposition 4 *Denote* $\pi_t = \omega_t^1 + \tau_t$ *agent 1's income at date* t, *and* T_t *the vector of formal transfers. There exists a function* $\bar{\mu}(\omega, \pi)$ *with values in* $\left[-\hat{P}'(\bar{V}), +\infty \right[$, *and a correspondence*

$\bar{T}(\mu)$, *with values in* $\times_e[\underline{\tau}(e), 0]$ *such that an optimal allocation verifies:*

i)

$$T_t \quad \in \quad \bar{T}(\mu_t); \tag{17}$$

$$\frac{u'(c_t^1)}{v'(c_t^2)} \quad = \quad \min\{\mu_t, \bar{\mu}(\omega_t, \pi_t)\}; \tag{18}$$

$$\mu_{t+1} \quad = \quad \frac{\eta_t}{\eta_{t+1}} \frac{u'(c_t^1)}{v'(c_t^2)} \ if \ V_{t+1} < \overline{V}. \tag{19}$$

ii) $\bar{\mu}(\omega, \pi)$ *is decreasing in* π

iii) $\bar{T}(\mu)$ *is non-increasing (according to the strong order set).*[4]

The condition (19) follows from conditions (14) and (16) and $-P'(V_{t+1}) = \mu_{t+1}$.

For a given formal contract, this Proposition is a generalization of Thomas and Worrall (1988) where one agent is risk neutral and no formal contract is allowed. When agent $1's$ resources are large and the weight put on agent 2 is large, incentive compatibility and efficient risk sharing are conflicting. This is alleviated in two ways. First, the consumption profile of agent 1 that follows such a state is shifted upwards. Second the problem can be alleviated ex-ante by using the formal contract. As the utility V of agent 2 increases, it becomes more difficult to maintain the incentives of agent 1. Formal contracts are thus more extensively used.

The dynamics can be described by mean of the evolution of the weight μ_t associated with the utility V_t chosen after history H_t. At this stage, agents sign a contract $\{\tau(e)\} \in \bar{T}(\mu_t)$. Consumption is given as a function of μ_t, the transfer τ_t and the state of nature s_t. This also defines the slope $Q'(E(V_{t+1} \mid H_t, s_t))$ at the end of period t. Then at date $t+1$, H_{t+1} is realized and condition (14) gives the new value of the weight μ_{t+1}.

Whenever the Pareto frontier is locally strictly concave the proposition defines exactly the whole dynamics because $\bar{T}(\mu)$ is single valued. If $P(.)$ is not strictly concave $\bar{T}(\mu)$ can be multi-valued. Notice that it is single valued for all values μ_t where program (15) has a unique solution, i.e. where $-\hat{P}'(V_t) = \mu_t$ has a unique solution V_t. It is shown in Lemma 9 (Appendix A) that this occurs with probability 1 due to the effect of the shock η_t. Thus in equilibrium $\bar{T}(\mu_t)$ is single valued with probability 1.

When there is no formal contract, the variable $\bar{\mu}(\omega_t, \pi_t)$ is exogenous. The dynamics of consumption after a date t where the incentive constraint binds is independent of the past history and depends only on state s_t. Introducing the possibility of formal contracts dramatically changes this conclusion. Indeed the resources of the agent depend on the past history of shocks

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[4]The transfer is strictly decreasing in all events such that the incentive constraint binds with a positive probability and income is positive in some state. If the incentive constraint doesn't bind in all states of event e, then the transfer is $\tau(e) = 0$.

through the contract T_t, even after a period where the incentive constraint binds. Notice that the proposition implies that income π_t is monotonic in the parameter μ_t.

Corollary 5 *Income π_t decreases (weakly) with μ_t in all states s_t.*

Proof. Condition iii) implies that transfer τ_t decreases in all events when μ_t increases. Thus income decreases uniformly. ∎

The same conclusion holds for the allocation of consumptions and future utilities (c_t^1, U_{t+1}).[5] Notice also that if there were no shock η_t, the weight μ_t would be decreasing over time. It would converge to some fixed value and the contract should be eventually backloaded in favour of the agent who cannot commit. But due to the shock η_t, there is a positive probability that μ_t increases. Thus starting from any point, there is a positive probability that the incentive compatibility constraint binds in finite time, and the asymptotic distribution of the allocations is not degenerate. For our argument to make sense empirically there must be enough variability in the shocks η_t.[6]

2.6   Additional sources of observed heterogeneity

In what precedes we assume time-invariant utility functions (except η_t) and a stationary resource process. Suppose now that the utility is $u(c^1; x_t^1)$ and $v(c^2; x_t^2)$. Suppose also that resources depend on state s_t and that a state variable q_t describes the information set. Letting $x_t = (x_t^1, x_t^2)$, suppose that (x_t, q_t) follows a Markov process whose realization is revealed to the agents at the beginning of period t. Then the value functions at date t are functions $P(V; x_t, q_t)$ and $Q(V; x_{t-1}, q_{t-1})$. The value function Q is the maximum

$$E\left\{ \chi_t \hat{P}(V_t; x_t, q_t) \mid x_{t-1}, q_{t-1} \right\}$$

subject to $E\left\{ V_t \mid x_{t-1}, q_{t-1} \right\} \geq V$. The value function $P(V; x_t, q_t)$ is equal to the maximum of

$$E\left[u\left(c^1(x_t, q_t, s_t); x_t^1 \right) + \rho Q\left(V(x_t, q_t, s_t); x_t, q_t \right) \mid x_t, q_t \right]$$

subject to incentive and participation constraints. In this set-up all proofs generalize. The function $\bar{\mu}$ depends only on ω_t, π_t and x_t, q_t, but not on χ_t, while the ratio of marginal utilities has to be conditioned on x_t^i only. We thus obtain

$$\frac{u'(c_t^1; x_t^1)}{v'(c_t^2; x_t^2)} = \min\left\{ \mu_t, \bar{\mu}\left(\omega_t, \pi_t; x_t, q_t \right) \right\}.$$

The contract $\bar{T}(\mu_t; x_t, q_t)$ depends on μ_t and x_t, q_t and the dynamics of the multiplier μ_t is unchanged.

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[5] This follows from the fact that at the solution of the sub-program with a given contract, both c_s^2 and $V(s)$ are non-decreasing with μ and non-increasing with $\tau(e)$.

[6] See Thomas and Worrall (1988), Beaudry and DiNardo (1991) and Ray (2002). We thank a referee for pointing this and the references.

3   The Structural Econometric Model

The main purpose in constructing an empirical model from these theoretical underpinnings is to test whether it leads to a credible empirical alternative to the perfect insurance and the limited commitment cases and to assess the degree of discrepancy with them. The feature that our theoretical set-up brings forth is that we should analyze the income process in addition to consumption growth. If income were exogenous, the hypothesis of complete markets would be tested by looking at the significance of the correlation between the residuals of the consumption growth equation and income. This procedure is correct since income is excluded from the marginal utility of consumption if leisure and consumption are separable. In our case, the choice of formal contracts makes income endogenous and there are indeed three endogenous variables (consumption, formal contracts and income). A test procedure that markets are complete should now be constructed by looking for exogenous or predetermined variables that affect formal contracts, thus income, and are independent of random preference shocks.[7] Moreover, the absence of formal contracts, the case of pure limited commitment, is evaluated by testing that income is exogenous. Nevertheless, we want to achieve more than simply testing null hypotheses and this section focuses on formalizing an econometric model suited to the theory developed in the previous section and that can be identified using household panel data on income and consumption. There are alternatives that have been proposed in the literature to understand the empirical evidence that we obtain but we postpone the discussion of their relevance until the empirical section.

Some discussion of the most striking omissions is nonetheless in order. The first issue concerns the adaptation of the two-agent framework to a village i.e. a multi-agent framework. We follow Ligon *et al.* (2002) by assuming that each household plays a two-person game with the rest of the village or with a pivot person in the village. The characteristics and the equilibrium realizations of endogenous variables of the other agent are written as village-and period indicators.

The second issue is that we do not model individual savings or "investment" technologies that enable households to transfer resources from one period to the next in an idiosyncratic way. As demonstrated by Ligon *et al.* (2000), the manipulation of incentive constraints becomes intractable in empirical applications. The level of aggregate savings is assumed to be determined by some unmodeled device and captured by village-and-period dummy variables in the empirical model.

Third, we use information on consumption and income only. It is difficult to tell in the data whether contracts are formal so that we only capture their impact through the income process.

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[7]The issue of income endogeneity is dealt with in some papers in a reduced-form setting (*e.g.* Jacoby and Skoufias, 1998, Jalan and Ravallion, 1999).

Furthermore, we refrain from using information on informal transfers as Foster and Rosenzweig (2001) do because it seems difficult to us to draw a precise line between truly exogenous transfers and those that result from the very endogenous informal contracts that we model here. Consumption growth would eventually include those informal transfers (Deaton, 1997).

For the sake of clarity and in lack of a better device, we shall adopt the convention that "observable" refers from now on to variables which we, the econometrician, observe and we reserve the expression "revealed at time t" to what households observe at time t. Moreover, we shall adopt the following notations:

Notations 3.1:

(i) All parameters indexed by vt control for village-and-period effects.

(ii) Household i's consumption at time t is denoted c_{it}.

(iii) Household i's agricultural profits at time t, net of input costs including non family labor costs, are denoted π_{it}^a. Household i's income at time t, including income from formal contracts, is denoted $\pi_{it} = \pi_{it}^a + \pi_{it}^e$ where π_{it}^e is exogenous income. The latter stands for other non-labor income or exogenous transfers such as exogenous remittances from abroad excluding informal transfers.

(iv) Observed preference shifters (household demographics for instance) and unobserved preference shocks, respectively denoted x_{it} and η_{it}, are revealed to agents at the beginning of period t.

(v) Besides taste shifters, the information set at the beginning of the period contains observed variables q_{it} (such as owned land) that are revealed to agents at the beginning of period t.

(vi) Income shocks are revealed at the mid-period. Observed shocks[8] are denoted z_{it} (days of illness and craft income for instance), unobservable shocks are denoted ξ_{it}. The set of all observables at mid-period is denoted $m_{it} = (x_{it}, q_{it}, z_{it})$.

We start below with the specification of utility functions, of formal contracts and of income processes. We continue by stating the identifying restrictions. All specification assumptions and restrictions are piled up into different items of Assumption 3.2. A proposition stating semi-parametric identification follows and the section closes by explaining the estimation method by penalized splines.

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[8]In the theoretical model, we should assume that z_{it} and (x_{it}, q_{it}) are independent. If they are not, we would replace z_{it} by the innovation in z_{it} independent of (x_{it}, q_{it}) that is by the unanticipated income shock.

3.1 The Specification of Consumption and Income Equations

3.1.1 Consumption Dynamics

Following most papers in this literature (noticeable exceptions are Ogaki and Zhang, 2001, for HARA estimation and Dubois, 2000, for heterogenous CRRA functional forms) we assume that households have constant relative risk aversion and that other goods, such as leisure, are excluded:

Assumption 3.2(i): *The ratio of marginal utility of consumption for household i at time t, relatively to the marginal utility of the village, v, is written as*

$$\eta_{it}\frac{u'_{it}(c_{it})}{u'_{vt}(c_{vt})} = \eta_{it}\exp(\delta_{vt})\exp(\sigma x_{it}\beta).c_{it}^{-\sigma} \tag{20}$$

where $\sigma > 0$ and where the log of the marginal utility of the village is captured by the village–and–period effect δ_{vt}. We denote $\lambda_{it}(\beta) = \ln(c_{it}) - x_{it}\beta$ the adjusted consumption index.

