DISCUSSION PAPER SERIES

IZA DP No. 2129

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April 2006

Forschungsinstitut
zur Zukunft der Arbeit
Institute for the Study
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IZA Discussion Paper No. 2129
April 2006

ABSTRACT

Choosing Monetary Sequences: Theory and Experimental Evidence*

In this paper we formulate and investigate experimentally a model of how individuals choose between time sequences of monetary outcomes. The theoretical model assumes that a decision-maker uses, sequentially, two criteria to screen options. Each criterion only permits a decision between some pairs of options, while the other options are incomparable according to that criterion. When the first criterion is not decisive, the decision maker resorts to the second criterion to select an alternative. This type of decision procedures has encountered the favour of several psychologists, though it is quite under-explored in the economics domain. In the experiment we find that: 1) traditional economic models based on discounting alone cannot explain a significant (almost 30%) proportion of the data no matter how much variability in the discount functions is allowed; 2) our model, despite considering only a specific (exponential) form of discounting, can explain the data much better solely thanks to the use of the secondary criterion; 3) our model explains certain specific patterns in the choices of the 'irrational' people. We can safely reject the hypothesis that anomalous behaviour is due simply to random 'mistakes' around the basic predictions of discounting theories: the deviations are not random and there are clear systematic patterns of association between 'irrational' choices.

JEL Classification: C91, D9

Keywords: time preference, time sequences, negative discounting

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* Part of this work was carried out while Manzini and Mariotti were visiting the University of Trento, and funding for the experiments was provided by the ESRC under grant RES-000-22-0866. We wish to thank both institutions for their support. We are also grateful to Alan Agresti, Stepana Lazarova, Daniel Read, Bob Sugden and seminar audiences at the LSE Choice Group and the University of Birmingham for insightful discussions and comments, as well as to the tireless staff of the CEEL lab in Trento, in particular to Marco Tecilla for superb programming support. All errors are our own.

1. Introduction

Most economic decisions involve a time dimension, hence the need for a reliable model of time preferences. The standard exponential discounting model for time preferences has been the object of strong, evidence-led criticisms in the last few years. Various discounting 'anomalies' have been identified. Some of these anomalies are in a sense 'soft': they do not contradict basic tenets of economic theory, and they can be addressed simply by changes in the functional form of the objective function which agents are supposed to maximize. The best-known example of a soft anomaly in choice over time is preference reversal[2] between date-outcome pairs, which can be explained by the now popular model of hyperbolic discounting, as well as by other models.[3]

But other observed violations of the standard model are more fundamental, because they seem to contradict the basic assumption of maximization of *any* economically reasonable objective function. One of these hard anomalies is the striking phenomenon of *negative time preferences*. Notably Lowenstein and Prelec (1991) (also Lowenstein and Prelec, 1993) have argued that there is evidence of negative time preferences when individuals choose between *sequences* of outcomes (e.g. wage profiles in a survey by Lowenstein and Sicherman (1991) and discomfort sequences in Varey and Kahneman, 1992). The empirical findings leads Lowenstein and Prelec to conclude that "To most persons, a deteriorating series of utility levels is a rather close approximation to the *least* attractive of all possible patterns" (p. 347). In a more recent survey, Frederick, Lowenstein, O'Donoghue (2002) emphasise once again that "In studies of discounting that involve choices between two outcomes... positive discounting is the norm. Research examining preferences over *sequences* of outcomes, however, has generally found that people prefer improving sequences to declining sequences" (p. 363). These findings obviously could not be explained by hyperbolic discounting, or indeed by any other form of positive discounting. Therefore they pose a more formidable challenge for the economic modeler of decision-making over time sequences.

This paper has three main aims:

1) We propose a theory of preferences over monetary time sequences that can account for the observed anomalies, while at the same time keeping simple exponential (positive) discounting as one of its core elements.

[2]Preference reversal is the phenomenon whereby the preference for a smaller reward obtained at an earlier date over a larger reward obtained at a later date is reversed when the obtainment of both rewards is pushed forward by the same amount of time. In a nutshell: one apple today is better than two apples tomorrow but two apples in a year and one day are better than one apple in one year.

[3] E.g. Manzini and Mariotti (2002), Read (2001), Rubinstein (2001).

2) We investigate to what extent conclusions on the preference for increasingness based on survey findings (e.g. Loewenstein and Sicherman, 1991) are supported in a laboratory experiment in which subjects received real money payments – we find mixed evidence on this.

3) We uncover some new clear patterns in choice (beside preference for increasing sequences) which are consistent with our theory but inconsistent with pure discounting models.

While experimental investigations of choices over date-outcome pairs form a small but non-negligible literature, experimental investigations of choices over reward sequences are extremely thin on the ground in the economics literature, especially with financially motivated subjects[4]. In the experimental part of the paper, we ask subjects to make binary choices among all possible pairs of monetary sequences, with an increasing, constant, decreasing or 'jump' (i.e. end effect) pattern, both in a paid (where subjects do indeed receive the sums corresponding to the sequence chosen) and an unpaid condition (where choices are hypothetical).

The fundamental hypothesis in the theoretical model we propose is the following. In order to rank monetary reward sequences, the decision maker looks first at the standard exponential discounting criterion. However, sequences are only *partially* ordered by the criterion. In other words, whenever the decision maker is able to make a trade-off between the time and the outcome dimension, he will do so in a time-consistent way; but he will not always be able to make such a trade-off. Preferences are *incomplete*.[5] We postulate a very simple (two-parameter, in the specification used for the experiment) *interval order* structure to formalize preference incompleteness. In this structure, preferences are described by two functions, a *utility function* u and a *vagueness function* σ, which combine additively. A sequence x is definitely chosen over another sequence y if the discounted utility of x exceeds the discounted utility of y by at least $\sigma(y)$. When sequences cannot be compared by means of discounted utilities, the decision-maker is assumed to focus on one prominent attribute of the sequences. This prominent attribute ranks (maybe partially) the sequences and allows a specific choice to be made. This latter aspect of the model is in the spirit of Tversky, Sattath and Slovic's *prominence hypothesis* (1988). The attribute may be context dependent. In the date-outcome pairs case,

[4] The literature includes Chapman (1996), Gigliotti and Sopher (1997), Guyse, Keller and Epple (2002). The last two papers in particular show that the domain of choice is rather important, in the sense that there are differences in observed choices depending on whether or not the sequences are of money, or health or environmental outcomes. On this point see also Read and Powell (2002).

[5] This idea originated in Manzini and Mariotti (2002)'s model of choice between date/outcome pairs. We showed that in that context a simple model can account for major anomalies both soft and hard, such as preference reversal and cyclical choice patterns.

for example, objects have two obvious attributes that may become prominent, the date and the outcome: in that case two natural models emerge according to whether date or outcome is looked at first. We stress that, at the abstract level, the only departure from the standard choice theoretic approach is that our decision maker's behavior is described by combining sequentially two possibly incomplete preference orderings, instead of using directly a complete preference ordering.

In the case of reward sequences the previous experimental evidence highlighted before suggests that the general trend of the sequence (increasing or decreasing) is relevant to make decisions. However, in our case the data provide much weaker evidence than Loewenstein and Prelec's (1991) in support of their view that 'sequences of outcomes that decline in value are greatly disliked' (p. 351). We find that, even in the simple decision problems we study, where monetary sequences can be clearly ordered according to their trends, simply choosing according to the heuristics that favors the 'increasingness' of the trend does a rather poor job at explaining the data. The modal subject and choice is 'rational', in the sense of being compatible with positive time preference combined with preference for income smoothing (concave utility function).

In this sense, early news of the death of positive discounting seem to have been greatly exaggerated. There *is* a problem for standard theory, but its magnitude is not of the scale the existing literature suggests. When there are no affective factors involved (such as, for example, the sense of dread for choices relating to health, or the sense of failure involved in a decreasing wage profile), *some* theory of positive discounting can provide a rough approximation of the choice patterns.

Nonetheless, it is still true that a disturbingly high number of people (around 30%) choose in ways that are incompatible with *any form* of positive discounting (exponential, hyperbolic or otherwise). This proportion of people violating such a basic economic assumption (that good things should come early and bad things should come late) is unsatisfactory from the point of view of the descriptive adequacy of standard theory. It suggests that other mechanisms beyond discounting are at work. So we believe that Loewenstein and Prelec's pioneering findings do capture, beside affective factors, some of the heuristic considerations that people use when evaluating 'neutrally' (without affects) money sequences. However, those considerations become really effective in explaining the deviant choices only when used as a 'secondary criterion', rather than directly. The very basic two-parameter version of our model is far superior, in order to explain observed choices, both to any pure discounting model and to a direct heuristics-based model. In addition, when specialized to date-outcome pairs comparisons, it can also explain other observed anomalies both soft and hard.

One important feature of our analysis is that we delve quite deeply into the analysis of "irrational" choices. We find that the observed pattern of irrationality is systematic. In general, our data reveal some interesting and non-obvious patterns of association in choice, on which standard theory (and simple increasingness heuristics) are completely silent. Among our findings are the following two: (i) *there is association between certain types of rational choices and irrational choices* (those who prefer a decreasing to a constant sequence are disproportionately concentrated among those who also prefer a constant to an increasing sequence); (ii) *there is association between irrational choices of a different type* (choosing an increasing over a decreasing sequence is very strongly associated with choosing an increasing over a constant sequence). Such patterns are what one would expect if our model were true. They cannot be generated by any discounting model, nor by such a model augmented with random independent mistakes.

Finally, our data also provide the basis for some preliminary and speculative insights in what other types of heuristics may guide agents' decisions when sequences exhibit some other prominent attributes, beside being simply decreasing or increasing, and they highlight some interesting differences in choice behavior between sexes.

2. A Model of Intertemporal Choice

Let X indicate a set of money amounts and let u: $X \rightarrow R$ be an instantaneous monotonic increasing utility function . Let $T=\{0,1,2,\ldots,T^*\}$ be a finite set of dates. The set of alternatives A is a subset of the set of finite sequences of outcomes. The typical alternative is denoted $a = \left((a_1, t_1);\ldots;(a_k, t_k)\right)$, with $a_1,\ldots,a_k \in X$ and $t_1,\ldots,t_k \in T$, and $t_i > t_{i'}$ for $i > i'$.

Let us recall that in discounting models sequences are evaluated by means of a discounting function δ: $T \rightarrow (0,1)$. The discounted utility at time 0 of

$$a = \left((a_1, t_1);\ldots;(a_k, t_k)\right)$$

is

$$\Sigma_{t=t_0}^{t_k} \delta(t)\, u(a_t)$$

With exponential discounting we have that $\delta(t) = \delta^t$ for some $\delta \in (0,1)$. With hyperbolic discounting instead $\delta(t)$ is a hyperbolic function of the type $(1+at)^{(-g/a)}$, with *g* and *a* being two preference parameters, and in the very popular (β-δ) version of hyperbolic discounting the following specification is used: $\delta(0)=1$ and $\delta(t)= \beta\delta^t$ for $t>0$, with $\beta,\delta \in (0,1)$.

The model we propose uses as a primitive a binary preference relation $\succeq^*$ of the individual on A constructed as follows:

Primary criterion. There exists a primary criterion P_1, which is a possibly incomplete strict ordering. On the basis of the primary criterion the individual makes (possibly partial) comparisons between sequences. We interpret P_1 as resolving comparisons for which the trade-off between outcomes and time yields, in the perception of the individual, a decisive advantage to one of the alternatives. We assume the trade-offs involved are resolved by P_1 in a 'time-consistent' way: they coincide with the standard ones based on exponentially discounted utility, with discount factor $\delta \in (0,1)$.

Vagueness function. There may be pairs of alternatives for which the primary criterion alone is not discriminating enough. This lack of discrimination is captured by a 'vagueness function' $\sigma: A \rightarrow R_+$. If the present, exponentially discounted utility of the higher value alternative *a* does not exceed the utility of the lower value alternative *b* by at least σ (b), we say that the decision maker is *vague*. In this case *a* and *b* are not related by P_1.

Secondary criterion. In the case where the decision maker is vague, and only in this case, a secondary criterion is used. The secondary criterion P_2 is just a (possibly partial) strict ordering on A. We interpret it as being based on one prominent attribute of the elements of A. In some contexts, as in the case of date-outcome pairs or in the experiment presented below, the relevant attributes are obvious; in other cases less so and the issue of what is an appropriate secondary criterion is essentially empirical. We view the secondary criterion P_2 as a primitive of the model just as u, β or δ are primitives in the simple version of the hyperbolic discounting model.

The relation $\succeq^*$ is derived by the combination of primary and secondary criterion.

In summary, let $\succ^*$ denote the strict binary preference relation on A. We propose the following general model, for given u, δ, σ and P_2:

For all $a, b \in A$, we have $a \succ^* b \Leftrightarrow$

1. $\Sigma_t \delta^t u(a_t) > \Sigma_t \delta^t u(b_t) + \sigma(b_t)$ (Primary Criterion P_1), or
2. $\Sigma_t \delta^t u(b_t) < \Sigma_t \delta^t u(a_t) \leq \Sigma_t \delta^t u(b_t) + \sigma(b_t)$, and aP_2b (Secondary Criterion P_2)

(where the summations are taken over the appropriate range). A brief discussion of some general properties of this model follows. The reader who is interested mainly in the application to our experiment may jump directly to section 3.

Cycles

Even if *both* the primary and secondary criteria are transitive, it is not difficult to verify that the preference $\succ^*$ obtained from their sequential application is not necessarily transitive. For example, suppose that for a decision maker the following holds:

$$\Sigma_t \delta^t u(b_t) > \Sigma_t \delta^t u(a_t) > \Sigma_t \delta^t u(c_t)$$
$$\Sigma_t \delta^t u(b_t) > \Sigma_t \delta^t u(c_t) + \sigma(c_t)$$
$$\Sigma_t \delta^t u(a_t) + \sigma(a_t) \geq \Sigma_t \delta^t u(b_t)$$
$$\Sigma_t \delta^t u(c_t) + \sigma(c_t) \geq \Sigma_t \delta^t u(a_t)$$
$$c\ P_2\, a\ P_2 b$$

then the resulting complete preference relation $\succ^*$ is cyclical and given by

$$a \succ^* b \succ^* c \succ^* a$$

Tversky (1969) first showed experimentally evidence of cycles in intertemporal choice. More recent and striking evidence of cyclical choices in an intertemporal context is due to Roelofsma and Read (2000).

Rubinstein's experiment

Rubinstein (2003) reports that subjects exhibited the following type of behavior: they chose x to be received at some date t* versus x+z to be received at date t*+1 (they were impatient and preferred smaller reward earlier rather than larger reward later) but chose the sequence

$$a = (x+z, t'+1; x+z, t'+2; x+z, t'+3; x+z, t'+4)$$

over

$$b = (x, t'; x, t'+1; x, t'+2; x, t'+3)$$

where $t'+4 \leq t^*+1$. This contradicts not only exponential discounting but also hyperbolic discounting and in fact any model of discounting based on diminishing impatience (declining discount rates): if the subject were impatient at the late date t* and not willing to trade off one unit of delay for an additional reward z, he should have been unwilling to perform all four trade-offs of this type involved in the comparisons between sequences. Our simple model can explain these preferences. Suppose for example that the secondary criterion is the natural one proposed by Rubinstein himself, namely 'Pareto dominance' between the outcome sequences, and that

$$\delta^{t^*}u(x)> \delta^{t^*+1}u(x+z)+\sigma(x+z,t)$$

$$\Sigma_{i=0}^{3}\delta^{t'+i}u(x)\leq \Sigma_{i=1}^{4}\delta^{t'+i}u(x+z)+\sigma(a)$$

In this case the preference between date-outcome pairs can be explained by present discounted utility (primary criterion) and the preference between sequences can be explained by the secondary criterion.

