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ABSTRACT

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A major criticism against staggered nominal contracts is that they give rise to the so called "persistency puzzle" – although they generate price inertia, they cannot account for the stylised fact of inflation persistence. It is thus commonly asserted that, in the context of the new Phillips curve (NPC), inflation is a jump variable. We argue that this "persistency puzzle" is highly misleading, relying on the exogeneity of the forcing variable (e.g. output gap, marginal costs, unemployment rate) and the assumption of a zero discount rate. We show that when the discount rate is positive in a general equilibrium setting (in which real variables not only affect inflation, but are also influenced by it), standard wage-price staggering models can generate both substantial inflation persistence and a nonzero inflation-unemployment tradeoff in the long-run. This is due to frictional growth, a phenomenon that captures the interplay of nominal staggering and permanent monetary changes. We also show that the cumulative amount of inflation undershooting is associated with a downward-sloping NPC in the long-run.

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1 Introduction

A major criticism against staggered nominal contracts is that, although they can account for price inertia, they do not generate inflation inertia. This proposition is referred to as the "persistency puzzle" in recent studies .[1] In their influential paper, Fuhrer and Moore (1995) argued that there is no inflation persistence independent of the persistence in the shocks. The "persistency puzzle" is widely recognised as a deficiency of the new Phillips curve (NPC) that rests on the contracting model; it cannot account for the high degree of inflation persistence commonly described by the empirical evidence. This insight has spawned a large literature that attempts to provide new explanations for inflation persistence (Blanchard and Gali (2005), and Mankiw and Reis (2002) are two prominent recent examples).[2]

It is important to emphasize that the critique against staggered nominal contracts mainly refers to the persistence of inflation in response to permanent shocks.[3] Specifically, the seminal contributions of Phelps (1978) and Taylor (1980a) imply that inflation responds instantaneously to exogenous macroeconomic shifts - hence the jargon "inflation is a jump variable".

In this paper we revisit this debate and show that, under staggered nominal contracts, the "persistency puzzle" proposition is highly misleading - inflation is generally not a jump variable after all. In fact, we show that

- the standard versions of the contract model can generate substantial inflation persistence (i.e. inflation persistence is an inherent feature of wage/price staggering), and

- the cumulative amount of inflation undershooting and overshooting is intimately related with the inflation-unemployment tradeoff in the long-run.

In particular, we show that when "inflation is a jump variable" the Phillips curve is vertical even in the short-run. Naturally, many economists find the absence of a short-run inflation-unemployment tradeoff hard to accept. For example, Mankiw (2001, p. C59) concludes 'Almost all economists today agree that monetary policy influences unemployment, at least temporarily......the so called new Keynesian Phillips curve is ultimately a failure'. Our analysis reveals that the verticality of the short-run NPC and the "persis-

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[1] See, for example, Westelius (2005).

[2] See also Christiano, Eichenbaum, and Evans (2001), Dotsey, King, and Wolman (1997), Estrella and Fuhrer (1998), Galí, Gertler, and López-Salido (2001), Huang and Liu (2001), Roberts (1997), and many others.

[3] Note that inflation persistence depends on the type of the exogenous macroeconomic shocks. Given a temporary shock, the longer it takes inflation to return to its equilibrium the higher the persistence. Given a permanent shock, the longer it takes inflation to reach its new equilibrium the higher the persistence.

tency puzzle" are the two sides of the NPC deficiency coin which manifests itself under the assumption of a zero discount rate.

Our arguments may be summarised as follows. The NPC postulates that current inflation depends linearly on expected future inflation and some real variable, x_t, such as output, the output gap, real marginal costs, or the unemployment rate. From this, it is commonly inferred that there is no inflation persistence independent of the persistence in x_t. After all, a one-period shock to x_t affects inflation for only one period. For this argument to hold, the real variable x_t must be viewed as exogenous. But in the context of all reasonable macro models of the Phillips curve, x_t is not exogenous. Rather, inflation and, say, unemployment are both endogenous. Commonly, unemployment (or output, etc.) depends, among other things, on real money balances (or some other relation between money and a nominal variable). And real money balances, in turn, depend on prices, whose evolution is given by the inflation rate. Once the influence of inflation on unemployment is taken into account in a general equilibrium context, inflation recovers only gradually from temporary shocks.

Although some recent studies (e.g. Mankiw and Reis (2002)) acknowledge the endogeneity of the "forcing" variable x_t, and, thus, the persistent inflation effects of a temporary shock, they still hold the view that inflation behaves as a jump variable when the shock is permanent. In other words, the NPC generates inflation persistence when the shock is temporary, but not permanent. We show that this discrepancy arises because the discount rate is assumed to be zero.

When the discount rate is zero, i.e. the discount factor is unity, equal weights are attached to the backward- and forward-looking components of the wage/price contract underlying the NPC. It can be shown that a positive discount rate is associated with "intertemporal weighting asymmetry" in the pricing behaviour, in the sense that a larger weight is attached to the backward-looking component than to the forward-looking one. Since the discount factor is close to unity in actual terms, the conventional wisdom dismisses the intertemporal weighting asymmetry as mere theoretical nicety.

However, we show that for plausible parameter values, the intertemporal weighting asymmetry leads to inflation undershooting and a nonvertical Phillips curve in the long-run.[4] This is because a positive (albeit low) discount rate enables the interplay of nominal staggering and permanent monetary changes - a phenomenon we call *frictional growth*.

In the context of NPC models, the necessary and sufficient conditions for the existence of frictional growth can be summarised as follows. While nominal frictions (due to wage/price staggering), and growth (i.e. permanent shocks like a change in the inflation target) are the necessary conditions, the intertemporal weighting asymmetry (due to a

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[4]Note that our analysis, in line with the NPC literature, contains no money illusion, no permanent nominal rigidities, and no departure from rational expectations.

positive discount rate) is the sufficient one. In the absence of frictional growth, inflation jumps immediately to the equilibrium dictated by the permanent shock. But in the presence of frictional growth, the NPC generates inflation persistence and a long-run tradeoff between inflation and the real variable x_t.

The paper is organised as follows. Section 2 gives the details of the standard staggered nominal contracts, and discusses the critique against the new Phillips curve. Section 3 derives the inflation dynamics implied by the workhorse model of the NPC, first under a temporary money growth shock, and then under a permanent one. The associated impulse response functions and measures of persistence are also obtained. Section 4 derives the slope of the Phillips curve and shows that it is intimately related to inflation undershooting. Section 5 extends the workhorse model of the NPC in various standard ways. Section 6 presents an overview of our analysis. Finally, Section 7 concludes.

2   Scanning the new Phillips curve

The staggered wage contracts proposed by Phelps (1978) and Taylor (1979, 1980a) paved the way for the new Phillips curve by accommodating monetarist and rational expectations elements in the wage-price setting. Calvo's (1983) particularly popular model of time-contingent nominal contracts is commonly used as a convenient algebraic shorthand for the Taylor model.[5]

The pioneering contribution of wage/price staggering was that it strengthened the case against the view that the dynamic nature of the unemployment rate is merely a statistical one - if one could observe and include in the model all the relevant exogenous variables, lagged unemployment terms would simply become statistically insignificant. It is now widely understood that in a standard macro model with rational expectations, wage/price staggering alone induces unemployment to depend on its own lags.