From equations (19) and (20), define the multiplier μ_{it} as:

$$\ln\mu_{it} = -\Delta\ln\eta_{it} - \sigma\lambda_{it-1}(\beta) + \delta_{vt-1}, \tag{21}$$

where first differences, Δ, are defined as, $\Delta\ln\eta_{it} = \ln\eta_{it} - \ln\eta_{it-1}$. Consumption dynamics is derived from equations (18) and (21). Whether the incentive constraint facing household i is binding or not defines two regimes:

$$-\sigma\lambda_{it}(\beta) + \delta_{vt} = \ln\bar{\mu}_{vt}(\pi_{it}, x_{it}, q_{it})$$
$$\text{if } -\Delta\ln\eta_{it} - \sigma\lambda_{it-1}(\beta) + \delta_{vt-1} > \ln\bar{\mu}_{vt}(\pi_{it}, x_{it}, q_{it})$$

$$-\sigma\lambda_{it}(\beta) + \delta_{vt} = -\Delta\ln\eta_{it} - \sigma\lambda_{it-1}(\beta) + \delta_{vt-1}$$
$$\text{if } -\Delta\ln\eta_{it} - \sigma\lambda_{it-1}(\beta) + \delta_{vt-1} < \ln\bar{\mu}_{vt}(\pi_{it}, x_{it}, q_{it})$$

where the notation $\bar{\mu}_{vt}$ replaces $\bar{\mu}$ in equation (18) because aggregate resources are now captured by the village-and-period index (Notations 3.1).

The functional form of the bound, $\ln\bar{\mu}_{vt}(\pi_{it}, x_{it}, q_{it})$, will be specified later on. Yet, note readily that this function is deterministic. It could implicitly be made random if (x_{it}, q_{it}) was allowed to include some unobserved heterogeneity components. The structure of stochastic shocks however is already sufficiently rich because of the income variable π_{it}. As we allow for measurement errors in income, the absence of unobserved heterogeneity in (x_{it}, q_{it}) does not seem to be such a tight assumption in this model.[9]

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[9]A more difficult issue that we do not deal with here, is the presence of unobserved household effects in x_{it}, q_{it}. Yet, household effects are notoriously difficult to handle in non-linear dynamic settings although some advances could be made along the lines proposed by Cunha, Heckman and Navarro (2005).

Whether the incentive constraint is binding, is not observable to us so that the two regimes cannot be distinguished. As a consequence, the system of equations above is observationally equivalent to a single equation describing the dynamics of consumption:

$$\sigma \Delta \lambda_{it}(\beta) = \Delta \delta_{vt} + \phi_{it}.\mathbf{1}\{\phi_{it} \geq 0\} + \Delta \ln \eta_{it} \tag{22}$$

where:

$$\phi_{it} = \delta_{vt-1} - \sigma \lambda_{it-1}(\beta) - \Delta \ln \eta_{it} - \ln \bar{\mu}_{vt}(\pi_{it}, x_{it}, q_{it}). \tag{23}$$

Two remarks are in order. First, complete markets is an interesting special case where the incentive constraints never bind. We have, $\phi_{it} < 0$, so that the usual Euler equation holds:

$$\Delta \ln c_{it} = \Delta x_{it}\beta + (\Delta \delta_{vt} + \Delta \ln \eta_{it})/\sigma.$$

More generally, the existence of self-enforcing constraints implies that dynamics is at times consistent with the predictions derived from the hypothesis of complete markets and at times inconsistent, according to the sign of ϕ_{it}.

Secondly, when the incentive constraint is binding, the second term in the right hand side of equation (22) is positive. It means that consumption growth is more than what would be expected under full insurance. It is the "rent" to pay to keep the household in the self-enforced informal arrangement when the income shock in the period is "too" favorable for the household.

Variable ϕ_{it} depends on the income process π_{it} that we now detail.

3.1.2 Formal Contracts and the Income Process

A formal contract is described by equation (17) in Proposition 4. The vector of formal transfers (*i.e.* in any state e) is a function of the multiplier μ_{it} and of variables belonging to the information set (x_{it}, q_{it}):

$$\tau_{it} = \tau_{vt}(-\Delta \ln \eta_{it} - \sigma \lambda_{it-1}(\beta) + \delta_{vt-1}, x_{it}, q_{it}).$$

when the multiplier $\ln \mu_{it}$ is replaced by its specification using equation (21). We have in mind that formal transfers τ_{it} can be supported by land-leasing contracts (in or out) such as share-cropping contracts and their sharing rules or fixed-rent contracts where the rent is fixed at the village level. Reduced-form estimation indeed showed that such contracts do help sharing risks (Dubois, 2000). In contrast, it seems dubious that labor contracts can be formally enforced since permanent labor contracts are infrequently encountered in village economies (see Bliss and Stern, 1982).

In consequence, agricultural profits is a function of the characteristics of these contracts and in particular the marginal utility of consumption μ_{it} and variables in the information set (x_{it}, q_{it}). It is also a function of observed and unobserved income shocks z_{it} and ξ_{it} that are

revealed after the signature of the contracts so that the reduced form of agricultural income is (using $m_{it} = (x_{it}, q_{it}, z_{it})$, see Notations 3.1):

$$\pi_{it}^a = \pi^a(-\Delta \ln \eta_{it} - \sigma \lambda_{it-1}(\beta) + \delta_{vt-1}, m_{it}, \xi_{it}).$$

We could have specified a structural production function or the structural relationships between the quantities of land under sharecropping and fixed-rent contracts and marginal utility. Given the complicated endogenous structure of land exploitation, we thought that such a route would not be robust.

Household net income is not only composed of agricultural profits but also of non-agricultural profits or other exogenous income, π_{it}^e. For more generality, we include π_{it}^e among variables z_{it} (Notation 3.1(vi)), instead of just adding it to π_{it}^a. By adding some measurement errors, ς_{it}, and adopting the semiparametric assumption that agricultural profits are a linear function, we finally obtain measured income $\tilde{\pi}_{it} \equiv \pi_{it}^a + \pi_{it}^e + \varsigma_{it}$ as:

Assumption 3.2(ii):

$$\tilde{\pi}_{it} = \pi_{vt} + \pi_0 \sigma \lambda_{it-1}(\beta) + m_{it} \pi_m + \pi_0 \Delta \ln \eta_{it} + \xi_{it} + \varsigma_{it}. \tag{24}$$

where the village-and-period effect, π_{vt}, absorbs parameters δ_{vt-1}.

Corollary 5 implies that parameter π_0 is positive which provides the first restriction of our model.

The equations describing consumption growth (22) and income (24) form our structural model. Assumptions about shocks and incentive constraints can now be completed.

3.2 Identifying Restrictions and Reduced Forms

Notations 3.1 and equations (22) and (24) lead to the following list of covariates, $w_{it} = (\ln c_{it-1}, x_{it-1}, x_{it}, q_{it}, z_{it})$ respectively standing for lagged consumption, lagged and current preference shifters, information variables and exogenous shocks. Identifying restrictions on heterogeneity terms, measurement error and predetermined variables are stated as:

Assumption 3.2 (ct'd): *(iii) The vector of unobserved shocks in preferences and income $(\Delta \ln \eta_{it}, \xi_{it})$ is independent of covariates w_{it} and of measurement errors, ς_{it}, and is identically distributed and independent across households and periods. It has an absolutely continuous distribution and its support is a compact set of $\mathbb{R}^2$.*

(iv) Measurement error ς_{it} is mean-independent of covariates, w_{it}, and is independent across households and periods.

(v) Variables in $w_{it} = (\ln c_{it-1}, x_{it-1}, x_{it}, q_{it}, z_{it})$ have a compact support in $\mathbb{R}^r$. If $w_{vt} = E(w_{it} \mid v, t)$ denotes their expectations within a village-and-period, there is full within-variation

of the covariates i.e.:

$$rank\left(E\left[(w_{it} - w_{vt})(w_{it} - w_{vt})'\right] \mid v, t\right) = r$$

The independence Assumption 3.2(iii) is slightly stronger than the ones generally used in linear dynamic models. Non-linearities, due here to the presence of a bound, require more than mean-independence assumptions. We also used the assumption of compact support in order to facilitate the proof of identification. Assumption 3.2(iv) is weaker as it takes advantage of linearity. Both Assumptions 3.2 (iii & iv) make the covariate process w_{it} weakly exogenous. Assumption 3.2 (v) implies that the distribution function of predetermined variables w_{it} is not degenerate. This assumption is substantive since it would be violated if the information set does not include at least one variable q_{it} which is excluded from preferences (an asset variable for instance). Such variables provide exogenous variability in income.

The final step to complete the specification of consumption dynamics is to specify the bound ϕ_{it} appearing in equation (23). As income π_{it}, without measurement errors as described in Assumption 3.1(ii), depends on marginal utility, $\Delta \ln \eta_{it} + \sigma \lambda_{it-1}(\beta) - \delta_{vt-1}$ and on the index, $\pi_{vt} + m_{it}\pi_m + \xi_{it}$, the unobserved variable ϕ_{it} defined in equation (23) can be written as a function of the following arguments:

$$\phi_{it} = \phi(\Delta \ln \eta_{it} + \sigma \lambda_{it-1}(\beta) - \delta_{vt-1}, \pi_{vt} + m_{it}\pi_m + \xi_{it}, x_{it}, q_{it}, \phi_{vt}),$$

where ϕ_{vt} is a village-and-period effect. Using Assumption 3.2 (iii), we can evaluate the conditional expectation of the non-linear term $\phi_{it}\mathbf{1}\{\phi_{it} \geq 0\}$ of equation (22) :

$$E(\phi_{it}\mathbf{1}\{\phi_{it} \geq 0\} \mid \ln c_{it-1}, x_{it-1}, x_{it}, q_{it}, z_{it}) \equiv$$
$$H(\lambda_{it-1}(\beta) - \delta_{vt-1}/\sigma, \pi_{vt} + m_{it}\pi_m, x_{it}, q_{it}, \phi_{vt}),$$

where the positive function H is derived from the joint distribution function of $(\Delta \ln \eta_{it}, \xi_{it})$.