Preference reversal between date-outcome pairs

This is one of the best documented and most discussed anomalies in intertemporal choice (see e.g. Frederick, Lowenstein, O'Donoghue, 2002), and it concerns the special case of one element sequences (date-outcome pairs). Letting with $x,y\in X$, $y>x$ and $t'>t^*$, the general pattern of a preference reversal is when an alternative (x,t^*) is chosen over an alternative (y,t'), but at the same time $(y,t'+k)$ is chosen over (x,t^*+k). That is, the decision maker prefers the worse outcome earlier (time t^*) than the better outcome later (time t'); but he reverses his preferences if the same time-money trade-off is pushed further away in time by k. This is compatible with our model whenever

$$u(x,t^*)>u(y,t')+\sigma(y,t')$$
$$u(x,t^*+k)\leq u(y,t'+k)+\sigma(y,t'+k)$$
$$u(y,t'+k)\leq u(x,t^*+k)+\sigma(x,t^*+k)$$
$$(y,t')P_2(x,t^*)$$

In Manzini and Mariotti (2002) we suggested specific secondary criteria for the case of date-outcome pairs, with two resulting models according to whether the decision maker looks first at time (time prominence model) or first at outcome (outcome prominence model) when he is vague, in order to order sequences. We find that the outcome prominence model explains the observed preference reversal while the time prominence model does not.

Foundation for the additive form of the primary criterion.

One convenient aspect of the proposed primary criterion is the additive form in which the vagueness term enters the formula. This is equivalent to assuming that the following property on the primary criterion holds. For $a\in A$, let L(a) be the lower contour set of *a*, that is the set of all sequences to which *a* is preferred according to the primary criterion:

$$L(a)=\{b\in A \mid aP_1b\}$$

Discrimination: For a, b$\in$ A, either $L(a) \subset L(b)$ or $L(b) \subset L(a)$.

The proof of equivalence and the additivity of P_1 follows from a simple adaptation of Manzini and Mariotti (2002, Proposition 1) or even from the classical results of Fishburn (1970). The assumption of Discrimination is tantamount to assuming that there is a hidden 'directionality' in the preferences expressed by P_1: if P_1 is a complete relation, Discrimination follows automatically from transitivity; otherwise it means that if a sequence *a* is not related by P_1 to another sequence *b*, either one 'improves' in moving from *a* to *b* in the sense if increasing the number of dominated alternatives, or viceversa: it does not happen that new alternatives become dominated moving in one direction (*a* to *b*), and the same happen when moving in the other 'direction' (*b* to *a*). Note that this assumption makes P_1 more rational rather than less rational, so it does not particularly help *per se* in explaining irrational choices. The additional assumption of time-consistency embodied in the choice of the exponential discounting functional form makes the criterion P_1 even more standard.

Foundation for the two-stage decision procedure

The existence of two criteria P_1 and P_2, applied sequentially to arrive at a choice can be justified at a more abstract level. Manzini and Mariotti (2005) provide an axiomatic foundation for an abstract choice function (taken as a primitive) to be 'rationalizable' by a two-stage procedure of the type specified in this paper.

3. Experimental Design

The experiments were carried out using the facilities of the Computable and Experimental Economics Laboratory at the University of Trento, in Italy. In all, we ran 13 sessions between July 2005 and February 2006. Experimental subjects were recruited through bulletin board advertising from the students of the University of Trento. Each sessions consisted of both male and female participants in roughly equal proportions. The experiment was computerised, and each participant was seated at an individual computer station, using separators so that subjects could not see the choices made by other participants. Experimental sessions lasted an average of around 26 minutes, of which an average of 18 minutes of effective play, with the shortest one lasting approximately 16 minutes and the longest around 37 minutes. We considered two treatments, one in which subjects were paid only a 5€ showup fee (a total 56 subjects in 4 sessions), and one with payments based on choice, where as we explain more in detail below an additional 48€ were made available to each subject (a total of 102 subjects in 9 sessions)[6]. We will refer to these two treatments as the **HYP** (for hypothetical) and **PAY** (for paid), respectively[7]. At the beginning of the experiment a screenshot with instructions appeared on each monitor, and at the same time an experimenter read the instructions aloud to the participants[8]. In each treatment, each experimental subject was presented with 23 different screens. Each screen asked the subject to choose the preferred one among a set of alternative remuneration plans in instalments to be received staggered over a time horizon of nine months, each consisting of €48 overall. Instructions were the same in both treatments, bar for one sentence, which in the HYP treatment clarified that choices were purely hypothetical, so that the only payment to be received would be the show up fee; whereas for the PAY treatment it was explained that at the end of the experiment one screen would be selected at random, and the preferred plan for that screen would be delivered to the subjects.[9]

Choices were based on two sets - depending on the number of instalments - of four plans each, namely an increasing (I), a decreasing (D), a constant (K) and a jump (J) series of payments, over either two or three instalments, as shown below. Though in both cases payments extended over nine months, because of the different number of instalments we abuse terminology and refer to 'two-*period*' (or also 'short') sequences and 'three-*period*' (or also 'long') sequences rather than two/three-*instalment* sequences:

Two period sequences

	I	**D**	**K**	**J**
in 3 months	16	32	24	8
in 9 months	32	16	24	40

Three period sequences

I	**D**	**K**	**J**	
8	24	16	8	In 3 months
16	16	16	8	In 6 months
24	8	16	32	In 9 months

Table 1 : the base remuneration plans

Pairwise choices, which are the interest of this paper, were interspersed with choices between larger sets of sequences (to be analysed elsewhere). In fact each subject had to make a selection from each possible subset of plans within each group (making up 11 choices per group). In addition, in a 23rd

6 In both treatments the show up fee alone, for an average of less than thirty minutes long experimental session, was higher than the hourly pay on campus, which is 8€. At the time of the experiments the exchange rate of the Euro was approximately 1€=1.2$=0.7£. In terms of purchasing power €1 was approximately equivalent to £1.

7 Distinguishing by treatment, sessions lasted an average of about 28 minutes for the PAY treatment, of which an average of just above 19 minutes of effective play; and an average of around 22 minutes for the HYP treatment, of which an average of about 16 minutes of effective play.

8 The translation of the original instructions (in Italian) can be found in the appendix.

9 As the experimental lab has a long tradition, issues of trust in receiving delayed payments were not relevant. At the time of writing all subjects have been paid.

question subjects were asked to choose between the three period sequences SJ=(8,32,8) and SI=(24,8,16). We discuss the reasons for this additional question in section 6.

Once made, each choice had to be confirmed explicitely, so as to minimize the chance of errors. Both the order in which the twenty-three questions appeared on screen and the position of each plan on the screen was randomised. Figure 1 displays sample screenshots. The participants made their choice by clicking with their mouse on the button corresponding to the preferred remuneration plan.

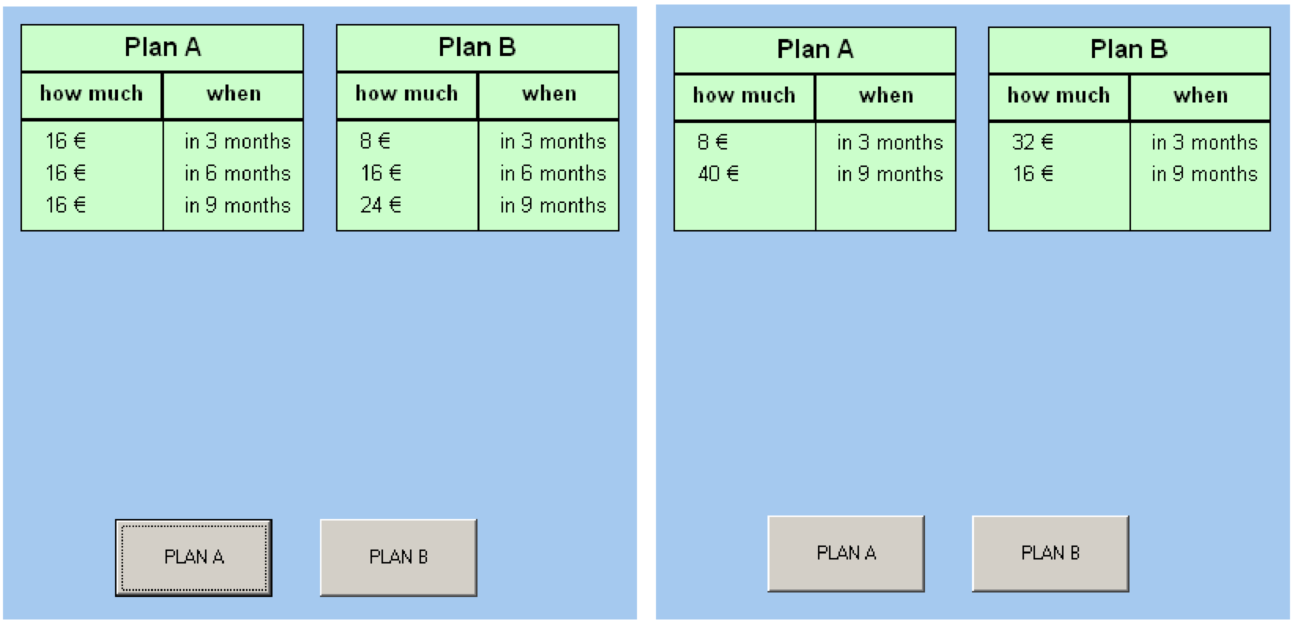

Figure 1 : sample screenshots

4. The (σ-δ) model and the experimental framework

In this section we present a simplified version of the 'vagueness' model where we constrain the σ function to be just a constant. We refer to this two-parameter version as the '(σ-δ) model'. Note that this restriction if anything limits the ability of our model to fit the data. Before deriving its predictions for choice in our experimental setup, though, we review briefly what the predictions of standard discounted utility theories are in this context. To distinguish them from the (σ-δ) model, in which the primary criterion is also based on discounting, we will refer to the large family of standard discounted utility theories (which includes e.g. hyperbolic discounting) as *pure discounting theories*. We make a number of standard assumptions on preference representation. As above, we assume the utility over date-outcome pairs to be separable in the time and outcome components, that is $U(x,t_x)=\delta(t_x)u(x)$, and let the utility for a stream of date-outcome pairs be given by $U\left(\left(x_1,t_{x_1}\right),\left(x_2,t_{x_2}\right),\ldots\left(x_n,t_{x_n}\right)\right)=\sum_{i=1}^{n}\delta\left(t_{x_i}\right)u\left(x_i\right)$, i.e. additively separable. Let u be monotonic increasing in outcome, concave and with positive third derivative[10]. Finally, let us assume that the discounting function is monotonically non increasing, i.e. $t_{x_1} > t_{x_2} \Rightarrow \delta\left(t_{x_1}\right) \leq \delta\left(t_{x_2}\right)$.

Now fix the times at which outcomes are received as 0, 1, 2, so that sequences can be defined in terms of the ordered outcomes with no mention of the time. To reduce notation let u_1=u(8), u_2=u(16), u_3=u(24) and u_4=u(32); and for the discounting function let $\delta(0)=\delta_0$, $\delta(1)=\delta_1$, and $\delta(2)=\delta_2$. Furthermore, normalize the utility function u (by dividing it by δ_0) so that we can set $\delta_0=1$. With the four three period sequences described in the right hand panel of Table 1, any discounting criterion for choice $\succ_d$ should order them as either D $\succ_d$ K $\succ_d$ I $\succ_d$ J or K $\succ_d$ D$\succ_d$ I $\succ_d$ J (to see this note that I $\succ_d$ J since sequence J shifts some outcome from the second to the last period, so that any discounting criterion is going to favor I over J).

In fact, use $\succ_d$ to denote the preference relation of a decision maker who discounts utility available at time t by some discount function (i.e. a 'pure discounter'). We indicate each sequence by the letter and the number of installments in which it was paid; for example J3 refers to the three period jump sequence (when we do not want to emphasize the length, we just use the letters). Then[11]:

$$D3 \succ_d K3 \Leftrightarrow \delta_2 < \frac{u_3 - u_2}{u_2 - u_1} \equiv \delta^* \text{ and } D2 \succ_d K2 \Leftrightarrow \delta_2 < \frac{u_4 - u_3}{u_3 - u_2} \equiv \hat{\delta}^* \tag{1}$$

Secondly, regardless of sequence length:

$$D \succ_d I \text{ always} \tag{2}$$

and

$$K \succ_d I \text{ always} \tag{3}$$

Thirdly

$$I \succ_d J \text{ always} \tag{4}$$

Consequently the patterns of choice which can be observed when a decision maker has utility for monetary streams which are additively separable and who has a non-decreasing discount function are either of the following two:

- D$\succ_d$ K$\succ_d$ I$\succ_d$ J, or
- K$\succ_d$ D$\succ_d$ I$\succ_d$ J.

This is true even if the preferences of the decision maker conform to hyperbolic discounting, since

[10] Note that these are mild assumptions, satisfied by the most common functional forms used in economics to describe an agent's utility function, such as for instance the constant risk aversion class of utility functions.

[11] Please refer to appendix A.1 for the full derivation of this and the other results in this section.

what matters is only the assumption that the discount function is monotonically nondecreasing. For instance, in the case of the (β-δ) model we would have $\delta_1 = \beta\delta$ and $\delta_2 = \beta\delta^2$, which would not affect the analysis above.

Consider now the (σ-δ) model. The primary criterion compares the present discounted utility of monetary streams. As explained before, we impose strong conditions on the discount function by letting $\delta_t = \delta^t$. We assume that, as natural, the secondary criterion orders by *increasingness*. For the simple sequences of payments K, I and D listed above, if a decision maker is vague between any two sequences, by the secondary criterion it must be that

$$I \succ_2 K \succ_2 D \qquad (5)$$

with $\succ_2$ transitive.

In general, putting both primary and secondary criterion together, it can be shown[12] that for the (σ-δ) model:

A) Comparison between series I and K:

$$I3 \succ^* K3 \Leftrightarrow \sigma \geq (u_2-u_1)-\delta^2(u_3-u_2) \equiv A \text{ and } I2 \succ^* K2 \Leftrightarrow \sigma \geq (u_3-u_2)-\delta^2(u_4-u_3) \equiv a \qquad (6)$$

B) Comparison between series I and D:

$$I3 \succ^* D3 \Leftrightarrow \sigma \geq (1-\delta^2)(u_3-u_1) \equiv B \text{ and } I2 \succ^* D2 \Leftrightarrow \sigma \geq (1-\delta^2)(u_4-u_2) \equiv b \qquad (7)$$

C) Comparison between series K and D:

$$K3 \succ^* D3 \Leftrightarrow \sigma \geq (u_3-u_2)- \delta^2(u_2-u_1) \equiv C \text{ and } K2 \succ^* D2 \Leftrightarrow \sigma \geq (u_4-u_3)- \delta^2 (u_3-u_2) \equiv c \qquad (8)$$

Few lines of algebra[13] show that it is always the case that $c<a,b$ and $C<A,B$. Moreover it is the case that $A>a$ and $B>b$.