In its simplest form, wage staggering assumes that nominal wages are fixed for two periods and there are two contracts that are evenly staggered. The contract wage depends on past and expected future contract wages, as well as current and future excess demand:

$$W_t = \alpha W_{t-1} + (1 - \alpha) E_t W_{t+1} + \gamma \left[\alpha x_t + (1 - \alpha) E_t x_{t+1} \right], \qquad (1)$$

where the contract wage W_t is set at the beginning of period t for periods t and $t + 1$, x_t denotes excess demand, and $E_t(\cdot)$ is the expectation of the variable conditional upon information available at time t, and the supply shock ω_t is a white noise process. (All variables are in logs; we ignore supply shocks for simplicity.) The demand sensitivity

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[5]Goodfriend and King (1997, p.254) show that under intertemporal optimisation, and with low inflation, constant elasticity of demand and small variations in adjustment patterns, Calvo's setup broadly resembles that of Taylor.

parameter γ describes how strongly wages are influenced by demand. Note that the only restriction that needs to be imposed on the backward- and forward-looking weights is that they add up to unity - they do not have to be equal to one another.[6]

The fundamental principle of finance that 'a dollar today worths more than a dollar tomorrow', implies that the coefficient α is a discounting parameter equal to $\frac{1+r}{2+r}$, where r is the discount rate. This can be seen as follows. The one-period ahead wage (W_{t+1}) needs to be discounted by the factor $\beta = \frac{1}{1+r}$ so that it is used in the wage-staggering equation (1) alongside with the wage set in the previous period (W_{t-1}) that still applies in period t. Given that wage staggering requires that the wage set at period t is a weighted average of past and future wages and their respective weights add up to $1 + \beta$, we need to rescale them by the parameter $\alpha = \frac{1}{1+\beta}$ so that they add up to unity. It then follows that time discounting and a nonzero interest rate (so that $\beta < 1$ and $\alpha > 1/2$) give rise to an asymmetry in wage determination: the current wage W_t is affected more strongly by the past wage W_{t-1} than the future expected wage $E_t W_{t+1}$. This may be called the *intertemporal weighting asymmetry.*

This result is also well known from the microfoundations of Taylor-type contract equations under time discounting. Recent contributions to the microfoundations of wage-price setting under time-contingent staggered nominal contracts have shown that when agents discount the future (viz., they have a positive rate of time preference), then the backward-looking variables are weighted more heavily than the forward-looking ones, i.e. $\alpha > 1/2$.[7] However, since the discount factor β is almost unity, this result is largely ignored in the empirical and policy literature which sets $\alpha = 1/2$ in the price staggering equation (3).

Taylor's and Calvo's wage/price-setting models were subsequently reformulated into what has become known as the workhorse model of the new (Keynesian) Phillips curve.[8] The so called sticky-price model of the NPC explains current inflation π_t by expected inflation one period ahead and a forcing variable x_t:

$$\pi_t = \beta E_t \pi_{t+1} + \gamma \left(1 + \beta\right) x_t, \tag{2}$$

where inflation (π_t) is the first difference of the log price level, $\pi_t \equiv P_t - P_{t-1}$, and the "forcing variable" (x_t) denotes (log) output gap, or (log) wage share, or the unemployment rate.

The new Phillips curve (2) is simply a reparameterisation of the following price-setting

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[6] However, the wage-staggering specification in Taylor (1980a) attaches equal weights to the backward- and forward-looking variables.

[7] Ascari (1998, 2000), Graham and Snower (2002, 2004), Helpman and Leiderman (1990), Huang and Liu (2002), and others.

[8] See, for example, Roberts (1995), Gali and Gertler (1999), and Mankiw and Reis (2002).

equation:[9]

$$P_t = \alpha P_{t-1} + (1 - \alpha) E_t P_{t+1} + \gamma x_t, \tag{3}$$

where, as explained above, the discount parameter $\alpha = \frac{1}{1+\beta}$ (the discount factor $\beta = \frac{1}{1+r}$, and r is the discount rate), and the "demand sensitivity parameter" γ is a constant.[10] The lagged price term captures nominal rigidities and so equation (3) clearly implies price inertia: a demand shock affects the price level for many periods.

Note that the use of term "forcing" variable in the NPC models suggests the exogeneity of x_t. However, in the context of all reasonable macro models of the Phillips curve, x_t is not exogenous.[11] Rather, inflation π_t and the real variable x_t are both endogenous responding to economic policy changes. Furthermore, as we show in the next section, a key element in deriving the properties of the new Phillips curve is whether the wage/price staggered contract displays *intertemporal weighting asymmetry* ($\alpha > 1/2$), or attaches equal weights to the backward- and forward-looking components ($\alpha = 1/2$).

2.1 The Deficiency of the NPC

To elucidate the viewpoint of Fuhrer and Moore (1995, p. 129) that 'All of the persistence in inflation derives from the persistence in the driving term', we use recursive substitution and express eq. (2) as

$$\pi_t = \gamma (1 + \beta) \sum_{j=0}^{\infty} \beta^j E_t x_{t+j}. \tag{4}$$

The above equation shows a one-off change in the driving force variable in period t cannot affect inflation beyond that period. Clearly, the critique against the NPC for not generating inflation persistence simply relied on eye inspection of eq. (4). Subsequent studies (e.g. Mankiw and Reis (2002)) analysed inflation persistence by first specifying an equation for the "forcing" variable and then deriving the closed-form rational expectations solution of the model. Commonly, the "forcing" variable depends, among other things, on real money balances and so shocks refer to money growth changes. These closed-form solutions of the NPC models show that

1. the effects of a temporary (one-period) shock on inflation gradually die out with the passage of time, and

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[9] To obtain the New Keynesian Phillips curve (2), subtract from both sides of the price-setting eq. (3) (i) P_{t-1} to get $\pi_t - (1 - \alpha) P_{t-1} = (1 - \alpha) E_t P_{t+1} + \gamma x_t$, and (ii) $(1 - \alpha) P_t$ so that $\alpha \pi_t = (1 - \alpha) E_t \pi_{t+1} + \gamma x_t$.

[10] Note that γ is positive when x_t denotes output or the wage share, and negative when x_t denotes unemployment.

[11] Bårdsen, Jansen and Nymoen (2002, 2004) put forward an econometric evaluation of the NPC and emphasize the importance of modelling a system that includes the forcing variable as well as the rate of inflation.

2. a permanent shock causes inflation to adjust instantly to its new equilibrium.

Therefore, a major weakness of the NPC (or sticky-price Phillips curve) is that it implies that inflation is a jump variable - following a permanent increase (decrease) in money growth at period t, inflation jumps up (down) instantaneously to its new long-run value. The need for a model that did not feature this "persistency puzzle" led to the development, among others, of the sticky-information Phillips curve by Mankiw and Reis (2002), and a Phillips curve that incorporates real wage rigidities by Blanchard and Gali (2005).

3   Inflation Dynamics

In what follows we show that the interaction of the intertemporal weighting asymmetry and the endogeneity of the "forcing" variable plays a crucial dual role: (i) it generates inflation persistence, i.e. inflation is not a jump variable, and (ii) it gives rise to a long-run tradeoff between inflation and unemployment.