We did not follow this non-parametric route[10] and chose to specify the bound as a linear index:

Assumption 3.2 (ct'd): *(vi)* $\ln \bar{\mu}_{vt}(\pi_{it}, x_{it}, q_{it})$ *is linear in all its arguments.*

We then have:

Lemma 6 *Under Assumption 3.2 (vi), ϕ_{it} is linear in all its arguments:*

$$\phi_{it} = -\sigma(1 + \pi_0 \frac{\partial \ln \bar{\mu}_{vt}}{\partial \pi_{it}})(\lambda_{it-1}(\beta) + m_{it}\phi_m + \phi_{vt}) + \varepsilon_{it}^0(\Delta \eta_{it}, \xi_{it})$$

---

[10]The proof of identification remains however very similar in this more general case. It suffices to condition on m_{it} and village-and-period effects in the following.

where $\frac{\partial \ln \bar{\mu}_{vt}}{\partial \pi_{it}} < 0$ *(Proposition 4, ii). Furthermore:*

$$E(\phi_{it}\mathbf{1}\{\phi_{it} \geq 0\} \mid \ln c_{it-1}, x_{it-1}, x_{it}, q_{it}, z_{it}) = H(\lambda_{it-1}(\beta) + m_{it}\phi_m + \phi_{vt}) \qquad (25)$$

where $H(.)$ is an unknown positive, decreasing, convex and twice differentiable function, where:

$$H'\left(\lambda_{it-1}(\beta) + m_{it}\phi_m + \phi_{vt}\right) = -\Pr(\phi_{it} > 0).\sigma(1 + \pi_0\frac{\partial \ln \bar{\mu}_{vt}}{\partial \pi_{it}}) < 0 \qquad (26)$$

and if there is more than one z variable,

$$\phi_{z^{(j)}}/\phi_{z^{(1)}} = \pi_{z^{(j)}}/\pi_{z^{(1)}} \text{ for any } j > 1. \qquad (27)$$

Proof. See Appendix B. ∎

Summarizing those results, the structural model consists first in the consumption growth moment condition derived from equation (22) and equation (25):

$$E(\sigma.\Delta\lambda_{it}(\beta) - \Delta\delta_{vt} \mid \ln c_{it-1}, x_{it-1}, x_{it}, q_{it}, z_{it}) = H(\lambda_{it-1}(\beta) + m_{it}\phi_m + \phi_{vt}). \qquad (28)$$

Second, using equation (24), the income moment condition is:

$$E(\tilde{\pi}_{it} \mid \ln c_{it-1}, x_{it-1}, x_{it}, q_{it}, z_{it}) = \pi_{vt} + \pi_0\sigma\lambda_{it-1}(\beta) + m_{it}\pi_m \qquad (29)$$

We allow for correlation between these equations. This correlation is not informative about the parameters of interest since it depends on the unrestricted joint distribution of random shocks.

We now study the identification of the system of structural equations (28) and (29).

3.3 Semi-Parametric Identification

The model depends on the following population parameters:

$$\theta = (\sigma, \beta, \pi_0, \pi_m, \phi_m) \in \Theta, \quad \varrho = (\{\delta_{vt}, \pi_{vt}, \phi_{vt}\}_{vt}) \in \Xi, \quad H(.) \in \mathcal{H}.$$

where θ is the parameter of interest. Village-and-period effects, ϱ, and the functional parameter $H(.)$ are treated as nuisance parameters. The identification analysis proceeds as follows.

First, parameter $\sigma > 0$ is not identified since the intertemporal elasticity of substitution is not identified in an Euler equation when the interest rate is unknown. The two equations of interest are invariant to transforming the vector of parameters $(\sigma, \pi_0, \Delta\delta_{vt}, H(.))$ into $(1, \pi_0/\sigma, \Delta\delta_{vt}/\sigma, H(.)/\sigma)$ (holding other parameters constant). We shall therefore normalize $\sigma = 1$ and change the interpretation of other parameters accordingly.

Second, the parameters of the reduced form of the income equation (29), $\pi_{vt}, \pi_0, -\beta\pi_0, \pi_m$ are identified because of Assumption 3.2(v). This is the most one can get from the income equation.

As π_0 and $-\beta\pi_0$ are identified, β is also identified and using equation (28), the dependent variable $\lambda_{it}(\beta) = \ln c_{it} - x_{it}\beta$ is identified. The consumption equation,

$$E(\Delta\lambda_{it}(\beta) \mid \lambda_{it-1}(\beta), m_{it}) = \Delta\delta_{vt} + H(\lambda_{it-1}(\beta) + m_{it}\phi_m + \phi_{vt}), \tag{30}$$

is an index model which identification is analyzed by Ichimura and Lee (1991). We adapt their assumptions to our special case as H is a positive, decreasing and convex function (Lemma 6) and as all regressors are bounded. The main problem we have to face is that the intercept term is identified at infinity only (Heckman, 1990). We restrict function H and its argument as follows:

Assumption 3.2 (ct'd):

(vii) For any (v,t) and at the true parameter value, the support of $\lambda_{-1} + m\phi_m$, is a connected interval of $\mathbb{R}$. Denote a_{1vt} the value such that:

$$Median(\lambda_{-1} + m\phi_m + a_{1vt}) = 0,$$

The support of $\lambda_{-1} + m\phi_m + a_{1vt}$ is denoted $S_I^{(v,t)}$.

(viii) $\mathcal{H}$ is the set of bounded, decreasing, convex and twice differentiable functions taking values on $\cup_{v,t} S_I^{(v,t)}$ and such that $H(0) = 0$.

As shown in the proof of the next Proposition, Assumption (vii & viii) are normalizations if we observe one village at one period. The identification of H follows from the normalization of its location and the location of its argument. Imposing these conditions for any village at any period is testable although the assumption is weak. In its absence, identification would not be robust to small departures from the other assumptions that $H(.)$ does not directly depend on v, t which would be generated by allowing for heteroskedasticity in ξ_{it} for instance. Finally, we cannot impose that function $H(.)$ is positive.

All these conditions are sufficient to obtain:

Proposition 7 *Under Assumption 3.2 (i-viii), (θ, H) is identified in $\Theta \times \mathcal{H}$.*

Proof. See Appendix B ∎

3.4 Estimation using Penalized Splines

To estimate the parameters of the consumption growth equation (30), we use penalized spline regression following Yu and Ruppert (2002). Letting $u = \lambda_{it-1}(\beta) + m_{it}\phi_m$, properly normalized using Assumption 3.2 (vii), we choose function $H(.)$ in the following set:

$$H(u; \gamma) = \gamma_1 u + ... + \gamma_p u^p + \sum_{k=1}^{K-1} \gamma_{p+k} S_p(u - \kappa_k)$$

21

where

$$S_p(u - \kappa_k) = \begin{cases} (u - \kappa_k)^p \mathbf{1}\{u > \kappa_k\} & \text{for } k > p/2 \\ (\kappa_k - u)^p \mathbf{1}\{u < \kappa_k\} & \text{for } k \leq p/2 \end{cases}$$

where $\mathbf{1}\{A\}$ is the indicator function of A.

Parameters to estimate are $(\beta, \phi_m, \{\Delta\delta_{vt}\}, \gamma)$ where $\{\Delta\delta_{vt}\}$ is the collection of village-and-period effects and where γ are the parameters of the spline. The number of knots $K - 1$ is considered as fixed (see Ruppert, 2002, for a procedure of selection) and the locations of the knots κ_k are supposed to be given by the $1/K$-quantiles of u. By construction $H(0) = 0$.

The method of estimation consists in finding the global minimum of:

$$Q(\beta, \phi_m, \{\delta_{vt}\}, \gamma) = \sum_{i,t} (\Delta\lambda_{it}(\beta) - \Delta\delta_{vt} - H(\lambda_{it-1}(\beta) + m_{it}\phi_m; \gamma))^2 + \nu.\gamma'G\gamma$$

where ν is a penalty weight. We chose to penalize equally the elements γ_{p+1} to γ_{p+K-1} writing:

$$\gamma'G\gamma = \sum_{k=1}^{K-1} \gamma_{p+k}^2$$

so that we do not impose that function $H(.)$ is decreasing and convex. The reader is referred to Yu and Ruppert (2002) for all proofs of asymptotic properties including the sandwich formula for the variance-covariance matrix of estimates.[11]

4   The Empirical Analysis

4.1   Data Description and Economic Analysis

The data come from a survey conducted by IFPRI (International Food Policy Research Institute) in Pakistan between 1986 and 1989 (see Alderman and Garcia, 1993, for a thorough presentation) whose information is roughly equivalent to the famous ICRISAT dataset. The survey consists of a random sample of around 900 households interviewed 12 times and coming from four districts in three regions. This rich data set contains information on household demographics, incomes from various sources, individual labor supply, endowments and owned assets, agrarian structure, crops and productions, and land contracts such as sharecropping and fixed rent.

Our endogenous variables are income and consumption growth. Sources of income are wages, agricultural profits, rents from property rights and pensions. Agricultural profits are computed at each survey round using the information since the previous round and consist of cash income from staples, milk products, animal poultry and livestock production, net of total input expenditures including non-family wage costs, feeding costs of productive animals, fertilizers and pesticides.

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[11]Other details concerning the algorithm, the choice of the smoothing parameter ν by generalized cross-validation and the choice of other regularization parameters are presented in the working paper version (Dubois *et al.*, 2005).

Other sources of income such as household's handicraft income, asset income – including property rents, fixed pensions regularly received from the government and rentals of other assets – and non-agricultural labor income are considered as exogenous income. Household wage income in off-farm agricultural tasks are considered as one of the support of informal transfers and is not included in income. Concerning the consumption variable, we choose food expenditures as our consumption variable because they represent the largest part of non-durable expenditures and are well measured. More details on the consumption variables are given in Appendix C and descriptive statistics are presented in Table 1.

In our model, observed heterogeneity between households are of different types: Preferences, x, other variables in the information set, q or unanticipated shocks z. First, Dubois (2000) using the same data showed that household size is the main preference shifter. Other variables such as the number of children never proved to be significant. As for variables in the information set, various empirical analyses (see Jalan and Ravallion, 1999, for instance) report evidence that contrasting rich and poor households is the main relevant differentiation when the complete markets hypothesis is evaluated. As income is endogenous in our model, the quantity of owned land seems to be a good indicator of household's productive assets and a predetermined predictor of income. The quantity of owned rainfed land gives additional information about the quality and price of productive land. These two variables constitute the set of variables, q_{it}, in the information set. Third, craft or trade profits or spells of illness of household members are unlikely to be contractible and are considered as a component of unexpected income shocks, z_t.

Decomposing the variance of agricultural income into the village-and-period variance, the household variance and a residual one, confirms that rural households are mostly affected by large idiosyncratic risks. It is also true for other sources of income like asset income, wages and exogenous income. In contrast, 52.7% of the variance of the logarithm of food expenditures comes from village-and-period effects and 23.7% from household effects. The first component refers to aggregate shocks, the second to the relative economic status of the household in the village i.e. its Pareto weight. We find the same pattern when looking at expenditures corrected by household size or at total expenditures. Turning to first differences in the logarithm of food expenditures, we find substantial though quite imperfect risk sharing since village-time effects explain 24.1% of these changes, and household effects explain 2.2%[12]. Of course, the variance decomposition of marginal utility also depends on preference shifters and first differencing magnifies measurement errors. Nevertheless, testing whether income shocks affect consumption growth controlling for preference shifters allowed one of the authors to reject perfect risk sharing using this dataset (Dubois, 2000).

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[12]At the district level, risk sharing seems to be significantly less effective and it is why we decided to pursue the analysis at the village level.

Finally, the land rental market is very active in these villages since we observe in the data that the per period rate of change of area under contract such as sharecropping and fixed rent is on average 20%. This statistic provides a lower bound for the frequency of contract changes because we are not able to observe with whom the household contracted or contractual changes that did not involve a change in the land area under contract. Thereby, it confirms that these contracts, which provide the natural candidates for our formal contracts, are the object of frequent renegotiations.

4.2   Structural Estimates

In the estimation of the income and consumption growth equations whose results we successively present, we shall examine the empirical relevance of the testable structural restrictions that we summarize as:

$C1 : \pi_0 \geq 0$: monotonicity of the contract from Corollary 5

$C2$: The coefficients of x_{it-1} in the income equation, and in both linear indices of the consumption growth equation, are all equal to β.

$C3 : H$ is decreasing.