5. Experimental Results

We start by analyzing the PAY treatment in some depth, and then contrast these results with those

[12] See Appendix A.1.
[13] See Appendix A.1.

for the HYP treatment.[14] To indicate the choice of one plan over another we use the "$\succ$" symbol, e.g. K2 $\succ$ D2 indicates that in two periods sequences, the constant one was chosen over the decreasing one. We will use the "$\succ^*$" and "$\succ_d$" notation when discussing the predictions for choice behavior according to the (σ-δ) model and pure discounting theories, respectively.

5.1 PAY treatment

The sample consisted of 102 experimental subjects, roughly in equal proportions across sexes.

Aggregate data

Frequency distributions for binary choices involving the base sequences of payments I, K, J and D are reported in Table 2.

Two period sequences				Three period sequences		
Value	Freq.	%		Value	Freq.	%
K2 $\succ$ I2	94	92.2	K versus I	K3 $\succ$ I3	95	93.1
I2 $\succ$ K2	8	7.8		I3 $\succ$ K3	7	6.9
D2 $\succ$ K2	68	66.7	D versus K	D3 $\succ$ K3	66	64.7
K2 $\succ$ D2	34	33.3		K3 $\succ$ D3	36	35.3
I2 $\succ$ D2	21	20.6	I versus D	I3 $\succ$ D3	19	18.6
D2 $\succ$ I2	81	79.4		D3 $\succ$ I3	83	81.4
I2 $\succ$ J2	94	92.2	I versus J	I3 $\succ$ J3	93	91.2
J2 $\succ$ I2	8	7.8		J3 $\succ$ I3	9	8.8
K2 $\succ$ J2	91	89.2	K versus J	K3 $\succ$ J3	93	91.2
J2 $\succ$ K2	11	10.8		J3 $\succ$ K3	9	8.8
J2 $\succ$ D2	10	9.8	J versus D	J3 $\succ$ D3	16	15.7
D2 $\succ$ J2	92	90.2		D3 $\succ$ J3	86	84.3

Table 2: frequency distribution of binary choice, aggregate data

[14] All the exact statistical analysis has been carried out usting *StatXact*, v.7. For a thorough reference on exact methods in categorical data analysis see Agresti (2002).

These aggregate data show the following:

1. **Sequence length does not matter:** the only difference in binary choice behavior when moving from two to three period sequences which is statistically significant is for the choice between J and D, where the proportion of subjects preferring J over D increases from 9.8% to 15.7% (the p-value for the corresponding Mc Nemar test is 0.035). This seems to suggest that when the 'jump' aspect of the J sequences kicks in (i.e. for the two period sequences J is simply steeper than I), it does affect choice behavior;

2. **A majority of subjects prefers decreasing to increasing sequences**: this is in sharp contrast with the alleged preference for increasing sequences discussed in the introduction;

3. **A majority of subjects prefers rational to irrational sequences**: the constant sequence is preferred to both the increasing and the jump sequence more than 90% of the times, and the decreasing sequence is preferred to both the increasing and the jump ones, though somewhat less decisively (more than 80% of the times). Indeed, regardless of length, the subjects who chose I over D are almost thrice as many as those choosing I over K[15] (the corresponding proportions are 20.6% against 7.8% for the short sequences and 18.6% against 6.9% for the long sequences);

4. **Endpoint effect?** For the long sequences, subjects choosing the jump series over the decreasing one are almost twice as many than those choosing the jump sequence over the constant one[16] (15.7% against 8.8%), whereas for the two period sequences the frequency is approximately the same (recall that for short sequences the J and I sequences are in fact both increasing ones, with the J sequence steeper than the I one. For three period sequences, though, the end effect in the J sequence comes to the fore).

The data above already suggest that decision making is unlikely to be guided by a clear-cut discounted utility rule: choice of either the increasing or jump sequence over either the constant or the decreasing one is a sign of 'irrationality', so that any individual choosing the increasing sequence over either the constant or the increasing sequence displays choice behavior which is incompatible with pure discounting theories.

Given that a significant proportion of subjects chose irrationally (that is not in accordance with pure discounting theories), we can look a bit deeper into the pattern of irrationality.

[15] This difference is statistically significant: a McNemar test of the difference between the proportion of subjects choosing I over K and the proportion of subjects choosing I over D returns a p-value of 0.001 for the short sequences and 0.002 for the long sequences.

[16] This difference is statistically significant: a McNemar test of the difference between the proportion of subjects

Association patterns for irrational choices

Can departures from irrationality be generated by independent mistakes or 'trembles' at the moment of making a decision? To answer this question, let us consider first the cross-tabulation between the choices involving one rational and one irrational sequence:

Two period sequences

	I2 $\succ$ K2	K2 $\succ$ I2
I2 $\succ$ D2	6	15
D2 $\succ$ I2	2	79

Three period sequences

	I3 $\succ$ K3	K3 $\succ$ I3
I3 $\succ$ D3	5	14
D3 $\succ$ I3	2	81

Table 3: rational versus increasing sequence

Two period sequences

	J2 $\succ$ K2	K2 $\succ$ J2
J2 $\succ$ D2	5	5
D2 $\succ$ J2	6	86

Three period sequences

	J3 $\succ$ K3	K3 $\succ$ J3
J3 $\succ$ D3	6	10
D3 $\succ$ J3	3	83

Table 4: rational versus jump sequence

The data show that the answer is negative. Table 3 and Table 4 make it clear at first sight that there is a strong association between choices between I and D and between I and K. The sample odds ratios are 15.8 and 14.46 in Table 3 and 14.3 and 16.6, respectively, in Table 4, with 95% exact confidence intervals[17] whose lower bounds are all above 2: in other words, the odds of choosing an irrational sequence over a rational one are at least twice as large when an irrational choice has been made between a different rational/irrational pair. Thus the hypothesis of independence in choices in the two situations can be rejected (Fisher's exact test yields p-values of 0.001, 0.002, 0.001 and 0.001). This suggests that there is some systematic mechanism generating the irrational choices that makes irrational choices in one context (e.g. I versus D) strongly associated with irrational choices in another context (I versus K).

Whatever this mechanism is, as we saw above it makes the proportion of 'mistakes' in the I versus D context significantly higher than the proportion of mistakes in the I versus K context (see footnotes 15 and 16). To explore further the structure of the observed deviations from rationality, let us for the moment focus on just the I, K and D sequences. Both irrational choices seem to be associated in turn with the choice between D and K. More precisely, consider Table 5:

---

choosing K over J and the proportion of subjects choosing D over J returns a p value of 0.046.

	D2≻K2	K2≻D2
I2≻K2	1	7
K2≻I2	67	27

	D2≻K2	K2≻D2
I2≻D2	4	17
D2≻I2	64	17

	D3≻K3	K3≻D3
I3≻K3	1	6
K3≻I3	65	30

	D3≻K3	K3≻D3
I3≻D3	2	17
D3≻I3	64	19

Table 5: choices with and without 'irrational' sequences

In the top (respectively. bottom) panel, for the table on the left, the sample odds ratio is 0.057 (resp. 0.076). In other words, the odds of being rational by preferring the constant over the increasing sequence of payments when the decreasing sequence is preferred to the increasing sequence are over 17 times (resp. 13 times) the odds of being rational when the constant sequence is preferred to the decreasing one). Independence is strongly rejected; Fisher's exact p-value is 0.001 (resp. 0.007). Observe the particularly counterintuitive nature of this association: the fact that K is chosen against D makes it *less* likely that it will be chosen against I! This would be very hard to explain in any 'preference ordering plus error' model even with a special, non-independent error structure.

Similar patterns are found for the right-hand tables. In the right table the odds ratio are 0.062 for the two-period sequences and 0.035 for the three period sequences. Again independence is clearly rejected in both cases (Fisher's exact p-values are 0.005 and 0.003).

In summary, *subjects who make an irrational choice (either I over K or I over D) are disproportionately concentrated among those who prefer the constant to the increasing sequence.*

According to pure discounting theories, any choice between K and D should give no information about the distribution of the other binary choices. Thus if one were to cross-tabulate the choice between K and D against the other choices, there should be no association. Yet, the data are as follows:

Two period sequences

	K2 ≻ I2, I2 ≻ D2	I2 ≻ K2, I2 ≻ D2	K2 ≻ I2, D2 ≻ I2	I2 ≻ K2, D2 ≻ I2
D2 ≻ K2	4	0	63	1
K2 ≻ D2	11	6	16	1

[17] The four 95% exact confidence intervals for the odds ratios are [2.42,167.4], [2.03, 159.1], [2.43, 80.3] and [2.86, 113].

Three period sequences

	K3 $\succ$ I3, I3 $\succ$ D3	I3 $\succ$ K3, I3 $\succ$ D3	K3 $\succ$ I3, D3 $\succ$ I3	I3 $\succ$ K3, D3 $\succ$ I3
D3 $\succ$ K3	2	0	63	1
K3 $\succ$ D3	12	5	18	1

Tables 6: choices including and excluding the increasing sequence

The percentage of subjects choosing D over I and I over K, corresponding to the last column in each of the tables above, is tiny (just 2 subjects for both sequence lengths, that is less than 2%). On the other hand, the percentage of subjects choosing I over D and K over I, corresponding to the second column in each of the tables above, is around 14% (15 and 14 subjects in the two and three period sequences, respectively). Of these, the overwhelming majority (11/15 and 12/14) lies in the second row of each table, i.e. subjects who also prefer the constant to the decreasing sequence. The effect is stronger for the longer sequences (where arguably the 'sequence' feature is more apparent).

Each of the cells in the two tables corresponds to one of the eight possible profiles of choice generated by the three binary comparisons involving the I, K and D sequences:

Profile 1: D $\succ$ K $\succ$ I;

Profile 2: K $\succ$ D $\succ$ I;

Profile 3: D $\succ$ I $\succ$ K $\succ$ D (cycle);

Profile 4: I $\succ$ K $\succ$ D;

Profile 5: K $\succ$ I $\succ$ D;

Profile 6: D $\succ$ K $\succ$ I $\succ$ D (cycle);

Profile 7: I $\succ$ D $\succ$ K;

Profile 8: D $\succ$ I $\succ$ K.

The first two profiles are the only admissible ones in any model of pure discounting, and correspond to the third column in Tables 6 above. Observations in any of the other cells of the columns could only be due to mistakes. Yet, the association between rows and columns in Tables 6 is immediately apparent. In fact the Fisher-Freeman-Halton exact test strongly rejects the null hypothesis of independence (the exact p-value is less than 0.001 for both tables). We conclude that no model of positive discounting is compatible with our data.

As we shall see, on the contrary the association in Tables 6 is to be expected in the (σ-δ) model, as the choice between D and K *is* informative on the distribution of the other choices.

Indeed, as we saw in section 4, the (σ-δ) model establishes that:

$I2 \succ^* K2 \Leftrightarrow \sigma \geq a$	$I3 \succ^* K3 \Leftrightarrow \sigma \geq A$
$I2 \succ^* D2 \Leftrightarrow \sigma \geq b$	$I3 \succ^* D3 \Leftrightarrow \sigma \geq B$
$D2 \succ^* K2 \Leftrightarrow \sigma < c$	$D3 \succ^* K3 \Leftrightarrow \sigma < C$

where a, A, b, B, c and C are real numbers whose precise values depend on the utility function and on the discount factor. Regardless of these values, however, it is always the case that c<a,b and C<A,B. Thus whenever a decision maker chooses D over K, it **must** be the case that he chooses K over I and D over I. On the other hand, a choice of K over D imposes no such restrictions on the choice between either I and K or I and D.

In short, there are only five (out of the eight possible) preference profiles which are compatible with the (σ-δ) model, as in Figure 2. Incidentally, given our observation that the proportion of subjects choosing I over D is much higher than the proportion of subjects choosing I over K, this seems to indicate that A≻B, so that the bottom portion of Figure 2 should be the one that applies, i.e. we should expect to find no subjects whose choices conform to profile 3.

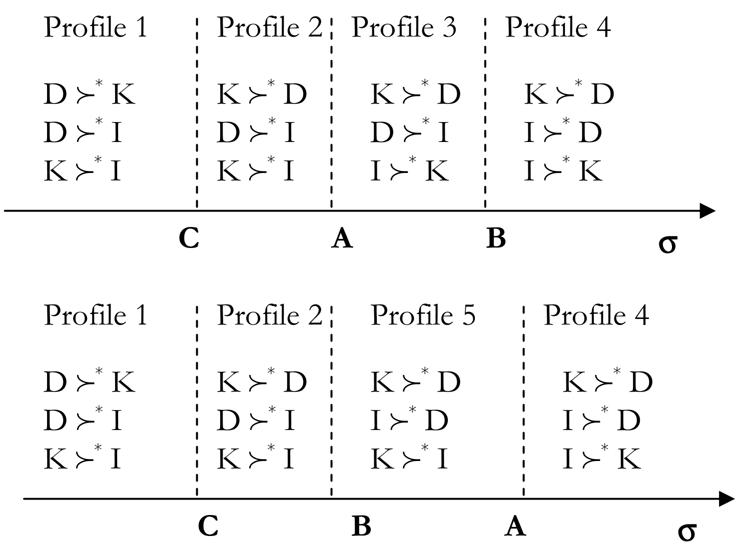

Figure 2 : admissible profiles in the (σ-δ) model

One can proceed similarly for the two period profiles, exchanging a for A, b for B and c for C.

Combining two and three period choices

In addition, we can examine the relationship that our model postulates between choice profiles in the two and three period cases. As we show in the appendix, the choice profile for sequences of a given length may determine the choice profile for the sequences of other length. In particular, the juxtaposition of the two graphs for the choice profiles in the case of two and three period sequences reveals that a switch either from $D \succ^* K \succ^* I$ to $K \succ^* D \succ^* I$ or the opposite switch from $K \succ^* D \succ^* I$ to $D \succ^* K \succ^* I$ with sequence length is possible. This point is visualized in Figure 3.

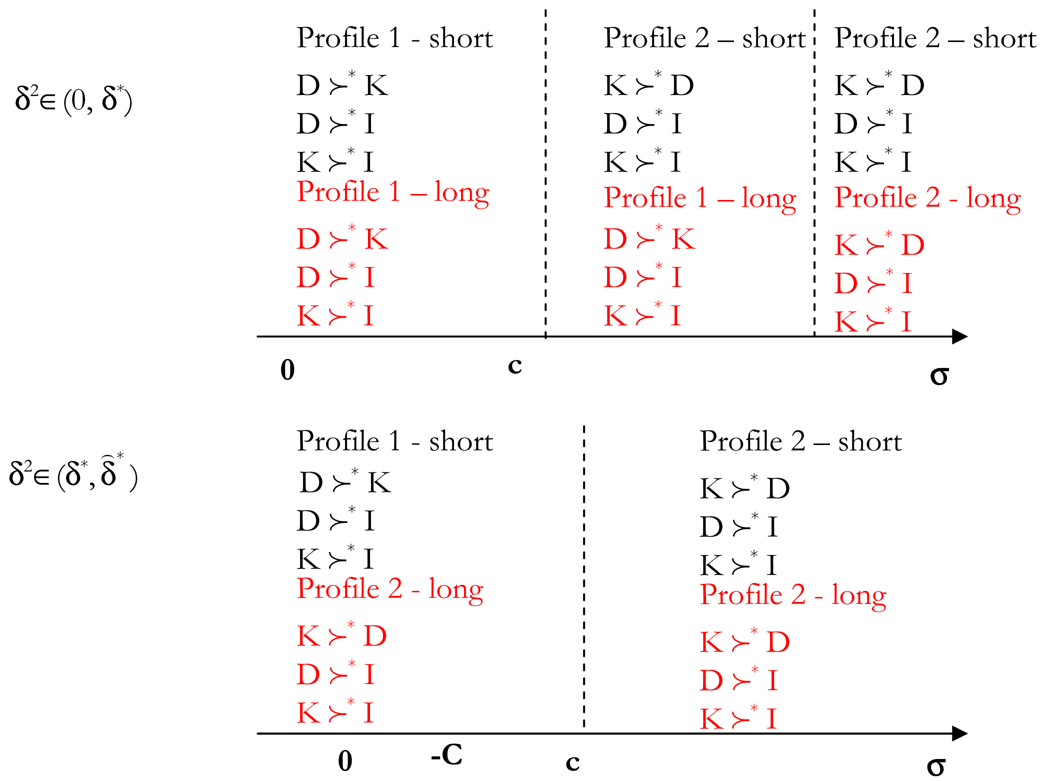

Figure 3: compatible profiles in choices for two and three period sequences

Recall that $\delta^* \equiv (u_3-u_2)/(u_2-u_1)$ and $\hat{\delta}^* \equiv (u_4-u_3)/(u_3-u_2)$, so that with our assumption on the concavity of the utility function it is the case that $\delta^* < \hat{\delta}^*$ always. In the top panel of Figure 3, which applies whenever $\delta^2 \in (0, \delta^*)$, a $(\sigma\text{-}\delta)$ decision maker whose σ is greater than c will exhibit the choice profile $K \succ^* D \succ^* I$ when choosing among two period sequences, and the profile $D \succ^* K \succ^* I$ when choosing among three period sequences. An opposite switch will be displayed by an individual whose discount factor is such that $\delta^2 \in (\delta^*, \hat{\delta}^*)$, and for whom the value of σ is smaller than c.