In the standard macro models, output (unemployment rate) usually depends positively (negatively) on real money balances. So, for simplicity, we write:

$$x_t = M_t - P_t, \tag{5}$$

where M_t denotes the money supply. Substituting this equation into equation (3), we obtain the following price equation:[12]

$$P_t = \phi P_{t-1} + \theta E_t P_{t+1} + \left(\frac{\gamma}{1+\gamma} \right) M_t, \tag{6}$$

where $\phi = \frac{\alpha}{1+\gamma}$, $\theta = \frac{1-\alpha}{1+\gamma}$. The corresponding inflation equation is[13]

$$\pi_t = \phi \pi_{t-1} + \theta E_t \pi_{t+1} + \left(\frac{\gamma}{1+\gamma} \right) \mu_t + \theta v_t, \tag{7}$$

where $\mu_t \equiv M_t - M_{t-1}$ is the money growth rate and $v_t = P_t - E_{t-1} P_t$ is an expectational error.[14]

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[12] To derive this equation, observe that $P_t = \alpha P_{t-1} + (1 - \alpha) E_t P_{t+1} + \gamma (M_t - P_t) \Rightarrow P_t = \left(\frac{\alpha}{1+\gamma} \right) P_{t-1} + \left(\frac{1-\alpha}{1+\gamma} \right) E_t P_{t+1} + \left(\frac{\gamma}{1+\gamma} \right) M_t.$

[13] To derive the inflation equation, lag eq. (6) once: $P_{t-1} = \phi P_{t-2} + \theta E_{t-1} P_t + \left(\frac{\gamma}{1+\gamma} \right) M_{t-1}$, and subtract it from (6) to get $\pi_t = \phi \pi_{t-1} + \theta E_t P_{t+1} - \theta E_{t-1} P_t + \left(\frac{\gamma}{1+\gamma} \right) \mu_t$. Now add and subtract θP_t on the right-hand side of the above to obtain the inflation staggered equation in terms of the *exogenous* growth rate of money: $\pi_t = \phi \pi_{t-1} + \theta E_t \pi_{t+1} + \left(\frac{\gamma}{1+\gamma} \right) \mu_t + \theta (P_t - \theta E_{t-1} P_t).$

[14] The error term $v_t = P_t - E_{t-1} P_t$ is included in Roberts (1995, 1997), but ignored by Fuhrer and

In this equation, current inflation depends on past inflation, as well as on expected future inflation, and thus the possibility of inflation persistence reemerges. The degree of persistence is of course related to the stochastic process generating the money supply. To analyse the inflation dynamics, it is convenient to rewrite the price equation (6) as[15]

$$P_t = \lambda_1 P_{t-1} + \frac{\gamma}{\lambda_2 (1 - \alpha)} \sum_{j=0}^{\infty} \left(\frac{1}{\lambda_2} \right)^j E_t M_{t+j}, \tag{8}$$

where λ_1 and λ_2 are the roots of equation (6):

$$\lambda_{1,2} = \frac{1 \mp \sqrt{1 - 4\phi\theta}}{2\theta} = \frac{1 \mp \sqrt{1 - 4\frac{\alpha(1-\alpha)}{(1+\gamma)^2}}}{2 \left(\frac{1-\alpha}{1+\gamma} \right)} \tag{9}$$

and $0 < \lambda_1 < 1$ and $\lambda_2 > 1$. In words, prices depend on past prices and expected future money supplies. Thus different stochastic monetary processes give rise to different price dynamics. We now consider two such processes in turn.

- A temporary money growth shock: The persistent after-affects of inflation to this temporary shock we refer to as *inflation persistence*. The greater the inflation effect after the shock has disappeared, the greater is inflation persistence.

- A permanent money growth shock: Since this shock leads to a permanent change in inflation, it is desirable to have a different name for the inflation effects. Thus the delayed inflation effects of a permanent monetary shock we call *inflation under-responsiveness*. The more slowly inflation responds to a permanent shock, the more under-responsive inflation is.

Although the persistent after-effects of a temporary money growth shock and the delayed after-effects of a permanent money growth shock are two distinct phenomena, they are, rather confusingly, both denoted by the word "persistence" in the prevailing literature.

3.1 A Temporary Money Growth Shock - Persistence

Let the money growth be stationary, fluctuating randomly around its mean (μ):

$$\mu_t = \mu + \varepsilon_t, \text{ where } \varepsilon_t \sim iid\left(0, \sigma^2\right). \tag{10}$$

Moore (1995) and much of the subsequent literature. It can be shown that, in the above price staggering model, this error term does not affect the dynamic structure of inflation; it only rescales its impulse response function to a temporary monetary shock.

[15]To see this, write (6) as $(1 - \lambda_1 B)(1 - \lambda_2 B) E_t P_t = \frac{-\gamma B M_t}{(1-\alpha)}$, where B is the backshift operator. This gives $(1 - \lambda_1 B) E_t P_t = \frac{\gamma}{\lambda_2(1-\alpha)} \sum_{j=0}^{\infty} \left(\frac{1}{\lambda_2} \right)^j E_t M_{t+j}$ which leads to (8) since $E_t P_t = P_t$.

A positive shock ε_t represents a temporary rise in money growth or, equivalently, a sudden, permanent increase in the money supply. The money supply is a random walk: $M_t = \mu + M_{t-1} + \varepsilon_t$, so that $E_t M_{t+j} = M_t + j\mu$, for $j \geq 0$. Substituting this last expression into the price equation (8), we obtain the closed form rational expectations solution of price:[16]

$$P_t = \lambda_1 P_{t-1} + (1 - \lambda_1) M_t + \frac{(1 - \lambda_1)}{(\lambda_2 - 1)} \mu. \tag{11}$$

The first difference of this equation yields the closed form rational expectations solution of inflation:

$$\pi_t = \lambda_1 \pi_{t-1} + (1 - \lambda_1) \mu + (1 - \lambda_1) \varepsilon_t. \tag{12}$$

(In the long-run $\pi = \mu$, i.e. there is no money illusion, as for the other models below.)

A one-period shock to money growth $\varepsilon_t = 1$, $\varepsilon_{t+j} = 0$ for $j > 0$ (i.e. a permanent increase in the level of money supply) is associated with the following impulse response function (IRF) of inflation:

$$R^\pi_{t+j} = \lambda_1^j (1 - \lambda_1), \ \ j = 0, 1, 2, ... \tag{13}$$

Thus the responses die out geometrically (recall that $0 < \lambda_1 < 1$), and the rate of decline is given by the autoregressive parameter λ_1. In this context, we measure inflation persistence (σ) as the "future" impact of the monetary shock to inflation, i.e. the sum of the inflation responses for all periods after the shock has occurred ($t + j$, $j \geq 1$):[17]

$$\sigma \equiv \sum_{j=1}^{\infty} R^\pi_{t+j} = \lambda_1. \tag{14}$$

By equation (9), we see that the degree of persistence rises with the discount rate (and α) and falls with the demand sensitivity parameter γ. It can be shown that inflation has this qualitative pattern of persistence when money growth follows any stationary ARMA process.

It is worth noting that, by eq. (12), the immediate impact ("current" response), $1 - \lambda_1$, can also be interpreted as the short-run slope, e_{SR}, of inflation with respect to money growth. Furthermore, the total impact of this monetary shock to inflation (i.e. the sum of persistence and immediate impact), in this case unity, is simply the long-run

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[16]The associated real money balances equation is

$$(M_t - P_t) = \lambda_1 (M_{t-1} - P_{t-1}) + \lambda_1 \mu + \lambda_1 \varepsilon_t.$$

[17]Other measures of persistence are the half life of the shock, the sum of the autoregressive parameters, and the largest autoregressive root. The virtues and faults of these measures are pointed out in a recent application by Pivetta and Reis (2004).

slope of inflation with respect to money growth:

$$e_{LR} = e_{SR} + \sigma. \tag{15}$$

In other words, in general, the long-run slope (or elasticity)[18] can be decomposed into the short-run slope (or elasticity) and our measure of persistence (14).