There are also other restrictions derived from auxiliary econometric assumptions in Assumptions 3.2 (i-viii) that we directly describe in the text.

4.2.1   The Income Equation

We present several sets of estimation results of the income equation (24) in Table 2. As predicted by our model, the significance of the coefficient of lagged consumption in the income equation yields some evidence that households smooth the income process. In column 1, we report two stage least squares estimates. This procedure controls for the presence of measurement errors in lagged consumption by introducing residuals of the instrumental regression of lagged consumption on other variables of the regression and a further lag of consumption. The estimated coefficient of those residuals is indeed significant and is three times larger than the OLS estimate of the coefficient of lagged consumption. We also tested for the presence of measurement errors in the other variables using lags and we did not reject their exogeneity. A robustness analysis to econometric assumptions is detailed in Section 4.2.3 below.

In conformity with restriction C1, coefficient π_0 is positive and significant. We also tried to use total expenditures with quite similar results. Yet, competing explanations that we now detail could be consistent with this result.

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Household superior information If all relevant information is not included in the income equation, the significantly positive coefficient of lagged consumption in the income equation could also reflect that lagged consumption is a good indicator of the information that households have about future income shocks that we, as econometricians, do not observe. If households have superior information, income and lagged consumption are indeed positively correlated.

In order to investigate the importance of this effect, we use, as proposed by Campbell (1987), another decision variable that might reveal the superior information that households have on future income shocks under the assumption that this information is single dimensional. This decision should be coincident with the signature of contracts to be on par in terms of information. Net-rented in area[13] is such a candidate variable since it is a productive factor. We regress net rented-in area on variables that are known at the beginning of the period and we interpret residuals as a measurement of the superior information that households have. Table A.1 reports results of this estimation. The regression is very well determined and lagged consumption, which is instrumented as before, negatively affects rented-in area. Note that it is because lagged consumption is instrumented that residuals indeed contain new superior information if any.

In the second column of Table 2, the residual of the rented-in land equation used as information controls is found to be significant. Households have superior information and, as the coefficient is negative, tend to rent-in more land in anticipation of an adverse shock in income either in off-farm or on-farm activities. Our prediction C1 of a positive effect of lagged consumption is nevertheless robust to the presence of superior information. The estimated coefficient of lagged consumption remains significant and positive although it decreases marginally.

Nutrition effects on productivity The positive effect of lagged consumption on income could also be due to nutrition effects on future productivity since more food consumption can help improve the productivity of agricultural laborers as analyzed by Behrman, Foster and Rosenzweig (1997) and Fafchamps and Quisumbing (1999). Both articles use working samples extracted from the same survey than ours.

When planting labor effort is incompletely rewarded in the labor market and the productivity of labor is assumed to depend on current consumption, Behrman et al. (1997) argue that planting stage calories positively affect harvest stage profits. Using a subsample of wheat producers at two pairs of planting and harvest periods, they report empirical evidence about this positive relationship.

The availability of data on calorie intakes at the household level actually allows us to distinguish a nutrition effect from the mechanism of risk sharing though formal contracting. As

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[13]or equivalently cultivated land as owned land is controlled for.

expected, calorie consumption and food expenditures are correlated although we found no effect of lagged calorie intakes on income when we use this variable along with or instead of food expenditures. For instance, column 3 of Table 2 shows that when lagged food expenditures and lagged calorie intakes are both included in the estimation, the coefficient of lagged expenditures only is significantly different from zero. When using only lagged calorie intakes without food expenditures, the former estimated coefficient is not significant. The discrepancy with the results cited above might come from differences in the sample selection and construction of the data. In particular, we are using all periods and harvest stage consumption has an impact on the next planting stage profits as well, which is difficult to reconcile with a pure nutrition effect on productivity.

As the effect of nutrition could go through longer term effects, we also use information about the bio-mass index – weight over height to the square – of household members. This variable is positively correlated with food calorie intakes. Introducing the average or total bio-mass index of household adults or of male adults only as additional variables does not change the main results and shows a small positive but insignificant effect on productivity (column 4 of Table 2). This is in line with Fafchamps and Quisumbing (1999) who find that such impacts are small.

Other determinants Regarding the estimated coefficients of other variables in Table 2, household demographics should partly compensate the effect of lagged consumption, and lagged household size indeed does, though insignificantly. The quantity of owned land affects household income although rainfed land compensates it so that their difference, irrigated land, only seems to matter. A larger number of days of illness for males and females decreases income as expected though insignificantly. Informal transfers could take the form of additional labor from the village that compensate for the sickness of household members since it might be less costly to intervene at that stage than afterwards (Fafchamps, 1992). Sickness would not affect household income in that case but the way we construct income would be contaminated by informal transfers that are implicitly already smoothing it. Errors in self-reporting wealth is another argument (Strauss and Thomas, 1998) but our data limitations preclude any further analysis of these arguments. Finally, exogenous income is positive and very significant.

4.2.2 Consumption Growth Equation

In Table 3, we report the estimates of the consumption growth equation using penalized spline regressions. The number of knots of the spline is small and equal to 8 and the penalization parameter is optimally chosen by Generalized Cross Validation for the specification corresponding to column 1.

We estimate the system of income and consumption growth equations by allowing for some

correlation between equations. This correlation is insignificant in magnitude (0.001) and statistically. The likelihood ratio statistic contrasting estimations allowing or not for correlated equations is equal to 0.4 and has one degree of freedom. It shows that even if residual shocks in income are correlated with the residuals of consumption growth through incentive constraints, the influence of these shocks is small relative to measurement errors in both equations and preference shocks in consumption dynamics. We do not report the estimates of the income equation because they are very similar to what is reported in Table 2.

As before, we follow a control function approach in non-linear models (see Blundell and Powell, 2003) and we introduce, as an explanatory variable, the residual of the regression of lagged consumption on its second lag and all other variables to take into account the measurement errors in lagged consumption. Consumption exogeneity is rejected very strongly indeed and the magnitude of the estimated coefficient denotes the presence of large measurement errors in consumption. In column 2 we also introduce the residual of the net rented-in area equation (Table A-1) as a control for superior information. It is very significant (the likelihood ratio statistic for the system of equations is around 60 and has two degrees of freedom) though it does not affect the estimation of the other coefficients.

We now consider testing the restrictions C2 and C3. First, the hypothesis that the coefficients of preference variables in the consumption and income equations are equal (C2) cannot be rejected. The likelihood ratio statistic is equal to 1.4 in column 2 of Table 3. It presumably has two degrees of freedom although we should account for the fact that we were unable to precisely estimate the coefficients of lagged household size in both linear indices because function $H()$ is close to a linear function (see below). Nevertheless, even if the statistic has one degree of freedom only, we cannot reject restriction (C2) at any reasonable level of confidence.

Figure 1 reports estimates of function $H(.)$ using optimal smoothing. Although it should not be taken as a formal test, its shape is not in contradiction with hypothesis (C3) as it is decreasing. Incentive constraints bind with some strictly positive probability and the model is statistically able to capture markets incompleteness at work in this economy. Moreover, recall from equation 26 that the slope of $H(.)$ is the product of the probability that an incentive constraint is binding multiplied by $1 + \pi_0 \frac{\partial \ln \bar{\mu}_{vt}}{\partial \pi_{it}} > 0$. As $\frac{\partial \ln \bar{\mu}_{vt}}{\partial \pi_{it}} < 0$, the slope is a lower bound on this probability and a rough estimate is equal to 0.15.

We analyze the most complete results of Table 3, Column 2, by starting with parameters in the linear index of the equation of consumption dynamics. As expected, household size has a positive and significant effect on the marginal utility of consumption in a way that is comparable to what was obtained in a more usual setting (Dubois, 2000). It is more difficult to analyze the effects of variables in the linear index which is the argument of function $H(.)$. Remember

that the larger this index is, the smaller is the probability that the incentive constraint not to renege the informal contract, binds. Positive coefficients indicate that we are moving away from the constraint. It is only the case for variables such as male days of illness (though the effect is insignificant) and for information controls that we have seen to be negatively correlated to income (Table 2). It is negative for household size, land assets, female days of illness and exogenous income. Intuitively, we should expect that beneficial income shocks should increase the probability that the incentive constraint binds.

Nevertheless, theory does not completely comfort these insights because there could be compensating effects due to the information, correlated with these variables that affect the contracting choices of the household. It is what is observed however for all variables except for female days of illness which is an intriguing result. As already mentioned, the issue that days of illness can be immediately compensated by informal transfers of labor seems to us to be an interesting way to explore. Moreover days of illness could affect function $H(.)$ in a more complicated way if the assumption of independence (A3.2(iii)) in this regressor is violated. As a matter of fact we did test another consequence of independence (A3.2(iii)) in unexpected income shocks (equation (27)) and we were able to reject this restriction. More complicated dependence of H on z_t, stemming from heteroskedasticity of ξ_{it+1} for instance, should thus be allowed for.

As residual variances in income are larger after the first half of the sample period (see below), we split the sample into two periods and re-estimated the consumption growth equation. Results are reported in the two last columns of Table 3 and are very stable across sub-samples although many estimates lose their significance. In Table A-2, we report tests of the separability of consumption and leisure obtained by regressing the residuals of the consumption growth equation on male and female labor supply variables. None are significant and we do not reject separability.

4.2.3 Robustness analysis

A thorough analysis of serial correlation and sensitivity to auxiliary parameters used in penalized spline regression is presented in Dubois et al. (2005) and we briefly summarize it now.

Serial correlation in Incomes We first analyzed the autocorrelation of unobservables affecting income. We found that during the sample period, two subperiods, can be distinguished, the first one with small variances (till round 7), the second one with a lot more variance. Alderman and Garcia (1993) report that the sample period was a quite uncertain and troubled period for agricultural activities, yet it seems to be true above all for the second half of the sample period.

This overall view does not indicate the presence of strong household specific effects. We also tried to first-difference the income equation but we lost all significance. The main question that remains is whether households are aware of the structure of autocorrelation because we use

econometric restrictions that they are not. More prosaically, the presence of serial correlation might also modify standard errors of the estimates of the income equation. We did try to correct for serial correlation by assuming that it is unrestricted over time but constant across individuals. It indeed modified standard errors of coefficients though by a maximal factor of 50% without affecting any diagnostic of significance that was used using the robust to heteroskedasticity White correction (Table 2).

Serial Correlation in Consumption Growth We computed the residual variances of the consumption growth equation along with their autocorrelations. In contrast to income residuals, there is no clear time pattern for consumption residual variance, only two peaks at rounds 6 and 9. They might be attributed to measurement errors since the length of the period is somehow variable across rounds even though we considered weekly income and consumptions. The pattern of autocorrelations is more surprising since they vary at order 1 between -0.2 and 0.05, at order 2 between -0.17 and 0, at order 3 between 0 and 0.10 and at order 4 between 0.05 and 0.15. In the case where the function $H(.)$ is linear, we develop in Dubois et al. (2005) a model with i.i.d. preference shocks and classical measurement errors and we calibrate it on our data. Autocorrelation coefficients are broadly in line with the empirical ones. The calibration also shows that the magnitude of preference shocks and measurement errors are roughly equal in the residuals of consumption growth.

Other auxiliary parameters The auxiliary parameters in penalized spline regression are the penalization parameter, the number of knots and the order of the spline. The income equation is not affected by the variations in these parameters as correlation between equations is virtually equal to zero.