In contrast, only one of these switches is admissible according to pure discounting theories. Recall that D is chosen over K if and only if the two period discount function is sufficiently small, with the smaller threshold δ^* applying to the case of three period sequences, and the larger threshold

$\widehat{\delta}^*$ applying to two period sequences. We show this in Figure 4 below, only the switch $K \succ_d D \succ_d I$ to $D \succ_d K \succ_d I$ is possible when increasing sequence length.

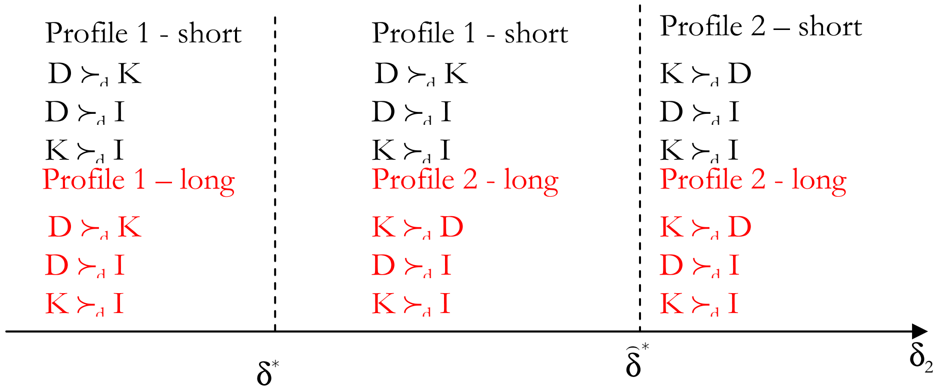

Figure 4: choice profiles admissible by discounting models

Remarkably, then, despite the fact that the (σ-δ) model uses exponential discounting, in this experimental setup it necessarily explains more choice profiles than *any* pure discounting theory.[18]

How about the data? We display in Table 7 the cross-tabulation of the choice profiles observed in two period (columns) and three period (rows) sequences. Number in parentheses refer to the overall percentage of cases. For legibility, diagonal observations (where no change in choice profiles is observed with sequence length) are in bold; groups of subjects whose preference profile amount to at least 5% of the total are highlighted in italics. Combinations of choice profiles compatible with the (σ-δ) model are underlined.

3\2	D≻K≻I	K≻D≻I	(D≻I≻K)	I≻K≻D	K≻I≻D	(D≻K≻I)	I≻D≻K	D≻I≻K	Total
D≻K≻I	***55 (53.9%)***	*6 (5.9%)*	0	0	0	1	0	1	63 (61.8)
K≻D≻I	*8 (7.8%)*	***9 (8.8%)***	1	0	0	0	0	0	18 (17.6)
(D≻I≻K)	0	0	**0**	1	0	0	0	0	1 (1)
I≻K≻D	0	0	0	**2**	2	1	0	0	5 (4.9)
K≻I≻D	0	1	0	3	***7 (6.9%)***	1	0	0	12 (11.8)
(D≻K≻I)	0	0	0	0	1	**1**	0	0	2 (2)
I≻D≻K	0	0	0	0	0	0	**0**	0	0
D≻I≻K	0	0	0	0	1	0	0	**0**	1 (1)
Total	63 (61.8)	16 (15.7)	1 (1)	6 (5.9)	11 (10.8)	4 (3.9)	0	1 (1)	102 (100)

Table 7: choice profiles for two and three periods sequences, PAY treatment

[18] Obviously this is not the case in general.

The 6 subjects whose preferences fall in the (1, 2) cell cannot be accommodated within any pure discounting model, nor can the other 20 subjects whose choice profiles fall anywhere in the table apart from the first 2 by 2 submatrix.

All in all, then, about 30% of subjects (i.e. 30 out of 102) display a pattern of choice incompatible with *any* discounting model. To the contrary, only 7 subjects' choice behavior is incompatible with the (σ-δ) model.[19]

We summarize the ability of pure discounting theories and of the (σ-δ) model to explain data in the following table, where the proportion of subjects whose choices cannot be accounted in standard and non-standard (e.g. hyperbolic) discounted utility frameworks is more than three and a half times than in the (σ-δ) model:

	explained	Unexplained	Total
Any discounting	72 (70.6%)	30 (29.4%)	102 (100%)
(σ-δ) model	94 (92.2%)	8 (7.8%)	102 (100%)

Table 8: explanatory power of different theories, PAY treatment

Since the two models are nested, in order to compare differences in explanatory power we take a conservative approach, and look at the 95% exact confidence intervals[20] for the proportion of choice profiles compatible with each model. The 95% exact confidence interval for the proportions of subjects whose choice profiles are compatible with the two theories are [0.610, 0.792] for the pure discounting model and [0.851, 0.965] for the (σ-δ) model. Since the latter confidence interval lies entirely above the one for the pure discounting model, we take it as compelling evidence that the explanatory power of the (σ-δ) model is significantly higher than any other discounting theory.

A role for sex?

We will show shortly that the two sexes exhibit distinct patterns of choice: women tend to prefer the constant sequence to the decreasing one significantly more often than men; in addition, women tend to choose steeper (i.e. more irrational) sequences significantly less often than men. Yet, in spite of this heterogeneity among the sexes, the (σ-δ) model describes well choice behavior for both sexes.

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[19] Arguments similar to those used in the main text to explain preference shift with sequence length can be used to show that for the 3 subjects exhibiting the shift from $I2 \succ K2 \succ D2$ to $K3 \succ I3 \succ D3$, the change is compatible with the (σ-δ) model if $\delta^2 \in (\delta^*, \overline{\delta})$ and $\sigma \in (a,A)$. For the 2 subjects with the opposite shift one needs $\delta^2 \in (\overline{\delta}, \overline{\gamma})$ $\sigma \in (b,A)$. For the subject with profiles ($D2 \succ I2 \succ K2 \succ D2$) and $K3 \succ D3 \succ I3$ we need $\delta^2 < \delta^*$ and $\sigma \in (a, A)$, and for the subject with choice profiles $I2 \succ K2 \succ D2$ and ($D3 \succ I3 \succ K3 \succ D3$) we need $\delta^2 < \delta^*$ and $\sigma \in (b, B)$.

[20] These are computed with the Blyth-Still-Casella method, which, though less conservative than others, is still quite conservative (in the sense that the coverage probability can be much greater than the nominal confidence level in small samples).

The key seems to lie in the fact that for both sexes there is a non negligible proportion of subjects whose irrational choices (i.e. preference for increasing sequences) is associated with a preference for the constant to the decreasing sequence, a pattern which can be accommodated within the (σ-δ) model but not within any pure discounting theory of time preferences. The frequency distribution of pairwise choices in Table 9. Out of the 102 experimental subjects in this treatment, 55 (i.e. around 54%) were males and 47 (i.e. around 46%) females.

In addition:

a. *Women prefer flatter sequences:* For the two period sequences, the proportion of women preferring I to D is much higher than those preferring the steeper sequence J to D (these proportions are 27.7.3% and 6.4%, respectively). This difference is statistically significant (McNemar's p-value is smaller than 0.001 in both cases). For three period sequences, though, where the 'jump' dimension of sequence J kicks in, the difference in the proportion of women choosing I over D and choosing J over D (25.5% and 14.9%, respectively) is not significant;

b. *Women like K better than D more than men do:* the proportion of women preferring the K over the D sequence is much larger than for men (38.3% against 29.1% for the short sequences and 44.7% against 27.3% in the long sequences); this difference is significant for the longer sequences (Fisher's mid p-values are 0.169 and 0.037 and for short and longer sequences, respectively).

c. *Women like I better than D more than men do*: the proportion of women preferring I over D is roughly twice the corresponding proportion of men independently of sequence length (i.e. 27.7% for women and 14.5% for men in the case of two period sequences, and 25.5% and 12.7% for women and men, respectively, in the case of three period sequences), and this difference is statistically significant at 10% confidence level (Fisher's mid-p values are 0.057 and 0.055 for short and long sequences, respectively).

d. *Both women and men choose I over D more often than they choose I over K*: the proportion of women preferring I to K is much smaller than the proportion of women choosing I over D, and this difference is statistically significant (McNemar's p-values are 0.002 and 0.019 for short and long sequences, respectively). For men this difference is not statistically significant (McNemar's p-values are and 0.144 and 0.062 for the short and long sequences, respectively). The difference in the proportions of men and women choosing I over D is not statistically significant at 5% level.

e. *The two sexes respond to end effects in a different way.* When the comparison is between J and D, there is no significant difference in proportions of men preferring J over D in short and long sequences; for women this proportion increases from 6.4% to 14.9%, and this difference is statistically significant at 10% level (McNemar's p-value is 0.062). However, when considering the comparison between J and K, about twice as many men choose J over D than they choose J over K in the case of three period sequences. This difference is significant at 10% level (Mc Nemar's p-value is 0.062).

Two period sequences						Three period sequences				
	Females		Males				Females		Males	
Value	Freq.	%	Freq.	%		Value	Freq.	%	Freq.	%
$K2 \succ I2$	43	91.5	51	92.7	K versus I	$K3 \succ I3$	42	89.4	53	96.4
$I2 \succ K2$	4	8.5	4	7.3		$I3 \succ K3$	5	10.6	2	3.6
$D2 \succ K2$	29	61.7	39	70.9	D versus K	$D3 \succ K3$	26	55.3	40	72.7
$K2 \succ D2$	18	38.3	16	29.1		$K3 \succ D3$	21	44.7	15	27.3
$I2 \succ D2$	13	27.7	8	14.5	I versus D	$I3 \succ D3$	12	25.5	7	12.7
$D2 \succ I2$	34	72.3	47	85.5		$D3 \succ I3$	35	74.5	48	87.3
$I2 \succ J2$	43	91.5	51	92.7	I versus J	$I3 \succ J3$	45	89.4	48	87.3
$J2 \succ I2$	4	8.5	4	7.3		$J3 \succ I3$	2	10.6	7	12.7
$K2 \succ J2$	44	93.6	47	85.5	K versus J	$K3 \succ J3$	42	89.4	51	92.7
$J2 \succ K2$	3	6.4	8	14.5		$J3 \succ K3$	5	10.6	4	7.3
$J2 \succ D2$	3	6.4	7	12.7	J versus D	$J3 \succ D3$	7	14.9	9	16.4
$D2 \succ J2$	44	93.6	48	87.3		$D3 \succ J3$	40	85.1	46	83.6

Table 9: frequency distribution of binary choice by sex, PAY treatment

Putting together this evidence with the points on aggregate data from Table 2 which we dealt with previously, we conclude the following:

I. Points 3, c and d suggest that the higher frequency with which subjects choose I over D as compared to I over K in the aggregate data is driven by female subjects;

II. Points 1 and e suggest that the increase with sequence length in the proportion of subjects observed choosing J over D in the aggregate data is mainly driven by women, who seem to be more sensitive than men to end effects. In this same direction, note that – unlike men - the proportion of women choosing J over K increases with sequence length, when the end effect kicks in (though this change is not statistically significant).

III. Points 4 and e suggest that men are more sensitive to end effects (in a negative way) involving sequence J when the comparison is with the D rather than the K sequence.

IV. Points 3 and c suggest that the large proportion of subjects who prefer I to D in aggregate data is mainly due to female subjects.

Regarding the various forms of irrationality, the same kind of pattern that we saw for aggregate data is repeated within each sex: a preference for I over either K or D is far more frequent in subjects who also choose K over D:

Females

	K2≻ I2	I2≻ K2	D2≻ I2	I2 ≻ D2
D2≻ K2	29	0	25	4
K2≻ D2	14	4	9	9

Males

	K2≻ I2	I2≻ K2	D2≻ I2	I2≻ D2
D2≻ K2	38	1	39	0
K2≻ D2	13	3	8	8

Table 10 : rational and irrational choices for two period sequences

Females

	K3≻ I3	I3≻ K3	D3≻ I3	I3 ≻ D3
D3≻ K3	25	1	25	1
K3≻ D3	17	4	10	11

Males

	K3≻ I3	I3≻ K3	D3≻ I3	I3≻ D3
D3≻ K3	40	0	39	1
K3≻ D3	13	2	9	6

Table 11: rational and irrational choices for three period sequences

Finally, distinguishing by sex, preference profiles show similar patterns for both groups, namely there are more preference profiles compatible with the (σ-δ) model than with pure discounting theories. The preference profiles are summarized in Table 12.

Code	Profile	2 periods				3 periods			
		Females		Males		Females		Males	
		count	%	count	%	Count	%	count	%
1	D ≻ K ≻ I	25	53.2	38	69.2	24	51.1	39	70.9
2	K ≻ D ≻ I	9	19.2	7	12.7	10	21.3	8	14.6
3	D ≻ I ≻ K ≻ D	0	0	1	1.8	0	0	1	1.8
4	I ≻ K ≻ D	4	8.5	2	3.6	4	8.5	1	1.8
5	K ≻ I ≻ D	5	10.6	6	10.9	7	14.9	5	9.1
6	D ≻ K ≻ I ≻ D	4	8.5	0	0	1	2.1	1	1.8
7	I ≻ D ≻ K	0	0	0	0	0	0	0	0
8	D ≻ I ≻ K	0	0	1	1.8	1	2.1	0	0
	Total	47	100.0	55	100.0	47	100.0	55	100.0

Table 12: choice profiles for two and three periods sequences distinguishing by sex, PAY treatment

5.2 HYP treatment

In the HYP treatment a total of 56 subjects (in roughly equal proportion across sexes) were paid just for taking part to the experiment, i.e. the choice they made were purely hypothetical. We ran this treatment to control whether any differences would arise in comparison with the incentive compatible choices of the PAY treatment. Frequencies are reported in Table 13. The figures in parentheses refer to the corresponding frequencies in the PAY treatment.