3.2 A Permanent Money Growth Shock - Responsiveness

For simplicity, let money growth be a random walk:[19]

$$\mu_t = \mu_{t-1} + \varepsilon_t, \text{ where } \varepsilon_t \sim iid\left(0, \sigma^2\right). \tag{16}$$

In this case a positive one-period unit shock (ε_t) represents a permanent increase in money growth which, in the absence of money illusion, leads to a unit increase in the long-run inflation rate. Note that the case of a negative shock represents a sudden disinflation.

By the price equation (8) and the random walk (16), we obtain the following price dynamics:[20]

$$P_t = \lambda_1 P_{t-1} + (1 - \lambda_1) M_t + \left(\frac{1 - \lambda_1}{\lambda_2 - 1}\right) \mu_t. \tag{17}$$

The associated closed form rational expectations solution of inflation is

$$\pi_t = \lambda_1 \pi_{t-1} + (1 - \lambda_1) \mu_t + \left(\frac{1 - \lambda_1}{\lambda_2 - 1}\right) \varepsilon_t. \tag{18}$$

It can be shown that the corresponding impulse response function (IRF) of inflation to the permanent unit increase in money growth is:

$$\overline{R}^\pi_{t+j} = 1 - \lambda_1^j (1 - \lambda_1) \left(\frac{2\alpha - 1}{\gamma}\right), \ j = 0, 1, 2, ..., \tag{19}$$

Observe that, since λ_1 is positive and less than unity, the long-run response of inflation is $\lim_{j \to \infty} \overline{R}^\pi_{t+j} \equiv \overline{R}^\pi_{LR} = 1$, i.e., in the long-run inflation stabilises at the new level of money growth.

In this context, we measure the persistence of inflation as the cumulative inflation ef-

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[18] In a log-linear model the impulse response function gives the elasticities of the dependend variable through time.

[19] The qualitative conclusions of this analysis do not hinge on the random walk assumption. Any money growth process involving a permanent change in money growth (e.g. an $I(0)$ money growth process with a change in money growth regime, or a permanent change in the monetary authority's reaction function) would do.

[20] To see this, observe that $\sum_{j=0}^{\infty} \left(\frac{1}{\lambda_2}\right)^j E_t M_{t+j} = \left(\frac{\lambda_2}{\lambda_2-1}\right) M_t + \frac{\lambda_2}{(\lambda_2-1)^2} \mu_t$, and $\frac{\gamma}{(\lambda_2-1)(1-\alpha)} = 1 - \lambda_1$.

fect of the money growth shock that arises because inflation does not adjust immediately to the new long-run equilibrium. As we explained above, we call this measure *inflation responsiveness* to distinguish it from the persistence of inflation that results from a temporary shock.

In particular, suppose that the economy, in an initial long-run equilibrium,[21] is perturbed by a one-period money growth shock ($\varepsilon_t = 1$, $\varepsilon_{t+j} = 0$ for $j > 0$). The inflation responsiveness is the sum of the differences through time between the inflation rate responses (19) and the new (post-shock) long-run equilibrium inflation rate. In other words, inflation responsiveness is the cumulative amount of inflation undershooting and overshooting:

$$\rho \equiv \sum_{j=0}^{\infty} \left(\overline{R}_{t+j}^{\pi} - 1 \right). \tag{20}$$

If inflation responds to the permanent shock by instantaneously jumping to its new long-run equilibrium, then $\rho = 0$, i.e. inflation is *perfectly responsive*.[22] In this case inflation can be described as a jump variable. If, on the other hand, the cumulative amount of undershooting exceeds the cumulative amount of overshooting, then inflation is *under-responsive* and $\rho < 0$. Finally, if the cumulative amount of overshooting exceeds the total amount of undershooting, then inflation is *over-responsive* and $\rho > 0$.

Substitution of the impulse response function (19) into the responsiveness equation (20) gives

$$\rho = -\frac{2\alpha - 1}{\gamma}. \tag{21}$$

This result shows that the workhorse NPC model (2) has the following interesting implications for inflation dynamics:

1. If the discount rate r is zero (i.e. $\beta = 1$, so that $\alpha = 1/2$), then inflation is perfectly responsive. In other words, it is a jump variable, along the same lines as in the recent literature on "inflation persistence" under staggered nominal contracts.

2. If the discount rate is positive (i.e. $\beta < 1$, so that $\alpha > 1/2$), then inflation is underresponsive. In particular, it gradually approaches its new equilibrium from below at a rate that depends on the autoregressive parameter λ_1.

As shown in Section 5, the above implications hold only for staggered price setting, but not for staggered wage setting.

It is worth emphasizing that the temporary and permanent shocks are associated with the inflation dynamics equations (12) and (18), respectively, and thus give rise to IRFs

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[21] This assumption only serves expositional simplicity.

[22] Using the jargon of the prevailing literature, inflation displays no persistence.

with distinct properties.[23] A summary of these properties is provided by the distinct measures of persistence and responsiveness.

Next, we show that the cumulative amount of inflation undershooting and overshooting is closely related to the slope of the long-run Phillips curve.

4   The Slope of the Phillips Curve

In order to derive the Phillips curve, we need to consider the unemployment effects of permanent changes in money growth (corresponding to different long-run inflation rates).[24]

Recall that the forcing variable x depends on real money balances ($x_t = M_t - P_t$), which (by the price equation (17)) are

$$M_t - P_t = \lambda_1 \left(M_{t-1} - P_{t-1} \right) + (1 - \lambda_1) \left(\frac{2\alpha - 1}{\gamma} \right) \mu_t. \tag{22}$$

Since the unemployment rate, u_t, is negatively related to real money balances, $u_t = -(M_t - P_t)$, we have the following closed form rational expectations solution for unemployment:

$$u_t = \lambda_1 u_{t-1} - (1 - \lambda_1) \left(\frac{2\alpha - 1}{\gamma} \right) \mu_t. \tag{23}$$

The IRF of unemployment gives the responses through time of unemployment to a permanent unit increase in money growth:

$$\overline{R}^u_{t+j} = -(1 - \lambda_1) \left(\frac{2\alpha - 1}{\gamma} \right) \sum_{i=0}^{j} \lambda_1^i, \ j = 0, 1, 2, ... \tag{24}$$

Note that, since λ_1 is positive and less than unity, the long-run response of unemployment is $\lim_{j \to \infty} \overline{R}^u_{t+j} \equiv \overline{R}^u_{LR} = - \left(\frac{2\alpha-1}{\gamma} \right)$.

The Phillips curve tradeoff, at any point in time, is obtained by the ratio of the inflation response (19) to the unemployment response (24):

$$(\text{slope of the PC})_{t+j} = \frac{\overline{R}^{\pi}_{t+j}}{\overline{R}^u_{t+j}}, \ j = 0, 1, 2, ... \tag{25}$$

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[23]Note that, although (12) and (18) have identical autoregressive components and satisfy the restriction of no money illusion in the long-run, their inflation dynamics are distinct.

[24]These permanent changes may be motivated by changes in a central bank's inflation target or other policy rule.

The long-run inflation-unemployment tradeoff can be derived either from (25),[25] or via the long-run solution of the unemployment dynamics eq. (23):

$$u_t = -\left(\frac{2\alpha - 1}{\gamma}\right)\mu_t.$$

The latter implies that the long-run Phillips curve is given by

$$\pi_t = -\left(\frac{\gamma}{2\alpha - 1}\right)u_t, \tag{26}$$

since $\pi_t = \mu_t$ in the long-run.

Observe that, in the context of the workhorse model of the NPC (2), the slope of the long-run Phillips curve, $-\left(\frac{\gamma}{2\alpha-1}\right)$, is simply the inverse of inflation under-responsiveness, i.e. the cumulative amount of inflation undershooting, (21).