Making the penalization parameter varies between one tenth of its optimal value and ten times its optimal value produces no noticeable differences on the values of the estimated coefficients but only differences in the estimated standard errors for some coefficients. We tested structural restrictions by using likelihood ratio statistics which are more robust than Wald tests. We also did the same kind of experiment by varying the number of knots and the order of the splines. In all cases, the central results of this paper remain the same though standard errors seem affected by those auxiliary parameters. Regarding the estimates of function $H(.)$ when the order of the spline varies, it is much smoother using quadratic and cubic splines although the estimation using linear splines does not contradict our main empirical conclusions.

Spatial correlation We also estimated auxiliary regressions of cross-products of income and consumption growth residuals within a village on the absolute difference between households of

characteristics such as land assets or household size. These regressions are intended to investigate whether unexplained components of consumption growth and income are associated across households of similar characteristics in each village. Results are reported in Table A-3. Interestingly the cross-product of residuals is always a negative function of the distance of characteristics between households. Furthermore, while the distance in land assets have a strong influence on the cross-product between income residuals, their influence is much attenuated in the consumption growth equation. This is not the case for the distance in household size. In the absence of household fixed effects, it is tempting to interpret these results as consistent with partial risk sharing of productivity shocks attached to asset levels but no risk sharing of demographic shocks.

5   Discussion & Extensions

We elaborated a theoretical setting which nests the case of complete markets when all risks can be insured by formal contracts and the case where only informal agreements are available. Its empirical implementation is built up around the estimation of an Euler-type equation of consumption dynamics and an income equation using data from village economies in Pakistan.

Overall, our results indicate that those villages are far from perfect risk sharing where lagged consumption would affect neither income nor consumption growth or from limited commitment where lagged consumption would not affect income. Both cases are significantly rejected. The importance of formal contracts and limited commitment can be evaluated from the empirical estimates of the two structural equations. First, from results in Table 2, we can derive that 12% of the within-village variance of income is explained by lagged consumption and lagged preference shifters and we have been arguing that this comes from formal contracting. Specifically, while information variables including land assets explain four times as much as exogenous income, formal contracts are on par with exogenous income in terms of explaining the total within-village variance of agricultural profits. Income smoothing due to formal contracting is thus economically significant. Second, we can also evaluate the explanatory power provided by limited commitment and formal contracts in the consumption equation although it is low, as usual with consumption growth. Whereas 24% of the variance of consumption growth is explained by village-and-period effects, only 1.6% is explained by the non linear function $H(.)$ and its arguments and 0.3% by preference shifters. Nonetheless, as incentive constraints do not bind at all periods, one in seven being the estimated lower bound, the economic magnitude of such a result is still significant.

Policy evaluation by simulation would require full structural estimates, as discussed in the limitations below. Nonetheless, we can already draw interesting policy implications.

First, by providing evidence on the impact of informal agreements, our work points to important but often neglected mechanisms interlinking formal and informal agreements. This is in

line with evidence on contract interlinkages (Bell, 1988). One consequence is that short term land agreements react to future anticipated shocks in a non trivial way, since anticipations affect the incentives to renege on informal agreements. Our analysis also comforts evidence explaining in part the success of micro-credit that relies on similar informal mechanisms.

Second, the dimension of informal risk sharing should be incorporated both into the evaluation and into the design of development programs. In particular, the policy designer should evaluate whether an intervention complements informal agreements or weakens the ability of agents to cooperate. In the latter case, gains in terms of access to new instruments or markets should outweigh the cost in terms of lost commitment. As Albarran and Attanasio (2003) show, policies that aims at improving formal risk sharing could crowd out informal mechanisms.

Furthermore, the emphasis on commitment issues also suggests designing policies that improve the enforceability of contracts within communities, prior to extending the set of contractual parties (*e.g.* governments, banks, international institutions ...). It may also be desirable to adjust interventions at the local level, with field studies aiming at identifying and exploiting channels of informal risk sharing. Localized programs may in particular aim at identifying the credible parties (able to commit), and associating them to policy.

Much remains to be done before getting a complete picture of the role of informal agreements in village economies. In a companion paper (Dubois *et al.*, 2002), we analyze a model where no agent can commit. Conclusions about contract monotonicity are preserved although the dynamics of the ratio of marginal utilities is affected by a second threshold which corresponds to the incentive constraint of agent 2.

Moreover, although we here focused attention on short-term contracts, our theoretical and empirical conclusions are to extend to the case of longer but finite horizon contracts, provided that these contracts can be renegotiated every period at no cost. What really matters is the ability to use contracts to affect the distribution of the outside options of the agents. Long term contracts can be used to affect the continuation utility of deviant agents, and then can be renegotiated (including voluntary cancellation of contracts) to improve the efficiency of the future relationship.

It was out of the scope of the paper to try to identify structural correspondence functions between the space of contracts and the marginal rate of substitution of consumption between two successive periods. It is nevertheless high on the research agenda since it would lead to full structural modelling enabling the simulation of the impact of economic policies. Whereas we posited a reduced form for modeling the incentive compatibility constraint, the precise determination of which contracts can be enforced or not would lead to the determination of the full structure. This line of research would certainly involve simplifications in the description of risks

that households face and contracts that they can use. We preferred to proceed with evaluating first the general relevance of our setting before attempting any such construction.

The issue of saving, and more generally of any device linking one period to the next (nutrition, investment in agricultural technology,...) is difficult to address. Some recent progress has been made by Gobert and Poitevin (2006) in the case of public savings. They assume that breaching an informal agreement triggers the loss of the saving assets. It would make sense in our context if the asset is carried by the agent who is able to commit. Beyond consumption smoothing, savings provide resources that can be used as collateral of informal promises and that help relax incentive constraints. When savings are publicly monitored but privately owned, it would affect the outside options of a deviant agent and would also raise convexity issues (see Ligon et al., 2000). Another critical issue is asymmetric information when savings are not publicly observed. As pointed out by Cole and Kocherlakota (2001), hidden savings drastically limit the ability to use even formal contracts.

REFERENCES

Abreu, D. (1988): "On the Theory of Infinitely Repeated Games with Discounting", *Econometrica,* 56, 383-396.

Albarran, P. and O., Attanasio (2003): "Limited commitment and crowding out of private transfers: evidence from Mexican villages**",** *Economic Journal***,** 113, 77-C85.

Alderman, H., and M. Garcia (1993): *Poverty, household food security, and nutrition in rural Pakistan.* Research Report 96. Washington, D.C.: International Food Policy Research Institute.

Attanasio, O., and J. V. Ríos-Rull (2003): "Consumption Smoothing and Extended Families", *Advances in Economics and Econometrics: Theory and Applications,* Volume III, 209-242.

Beaudry, P., and J. DiNardo, (1991): "The effect of implicit contracts on the movement of wages over the business cycle: evidence from micro data", *Journal of Political Economy,* 665-688.

Behrman, J.R., A. Foster and M., Rosenzweig, (1997), "The dynamics of agricultural production and the calorie-income relationship: evidence from Pakistan", *Journal of Econometrics*, 77:187-207.

Bell, C. (1988), "Credit Markets, Contracts and Interlinked Transactions", in H. Chenery and T.N. Srinivasan, (eds.), *Handbook of Development Economics*, Amsterdam: North Holland Vol. 1, Chap. 16, 763-830.

Benveniste J., and J. Scheinkman (1979): "On the differentiability of the value function in dynamic models of economics", *Econometrica*, 47:727-32.

Bliss, C., and N. Stern (1982): *Palampur: The Economy of An Indian Village,* Oxford UP, Oxford.

Blundell, R., L. Pistaferri and I. Preston (2003): "Consumption Inequality and Partial Insurance", unpublished manuscript.

Blundell, R., and J. Powell (2003): "Endogeneity in Non Parametric and Semi Parametric Regression Models", *Advances in Economics and Econometrics: Theory and Applications,* Volume II:312-57.

Campbell, J. Y. (1987): "Does saving anticipate declining labor income?", *Econometrica,* 55, 1249-1273

Coate, S., and M. Ravallion (1993): "Reciprocity without commitment - Characterization and Performance of Informal Insurance Arrangements", *Journal of Development Economics,* 40, 1-24

Cochrane, J. (1991): "A Simple test of Consumption Insurance", *Journal of Political Economy,* 99, 957-976.

Cole, H., and N. R. Kocherlakota (2001): "Efficient Allocations with Hidden Income and Hidden Storage", *Review of Economic Studies,* 68, 523-542.

Cunha, F., J.J., Heckman and S., Navarro (2005): "Separating Uncertainty from Heterogeneity in Life-Cycle Earnings", NBER WP 11024.

Deaton, A. (1997): *The Analysis of Household Surveys,* The World Bank, John Hopkins: Baltimore.

Dercon, S. (2004): *Insurance against Poverty,* Oxford U.P.

Doepke M. and R. Townsend (2006): "Dynamic Mechanisms Design with Hidden Income and Hidden Actions", *Journal of Economic Theory,* 106, 235-285.

Dubois, P. (2000): "Assurance Parfaite, Hétérogénéité des Préférences et Métayage au Pakistan", *Annales d'Economie et de Statistique,* 59, 1-36.

Dubois, P., B. Jullien and T. Magnac (2002): "Risk Sharing with Formal and Informal Contracts: Theory, Semi-Parametric Identification and Estimation", unpublished manuscript.

Dubois, P., B. Jullien and T. Magnac (2005): "Formal and Informal Risk Sharing in LDCs: Theory and Empirical Evidence", IDEI Working Paper 351.

Fafchamps, M. (1992): "Solidarity Networks in Pre-Industrial Societies: Rational Peasants with a Moral Economy", *Economic Development and Cultural Change,* 41, 1, 147-174.

Fafchamps, M. (2003): *Rural Poverty, Risk and Development,* Edward Elgar Publishing, Cheltenham (UK)

Fafchamps, M., and A., Quisumbing, (1999): "Human Capital, Productivity and Labor Allocation in Rural Pakistan", *The Journal of Human Resources,* 34:369-406.

Foster, A. and M., Rosenzweig (2001): "Imperfect Commitment, Altruism and the Family: Evidence from Transfer Behavior in Low-Income Rural Areas", *Review of Economics and Statistics,* 83, 3, 389-407.

Gauthier, C., M., Poitevin, and P., Gonzalez (1997): "Ex-ante payments in self enforcing risk sharing contracts", *Journal of Economic Theory,* 76, 106-144.

Heckman, J.J. (1990): "Varieties of Selection Biases", *American Economic Review,* 80, 313-318.

Ichimura, H., and L.F. Lee (1991): "Semiparametric Least Squares Estimation of Multiple Index Models : Single Equation Estimation" in *Non Parametric and Semiparametric Methods in Econometrics and Statistics,* ed. by W.A. Barnett, J. Powell and G. Tauchen, Cambridge UP: Cambridge.

Jacoby, H. G., and E., Skoufias (1998): "Testing Theories of Consumption Behavior Using Information on Aggregate Shocks: Income Seasonality and Rainfall in Rural India", *American Journal of Agricultural-Economics*, 80, 1, 1-14.

Jalan, J., and M. Ravallion (1999): "Are the Poor Less Well Insured? Evidence on Vulnerability to Income Risk in Rural China", *Journal of Development Economics,* 58, 1, 61-81.

Kocherlakota, N. (1996): "Implications of Efficient Risk Sharing without Commitment", *Review of Economic Studies,* 63, 595-609.