As for the PAY treatment, sequence length seems not to matter much: the only change in proportions which is statistically significant (at 10% confidence level) concerns the choice between I and D, where the proportion of irrational choices increases with sequence length (McNemar's p-value is 0.089).

Two period sequences Value	Freq.	%		Three period sequences Value	Freq.	%
K2 ≻ I2	48	87.7 (92.2)	K versus I	K3 ≻ I3	49	87.5 (93.1)
I2 ≻ K2	8	14.3 (7.8)		I3 ≻ K3	7	12.5 (6.9)
D2 ≻ K2	36	64.3 (66.7)	D versus K	D3 ≻ K3	34	60.7 (64.7)
K2 ≻ D2	20	35.7 (33.3)		K3 ≻ D3	22	39.3 (35.3)
I2 ≻ D2	11	19.6 (20.6)	I versus D	I3 ≻ D3	16	28.6 (18.6)
D2 ≻ I2	45	80.4 (79.4)		D3 ≻ I3	40	71.4 (81.4)
I2 ≻ J2	55	98.2 (92.2)	I versus J	I3 ≻ J3	52	92.9 (91.2)
J2 ≻ I2	1	1.8 (7.8)		J3 ≻ I3	4	7.1 (8.8)
K2 ≻ J2	50	89.3 (89.2)	K versus J	K3 ≻ J3	51	91.1 (91.2)
J2 ≻ K2	6	10.7 (10.8)		J3 ≻ K3	5	8.9 (8.8)
J2 ≻ D2	6	10.7 (9.8)	J versus D	J3 ≻ D3	8	14.3 (15.7)
D2 ≻ J2	50	89.3 (90.2)		D3 ≻ J3	48	85.7 (84.3)

Table 13: frequency distribution of binary choices, HYP treatment

In comparisons *across* treatments, the percentage of subjects choosing I over either K or J is higher in the HYP than in the PAY treatment, regardless of sequence length. The proportions of subjects choosing I over D are similar across treatments in the case of short sequences, while for long sequences the proportions of agents preferring I over D in the HYP treatment is much higher than in the PAY treatment. These differences, though, are only significant at 10% level for the differences in proportions across treatments for choice ID3 and for choice IJ2 (Fisher's mid-p values are 0.08 and 0.063, respectively). All in all, then, there is some evidence that incentive compatible choices do make a difference, and that when real money is involved 'irrational' choices are less frequent. However this evidence is weak, and the fact remains that the overwhelming majority of subjects does prefer a 'rational' (i.e. either constant of decreasing) over an 'irrational' sequence. In addition, as for the PAY treatment, in the HYP treatment too the proportions of subjects choosing I is greater when the choice is against D than against K (proportions of I over D and I over K are 19.6% and 14%, respectively, for the short sequences and 28.8 and 12.5%, respectively, for the long sequences[21]). Finally, the end point effect observed in the PAY treatment is considerably 'damped down' in the HYP treatment, in the sense that there is a modest increase with sequence length in the percentage of subjects preferring J over D. There is no difference in the choice between J and K.

Regarding preference profiles, for the HYP treatment we find again that the distribution of choices between I and D and I and K is strongly associated to the choice between D and K. As we saw earlier, this is in perfect agreement with the (σ-δ) model, unlike pure discounting theories.

Two period sequences

	K2 ≻ I2, I2 ≻ D2	I2 ≻ K2, I2 ≻ D2	K2 ≻ I2, D2 ≻ I2	I2 ≻ K2, D2 ≻ I2
D2 ≻ K2	0	0	35	1
K2 ≻ D2	5	6	8	1

Three period sequences

	K3 ≻ I3, I3 ≻ D3	I3 ≻ K3, I3 ≻ D3	K3 ≻ I3, D3 ≻ I3	I3 ≻ K3, D3 ≻ I3
D3 ≻ K3	2	0	31	1
K3 ≻ D3	9	5	7	1

Tables 14: choices including and excluding the increasing sequence, HYP treatment

We report in Table 15 the cross-tabulation of choice profiles across sequence length, where as in the corresponding table for the PAY treatment we underline the combinations of choice profiles compatible with the (σ-δ) model:

[21] Statistically significant only for long sequences (McNemar's p-value is 0.01).

3\2	D ≻K≻I	K ≻D ≻I	(D≻I ≻K)	I ≻K≻D	K≻I≻D	(D≻K≻I)	I≻D≻K	D≻I≻K	Total
D≻K≻I	***30 (53.6%)***	1	0	0	0	0	0	0	31 (55.6)
K≻D≻I	2	***4 (7.1%)***	0	0	0	0	0	1	7 (12.5)
(D≻I≻K)	0	0	**0**	1	0	0	0	0	1 (1.8%)
I≻K≻D	0	0	0	***5 (8.9)***	0	0	0	0	5 (8.9)
K≻I≻D	2	*3 (5.3)*	0	0	***4 (7.1%)***	0	0	0	9 (16.7)
(D≻K≻I)	1	0	1	0	0	**0**	0	0	2 (3.6)
I≻D≻K	0	0	0	0	0	0	**0**	0	0
D≻I≻K	0	0	0	0	1	0	0	**0**	1 (1.8%)
Total	35 (62.5%)	8 (14.3)	1 (1.8%)	6 (10.7)	5 (8.9)	0	0	1	56 (100)

Table 15: choice profiles for two and three periods sequences, HYP treatment

As for the PAY treatment, we summarize below (see Table 16) the explanatory power of alternative theories.[22]

Although in this treatment both classes of explanations fare worse than in the PAY treatment, the ability of the (σ-δ) model to explain the data is far greater than that of any pure discounting theory (combinations of preference profiles incompatible with the (σ-δ) model are still less than half those incompatible with standard discounting theories):

	Explained	Unexplained	total
Any discounting	36 (64.3%)	20 (35.7%)	56 (100%)
(σ-δ) model	47 (83.9%)	9 (16.1%)	56 (100%)

Table 16: explanatory power of different theories, HYP treatment

For this treatment evidence between the different 'explanatory' power of each theory is less straightforward than for the PAY treatment, in the sense that the 90% Blyth-Still-Casella exact confidence intervals for the proportions of choice profiles compatible with the two models are [0.529, 0.747] for the pure discounting model and [0.747, 0.907] for the (σ-δ) model, that is they intersect just at one extreme.[23]

Finally, when distinguishing by sex (see Table 17) there are some statistically significant changes as compared to the PAY treatment at the 5% level.

[22] Please refer to the discussion of the PAY treatment for the conditions establishing which combinations of preference profiles are compatible with the (σ-δ) model

[23] The overlap is obviouusly larger with 95% confidence intervals, which are [0.514, 0.766] and [0.719, 0.916] for pure discounting and (σ-δ) model, respectively.

	Two period sequences						Three period sequences			
	Females		Males				Females		Males	
Value	freq	%	freq	%		Value	freq	%	freq	%
K2≻I2	27	93.1 (91.5)	21	77.8 (92.7)	K versus I	K3≻I3	27	93.1 (89.4)	22	81.5 (96.4)
I2≻K2	2	6.9 (8.5)	6	22.2 (7.3)		I3≻K3	2	6.9 (10.6)	5	18.5 (3.6)
D2≻K2	20	69 (61.7)	16	59.3 (70.9)	D versus K	D3≻K3	18	62.1 (55.3)	16	59.3 (72.7)
K2≻D2	9	31 (38.3)	11	40.7 (29.1)		K3≻D3	11	37.9 (44.7)	11	40.7 (27.3)
I2≻D2	4	13.8 (27.7)	7	25.6 (14.5)	I versus D	I3≻D3	8	27.6 (25.5)	8	29.6 (12.7)
D2≻I2	25	86.2 (72.3)	20	74.4 (85.5)		D3≻I3	21	72.4 (74.5)	19	70.4 (87.3)
I2≻J2	28	96.6 (91.5)	27	100 (92.7)	I versus J	I3≻J3	28	96.6 (89.4)	24	88.9 (87.3)
J2≻I2	1	3.4 (8.5)	0	0 (7.3)		J3≻I3	1	3.4 (10.6)	3	11.1 (12.7)
K2≻J2	26	89.7 (93.6)	24	88.9 (85.5)	K versus J	K3≻J3	28	96.6 (89.4)	23	85.2 (92.7)
J2≻K2	3	10.3 (6.4)	3	11.1 (14.5)		J3≻K3	1	3.4 (10.6)	4	14.8 (7.3)
J2≻D2	2	6.9 (6.4)	4	14.8 (12.7)	J versus D	J3≻D3	3	10.3 (14.9)	5	18.5 (16.4)
D2≻J2	27	93.1 (93.6)	23	85.2 (87.3)		D3≻J3	26	89.7 (85.1)	22	81.5 (83.6)

Table 17: frequency distribution of binary choice by sex

In particular, for male subjects the proportion of decision makers who prefer I over either 'rational' sequence (i.e. D or K) increases significantly at 5% confidence level for the ID3 choice (Fisher's mid-p value is 0.040), KI2 (mid p-value equal to 0.036) and KI3 (mid p-value equal to 0.005), and at lower confidence levels for IJ2 (mid p-value of 0.09). For female subjects there appear to be no statistically significant changes, with the exception of choice ID2, where the increase in the proportion of subjects choosing D over I is significant at the 10% confidence level (Fisher's mid p-value is 0.084). We report in parentheses the corresponding percentages observed in the PAY treatment.

6. Generalizing the secondary criterion: Some preliminary considerations

This section is highly exploratory and tentative. We investigate further the nature of the secondary criteria. The analysis of simple plans like K, I and D considered so far has advantages and disadvantages. The main advantage is that the attribute of the sequences to be used in the secondary criterion is pretty unambiguous. The main disadvantage is that many alternative criteria collapse and yield the same prediction, so that it is hard to get indications about what secondary criterion agents would use for more complicated sequences. In particular, it could be that decision makers do not really look directly at increasingness, but are rather sensitive to *total net cumulative changes* in utility. Both this criterion and the increasingness criterion would yield the same ranking when applied only to the sequences K, I and D.

With this in mind, we introduced two more sequences, while at the same time trying to keep 'noise' at a minimum. Plan J can perform several roles in our analysis. First of all, over two periods, plan J2 is simply an increasing sequence steeper than I2. On the other hand, for the longer plans the jump element of sequence J3 kicks in, and the role of an end effect would become apparent if the proportion of subjects choosing I over J were to change when moving from shorter to longer sequences. Similarly, if the preference for the increasingness of a sequence is an expression of the cumulative change secondary heuristics, one might hope to observe additional patterns of association beside those discussed so far. To address this point, in the 23rd question mentioned in the discussion of the experimental design, we asked our experimental subjects to choose between a permutation of the three period I and J sequences, where all terms had been 'shifted' forward. That is, subjects were asked to choose between the sequences SJ=(8, 32, 8) and SI=(16, 24, 8). The SJ sequence yields a total net cumulative change of 0, whereas the SI sequence yields a total net cumulative change of minus 8. So, *when vague* a decision maker whose preferences conform to the (σ-δ) model should prefer SJ to SI.

As we show in the appendix, the following implications hold for a subject whose preference conform to the (σ-δ) model and who uses the cumulative change secondary criterion:

1. The choice of J2 over I2 implies the choice of SJ over SI.
2. The choice of J2 over I2 implies the choice of J3 over I3.
3. The choice of J2 over D2 implies the choice of J3 over D3.

That is, knowledge of the choice between J2 and either I2 or D2 should help predict the choice in other binary comparisons. As a rough and ready rule of thumb, the first two predictions would be violated if the proportions of subjects choosing J2 over I2 were higher than proportion of subjects choosing the 'jump related' plan. Similarly, the third prediction would be violated if the proportions of subjects who chose J2 over D2 were higher than the proportion of subjects choosing J3 over D3.

In fact, as shown in Table 2, neither the first nor the third prediction is violated. Similarly, the proportion of individuals choosing SJ over SI is not smaller (indeed higher) than the proportion of subjects choosing J2 over I2):

Value	Freq.	%
SJ ≻ SI	15	14.7
SI ≻ SJ	87	85.3

Table 18 : frequency of choices for SJ against SI, PAY treatment

These conclusions are supported by a more formal investigation of the three hypotheses, where we study the association patterns between choices. Statistically, we interpret the hypotheses 1, 2 and 3 as statements of the form: 'the choice of x over y increases the odds of choosing w over z'.

Consider the three cross-tabulations between the two choices involved in each of the hypotheses, and let us check the null hypothesis of lack of association.

	I2≻ J2	J2≻ I2
SJ≻ SI	9	6
SI≻ SJ	85	2

	I2≻ J2	J2≻ I2
I3≻ J3	88	5
J3≻ I3	6	3

	J2≻D2	D2≻ J2
J3≻ D3	9	7
D3≻ J3	1	85

Table 19 : the three hypotheses involving choices involving sequence J2, PAY treatment

The odd ratios for the tables from left to right are 0.03529 (the exact p-value for a test of independence is 0.0001759), 8.8 (the exact p-value is 0.04332) and 109.3 8 (the exact p-value is 0.0031242), respectively. The 95% confidence intervals are [0.003272, 0.2484], [1.063, 57.94], and [11.21, 4839], respectively.

In summary, there is dramatic evidence to reject lack of association between the choice of SJ versus SI and of I2 versus J2. Similarly, there is very strong evidence against the lack of association between the choice of J3 versus D3 and the choice of J2 versus D2. There is weaker, but still strong, evidence against the lack of association between the choice of I3 versus J3 and the choice of I2 versus J2.

Results look remarkably similar when performing the same sort of analysis for the HYP treatment:

	I2≻ J2	J2≻ I2
SJ≻ SI	2	1
SI≻ SJ	53	0

	I2≻ J2	J2≻ I2
I3≻ J3	52	0
J3≻ I3	3	1

	J2≻D2	D2≻ J2
J3≻ D3	5	3
D3≻ J3	1	47

Table 20 : the three hypotheses involving choices involving sequence J2, HYP treatment

In this case, though, the sparseness of the table makes exact tests very discrete, so that independence can be rejected for the rightmost panel only. The odd ratios for the tables from left to right are infinity, (the exact p-value for the test of independence is 0.10), 0 (the exact p-value is 0.143) and 0.013 (the exact p-value is 0.0002), respectively. The 95% exact confidence intervals are [0.453, +∞), [0, 3], and [0.0003, 0.19], respectively.

As we stated above, these are only tentative findings. However, (1) they reinforce the general theme that departures from pure discounting cannot be explained by random errors and (2) they are consistent with the use on the part of decision makers of a more general secondary criterion than just increasingness.

7. Concluding remarks

We have proposed a simple model of choice between sequences of monetary rewards with exponential discounting as one of its core elements, the other core element being a 'secondary heuristics'. This 'hybrid model' which combines the traditional, consistent discounting, theory with a heuristics component, is very successful at explaining choices between time sequences with an obvious trend (increasing, constant or decreasing). Neither a pure discounting model (of any type) nor a pure heuristics model can explain the data well (though discounting alone does much better than heuristics alone).

Encouragingly, the general pattern of choice we have uncovered in our experiment is consistent with data found elsewhere. Notably, in Gigliotti and Sopher (1997), depending on treatment, the choice profiles of up to 80%-90% of their experimental subjects fall into the five choice profiles compatible with our (σ-δ) model.