When the discount rate is zero, i.e. $\alpha = 1/2$, inflation is a jump variable (perfectly responsive, $\rho = 0$) and, thus, the Phillips curve is vertical. This is an implausible, counter-factual special case, not just because the discount rate is zero, but also because - as equation (23) shows - it is not just the long-run Phillips curve that is vertical; the short-run Phillips curve is vertical as well.

By contrast, when the discount rate is positive ($\alpha > 1/2$), inflation is under-responsive ($\rho < 0$), and the long-run Phillips curve is downward-sloping. The higher is the under-shooting of inflation, the flatter the long-run Phillips curve.

As already mentioned, it is often casually asserted that, since the discount factor is close to unity in practice, the long-run Phillips curve must be close to vertical. Inspection of the long-run Phillips curve (26), however, shows this presumption to be false. As we can see, the slope of this Phillips curve depends on both the discount parameter α and demand sensitivity parameter γ. Table 1 presents the slope for various common values of α and commonly estimated values of γ:[26] It is clear that for a range of plausible parameter values the long-run Phillips curve is quite flat and, correspondingly, inflation displays significant undershooting.

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[25]In this case we have

$$\frac{\lim\limits_{j \to \infty} \overline{R}^{\pi}_{t+j}}{\lim\limits_{j \to \infty} \overline{R}^{u}_{t+j}} \equiv \frac{\overline{R}^{\pi}_{LR}}{\overline{R}^{u}_{LR}} = \frac{1}{-\left(\frac{2\alpha-1}{\gamma}\right)} = -\left(\frac{\gamma}{2\alpha - 1}\right).$$

[26]Taylor (1980b) estimates it to be between 0.05 and 0.1; Sachs (1980) finds it in the range 0.07 and 0.1; Gordon (1982) gives an estimate of 0.1; Gali and Gertler (1999) estimate it to be between 0.007 and 0.047; calibration of microfounded models (e.g. Huang and Liu, 2002) assigns higher values. The discount rate applies to a period of analysis which is half the contract span.

Table 1: Slope of the long-run Phillips curve

r (%)	β	α	$\gamma = 0.01$	$\gamma = 0.02$	$\gamma = 0.05$	$\gamma = 0.07$	$\gamma = 0.10$
					slope		
1.0	0.990	0.502	-2.01	-4.02	-10.1	-14.1	-20.1
2.0	0.980	0.505	-1.01	-2.02	-5.05	-7.07	-10.1
3.0	0.971	0.507	-0.68	-1.35	-3.38	-4.74	-6.77
4.0	0.962	0.510	-0.51	-1.02	-2.55	-3.57	-5.10
5.0	0.953	0.512	-0.41	-0.82	-2.05	-2.87	-4.10

Our analysis calls into question the conventional view that the long-run Phillips curve is either vertical or nearly vertical and that forward-looking Phillips curves are difficult to reconcile with substantial inflation persistence. The endogeneity of the forcing variable, on one hand, and the intertemporal weighting asymmetry (due to a positive discount rate), on the other, can generate sufficient inflation persistence and produce an inflation-unemployment tradeoff both in the short- and long-run. This is the result of *frictional growth*, a phenomenon that, in the context of the NPC model, encapsulates the interplay of frictions (nominal staggering) and growth (permanent shocks to money growth).

Table 2 summarises the properties of the NPC model in the absence of frictional growth ($\alpha = 1/2$), and in the presence of frictional growth ($\alpha < 1/2$).

Table 2: Inflation dynamics and the slope of the NPC

	Inflation dynamics	Short-run Phillips curve	Long-run Phillips curve
$\alpha = \frac{1}{2}$	jump variable	vertical	vertical
$\alpha > \frac{1}{2}$	undershooting	downward-sloping	downward-sloping

5 Extensions

To gain some perspective on the determinants of inflation persistence and responsiveness, we now examine these phenomena in the context of other forms of nominal staggering.

5.1 Price Staggering and Future Demand

Whereas the price setting equation (3) is common in the literature on inflation persistence, microfoundations of staggered price setting suggest that current prices (set over periods t and $t + 1$) depend not only on current demand (x_t) but also on future demand (x_{t+1}). Thus, let us consider the following price setting behavior:

$$P_t = \alpha P_{t-1} + (1 - \alpha) E_t P_{t+1} + \gamma \left[\alpha x_t + (1 - \alpha) E_t x_{t+1} \right]. \tag{27}$$

Substituting real money balances (5) into this equation, we obtain

$$P_t = \phi_p P_{t-1} + \theta_p E_t P_{t+1} + \left(\frac{\gamma}{1 + \gamma\alpha}\right)[\alpha M_t + (1 - \alpha) E_t M_{t+1}], \tag{28}$$

where $\phi_p = \frac{\alpha}{1+\gamma\alpha}$, and $\theta_p = \frac{(1-\gamma)(1-\alpha)}{(1+\gamma\alpha)}$. In this model the lead parameter is positive under the plausible assumption that $\gamma < 1$. The sum of both the lag and lead parameters is less than one.

Expressing this difference equation as

$$P_t = \lambda_{1p} P_{t-1} + \frac{\gamma}{\lambda_{2p}(1-\gamma)(1-\alpha)} \sum_{j=0}^{\infty}\left(\frac{1}{\lambda_{2p}}\right)^j E_t[\alpha M_{t+j} + (1 - \alpha) M_{t+1+j}], \tag{29}$$

where[27]

$$\lambda_{1p,2p} = \frac{1 \mp \sqrt{1 - 4\phi_p\theta_p}}{2\theta_p} = \frac{1 \mp \sqrt{1 - 4\frac{\alpha(1-\gamma)(1-\alpha)}{(1+\gamma\alpha)^2}}}{2\frac{(1-\gamma)(1-\alpha)}{(1+\gamma\alpha)}}, \tag{30}$$

$0 < \lambda_{1p} < 1$, and $\lambda_{2p} > 1$, we find how price dynamics depend on the stochastic monetary process. Once again, we examine inflation persistence arising from a temporary money growth shock and inflation responsiveness arising from a permanent money growth shock.

We begin with a **temporary money growth shock.** When money growth follows the stationary process (10), the rational expectations solution of (29) is

$$P_t = \lambda_{1p} P_{t-1} + (1 - \lambda_{1p}) M_t + \kappa(1 - \lambda_{1p})\mu, \tag{31}$$

where $\kappa = \lambda_{2p}/(\lambda_{2p} - 1) - \alpha$. Consequently inflation is given by

$$\pi_t = \lambda_{1p}\pi_{t-1} + (1 - \lambda_{1p})\mu + (1 - \lambda_{1p})\varepsilon_t. \tag{32}$$

Observe that this inflation dynamics equation has the same form as the corresponding equation (12) in the previous model. Thus, the impulse response function $R(\pi_{t+j}) = \lambda_{1p}^j(1 - \lambda_{1p}), \; j = 0, 1, 2, ...,$ has the same form as well. Inflation persistence now is simply λ_{1p}. The magnitude of this autoregressive parameter is all that differentiates the inflation responses in the two models.

Now consider a **permanent money growth shock**. When money growth follows the random walk process (16), the rational expectations solution of (29) is

$$P_t = \lambda_{1p} P_{t-1} + (1 - \lambda_{1p}) M_t + \kappa(1 - \lambda_{1p})\mu_t, \tag{33}$$

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[27]It can be shown that $(1 - \lambda_{1p}) = \frac{\gamma}{(\lambda_{2p}-1)(1-\gamma)(1-\alpha)}$.