Koeppl, T. (2006): "Differentiability of the Efficient Frontier when Commitment to Risk Sharing is Limited", *Topics in Macroeconomics*, 6(1):Art. 10.

Ligon, E., J. Thomas and T. Worrall (2000): "Mutual Insurance, Individual Savings and Limited Commitment", *Review of Economic Dynamics,* 3, 1-47.

Ligon, E., J. Thomas and T. Worrall (2002): "Mutual Insurance and Limited Commitment: Theory and Evidence in Village Economies", *Review of Economic Studies,* 69, 209-244.

Mace, B. (1991): "Full Insurance in the Presence of Aggregate Uncertainty", *Journal of Political Economy*, 99, 5, 928-956.

Milgrom, P., and Shannon C. (1994): "Monotone Comparative Statics", *Econometrica*, 62, 1, 157-80.

Morduch, J. (1995): "Income Smoothing and Consumption Smoothing" *Journal of Economic Perspectives*; 9(3), 103-114.

Morduch, J. (1999): "Between the State and the Market: Can Informal Insurance Patch the Safety Net?" *World Bank Research Observer*; 14(2), 187-207.

Newbery, D. (1977) "Risk Sharing, Sharecropping and Uncertainty Labor Markets", *Review of Economic Studies*, 44, 3, 585-594

Ogaki, M., and Q. Zhang (2001): "Decreasing Relative Risk Aversion and Tests of Risk Sharing", *Econometrica,* 69, 515-526.

Ray, D. (2002): "The Time Structure of Self-Enforcing Agreement", *Econometrica,* 70, 547-582.

Rockafellar (1970): *Convex Analysis* Princeton, N.J., Princeton University Press

Ruppert, D. (2002): "Selecting the Number of Knots for Penalized Splines", *Journal of Computational and Graphical Statistics,* 11(4), 735-57.

Stiglitz, J. (1974): "Incentives and Risk Sharing in Sharecropping", *Review of Economic Studies*, 41, 2, 219-255

Strauss J. and D. Thomas, (1998)**,** "Health, Nutrition and Economic Development", *Journal of Economic Literature*, 36:766-817.

Thomas, J., and T. Worrall. (1988): "Self Enforcing Wage Contracts", *Review of Economic Studies*, 55, 541-554.

Thomas, J., and T. Worrall. (1990): "Income Fluctuation and Asymmetric Information: an Example of Repeated Principal-Agent Problem", *Journal of Economic Theory,* 51, 367-390.

Townsend, R. (1994): "Risk and Insurance in Village India", *Econometrica* 62(3), 539-591.

Udry, C. (1994): "Risk and Insurance in a Rural Credit Market: An Empirical investigation in Northern Nigeria", *Review of Economic Studies*, 61, 3, 495–526.

Yu, Y., and D. Ruppert (2002): "Penalized Spline Estimation for Partially Linear Single Index Models", *Journal of the American Statistical Association,* 97, 1042-54.

6 Tables & Figure

Table 1: Descriptive statistics[14]

Descriptive statistics (all periods, 8053 observations)		
Variable	Average	Std. Err.
Food consumption	176.29	103.21
Other non durable expenditures (heating, ..)	43.66	188.98
Total owned land area (acres)	9.42	21.81
Land owner (0:landless, 1:owner)	0.60	0.48
Total Land Owned in Village (acres)	6.66	13.2
Irrigated land (acres)	3.32	8.94
Non irrigated land (acres)	1.79	4.76
Household size	7.73	2.73
Number of children ($\leq$15years)	4.56	2.21
Number of days of illness per week (male)	0.52	1.75
Number of days of illness per week (female)	0.25	0.90
Pensions	18.76	199
Agricultural profits	-169.0	656.7
Transfers	87.03	408.2
Exogenous income	154.1	628.3
Total income (without transfers)	151.19	840.7
Sharecropping dummy variable (renting in)	0.35	0.47
Fixed rent dummy variable (renting in)	0.08	0.26
Total area under sharecropping (renting in), (acres)	2.65	4.96
Total area under fixed rent (renting in), (acres)	0.47	3.39
Total area under sharecropping (renting out), (acres)	2.55	9.84
Total area under fixed rent (renting out), (acres)	0.35	3.80

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[14]All income and expenditure variables are in Rupees per week.

Table 2: Income Equation[15]

Dependent variable: $\tilde{\pi}_{it}$	(2SLS)			
π_0	(1)	(2)	(3)	(4)
	0.194	0.177	0.169	0.166
	(0.060)	(0.062)	(0.061)	(0.061)
$-\pi_0\beta$				
lagged (log) household size	0.032	0.035	0.013	0.016
	(0.079)	(0.079)	(0.092)	(0.092=
π_q				
total land in village	0.149	0.150	0.149	0.148
	(0.016)	(0.016)	(0.016)	(0.016)
rainfed land in village	-0.195	-0.193	-0.195	-0.195
	(0.031)	(0.031)	(0.032)	(0.032)
π_x				
(log) household size	-0.129	-0.125	-0.133	-0.134
	(0.079)	(0.080)	(0.081)	(0.081)
π_z				
female illness days	-0.004	-0.005	-0.005	-0.005
	(0.011)	(0.011)	(0.011)	(0.011)
male illness days	-0.008	-0.008	-0.009	-0.008
	(0.005)	(0.005)	(0.005)	(0.005)
exogenous income shocks	0.159	0.157	0.157	0.157
	(0.038)	(0.036)	(0.036)	(0.036)
Consumption Residuals	-0.135	-0.116	-0.112	-0.109
(for $\ln c_{t-1}$ equation in lags)	(0.064)	(0.065)	(0.064)	(0.064)
Information controls		-0.005	-0.005	-0.005
(from rented-in land equation)		(0.002)	(0.002)	(0.002)
Lagged log-Calorie Consumption			0.040	0.04
			(0.078)	(0.078)
Lagged Adults Bio Mass Index				0.002
				(0.003)
Observations	7133	7133	7131	7131

[15]Robust standard errors in parentheses. Village-and-period effects (46 villages, 11 rounds) not shown. $\ln c_{it-1}$ is instrumented by $\ln c_{it-2}$ and all other variables.

Table 3: Consumption Growth[16]

Consumption Growth Linear index	System Estimation			
	(1)	(2)	(3) Round <= 7	(4) Round >= 8
$\Delta \ln(hh\_size)_t : \beta$	0.248	0.249	0.212	0.267
	(0.075)	(0.035)	(0.60)	(0.34)
Residual ($\ln c_{it-1}$)	-0.571	-0.565	-0.652	-0.505
	(0.024)	(0.024)	(0.075)	(0.040)
Linear argument of $H(.)$				
$\ln c_{it-1}$	1	1	1	1
	(-)	(-)	(-)	(-)
$-\ln(hh\_size)_{t-1} : \beta$	0.248	0.249	0.212	0.267
	(-)	(-)	(-)	(-)
$\ln(hh\_size)_t$	-0.466	-0.458	-0.332	-0.435
	(0.083)	(0.15)	(0.46)	(0.17)
Owned Land	-0.020	-0.015	-0.032	0.005
	(0.006)	(0.003)	(0.0157)	(0.007)
Rainfed Land	-0.041	-0.052	0.448	-0.173
	(0.014)	(0.0264)	(0.38)	(0.30)
Female illness	-0.231	-0.220	-0.193	-0.203
	(0.066)	(0.062)	(0.41)	(0.070)
Male illness	0.008	0.004	0.013	-0.038
	(0.0011)	(0.0085)	(0.0057)	(0.0480)
Exogenous income	-0.18	-0.20	-0.31	-0.17
	(0.14)	(0.14)	(0.65)	(0.025)
Information Control		0.011	0.034	0.007
		(0.0016)	(0.0166)	(0.0031)
Likelihood	-850.0	-829.6	204.8	-809.8
Observations	7133	7133	3704	3429

[16]Quadratic splines. 8 knots. Penalization parameter =1.69. Robust standard errors below estimates in parenthesis. Village-and-period effects (46 villages, 11 rounds) not shown. $\ln c_{it-1}$ is instrumented by $\ln c_{it-2}$ and all other variables. Coefficients whose standard errors are (-) are normalized or set by a structural restriction.
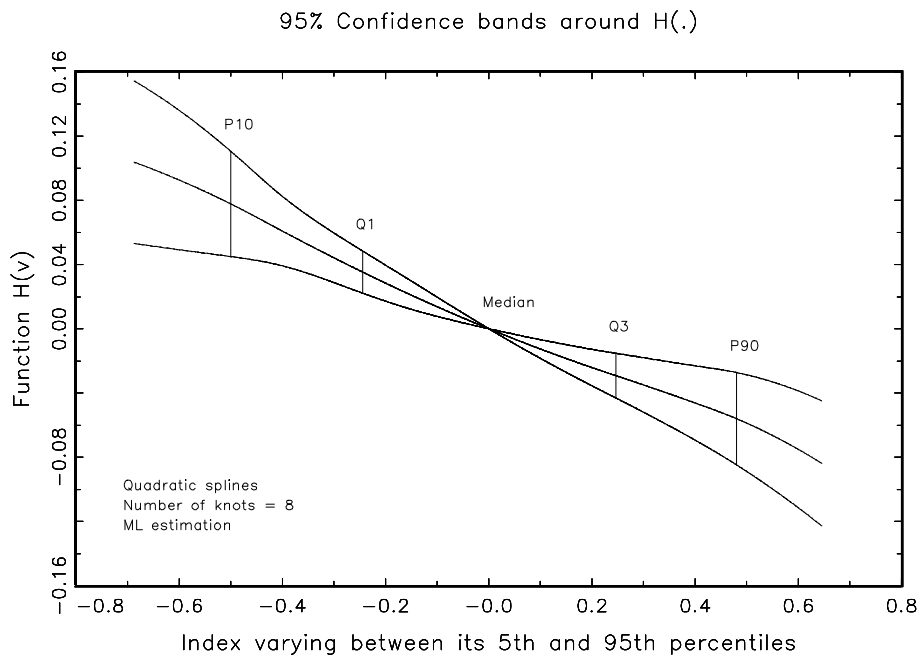
95% Confidence bands around H(.)

Figure 1: Quadratic Splines: Optimal Smoothing

A Appendix to Section 2

Lemma 8 $\hat{P}(V)$ *is differentiable on* $]\underline{V}, \bar{V}[$.