We conclude with a few comments on the 'context-dependence' of the secondary heuristics. Does that mean that our theory is 'ad hoc', because we are free to tailor the secondary criterion to the data set we are trying to explain? For example, we used outcome or time prominence when studying date-outcome pairs, we used Pareto dominance when studying Rubinstein's experiment, and we used sequence trend or cumulative gains when studying sequences. However, there is nothing specially 'ad hoc' about this. As we have already remarked, at the abstract level, our model departs from the standard choice theoretic model in just one way, by positing two sequential incomplete (but transitive) preference relations instead of a complete one. So, the secondary criterion is no more context-dependent than any preference relation is: different preference relations will apply (by definition) to different sets of objects. We are not arguing here that different rankings ought to apply to the *same* objects in different contexts (though we do not exclude this possibility). Nonetheless, it is true that - because we interpret the secondary criterion as a heuristic tied to some *salient* feature of the objects - it will generally be easier to glean intuition about an individual's secondary criterion than about his general preferences. We view this as a strength of the approach, since it makes the abstract model more easily adaptable to specific circumstances.

From our perspective, the search to uncover the nature of the secondary heuristic in cases different from those considered so far ought to be one of the main empirical developments of the theory proposed in this paper.

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Appendices

A1. Preference profiles in the (σ-δ) model

A1.i Three period sequences

We provide a full derivation of the results presented in sections 4 and 5 in the main text. Recall that we are making the following assumptions: u is monotonic increasing in outcome, concave and with positive third and the discounting function is monotonically non increasing.

Moreover, we are fixing the times at which outcomes are received as 0, 1 and 2, and for simplicity we denote u_1=u(8), u_2=u(16), u_3=u(24) and u_4=u(32); and for the discounting function let $\delta(0)= \delta_0$, $\delta(1)= \delta_1$, and $\delta(2)=\delta_2$. The three period sequences of payments considered by experimental subjects were the following:

I=(8, 16, 24)

K=(16, 16,16)

D=(24, 16, 8)

J=(8, 8, 32)

As we have already pointed out, any pure discounting criterion for choice $\succ_d$ should order them as either $D \succ_d K \succ_d I \succ_d J$ or $K \succ_d D \succ_d I \succ_d J$.

Denoting by $\succ_d$ the preference relation of a decision maker who discounts utility available at time t by some monotonically non decreasing discount function δ_t, then:

$$D \succ_d K \Leftrightarrow \delta_2 < \frac{u_3 - u_2}{u_2 - u_1} \tag{1}$$

holds since:

$$D \succ_d K \Leftrightarrow u_3 + \delta_1 u_2 + \delta_2 u_1 > u_2 + \delta_1 u_2 + \delta_2 u_2$$

$$\Leftrightarrow \delta_2 < \frac{u_3 - u_2}{u_2 - u_1}$$

Secondly,

$$D \succ_d I \text{ always} \tag{2}$$

holds since:

$$D \succ_d I \Leftrightarrow u_3 + \delta_1 u_2 + \delta_2 u_1 > u_1 + \delta_1 u_2 + \delta_2 u_3$$

$$\Leftrightarrow u_3 - u_1 > \delta_2 (u_3 - u_1)$$

and

$$K \succ_d I \text{ always}$$

holds since:

$$K \succ_d I \Leftrightarrow u_2 + \delta_1 u_2 + \delta_2 u_2 > u_1 + \delta_1 u_2 + \delta_2 u_3$$

$$\Leftrightarrow u_2 - u_1 > \delta_2 (u_3 - u_2)$$

Finally

$$I \succ_d J \text{ always} \tag{3}$$

holds (with a nondecreasing discount function) since:

$$I \succ_d J \Leftrightarrow u_1 + \delta_1 u_2 + \delta_2 u_3 > u_1 + \delta_1 u_1 + \delta_2 u_4$$

$$\Leftrightarrow \delta_1 (u_2 - u_1) > \delta_2 (u_4 - u_3)$$

Then, the only two patterns of choice consistent with pure discounting theories are $D \succ_d K \succ_d I \succ_d J$ and $K \succ_d D \succ_d I \succ_d J$.

We can now turn to check what preference profiles are compatible with the (σ-δ) model. We begin by enumerating them, then we show how the various restrictions on the parameters have been derived.

As we explained in the main text, in the (σ-δ) model the secondary criterion alone implies $I \succ_2 K \succ_2 D$, where $\succ_2$ is transitive.

The profiles of choices based on all pairwise comparisons involving the three series I, K and D which are compatible with the model of vague time preferences are as follows:

1. $D \succ^* K \succ^* I$: this can be if $\sigma<(u_3-u_2)- \delta^2(u_2-u_1)$ and $\delta_2 < \dfrac{u_3 - u_2}{u_2 - u_1}$. For this profile of preferences to obtain it is necessary that $D \succ_1 K \succ_1 I$ and $D \succ_1 I$. This corresponds to the standard preferences of a rational exponential discounting decision maker.

2. $K \succ^* D \succ^* I$: this can be if

(a) $\sigma<(1-\delta^2)(u_3-u_1)$ and $\delta^2 \in \left(\frac{(u_3-u_1)+(u_3-u_2)}{(u_2-u_1)+(u_3-u_1)},1\right)$

(b) $\sigma<\delta^2(u_2-u_1)-(u_3-u_2)$ and $\delta^2 \in \left(\frac{u_3-u_2}{u_2-u_1},\frac{(u_3-u_1)+(u_3-u_2)}{(u_2-u_1)+(u_3-u_1)}\right)$

(c) $\sigma\in[(u_3-u_2)-\delta^2(u_2-u_1),(u_3-u_2)-\delta^2(u_3-u_2))$ and $\delta^2 \in \left(0,\frac{u_3-u_2}{u_2-u_1}\right)$

For this profile of preferences to obtain it is necessary that $K \succ_{1,2} D \succ_1 I$ and $K \succ_1 I$.

3. $D \succ^* I \succ^* K \succ^* D$: this can be if $\sigma\in[(u_2-u_1)-\delta^2(u_3-u_2),(1-\delta^2)(u_3-u_1))$ and $\delta^2 < \frac{u_3-u_2}{u_2-u_1}$. For this profile of preferences to obtain it is necessary that $I \succ_2 K \succ_{1,2} D \succ_1 I$.

4. $I \succ^* K \succ^* D$: this can be if $\sigma \geq (1-\delta^2)(u_3-u_1)$ and $\delta^2 < \frac{u_3-u_2}{u_2-u_1}$. For this profile of preferences to obtain it is necessary that $I \succ_2 K \succ_{1,2} D$ and $I \succ_2 D$.

5. $K \succ^* I \succ^* D$: this can be if $\sigma\in[(1-\delta^2)(u_3-u_1),\delta^2(u_2-u_1)-(u_3-u_2))$ and $\delta^2 > \frac{u_3-u_2}{u_2-u_1}$. For this profile of preferences to obtain it is necessary that $K \succ_1 I \succ_2 D$ and $K \succ_{1,2} D$.

6. $D \succ^* K \succ^* I \succ^* D$: this is incompatible with the model (σ-δ) model, since it would require $K \succ_1 I \succ_2 D \succ_1 K$ which is impossible.

7. $I \succ^* D \succ^* K$: this is incompatible with the (σ-δ) model, since it would require $I \succ_2 D \succ_1 K$ and $I \succ_2 K$ which is impossible.

8. $D \succ^* I \succ^* K$: this is incompatible with the (σ-δ) model, since it would require $D \succ_1 I \succ_2 K$ and $D \succ_1 K$, which is impossible.

W now show how the conditions for the preference profiles above to be compatible or otherwise with the (σ-δ) model have been derived.

A) Comparison between series I and K: consider the following inequality (which, if true, would imply that I is preferred to K by the primary criterion):

$$u_1+\delta u_2+\delta^2 u_3>u_2+\delta u_2+\delta^2 u_2+\sigma \Leftrightarrow u_1+\delta^2 u_3>u_2+\delta^2 u_2+\sigma \Leftrightarrow \sigma<\delta^2(u_3-u_2)-(u_2-u_1)<0.$$

The last inequality follows from the fact that $\delta^2<1$ and from our assumption of decreasing marginal utility, so that $(u_2-u_1)>(u_3-u_2)>\delta^2(u_3-u_2)$. Consequently, given that σ must be positive, it can only be that $u_1+\delta u_2+\delta^2 u_3<u_2+\delta u_2+\delta^2 u_2+\sigma$: in other words, it can never be that I 'beats outright' K. Therefore the comparison between the two sequences hinges on whether or not the following holds:

$$u_2+\delta u_2+\delta^2 u_2\leq u_1+\delta u_2+\delta^2 u_3+\sigma \Leftrightarrow \sigma\geq(u_2-u_1)-\delta^2(u_3-u_2)>0$$

In short we can summarize the only possible mutually incompatible cases as follows:

$$I \succ_2 K \Leftrightarrow \sigma \geq (u_2-u_1)-\delta^2(u_3-u_2)\equiv A$$

$$K \succ_1 I \Leftrightarrow \sigma < (u_2-u_1)-\delta^2(u_3-u_2)\equiv A$$

B) Comparison between series I and D: consider the following inequality (which, if true, would imply that a decision maker chooses I over D by the primary criterion)

$$u_1+\delta u_2+\delta^2 u_3>u_3+\delta u_2+\delta^2 u_1+\sigma \Leftrightarrow u_1+\delta^2 u_3>u_3+\delta^2 u_1+\sigma \Leftrightarrow \sigma<-(1-\delta^2)(u_3-u_1)<0$$

As above since we require $\sigma\geq 0$, it can only be that $u_1+\delta u_2+\delta^2 u_3\leq u_3+\delta u_2+\delta^2 u_1+\sigma$, i.e. it can never be that I beats D outright. Therefore the comparison hinges on:

$$u_3+\delta u_2+\delta^2 u_1\leq u_1+\delta u_2+\delta^2 u_3+\sigma \Leftrightarrow u_3+\delta^2 u_1\leq u_1+\delta^2 u_3+\sigma \Leftrightarrow \sigma\geq(1-\delta^2)(u_3-u_1)>0$$

In short we can summarize the only possible mutually incompatible cases as follows:

$$I \succ_2 D \Leftrightarrow \sigma \geq (1-\delta^2)(u_3-u_1)\equiv B$$

$$D \succ_1 I \Leftrightarrow \sigma < (1-\delta^2)(u_3-u_1)\equiv B$$

C) Comparison between series K and D: consider the following inequality (which, if true, would imply that K is chosen over D by the primary criterion):

$$u_2+\delta u_2+\delta^2 u_2>u_3+\delta u_2+\delta^2 u_1+\sigma \Leftrightarrow u_2+\delta^2 u_2>u_3+\delta^2 u_1+\sigma \Leftrightarrow \sigma<\delta^2(u_2-u_1)-(u_3-u_2)$$

The value on the left hand side is positive provided that $\delta^2>\frac{u_3-u_2}{u_2-u_1}\equiv\delta^*$. So if the discount function in period 2 is sufficiently large, then K can beat D outright provided the vagueness term is sufficiently *small*. If instead $\delta^2\leq\delta^*$, then it can never be that K beats D outright, and as in the other cases the comparison hinges on whether:

$$u_3+\delta u_2+\delta^2 u_1 \leq u_2+\delta u_2+\delta^2 u_2+\sigma \Leftrightarrow u_3+\delta^2 u_1 \leq u_2+\delta^2 u_2+\sigma \Leftrightarrow \sigma \quad (u_3-u_2)-\delta^2(u_2-u_1) \quad 0$$

(where the last inequality follows from our assumption that $\delta^2 \leq \delta^*$).

In short we can summarize as follows:

- If $\delta^2 > \frac{u_3-u_2}{u_2-u_1} \equiv \delta^*$, then: $K \succ_1 D$ if $\sigma < \delta^2 (u_2-u_1) - (u_3-u_2) \equiv -C$ and

  $K \succ_2 D$ if $\sigma \geq \delta^2 (u_2-u_1) - (u_3-u_2) \equiv -C$

- If $\delta^2 < \frac{u_3-u_2}{u_2-u_1} \equiv \delta^*$, then: $K \succ_2 D$ if $\sigma \geq (u_3-u_2) - \delta^2 (u_2-u_1) \equiv C$ and

  $D \succ_1 K$ if $\sigma < (u_3-u_2) - \delta^2 (u_2-u_1) \equiv C$

Next we compare the relative magnitudes of A, B, C and -C to see which configurations of preferences of the eight listed above are compatible with the model. Recall:

$$A \equiv (u_2-u_1)-\delta^2(u_3-u_2)>0$$

$$B \equiv (1-\delta^2)(u_3-u_1)>0$$

$$C \equiv (u_3-u_2)-\delta^2 (u_2-u_1)$$

$$-C \equiv <\delta^2 (u_2-u_1) - (u_3-u_2)$$

Observe that

$$A>B \Leftrightarrow \delta^2 > \frac{u_3-u_2}{u_2-u_1} \equiv \delta^*$$

since

$$A>B \Leftrightarrow (u_2-u_1)-\delta^2(u_3-u_2)> (1-\delta^2)(u_3-u_1) \Leftrightarrow \delta^2(u_2-u_1)>(u_3-u_2)$$

Next observe that

A>C always

(since $A>C \Leftrightarrow (u_2-u_1)-\delta^2(u_3-u_2)> (u_3-u_2)-\delta^2 (u_2-u_1) \Leftrightarrow (1+\delta^2)(u_2-u_1)> (1+\delta^2)(u_3-u_2)$, which is always true given our assumptions on the shape of the utility function). Finally, observe that whenever $\delta<\delta^*$, the threshold C is negative. In this case comparing A and -C yields

A>-C always

since:

$$A>-C \Leftrightarrow (u_2-u_1)-\delta^2(u_3-u_2)> \delta^2(u_2-u_1) -(u_3-u_2) \Leftrightarrow (1-\delta^2)(u_2-u_1)> -(1-\delta^2)(u_3-u_2)$$

In the comparison between B and C:

$$B>C \text{ always}$$

since:

$$B>C \Leftrightarrow (1-\delta^2)(u_3-u_1)> (u_3-u_2)- \delta^2 (u_2-u_1) \Leftrightarrow (u_2-u_1) > \delta^2 (u_3-u_2)$$

and the last inequality holds by our assumption on the shape of the utility function. Finally,

$$B>-C \Leftrightarrow \delta^2 < \frac{(u_3-u_1)+(u_3-u_2)}{(u_2-u_1)+(u_3-u_1)} \equiv \bar{\delta} < 1$$

since

$$B>-C \Leftrightarrow (1-\delta^2)(u_3-u_1)> \delta^2(u_2-u_1)-(u_3-u_2) \Leftrightarrow (u_3-u_1)+(u_3-u_2)> \delta^2[(u_2-u_1)+(u_3-u_1)]$$

from which the condition on the discount factor follows.

Observe that given our assumption on diminishing increases in utility it is always the case that

$$\bar{\delta} > \delta^*$$

since $\bar{\delta} > \delta^* \Leftrightarrow \frac{(u_3-u_1)+(u_3-u_2)}{(u_2-u_1)+(u_3-u_1)} > \frac{u_3-u_2}{u_2-u_1} \Leftrightarrow u_2-u_1 > u_3-u_2$

where the last inequality holds because of our assumption on the shape of the outcome utility function.

Summing up:

1. If $\delta^2 \in (0, \delta^*)$, then B>A>C>0>-C.
2. If $\delta^2 \in (\delta^*, \bar{\delta})$, then A>B>-C>0>C.
3. If $\delta^2 \in (\bar{\delta}, 1)$, then A>-C>B>0>C.

We can now derive the conditions such that the various preference profiles listed at the beginning are compatible with the (σ-δ) model.