First differencing the above gives the following inflation equation:

$$\pi_t = \lambda_{1p}\pi_{t-1} + (1 - \lambda_{1p})\,\mu_t + \kappa\,(1 - \lambda_{1p})\,\varepsilon_t. \tag{34}$$

Once again, this equation has the same form as its counterpart (18) in the previous model. As above, inflation is perfectly responsive when $\alpha = 1/2$, it is under-responsive when $\alpha > 1/2$, and the degree of under-responsiveness is inversely related to the slope of the long-run Phillips curve.

5.2   Wage Staggering

Consider the following common wage staggering model:

$$W_t = \alpha W_{t-1} + (1 - \alpha)\,E_t W_{t+1} + \gamma x_t. \tag{35}$$

Assuming constant returns to labor, the price is a constant mark-up over the relevant wages:

$$P_t = \frac{1}{2}\,(W_t + W_{t-1}). \tag{36}$$

Substitution of the price mark-up (36) and real money balances (5) equations into the wage setting equation (35) gives

$$W_t = \phi_w W_{t-1} + \theta_w E_t W_{t+1} + \left(\frac{2\gamma}{2 + \gamma}\right) M_t, \tag{37}$$

where $\phi_w = \frac{2\alpha - \gamma}{2 + \gamma}$, $\theta_w = \frac{2(1-\alpha)}{2+\gamma}$. We can write the above second order difference equation as

$$W_t = \lambda_{1w} W_{t-1} + \frac{\gamma}{\lambda_{2w}\,(1 - \alpha)} \sum_{j=0}^{\infty} \left(\frac{1}{\lambda_{2w}}\right)^j E_t M_{t+j}, \tag{38}$$

where $\lambda_{1w,2w} = \frac{1 \mp \sqrt{1 - 4\phi_w \theta_w}}{2\theta_w}$, $0 < \lambda_{1w} < 1$, and $\lambda_{2w} > 1$.

In this context, consider the inflation effects of a **temporary money growth shock**. We substitute the money growth stochastic process (10) into (38) to obtain the wage dynamics equation:

$$W_t = \lambda_{1w} W_{t-1} + (1 - \lambda_{1w})\,M_t + \left(\frac{1 - \lambda_{1w}}{\lambda_{2w} - 1}\right)\mu. \tag{39}$$

Insert this wage dynamics equation into the price mark-up eq. (36) to obtain the price dynamics equation:

$$P_t = \lambda_{1w} P_{t-1} + \frac{1}{2}\,(1 - \lambda_{1w})\,M_t + \frac{1}{2}\,(1 - \lambda_{1w})\,M_{t-1} + \left(\frac{1 - \lambda_{1w}}{\lambda_{2w} - 1}\right)\mu. \tag{40}$$

Therefore, inflation is given by

$$\pi_t = \lambda_{1w}\pi_{t-1} + (1 - \lambda_{1w})\mu + \frac{1}{2}(1 - \lambda_{1w})\varepsilon_t + \frac{1}{2}(1 - \lambda_{1w})\varepsilon_{t-1}. \tag{41}$$

The responses of inflation to a period-t unit money growth shock are:

$$R(\pi_t) = \frac{1}{2}(1 - \lambda_{1w}), \tag{42}$$

$$R(\pi_{t+j}) = \frac{\lambda_{1w}^{j-1}}{2}\left(1 - \lambda_{1w}^2\right), \; j = 1, 2, 3, ...$$

Thus inflation persistence is

$$\sigma = \frac{1 + \lambda_{1w}}{2}. \tag{43}$$

Now turning to the inflation effects of a **permanent change in money growth**, the rational expectations solution of the model gives the following dynamics equation:

$$W_t = \lambda_{1w}W_{t-1} + (1 - \lambda_{1w})M_t + \left(\frac{1 - \lambda_{1w}}{\lambda_{2w} - 1}\right)\mu_t, \tag{44}$$

$$P_t = \lambda_{1w}P_{t-1} + (1 - \lambda_{1w})M_t + \frac{1}{2}(1 - \lambda_{1w})\left(\frac{2 - \lambda_{2w}}{\lambda_{2w} - 1}\right)\mu_t + \frac{1}{2}\left(\frac{1 - \lambda_{1w}}{\lambda_{2w} - 1}\right)\mu_{t-1}, \tag{45}$$

$$\pi_t = \lambda_{1w}\pi_{t-1} + (1 - \lambda_{1w})\mu_t + \frac{1}{2}(1 - \lambda_{1w})\left(\frac{2 - \lambda_{2w}}{\lambda_{2w} - 1}\right)\varepsilon_t + \frac{1}{2}\left(\frac{1 - \lambda_{1w}}{\lambda_{2w} - 1}\right)\varepsilon_{t-1}. \tag{46}$$

It can be shown that the responses through time of inflation to a period-t permanent unit money growth shock are:

$$R(\pi_t) = 1 - \frac{1}{2}\left[(1 - \lambda_{1w})\left(\frac{2\alpha - 1}{\gamma}\right) + \frac{1 + \lambda_{1w}}{2}\right] < 1, \tag{47}$$

$$R(\pi_{t+j}) = 1 - \lambda_{1w}^{j-1}\frac{\left(1 - \lambda_{1w}^2\right)}{2}\left(\frac{2\alpha - 1}{\gamma} - \frac{1}{2}\right), \; j = 1, 2, ...,$$

$$\lim_{j \to \infty} R(\pi_{t+j}) = 1.$$

As for the price staggering model, inflation responsiveness is $\rho \equiv -\frac{2\alpha - 1}{\gamma}$. By this measure, again, inflation is perfectly responsive when the discount rate is zero ($\alpha = 1/2$) and under-responsive when the discount rate is positive ($\alpha > 1/2$). However, in neither case does inflation jump immediately to its long-run equilibrium value. Specifically, the instantaneous (period-t) response of inflation is to undershoot both when $\alpha = 1/2$ and $\alpha > 1/2$.

In period-1, when $\alpha = 1/2$, inflation overshoots and thereafter converges geometrically to its long-run equilibrium. In this case inflation undershooting and overshooting cancel out, inflation is perfectly responsive, $\rho = 0$, and the long-run Phillips curve is vertical.

On the other hand, when $\alpha > 1/2$, inflation can either remain below its new equilibrium level in period-1, if $\frac{2\alpha-1}{\gamma} > \frac{1}{2}$,[28] or overshoot if $\frac{2\alpha-1}{\gamma} < \frac{1}{2}$. Since $0 < \lambda_{1w} < 1$, period-2 onwards inflation converges to its equilibrium in a geometric fashion.

Finally, we consider a wage staggering model in which the nominal wage depends not only on current demand (x_t) but also on future demand (x_t), along the lines originally proposed by Taylor (1980a):

$$W_t = \alpha W_{t-1} + (1-\alpha) E_t W_{t+1} + \gamma \left[\alpha x_t + (1-\alpha) E_t x_{t+1}\right]. \tag{48}$$

It is straightforward to show that the associated impulse response functions of inflation to a temporary and permanent money growth shock have the same functional forms as in the previous model. The only difference between the impulse response functions of the two wage staggering models (35) and (48) lies in the autoregressive root of their rational expectations dynamic equations.[29]

6　Overview of our Analysis

We have examined four macro versions of the new Phillips curve:

⟨1⟩ *PS-*(x_t) stands for the price staggering model in which prices depend only on current demand - the workhorse model in the NPC literature.

⟨2⟩ *PS-*(x_t, x_{t+1}) is the model in which prices also depend on future demand.