Proof. >From Carathéodory's Theorem (see Rockafellar (1970), corollary 17.1.5), the optimum obtains with a two-point support distribution, say $V_\chi \in \{V_\chi^1, V_\chi^2\}$ with probability p and $1 - p$. Thus an optimal allocation is such that conditional on χ, the distribution of the future utility solves the program

$$\hat{P}(V_\chi) = \max_{p \in]0,1]} \left\{ pP(V_\chi^1) + (1-p)P(V_\chi^2) \right\} \ s.t. \ pV_\chi^1 + (1-p)V_\chi^2 = V_\chi.$$

For conciseness, we now focus on a point where $P(V) = \hat{P}(V)$ so that there **is** no randomization and $p = 1$, and consider the solution of program P. Notice that it must be the case that $\forall e \in \mathcal{E}$, $\tau(e) \leq 0$. Otherwise a reduction in $\tau(e)$ would raise the utility of agent 2 without affecting the other constraints. As a consequence, the constraint (10) is not binding in at least one state $\hat{s}$, since otherwise the agent 1 would receive less than the autarky level. Then, for x small, consider the allocation obtained by simply adjusting $c^2(\hat{s})$ by $\gamma(x)$ such that $p_{\hat{s}}(v(c^2(\hat{s}) + \gamma(x)) - v(c^2(\hat{s}))) = x$ and $c^1(\hat{s})$ to $c^1(\hat{s}) - \gamma(x)$. The new utility of agent 1 is

$$H(V + x) = \hat{P}(V) + p_{\hat{s}} \left(u\left(c^1(\hat{s}) - \gamma(x)\right) - u\left(c^1(\hat{s})\right) \right) \leq \hat{P}(V + x)$$

as the new allocation is feasible and yields $V + x$ to agent 2. Moreover H is concave and differentiable, and $H(V) = \hat{P}(V)$. From lemma 1 in Benveniste and Scheinkman (1979), the function $\hat{P}(.)$ is differentiable ∎

Lemma 9 $Q(.)$ *is strictly concave.*

Proof. First notice that for $V = 0$, the solution is to set $\tau_t = 0$ and $c_t^1 = \omega_t$ at all dates so that $\hat{P}(0) = P(0) = \frac{1}{1-\rho} \sum_s p_s u(\omega^1(s) + \omega^2(s))$ and no incentive constraint is binding. Given that $v'(0) = +\infty$, we obtain $\hat{P}'(0) = 0 > \hat{P}'(\bar{V})$. Consider the set B of slopes b such that $b = -\hat{P}'(V)$ occurs for more than one value V. The solution of $Q(V)$ is a function $\hat{V}(\chi)$ with

$$\chi \hat{P}'(\hat{V}(\chi)) = Q'(V) \text{ if } \chi > \frac{Q'(V)}{\hat{P}'(\bar{V})}, \ \hat{V}(\chi) = \bar{V} \text{ if } \chi \leq \frac{Q'(V)}{\hat{P}'(\bar{V})}.$$

This defines $\hat{V}(\chi)$ uniquely except when $\frac{Q'(V)}{\chi} \in B$, which occurs with zero probability since B is a countable set and χ is a continuous random variable. Consider now $V' > V$ with a solution $V'(\chi)$. It is impossible that $Q'(V') = Q'(V)$ because this would imply $V'(\chi) = V(\chi)$ with probability one and thus contradicts $E\{V'(\chi)\} = V' > V$. Thus $Q'(.)$ must be decreasing. ∎

A.1 Proof of proposition 4

We solve the program $\Phi(\mu) = \max_T \hat{\Phi}(\mu, T)$ where

$$\hat{\Phi}(\mu, T) = \max_{(c^1(s), c^2(s), V(s))_s,} \sum_{s \in \mathcal{S}} p_s \left(u\left(c^1(s)\right) + \rho Q(V(s)) + \mu \left[v\left(c^2(s)\right) + \rho V(s) \right] \right)$$

$$s.t \ (10), \ (11), \ (12).$$

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Lemma 10 *There exists a function $\bar{\mu}(\omega, \pi)$ with values in $[-\hat{P}'(\underline{V}), -\hat{P}'(\bar{V})[$, decreasing in π whenever interior, such that the solution verifies:*

$$\frac{u'\left(c^1(s)\right)}{v'\left(c^2(s)\right)} = -Q'(V(s)) = \min\{\mu, \bar{\mu}(\omega(s), \pi(s))\}$$

Proof. Given the separability in $\tau(e)$, we can optimize events by events: $\hat{\Phi}(\mu, T) = \sum_e p_e \Phi_e(\mu, \tau(e))$ where:

$$\Phi_e(\mu, \tau(e)) = \max_{(c^1(s), c^2(s), V(s))} E\left[u\left(c^1(s)\right) + \rho Q(V(s)) \mid e\right] + \mu E\left[v\left(\omega(s) - c^1(s)\right) + \rho V(s) \mid e\right]$$

s.t.

$$\left(\frac{p_s}{p_e}\lambda_s^1\right) \quad u\left(c^1(s)\right) + \rho Q(V(s)) \geq u(\pi(s)) + \rho\underline{v}^1 \quad \forall s \in e \tag{A.1}$$

$$\left(\frac{p_s}{p_e}\rho\bar{\gamma}_s\right) \quad V(s) \leq \bar{V} \tag{A.2}$$

$$\left(\frac{p_s}{p_e}\rho\underline{\gamma}_s\right) \quad \underline{V} \leq V(s) \tag{A.3}$$

where the terms in parentheses are Lagrange multipliers. The Lagrangian of the program is:

$$\sum_{s\in e} \frac{p_s}{p_e} \left\{ u\left(c^1(s)\right) + \rho Q(V(s)) + \mu\left[v\left(\omega(s) - c^1(s)\right) + \rho V(s)\right] \right.$$

$$\left. + \lambda_s^1\left[u\left(c^1(s)\right) + \rho Q(V(s))\right] + (\underline{\gamma}_s - \bar{\gamma}_s)\rho V(s) \right\}$$

As the program is strictly concave, the first order conditions are necessary and sufficient for optimality. After elimination of $\underline{\gamma}_s$, $\bar{\gamma}_s$, they reduce to:

$$\frac{u'\left(c^1(s)\right)}{v'\left(c^2(s)\right)} = \frac{\mu}{1 + \lambda_s^1}$$

$$-Q'(\bar{V}) \leq \frac{\mu}{1 + \lambda_s^1} \quad (= \text{ if } V(s) < \bar{V})$$

$$\mu v'(c^2(s)) = u'\left(c^1(s)\right)\left(1 + \lambda_s^1\right).$$

Let $\phi(.)$ be the (increasing) inverse function of $-Q'(.)$ and $\psi^1(\omega, \mu)$ be the solutions of

$$\frac{u'\left(\psi^1\right)}{v'\left(\omega - \psi^1\right)} = \mu.$$

The solution coincides with $c^1(s) = \psi^1(\omega(s), \mu)$ and $V(s) = \phi(\mu)$ in all states where the incentive constraint is not binding:

$$u\left(\psi^1(\omega(s), \mu)\right) + \rho Q(\phi(\mu)) \geq u(\pi(s)) + \rho\underline{v}_1. \tag{A.4}$$

The LHS of the condition decreases with μ. Define $\bar{\mu}(\omega, \pi)$ as the solution of equation (A.4) holding with equality. Then $\bar{\mu}(\omega, \pi)$ decreases with π, and the incentive constraint is not binding whenever $\mu \leq \bar{\mu}(\omega(s), \pi(s))$.

Now suppose that $\mu \geq \bar{\mu}(\omega(s), \omega^1(s) + \tau(e))$. Then $\frac{u'\left(c^1(s)\right)}{v'\left(c^2(s)\right)} = -Q'(V(s)) = \bar{\mu}(\omega(s), \pi(s)) = \frac{\mu}{1+\lambda_s^1}$ verifies the first order conditions and thus is the solution. ∎

Lemma 11 *The mapping* $\bar{T}(\mu) : \mu \rightarrow \arg\max_T \hat{\Phi}(\mu, T)$ *is a monotonic non-increasing correspondence in* μ *(according to the strong order set).*

Proof. We can restrict attention to the subset $\tau(e) \leq 0$ where $\frac{\partial \varphi_e}{\partial \tau(e)} \leq 0$. The result follows from Milgrom and Shannon (1994), Theorem 4. Given the separability in $\tau(e)$, $\hat{\Phi}(\mu, T)$ is quasi-supermodular in T. The following lemma shows that it also verifies the single crossing condition in $(T; \mu)$.

Lemma 12 *For a.e.* $\tau(e)$, $\frac{1}{\mu} \frac{\partial \hat{\Phi}(\mu, T)}{\partial \tau(e)}$ *is non-increasing with* μ, *decreasing if* (10) *is binding in some state of event* e.

∎

Proof. >From the envelop theorem, at any point where $\Pr\{\omega^1(s) + \tau(e) = 0 \mid e\} = 0$, and thus for all except a finite number of values of $\tau(e)$, we have[17]

$$\frac{\partial \Phi_e(\mu, \tau(e))}{\partial \tau(e)} = -E\left\{ \mu v'(c^2(s)) \frac{\partial \varphi(e, \tau(e))}{\partial \tau(e)} + \lambda_s^1 u'\left(\omega^1(s) + \tau(e)\right) \mid e \right\}.$$

Denote by $\hat{\psi}(\mu, \omega)$ the solution of

$$\hat{\psi} = u'(x) = \mu v'(\omega - x)$$

When $\mu > \bar{\mu}$, then $\lambda_s^1 = \frac{\mu}{\bar{\mu}(\omega(s), \pi(s))} - 1$ and $\mu v'(c^2(s)) = \frac{\mu}{\bar{\mu}} \hat{\psi}(\bar{\mu}, \omega(s))$. When $\mu < \bar{\mu}$, then $\lambda_s^1 = 0$ and $\mu v'(c^2(s)) = \hat{\psi}(\mu, \omega(s))$. Thus

$$\frac{\partial \Phi_e(\mu, \tau(e))}{\partial \tau(e)} = -E\left\{ \hat{\psi}(\mu, \omega(s)) \frac{\partial \varphi(e, \tau(e))}{\partial \tau(e)} \mid \mu < \bar{\mu}, e \right\} prob(\mu < \bar{\mu} \mid e)$$
$$- E\left\{ \frac{\mu}{\bar{\mu}} \hat{\psi}(\bar{\mu}, \omega(s)) \frac{\partial \varphi(e, \tau(e))}{\partial \tau(e)} + \left(\frac{\mu}{\bar{\mu}} - 1\right) u'\left(\omega^1(s) + \tau(e)\right) \mid \mu > \bar{\mu}, e \right\} prob(\mu > \bar{\mu} \mid e)$$

This function is differentiable in μ for all μ such that μ is not equal to $\bar{\mu}$ with probability 1. Otherwise the argument which follows applies to the right and left derivative in μ. We have:

$$\mu \frac{\partial}{\partial \mu}\left(\frac{\partial \Phi_e(\mu, \tau(e))}{\partial \tau(e)}\right) = -E\left\{ \mu \frac{\partial \hat{\psi}(\mu, \omega(s))}{\partial \mu} \frac{\partial \varphi(e, \tau(e))}{\partial \tau(e)} \mid \mu < \bar{\mu}, e \right\} prob(\mu < \bar{\mu} \mid e)$$
$$- E\left\{ \frac{\mu}{\bar{\mu}} \hat{\psi}(\bar{\mu}, \omega(s)) \frac{\partial \varphi(e, \tau(e))}{\partial \tau(e)} + \frac{\mu}{\bar{\mu}} u'\left(\omega^1(s) + \tau(e)\right) \mid \mu > \bar{\mu}, e \right\} prob(\mu > \bar{\mu} \mid e)$$

$$\mu \frac{\partial}{\partial \mu}\left(\frac{\partial \Phi_e(\mu, \tau(e))}{\partial \tau(e)}\right) = \frac{\partial \Phi_e(\mu, \tau(e))}{\partial \tau(e)}$$
$$+ E\left\{ \left(\hat{\psi}(\mu, , \omega(s)) - \mu \frac{\partial \hat{\psi}(\mu, \omega(s))}{\partial \mu}\right) \frac{\partial \varphi(e, \tau(e))}{\partial \tau(e)} \mid \mu < \bar{\mu}, e \right\} prob(\mu < \bar{\mu} \mid e)$$
$$- E\left\{ u'\left(\omega^1(s) + \tau(e)\right) \mid \mu > \bar{\mu}, e \right\} prob(\mu > \bar{\mu} \mid e)$$

---

[17]If $\omega^1(s) + \tau(e) < 0$, the convention we adopted implies that $u'\left(\omega^1(s) + \tau(e)\right) = 0$. Notice that $w^1(s) + \tau(e) \geq 0$ in at least one state where the incentive constraint binds (otherwise increasing $\tau(e)$ would raise efficiency).