1. Profile $D \succ_1 K \succ_1 I$ and $D \succ_1 I$: it requires σ<C with $\delta^2<\delta^*$ (to have $D \succ_1 K$), σ<A (to have $K \succ_1 I$) and σ<B (to have $D \succ_1 I$). Since we need $\delta^2<\delta^*$, then B>A>C>0>-C, so that σ<min{A,B,C}=C.

2. Profile $K \succ_{1,2} D \succ_1 I$ and $K \succ_1 I$: it requires either σ<-C and $\delta^2>\delta^*$, or $\sigma \geq$C and $\delta^2<\delta^*$ (to have $K \succ_{1,2} D$), σ<B (to have $D \succ_1 I$) and σ<A (to have $K \succ_1 I$). If $\delta^2>\delta^*$, then we require σ<min{A,B,-C}, which is either B or -C depending on how large δ^2 is (i.e. whether or not it

is greater than $\bar{\delta}$). If instead $\delta^2<\delta^*$, we know that B>A>C>0>-C, in which case the requirement is $\sigma\in[C,A)$.

3. Profile D $\succ_1$I $\succ_2$ K $\succ_{1,2}$ D: this requires $\sigma\geq A$ (to have I $\succ_2$K), either $\sigma<-C$ and $\delta^2>\delta^*$, or $\sigma\geq C$ and $\delta^2<\delta^*$ (to have K $\succ_{1,2}$ D), $\sigma<B$ (to have D $\succ_1$ I). Since at the very least we need $A\leq\sigma<B$, we must be in the situation where $\delta^2<\delta^*$, in which case we also need $\sigma\geq C$, which is however not binding. Consequently this preference profile is compatible with $\sigma\in[A,B)$ and $\delta^2<\delta^*$.

4. Profile I $\succ_2$ K $\succ_{1,2}$ D and I $\succ_2$ D : this would require $\sigma\geq A$ (to have I $\succ_2$K), either $\sigma<-C$ and $\delta^2>\delta^*$, or $\sigma\geq C$ and $\delta^2<\delta^*$ (to have K $\succ_{1,2}$ D), and $\sigma\geq B$ (to have I $\succ_2$ D). So, if $\delta^2<\delta^*$ as we saw above we have B>A>C>0>-C, so that in this case all conditions are satisfied provided that $\sigma\geq\max\{A,B,C\}=B$, whereas if $\delta^2>\delta^*$ there is no possibility that the profile can be justified, since the interval (A,C) is in this case empty.

5. Profile K $\succ_1$I $\succ_2$ D and K $\succ_{1,2}$ D: this requires $\sigma<A$ (to have K $\succ_1$I), either $\sigma<-C$ and $\delta^2>\delta^*$, or $\sigma\geq C$ and $\delta^2<\delta^*$ (to have K $\succ_{1,2}$ D), and $\sigma\geq B$ (to have I $\succ_2$ D). Since at the very least we must have $\sigma\in(B,A)$, it must be that $\delta^2>\delta^*$, so that A>B. Then the comparison between K and D is resolved on the basis of the primary criterion, and we need $\sigma<\min\{A,-C\}=-C$. In short, for this profile of preferences we need $\sigma\in[B,C)$ and $\delta^2>\delta^*$.

6. Profile D $\succ_1$K $\succ_1$I $\succ_2$D: this requires $\sigma<A$ (to have K $\succ_1$I), $\sigma\geq B$ (to have I $\succ_2$ D), and $\sigma<C$ with $\delta^2<\delta^*$ (to have D $\succ_1$K), so that B>A>C>0>-C. Since σ cannot be at the same time smaller than A and greater than B, this case is not compatible with the (σ-δ) model.

7. Profile I $\succ_2$D $\succ_1$K and I $\succ_2$ K: would require $\sigma\geq A$ (to have I $\succ_2$ K), $\sigma\geq B$ (to have I $\succ_2$ D), and $\sigma<C$ with $\delta^2<\delta^*$ (to have D $\succ_1$K), so that the interval (A, C) is empty, incompatible with the (σ-δ) model.

8. Profile D $\succ_1$ I $\succ_2$ K and D $\succ_1$ K: requires $\sigma<B$ (to have D $\succ_1$I), $\sigma\geq A$ (to have I $\succ_2$ K), and $\sigma<C$ with $\delta^2<\delta^*$ (to have D $\succ_1$K), implying B>A>C>0>-C, incompatible with the (σ-δ) model.

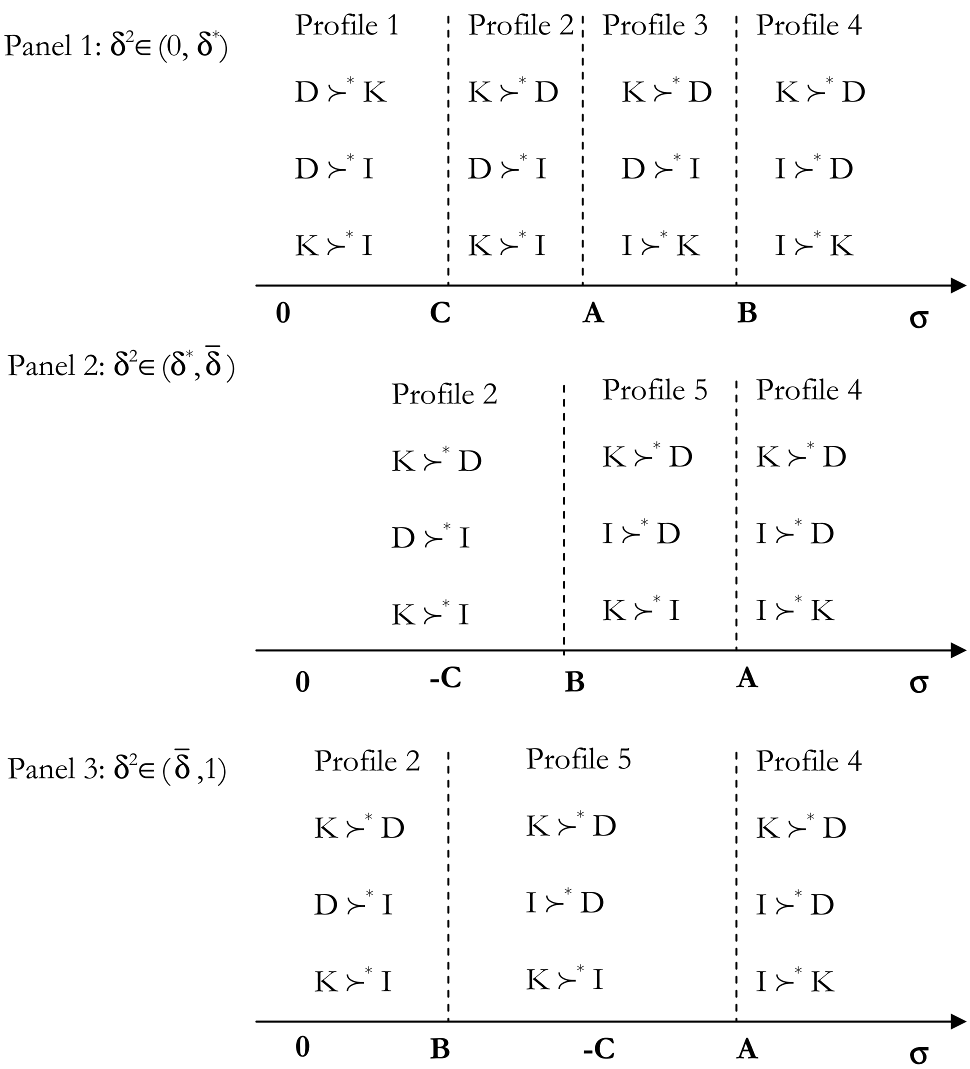

Figure 5: choice profiles and parameter values for three period sequences

Figure 5 depicts these choice profiles for given combinations of the preference parameters. Inspection of this figure shows that when C is negative the relative position of –C with respect to A and B is irrelevant. This is why in the main text we present the more compact version, ignoring –C.

A1.ii Two period sequences

The analysis of two period sequence is parallel to that carried out so far. Now the periods involved are only 0 and 2, where recall that u_2=u(16), u_3=u(24) and u_4=u(32); moreover we denote u_5=u(40). The two period sequences of payments considered by the experimental subjects were the following:

I=(16, 32)

K=(24, 24)

D=(32, 16)

J=(8, 40)

Here too any pure discounting criterion for choice $\succ_d$ should order them as either $D \succ_d K \succ_d I \succ_d J$ or $K \succ_d D \succ_d I \succ_d J$.

Denoting by $\succ_d$ the preference relation of a decision maker who discounts utility available at time t by some monotonically non decreasing discount function δ_t, then:

$$D \succ_d K \Leftrightarrow \delta_2 < \frac{u_4 - u_3}{u_3 - u_2} \tag{1}$$

holds since:

$$D \succ_d K \Leftrightarrow u_4 + \delta_2 u_2 > u_3 + \delta_2 u_3 \Leftrightarrow \delta_2 < \frac{u_4 - u_3}{u_3 - u_2}$$

Checking back the conditions for the choice between D and K for long sequences, recall that D is chosen over K if and only if $\delta_2 < \frac{u_3 - u_2}{u_2 - u_1} < \frac{u_4 - u_3}{u_3 - u_2}$, where the last inequality follows from our assumption of positive third derivative. Consequently, a choice of D over K for the long sequences implies a choice of D over K for the short sequences, too. Equivalently, a choice of K over D for the short sequences means that the subject is patient enough that he should also choose K over D in the long sequences. In short, then, a cross-tabulation of DK2 and DK3 should have at least a zero off-diagonal element (corresponding to the $D3 \succ K3/K2 \succ D2$ cell).

Secondly,

$$D \succ_d I \text{ always} \tag{2}$$

given that:

$$D \succ_d I \Leftrightarrow u_4 + \delta_2 u_2 > u_2 + \delta_2 u_4 \Leftrightarrow u_4 - u_2 > \delta_2 (u_4 - u_2)$$

while

$$K \succ_d I \text{ always}$$

holds since:

$$K \succ_d I \Leftrightarrow u_3 + \delta_2 u_3 > u_2 + \delta_2 u_4 \Leftrightarrow u_3 - u_2 > \delta_2 (u_4 - u_3)$$

Finally

$$I \succ_d J \text{ always} \tag{3}$$

holds since:

$$I \succ_d J \Leftrightarrow u_2 + \delta_2 u_4 > u_1 + \delta_2 u_5 \Leftrightarrow u_2 - u_1 > \delta_2 (u_5 - u_4)$$

Once more then the only two patterns of choice consistent with discounting theories are as for the three period sequences.

Turning now to the (σ-δ) model, the profiles of choices based on all pairwise comparisons involving the three series are derived in an analogous way as for the three period sequences. Define:

$$a \equiv (u_3-u_2) - \delta^2(u_4-u_3) > 0$$

$$b \equiv (1-\delta^2)(u_4-u_2) > 0$$

$$c \equiv (u_4-u_3) - \delta^2(u_3-u_2)$$

$$-c \equiv \delta^2 (u_3-u_2) - (u_4-u_3)$$

$$\hat{\delta}^* \equiv \tfrac{u_4-u_3}{u_3-u_2} \text{ and } \bar{\gamma} \equiv \frac{(u_4-u_3)+(u_4-u_2)}{(u_3-u_2)+(u_4-u_2)}$$

and observe that $\bar{\gamma} > \hat{\delta}^*$, and that[24]:

- If $\delta^2 \in (0, \hat{\delta}^*)$, then b>a>c>0>-c.
- If $\delta^2 \in (\hat{\delta}^*, \bar{\gamma})$, then a>b>-c>0>c.
- If $\delta^2 \in (\bar{\gamma}, 1)$, then a>-c>b>0>c

Then the choice profiles compatible with the (σ-δ) model are as follows:

1. $D \succ^* K \succ^* I$: this can be if σ<c and $\delta^2 \in (0, \hat{\delta}^*)$;
2. $K \succ^* D \succ^* I$: this can be if
   (a) σ<b and $\delta^2 \in (\bar{\gamma}, 1)$
   (b) σ<-c and $\delta^2 \in (\hat{\delta}^*, \bar{\gamma})$;
   (c) $\sigma \in [c, a)$ and $\delta^2 \in (0, \hat{\delta}^*)$;
3. $D \succ^* I \succ^* K \succ^* D$: this can be if $\sigma \in [a, b)$ and $\delta^2 \in (0, \hat{\delta}^*)$;.
4. $I \succ^* K \succ^* D$: this can be if $\sigma \geq b$ and $\delta^2 \in (0, \hat{\delta}^*)$;

[24] A full derivation of these values follows later in this appendix.

5. $K \succ^* I \succ^* D$: this can be if $\sigma \in [b, a]$ and $\delta^2 > \hat{\delta}^*$;

6. $D \succ^* K \succ^* I \succ^* D$;

7. $I \succ^* D \succ^* K$;

8. $D \succ^* I \succ^* K$.

where the last three profiles are not compatible with the (σ-δ) model. We derive these results below. Before doing so, though, observe the relationship that our model postulates between choice profiles in the two and three period cases. As we show below, it is the case that $\delta^* < \hat{\delta}^*$, $\bar{\delta} < \bar{\gamma}$, A>a, B>b and C>c.

Figure 3 in the main text illustrates this point, showing that a switch either from $D \succ^* K \succ^* I$ to $K \succ^* D \succ^* I$ or the opposite switch from $K \succ^* D \succ^* I$ to $D \succ^* K \succ^* I$ with sequence length is possible.

Next we show how the various thresholds have been derived for the case of choice between two period sequences.

A) Comparison between series I and K: sequence I is chosen over K by the primary criterion whenever $u_2 + \delta^2 u_4 > u_3 + \delta^2 u_3 + \sigma \Leftrightarrow \sigma < \delta^2(u_4-u_3)-(u_3-u_2) < 0$, where the last inequality follows from the fact that $\delta^2<1$ and from our assumption of decreasing marginal utility, so that $(u_3-u_2) > (u_4-u_3) > \delta^2(u_4-u_3)$. Consequently, given that σ must be non negative, it can only be that $u_2 + \delta^2 u_4 < u_3 + \delta^2 u_3 + \sigma$: in other words, I can never beat K outright. Therefore the comparison between the two sequences hinges on whether or not the following holds: $u_3 + \delta^2 u_3 \leq u_2 + \delta^2 u_4 + \sigma \Leftrightarrow \sigma \geq (u_3-u_2) - \delta^2(u_4-u_3) > 0$.

In short we can summarize the only possible mutually incompatible cases as follows:

$$I \succ_2 K \Leftrightarrow \sigma \geq (u_3-u_2) - \delta^2(u_4-u_3) \equiv a < A \equiv (u_2-u_1) - \delta^2(u_3-u_2)$$

$$K \succ_1 I \Leftrightarrow \sigma < (u_3-u_2) - \delta^2(u_4-u_3) \equiv a < A \equiv (u_2-u_1) - \delta^2(u_3-u_2)$$

where the comparison between a and A follows from our assumptions on the concavity of the utility function. This means that if an agent chooses K over I (by the primary criterion) when confronted with short sequences, he must do so when confronted with longer sequences too.