⟨3⟩ *WS-*(x_t), and

⟨4⟩ *WS-*(x_t, x_{t+1}) represent the corresponding wage staggering models.

We analysed the inflation dynamics implied by the above models by considering two types of monetary shocks: (i) a temporary shock, i.e. a one-off unit increase in money growth, and (ii) a permanent shock, i.e. a permanent unit increase in money growth.

To avoid confusion, we have used the terms of persistence and responsiveness to summarise the impulse response functions of inflation associated with a temporary and a permanent shock, respectively. In other words, in the context of the above macro models,

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[28] That is, when $\alpha > 1/2$, inflation undershoots and converges to its equilibrium from below if

$$r > \frac{2\gamma}{2-\gamma}, \text{ or } \gamma < \frac{2r}{2+r}.$$

[29] For the above Taylor model, it can be shown that $\lambda_{1w} = \frac{1-\sqrt{1-4\phi_w\theta_w}}{2\theta_w}$, $\phi_w = \alpha\left(\frac{2-\gamma}{2+\gamma}\right)$, $\theta_w = (1-\alpha)\left(\frac{2-\gamma}{2+\gamma}\right)$, and $0 < \lambda_{1w} < 1$. For a detailed analysis of this model see Karanassou, Sala and Snower (2005).

- *inflation persistence* denotes inflation inertia in the presence of the temporary shock, whereas

- *inflation under-responsiveness* denotes inflation inertia in the presence of a permanent shock.

Table 3 outlines our results on inflation persistence, over the NPC models $\langle 1 \rangle$-$\langle 4 \rangle$. As we have seen, the responses to a temporary shock can be divided into (i) the short-run slope (e_{SR}), i.e. the "current" response, (ii) the persistence (σ), i.e. the sum of "future" responses, and (iii) the long-run slope (e_{LR}), i.e. the sum of all responses ($e_{LR} = e_{SR} + \sigma$).[30] We find that a temporary money growth shock always has prolonged after-effects on inflation (regardless of whether the discount rate is zero or positive, or whether there is price or wage staggering).

**Table 3: Inflation persistence after a shift in the money supply**

Models	autoregressive coefficient λ			responses		
	$\lambda = \frac{1-\sqrt{1-4\phi\theta}}{2\theta}$	ϕ	θ	Short-run e_{SR} :	Persistence σ :	Long-run e_{LR} :
$PS\text{-}(x_t)$		$\frac{\alpha}{1+\gamma}$	$\frac{1-\alpha}{1+\gamma}$	$1-\lambda$	λ	1
$PS\text{-}(x_t, x_{t+1})$		$\frac{\alpha}{(1+\gamma\alpha)}$	$\frac{(1-\gamma)(1-\alpha)}{(1+\gamma\alpha)}$	$1-\lambda$	λ	1
$WS\text{-}(x_t)$		$\frac{2\alpha-\gamma}{2+\gamma}$	$\frac{2(1-\alpha)}{2+\gamma}$	$\frac{1}{2}(1-\lambda)$	$\frac{1}{2}(1+\lambda)$	1
$WS\text{-}(x_t, x_{t+1})$		$\frac{\alpha(2-\gamma)}{2+\gamma}$	$\frac{(1-\alpha)(2-\gamma)}{2+\gamma}$	$\frac{1}{2}(1-\lambda)$	$\frac{1}{2}(1+\lambda)$	1

The dependence of inflation persistence on the discount rate r and the demand sensitivity parameter γ, for our four macro models, are pictured in Figures 1. Observe that, for given values of r and γ, there is more inflation persistence (i) under wage staggering than under price staggering and (ii) when nominal variables depend on both present and future demands than when they depend on present demands alone. Furthermore, note that variations in the demand sensitivity parameter over the frequently estimated range have a strong effect on inflation persistence, whereas the discount rate (over the standard range) has a relatively weak effect.[31]

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[30] Strictly speaking, the short-run slope is the immediate impact, whereas the long-run slope is the total impact of the temporary shock.

[31] Since the demand sensitivity parameter (γ) is assumed positive and nonzero, the unit value of persistence in Figure 1b for $\gamma = 0$ represents a limiting case $\left(\text{i.e., } \lim_{\gamma \to 0} \sigma = 1 \right)$.

Figures 1

a. Inflation persistence and interest rate (gamma=0.05)

b. Inflation persistence and gamma (r=2%)
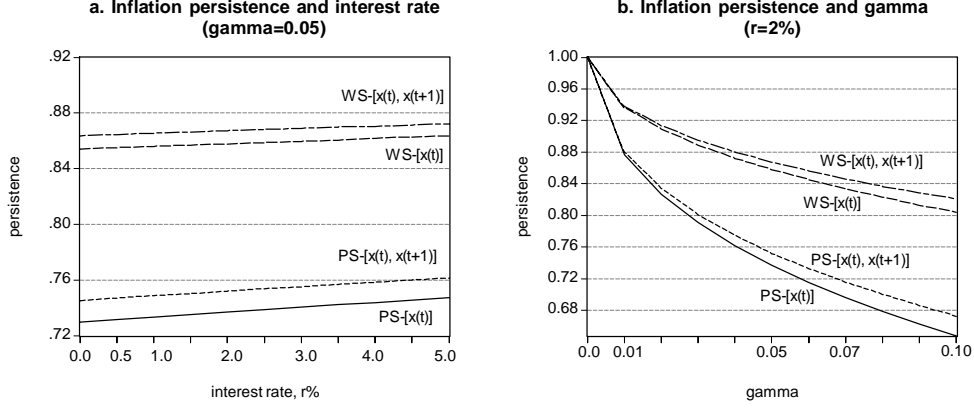
Tables 4a-b summarise our results on inflation responsiveness over the NPC models $\langle 1 \rangle$-$\langle 4 \rangle$. Recall that the "current" response of a permanent money growth shock is denoted by $\overline{R}_t^\pi$ and the "future" responses by $\overline{R}_{t+j}^\pi$, $j \geq 1$. The degree of inflation responsiveness ρ has been shown to be the inverse of the slope of the long-run Phillips curve. This measure of responsiveness is zero (denoting perfect responsiveness) when the discount rate is zero ($\alpha = 1/2$) and negative (denoting under-responsiveness) when the discount rate is positive ($\alpha > 1/2$). However, this does not imply that inflation necessarily jumps to its long-run value whenever the discount rate is zero. On the contrary, we have shown that under staggered wage setting inflation is never a jump variable, regardless of the discount rate.

Table 4a: Inflation responsiveness , $\alpha > 1/2$

Models	$\overline{R}_t^\pi$	$\overline{R}_{t+j}^\pi$ $j=1,2,...$	ρ
PS-(x_t)	< 1	< 1	$-\left(\frac{2\alpha-1}{\gamma}\right)$
PS-(x_t, x_{t+1})	< 1	< 1	$-\left(\frac{2\alpha-1}{\gamma}\right)$
WS-(x_t)	< 1	$\lessgtr 1$ if $r \gtrless \frac{2\gamma}{2-\gamma}$	$-\left(\frac{2\alpha-1}{\gamma}\right)$
WS-(x_t, x_{t+1})	< 1	$\lessgtr 1$ if $r \gtrless \frac{2\gamma}{2-\gamma}$	$-\left(\frac{2\alpha-1}{\gamma}\right)$

In all models, when $\alpha > 1/2$, the immediate response of inflation is to undershoot $\left(\overline{R}_t^\pi < 1\right)$. In the wage-staggering versions of the NPC (the bottom two rows in Table 4a), inflation will continue to undershoot its equilibrium after period-t if $\frac{2\alpha-1}{\gamma} > \frac{1}{2}$, i.e. if $r > \frac{2\gamma}{2-\gamma}$. Otherwise, inflation overshoots in period $t+1$ and then gradually converges (from above) to its new equilibrium.