As

$$\hat{\psi} - \mu\frac{\partial\hat{\psi}}{\partial\mu} = \mu v'\left(1 - \frac{u''}{u'' + \mu v''}\right) > 0$$

and $\frac{\partial\varphi_e}{\partial\tau(e)}\leq 0$, we have

$$\mu\frac{\partial}{\partial\mu}\left(\frac{\partial\Phi_e(\mu,\tau(e))}{\partial\tau(e)}\right) \leq \frac{\partial\Phi_e(\mu,\tau(e))}{\partial\tau(e)}.$$

Equality arises only if $prob(\mu < \bar{\mu} \mid e) = 1$ and $\tau(e) = 0$, which means that the incentive constraints is not binding.

This shows that for a.e. $\tau(e) : \frac{\partial}{\partial\mu}\left(\frac{1}{\mu}\frac{\partial\hat{\Phi}}{\partial\tau(e)}\right) = \frac{\partial}{\partial\mu}\left(\frac{1}{\mu}p_e\frac{\partial\Phi_e}{\partial\tau(e)}\right) = \frac{p_e}{\mu^2}\left(\mu\frac{\partial}{\partial\mu}\left(\frac{\partial\Phi_e}{\partial\tau(e)}\right) - \frac{\partial\Phi_e}{\partial\tau(e)}\right) \leq 0$, with equality only **if** the incentive constraint is never binding in event e. ■

Proof of proposition 4. Condition (17) follows from lemma 11, condition (18) follows from Lemma (10), and condition (19) follows from Lemma (10) and condition (14). ■

B Appendix to Section 3

B.1 Lemma 6

Using equation (23) and the linearity of $\ln\bar{\mu}_{vt}$ by Assumption 3.2(vi), simple algebra delivers the first statement:

$$\phi_{it} = -\sigma(1 + \pi_0\frac{\partial\ln\bar{\mu}_{vt}}{\partial\pi_{it}})\lambda_{it-1}(\beta) - x_{it}(\frac{\partial\ln\bar{\mu}_{vt}}{\partial x_{it}} + \pi_x\pi_0\frac{\partial\ln\bar{\mu}_{vt}}{\partial\pi_{it}}) \tag{B.1}$$

$$-q_{it}(\frac{\partial\ln\bar{\mu}_{vt}}{\partial q_{it}} + \pi_q\pi_0\frac{\partial\ln\bar{\mu}_{vt}}{\partial\pi_{it}}) - z_{it}(\pi_z\frac{\partial\ln\bar{\mu}_{vt}}{\partial\pi_{it}}\pi_0) + \tilde{\phi}_{vt} + \varepsilon_{it}^0(\Delta\eta_{it},\xi_{it}) \tag{B.2}$$

$$= -\sigma(1 + \pi_0\frac{\partial\ln\bar{\mu}_{vt}}{\partial\pi_{it}})(\lambda_{it-1}(\beta) + m_{it}\phi_m + \phi_{vt}) + \varepsilon_{it}^0 \tag{B.3}$$

where ϕ_{vt} is a linear combination of village-and-period effects and where ε_{it}^0 is a linear combination of $\Delta\eta_{it}$ and ξ_{it}. ε_{it}^0 is independent of any covariates w_{it} and is absolutely continuous by Assumption 3.2(iii). In consequence, the second statement follows:

$$E(\phi_{it}\mathbf{1}\{\phi_{it} \geq 0\} \mid w_{it}) = E(\phi_{it}\mathbf{1}\{\phi_{it} \geq 0\} \mid \phi_{it}^0) = H(\phi_{it}^0)$$

where $\phi_{it}^0 = \lambda_{it-1}(\beta) + m_{it}\phi_m + \phi_{vt}$ and function $H(x)$ is positive.

Furthermore $(1 + \pi_0\frac{\partial\ln\bar{\mu}_{vt}}{\partial\pi_{it}}) > 0$ (although $\frac{\partial\ln\bar{\mu}_{vt}}{\partial\pi_{it}} < 0$) because the ratio of marginal utilities tomorrow, $\lambda_{it+1}(\beta)$, is an increasing function of marginal utilities today, $\lambda_{it}(\beta)$, because of the strict concavity of function $Q(.)$.

Moreover, function $H(x)$ is equal (up to a positive scale) to:

$$H(x) \propto \int_{u>x}(-x + u)f(u)du,$$

and thus:

$$H'(x) \propto -\Pr(u > x).$$

which proves that $H(.)$ is decreasing, convex and twice differentiable. The scale parameter is given by the coefficient of $\lambda_{it-1}(\beta)$ in equation (B.2) which proves the third statement of the Lemma.

Finally, equation (B.1) implies the fourth statement that the coefficients of z_{it} are proportional to π_z.■

B.2   Proposition 7

The only remaining parameters of interest to identify ϕ_m and $H()$ are in the consumption equation. We first show that Assumptions 3.2 (vii,viii) are normalizations at the village-and-period level. Then we show how ϕ_m and $H()$ are identified on the support $S_I^{(v,t)}$ and thus are globally identified.

Fix v and t and drop those indices. Write the consumption equation as:

$$E(\Delta\lambda \mid \lambda_{-1}, m) = \Delta\delta + H(\lambda_{-1} + m\phi_m + \phi) \qquad (B.4)$$

As function H is positive, decreasing and convex and as its argument $u = \lambda_{-1} + m\phi_m + \phi$ is bounded, this moment condition is invariant to transformations of $(\Delta\delta, \phi, H(u))$ into $(\Delta\delta + a_0, \phi + a_1, -a_0 + H(u - a_1))$ where (a_0, a_1) are any pair of scalars such that $a_0 < \min_u H(u - a_1)$. Hence, $(\Delta\delta, \phi, H)$ is not identified and we can adopt any normalization (a_0, a_1, H_0) for the true $(\Delta\delta, \phi, H)$. More precisely, we can normalize the location of $H(.)$ and the location of its argument. Recall that by Assumption 3.2 (iii & v), the variable λ_{-1} is absolutely continuous and (λ_{-1}, m) are bounded so that $\lambda_{-1} + m\phi_m + a_1$ is a bounded index which distribution is continuous on the support S_I. The support is connected by Assumption 3.2 (vii). Any quantile normalization such as the median in Assumption 3.2 (vii) implies that 0 is an interior point of S_I. Assumption 3.2 (viii) then posits that, at that value 0, H takes value 0.

We now return to the proof of the Proposition. As function H is bounded, decreasing and twice differentiable and as the index is continuous on S_I, $E(\Delta\lambda \mid \lambda_{-1}, m)$ necessarily continuously decreases from c_H to c_L (say) where $c_L < c_H$.

Conversely, for any $c \in [c_L, c_H]$, the set $S_0 = \{(\lambda_{-1}, m) \text{ such that } E(\Delta\lambda \mid \lambda_{-1}, m) = c\}$ is defined and equal to a linear manifold $\lambda_{-1} + m\phi_m = b$. Therefore ϕ_m is identified because of Assumption 3.2 (v). By Assumption 3.2(vii), $a_1 = -Median(\lambda_{-1} + m\phi_m)$. Denote S_0 the set of values of (λ_{-1}, m), such that $\lambda_{-1} + m\phi_m + a_1 = 0$. Then, $a_0 = E(\Delta\lambda \mid (\lambda_{-1}, m) \in S_0)$ is identified. For any value $b = (\lambda_{-1} + m\phi_m + a_1)$ in support S_I, we can then construct function H as the function of b such as:

$$H(b) = E(\Delta\lambda \mid \lambda_{-1} + m\phi_m + a_1 = b) - a_0$$

and H is identified in S_I.

Finally, as (ϕ_m, H) are identified at the village-and-period level, they are identified globally, $H()$ on the union of the previous sets which all contain 0.

C   Data Appendix

The data provided by IFPRI consist of a sample of 927 households (first round) interviewed 12 times between 1986 and 1989. The survey is realized in four districts in three regions (Attock and Faisalabad in Punjab, Badin in the Sind, and Dir in the North West Frontier Province). In each of the four districts, villages were randomly chosen from a comprehensive list of villages classified in three sets according to their distances to two markets (*mandis*), and, households were randomly drawn from a comprehensive list of village households. Some attrition is observed in the data (887 households in the final survey) that seems to stem from administrative and political problems rather than from self-selection of households (Alderman and Garcia, 1993). We shall assume that attrition is exogenous.

In the survey, household demographics are directly available. Household food consumption is initially reported by food item, in quantity and value, or quantity and price. It comprises meals

at home including home-produced goods, and meals taken outside for all household members except meals that were the result of invitation or rewards in kind as the information on those items was not available. The non durable non food expenditures correspond mainly to heating expenditures. Other expenditures are related to travel, education, entertainment (very few), health, hygiene, clothes and tobacco, electricity and gas (some of which were missing in the sample for several periods). Transfers correspond to transfers received from relatives, friends and from solidarity funds of local mosques (zakat).

C.1 Tables

Table A.1 : Rented-in land area[18]

	OLS	2SLS
Net rented-in area	(1)	(2)
$\ln c_{t-1}$	-1.807	-5.323
	(0.275)	(0.851)
q_{it} : land owned		
total land in village	-5.192	-5.054
	(0.237)	(0.249)
rainfed land in village	-5.152	-4.875
	(0.785)	(0.837)
x_{it}: (log) household size	1.318	1.915
	(1.241)	(1.340)
x_{it-1}: lag (log) household size	1.735	2.523
	(1.256)	(1.327)
Constant	5.815	19.666
	(1.598)	(3.754)
Observations	8053	7133
R-squared	0.53	

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[18]Robust standard errors in parentheses. All village-and-period effects (46 villages, 11 rounds) not shown. In the case of 2SLS estimates, $\ln c_{it-1}$ is instrumented by $\ln c_{it-2}$ and all other variables.

Table A.2: Consumption & Leisure Separability

Dependent Variable	Consumption-growth residuals	
Explanatory Variables	Coeff.	Std. Error
Male labor (days)	-0.0009	(0.0010)
Female labor (days)	-0.0045	(0.0024)
Observations	7133	
R-squared	0.0007	

Table A.3: Within-village Correlations of Income and Consumption Growth[19]

	Within-village Income Correlations		Within-village Consumption Growth Correlations	
Dependent Variable	Cross-product of residuals (i,j)		Cross-product of residuals (i,j)	
Explanatory Variables $|x_{it} - x_{jt}|$	Coeff.	Std. Error	Coeff.	Std. Error
total land in village	-0.140	(0.0035)	-0.030	(0.0030)
rainfed land in village	-0.061	(0.010)	-0.023	(0.0084)
(log) household size	-0.19	(0.012)	-0.21	(0.0095)
Observations	137561		137561	
R-squared	0.024		0.0056	

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[19]This is the regression of all cross-products of residuals of the income equation (or consumption growth equation) within each village and each period on the distance between explanatory variables. No correction for first stage estimation. Village-and-period effects are included and not shown.