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When $\alpha = 1/2$ (see Table 4b), the inflation generated by the price staggering models is a jump variable and both the short- and long-run Phillips curves are vertical. In other words, there is no inflation "persistence" and the monetary policy has no real effects in the economy. With wage staggering, when $\alpha = 1/2$, inflation responsiveness remains zero but inflation does not immediately jump to its new value. Initially inflation undershoots, and then it overshoots before it starts approaching its new equilibrium. The net effect is zero and so $\rho = 0$. Thus, the Phillips curve is downwards sloping in the short-run and becomes vertical in the long-run.
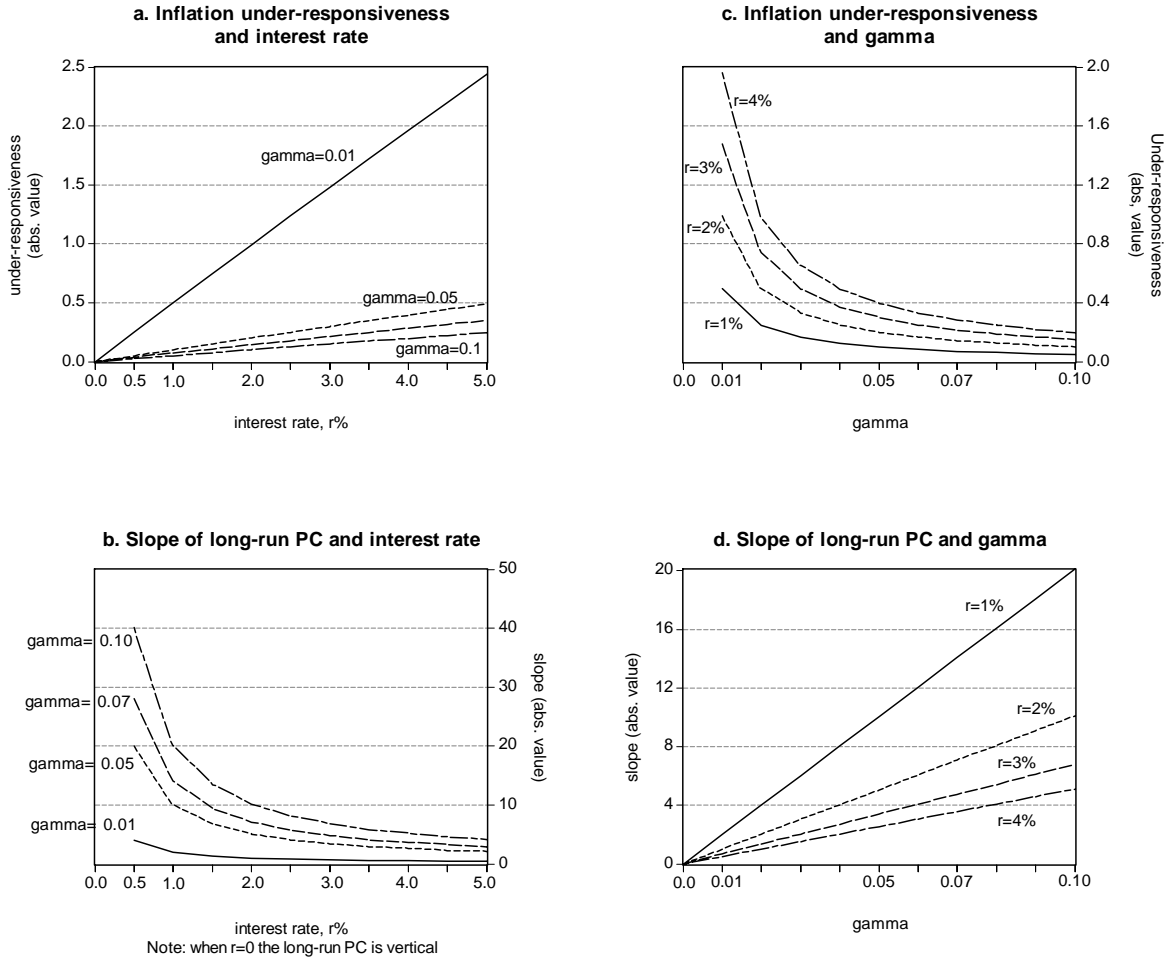
Table 4b: Inflation responsiveness , $\alpha = 1/2$

Models	$\overline{R}^\pi_t$	$\overline{R}^\pi_{t+j}$ $j=1,2,...$	ρ	PC (short-run)
PS-(x_t)	1	1	0	vertical
PS-(x_t, x_{t+1})	1	1	0	vertical
WS-(x_t)	< 1	> 1	0	downward-sloping
WS-(x_t, x_{t+1})	< 1	> 1	0	downward-sloping

Figure 2a pictures the relation between inflation under-responsiveness (in absolute value terms) and the interest rate; Figure 2b is the corresponding relation between the slope of the long-run Phillips curve and the interest rate. When the interest rate is zero the Phillips curve is vertical, while a positive interest rate produces a downward sloping PC. The higher is the interest rate, the more under-responsive inflation and the flatter the Phillips curve. Along the same lines, Figures 2c and 2d show how inflation under-responsiveness and the slope of the long-run Phillips curve depend on the demand sensitivity parameter γ. The lower is gamma, the more under-responsive inflation and the flatter the Phillips curve.

These results have one common thrust: the "persistency puzzle" proposition is highly misleading. Under plausible parameter values, high degrees of inflation persistence and under-responsiveness may arise in the context of standard wage-price staggering models.

Figures 2

**a. Inflation under-responsiveness
and interest rate**

**c. Inflation under-responsiveness
and gamma**

b. Slope of long-run PC and interest rate

Note: when r=0 the long-run PC is vertical

d. Slope of long-run PC and gamma

7 Concluding Remarks

It is commonly asserted that inflation is a jump variable in the new Phillips curve - this is at odds with the stylised fact of inflation persistence. We showed that this so called "persistency puzzle" is highly misleading, relying on the exogeneity of real variables and the assumption of a zero discount rate. When the discount rate is positive in a general equilibrium setting (in which real variables not only affect inflation, but are also influenced by it) inflation persistence re-emerges.

In the context of the standard models of the NPC, we first derived the closed form rational expectations solutions of inflation, real money balances, and the unemployment rate under one-off and permanent unit increases in money growth. We then measured inflation inertia by obtaining the impulse response function (IRF) of inflation with respect

to each type of shock. To distinguish between the persistence of inflation that results from a temporary shock and the persistence of inflation that results from a permanent shock, we called the latter *inflation responsiveness*.

Finally, we showed that the time-varying slope of the NPC is given by the ratio of the inflation and unemployment IRFs. We also found that the long-run slope is the inverse of inflation under-responsiveness (i.e., the cumulative amount of undershooting).

We showed that when the discount rate is zero, the conventional wisdom is confirmed: inflation is a jump variable and the workhorse NPC is vertical. In contrast, when the discount rate is positive, there is substantial inflation undershooting and the NPC is downwards sloping in the long-run. This result is a manifestation of *frictional growth*, a phenomenon that encapsulates the interplay of nominal staggering and permanent monetary changes.

The finding of a long-run inflation-unemployment tradeoff suggests that the development of an interactive dynamics framework that includes wage-price setting equations as well as labour market ones can enhance our understanding of the evolution of inflation and unemployment.

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