Comparison between series I and D: Sequence I is chosen over series D by the primary criterion if $u_2 + \delta^2 u_4 > u_4 + \delta^2 u_2 + \sigma \Leftrightarrow \sigma < -(1-\delta^2)(u_4-u_2) < 0$. Again, because of the restriction on σ, it can only be that $u_2 + \delta^2 u_4 \leq u_4 + \delta^2 u_2 + \sigma$, i.e. it can never be that I beats D outright. Therefore the comparison hinges on:

$u_4 + \delta^2 u_2 \leq u_2 + \delta^2 u_4 + \sigma \Leftrightarrow \sigma \geq (1-\delta^2)(u_4-u_2) > 0$

In short we can summarize the only possible mutually incompatible cases as follows:

$$I \succ_2 D \Leftrightarrow \sigma \geq (1-\delta^2)(u_4-u_2) \equiv b < B \equiv (1-\delta^2)(u_3-u_1)$$

$$D \succ_1 I \Leftrightarrow \sigma < (1-\delta^2)(u_4-u_2) \equiv b < B \equiv (1-\delta^2)(u_3-u_1)$$

where again the comparison between b and B follows from the assumptions on the third derivative of the utility function. As a consequence, if an agent chooses D over I in the short sequences, he must do so in the long sequences, too.

C) Comparison between series K and D: Sequence K is chosen over sequence D by the primary criterion if $u_3 + \delta^2 u_3 > u_4 + \delta^2 u_2 + \sigma \Leftrightarrow \sigma < \delta^2(u_3-u_2)-(u_4-u_3)$. The left hand side is positive provided that $\delta^2 > \frac{u_4-u_3}{u_3-u_2} \equiv \hat{\delta}^*$. So if the discount function in period 2 is sufficiently large, then K can beat D outright provided the vagueness term is sufficiently *small*. If instead $\delta^2 \leq \hat{\delta}^*$, then it can never be that K beats D outright, and as in the other cases the comparison hinges on whether or not $u_4 + \delta^2 u_2 \leq u_3 + \delta^2 u_3 + \sigma \Leftrightarrow \sigma \geq (u_4-u_3) - \delta^2(u_3-u_2) \geq 0$, where the last inequality follows from our assumption that $\delta^2 \leq \hat{\delta}^*$. Incidentally, observe that $\hat{\delta}^* > \delta^* \equiv \frac{u_3-u_2}{u_2-u_1}$.

In short we can summarize as follows:

If $\delta^2 > \frac{u_4-u_3}{u_3-u_2} \equiv \hat{\delta}^*$, then: $K \succ_1 D$ if $\sigma < \delta^2 (u_3-u_2) - (u_4-u_3) \equiv -c$ and

$K \succ_2 D$ if $\sigma \geq \delta^2 (u_3-u_2) - (u_4-u_3) \equiv -c$

If $\delta^2 < \frac{u_4-u_3}{u_3-u_2} \equiv \hat{\delta}^*$, then: $K \succ_2 D$ if $\sigma \geq (u_4-u_3)-\delta^2(u_3-u_2) \equiv c$ and

$D \succ_1 K$ if $\sigma <$ $u_4-u_3)-\delta^2(u_3-u_2) \equiv c$ $(u_3-u_2) - \delta^2 (u_2-u_1) \equiv c$

Now let us compare the relative positions of a, b, c and -c to see which configuration of preferences of the eight listed above are compatible with the model. Recall:

$$a \equiv (u_2-u_1)-\delta^2(u_3-u_2) > 0$$

$$b \equiv (1-\delta^2)(u_4-u_2) > 0$$

$$c \equiv (u_4-u_3)-\delta^2(u_3-u_2)$$

$$-c \equiv \delta^2 (u_3-u_2) - (u_4-u_3)$$

Observe that

$$a>b \Leftrightarrow \delta^2 > \hat{\delta}^* \equiv \frac{u_4-u_3}{u_3-u_2} > \frac{u_3-u_2}{u_2-u_1} \equiv \delta^*$$

That is, a>b implies that A>B, too. The derivation is analogous to that for three period sequences, and is thus omitted. Similarly, one can show that

$$a>\max\{-c,c)$$

while $b>c$

and

$$b>-c \Leftrightarrow \delta^2 < \frac{(u_4-u_2)+(u_4-u_3)}{(u_3-u_2)+(u_4-u_2)} \equiv \bar{\gamma} < 1$$

In this case too it is straightforward to verify that

$$\bar{\gamma} > \hat{\delta}^*$$

Similarly to what we saw for the three period sequences, then:

1. If $\delta^2 \in (0, \hat{\delta}^*)$, then b>a>c>0>-c.
2. If $\delta^2 \in (\hat{\delta}^*, \bar{\gamma})$, then a>b>-c>0>c.
3. If $\delta^2 \in (\bar{\gamma}, 1)$, then a>-c>b>0>c.

We can now derive the conditions such that the various preference profiles listed at the beginning are compatible with the model.

1. Profile $D \succ_1 K \succ_1 I$ and $D \succ_1 I$: this requires $\sigma<c$ with $\delta^2<\delta^*$ (to have $D \succ_1 K$), $\sigma<a$ (to have $K \succ_1 I$) and $\sigma<b$ (to have $D \succ_1 I$). Since we need $\delta^2<\delta^*$, then b>a>c>0>-c, so that it must be $\sigma<\min\{a,b,c\}=c$.

2. Profile $K \succ_{1,2} D \succ_1 I$ and $K \succ_1 I$: it requires either $\sigma<-c$ and $\delta^2>\delta^*$, or $\sigma \geq c$ and $\delta^2<\delta^*$ (to have $K \succ_{1,2} D$), $\sigma<b$ (to have $D \succ_1 I$) and $\sigma<a$ (to have $K \succ_1 I$). If $\delta^2>\delta^*$, then we require $\sigma<\min\{a,b,-c\}$, which is either b or -c depending on how large δ^2 is (i.e. whether or not it is greater than $\bar{\delta}$). If instead $\delta^2<\delta^*$, we know that b>a>c>0>-c, in which case the requirement is $\sigma \in [c,a)$.

3. Profile $D \succ_1 I \succ_2 K \succ_{1,2} D$: this requires $\sigma \geq a$ (to have $I \succ_2 K$), either $\sigma<-c$ and $\delta^2>\delta^*$, or $\sigma \geq c$ and $\delta^2<\delta^*$ (to have $K \succ_{1,2} D$), $\sigma<b$ (to have $D \succ_1 I$). Since at the very least we need $a \leq \sigma < b$, we must be in the situation where $\delta^2<\delta^*$, in which case we also need $\sigma \geq c$, which is however not

binding. Consequently this preference profile is compatible with $\sigma \in [a,b)$ and $\delta^2<\delta^*$.

4. Profile I $\succ_2$ K $\succ_{1,2}$ D and I $\succ_2$ D : this would require $\sigma \geq a$ (to have I $\succ_2$K), either $\sigma<-c$ and $\delta^2>\delta^*$, or $\sigma \geq c$ and $\delta^2<\delta^*$ (to have K $\succ_{1,2}$ D), and $\sigma \geq b$ (to have I $\succ_2$ D). So, if $\delta^2<\delta^*$ as we saw above we have b>a>c>0>-c, so that in this case all conditions are satisfied provided that $\sigma \geq \max\{a,b,c\}=b$, whereas if $\delta^2>\delta^*$ there is no possibility that the profile can be justified, since the interval [a,c) is in this case empty.

5. Profile K $\succ_1$I $\succ_2$ D and K $\succ_{1,2}$ D: this requires $\sigma<a$ (to have K $\succ_1$I), either $\sigma<-c$ and $\delta^2>\delta^*$, or $\sigma \geq c$ and $\delta^2<\delta^*$ (to have K $\succ_{1,2}$ D), and $\sigma \geq b$ (to have I $\succ_2$ D). Since at the very least we must have $\sigma \in [b,a)$, it must be that $\delta^2>\delta^*$, so that a>b. Then the comparison between K and D is resolved on the basis of the primary criterion, and we need $\sigma<\min\{a,-c\}=-c$. In short, for this profile of preferences we need $\sigma \in [b,c)$ and $\delta^2>\delta^*$.

6. Profile D $\succ_1$K $\succ_1$I $\succ_2$D: this requires $\sigma<a$ (to have K $\succ_1$I), $\sigma \geq b$ (to have I $\succ_2$ D), and $\sigma<c$ with $\delta^2<\delta^*$ (to have D $\succ_1$K). Since $\delta^2<\delta^*$ is necessary, we saw above that in this case b>a>c>0>-c, so that σ cannot be at the same time smaller than a and greater than b, so that this case is not compatible with the (σ-δ) model.

7. Profile I $\succ_2$D $\succ_1$K and I $\succ_2$ K: this would require $\sigma \geq a$ (to have I $\succ_2$ K), $\sigma \geq b$ (to have I $\succ_2$ D), and $\sigma<c$ with $\delta^2<\delta^*$ (to have D $\succ_1$K). But if $\delta^2<\delta^*$ the interval [a,c) is empty, so that this profile is not compatible with the (σ-δ) model.

8. Profile D $\succ_1$ I $\succ_2$ K and D $\succ_1$ K: this requires $\sigma<b$ (to have D $\succ_1$I), $\sigma \geq a$ (to have I $\succ_2$ K), and $\sigma<c$ with $\delta^2<\delta^*$ (to have D $\succ_1$K). Again, $\delta^2<\delta^*$ implies b>a>c>0>-c, so that this case is not compatible with the (σ-δ) model.

A.2 The role of plan J

We now show how the three implications for choices involving plan J discussed in section 6 in the main text have been derived. Recall that J2=(8, 40), I2=(16,32), SJ=(8, 32, 8) and SI=(16, 24, 8).

Then:

1. **the choice of J2 over I2 implies the choice of SJ over SI**. If a subject is vague between them and his preferences conform to our model, he will choose the former based on the secondary criterion as it is steeper than I2. Since $u(16)-u(8)>\delta^2(u(40)-u(32))$, a choice of J2 over I2 can occur only by the secondary criterion, so that it must be that $u(16)+ \delta^2 u(32)\leq u(8)+\delta^2 u(40)+\sigma$, that is the decision maker must have

$$\sigma\geq (u(16)-u(8))- \delta^2(u(40)-u(32)) \qquad (9)$$

Note that by our assumptions on the shape of the utility function, it is the case that $(u(16)-u(8))-\delta^2(u(40)-u(32))> (u(16)-u(8))- \delta^2(u(32)-u(24))$, so that condition (10) implies also

$$\sigma\geq (u(16)-u(8))- \delta^2(u(32)-u(24)) \qquad (10)$$

Rearranging, this yields $u(16) + \delta u(24)+\delta^2 u(8) \leq u(8) + \delta u(32)+\delta^2 u(8)+ \sigma$, that is the discounted utility of SI does not exceed the discounted utility of SJ by more than σ. In short, then, choosing J2 over I2 implies the choice of SJ over SI.

2. **the choice of J2 over I2 implies the choice of J3 over I3**. To show this just observe that condition (10) implies also

$$\sigma\geq\delta(u(16)-u(8))- \delta^2(u(32)-u(24)) \qquad (11)$$

which can be rearranged as $u(8)+\delta u(16)+\delta^2 u(24)\leq u(8)+\delta u(8)+\delta^2 u(32)+ \sigma$, so that the discounted utility of I3 does not exceed the discounted utility of J3 by more than σ. In short, then, choosing J2 over I2 implies the choice of J3 over I3.

3. **the choice of J2 over D2 implies the choice of J3 over D3**. Recall that J2=(8, 40) while D2=(32, 16). Since the choice of J2 over D2 can occur only because of the secondary criterion, it is necessary that $u(32) +\delta^2 u(16)\leq u(8)+\delta^2 u(40)+ \sigma$, which can be rearranged as

$$\sigma\geq u(32)-u(8) - \delta^2(u(40)-u(16))\equiv A \qquad (12)$$

Similarly for the longer series, that is it must be that

$u(24) +\delta u(16)+\delta^2 u(8)\leq u(8)+ \delta u(8)+\delta^2 u(40)+ \sigma$, which can be rearranged as

$$\sigma\geq u(24)-u(8)+\delta(u(16)-u(8))-\delta^2(u(32)-u(8)) \equiv B \qquad (13)$$

We now show that A>B, so that condition 13 implies condition 14, and our claim is proved.

Observe that A>B if and only if:

$$u(32)\text{-}u(8) - \delta^2(u(40)\text{-}u(16)) > u(24)\text{-}u(8)+\delta(u(16)\text{-}u(8))\text{-}\delta^2(u(32)\text{-}u(8)) \Leftrightarrow$$

$$\Leftrightarrow u(32)\text{-} u(24)\text{-} \delta(1\text{-}\delta)(u(16)\text{-}u(8))\text{-} \delta^2(u(40)\text{-}u(32))>0 \quad (14)$$

The last term is decreasing in the discount factor: the first derivative with respect to δ is given by $2\delta[(u(16)\text{-}u(8))\text{-} (u(40)\text{-}u(32))]\text{-} (u(16)\text{-}u(8))$, which is negative if and only if :

$$\delta < \frac{u(16) - u(8)}{\left(u(16) - u(8)\right) - \left(u(40) - u(32)\right)}$$

which is always verified since the right hand side is greater than unity. Consequently expression (15) is smallest in the limit as δ approaches unit. In this limit, however, (15) has value $u(32)\text{-} u(24) – (u(40)\text{-}u(32))>0$, which establishes our claim.

In order to compare plans I and J, observe that:

$$u_1 + \delta_1 u_2 + \delta_2 u_3 \geq u_1 + \delta_1 u_1 + \delta_2 u_4 + \sigma \Leftrightarrow \delta_1 u_2 + \delta_2 u_3 \geq \delta_1 u_1 + \delta_2 u_4 + \sigma \Leftrightarrow \sigma \leq \delta_1 \left(u_2 - u_1\right) - \delta_2 \left(u_4 - u_3\right)$$

Note that the value on the left hand side is positive provided that $\delta_1 \geq \delta_2$. In this case I can beat J outright provided the vagueness term is sufficiently *small*. If instead σ is large enough, then the two sequences are vague, and based on the secondary criterion our decision maker will prefer J to I.

In short we can summarize as follows:

$$I \succ J \Leftrightarrow \sigma < \delta(u_2\text{-}u_1)\text{-}\delta^2 (u_4\text{-}u_3) \equiv d > 0$$

A.3 Instructions

Please note: you are not allowed to communicate with the other participants for the entire duration of the experiment.

The instructions are the same for all you. You are taking part in an experiment to study intertemporal preferences. The project is financed by the ESRC.

Shortly you will see on your screen a series of displays. Each display contains various remuneration plans worth the same total amount of 48 Euros each, staggered in three, six and nine months instalments. For every display you will have to select the plan that you prefer, clicking on the button with the letter corresponding to the chosen plan. (HYP: These remuneration plans are purely hypothetical. At the end of the experiment you'll be given a participation fee of 5 €.) (PAY: At the end of the experiment one of the displays will be drawn at random and your remuneration will be made according to the plan you have chosen in that display).

In order to familiarise yourself with the way the plans will be presented on the screen, we shall now give you a completely hypothetical example, based on a total remuneration of 7 Euros.

Plan	A
How much	When
3 €	in one year
1 €	in two years
1 €	in three years
2 €	in four years

Plan	B
How much	When
1 €	in one year
2 €	in two years
3 €	in three years
1 €	in four years

In this example plan A yields 7 € in total in tranches of 3 €, 1 €, 1 € and 2 € in a year, two years, three years and four years from now, respectively, while plan B yields 7 Euros in total in tranches of 1 €, 2 €, 3 € and 1 € in a year, two years, three years and four years from now, respectively.

A.4 Raw data

We report our data in the table below. Variable names are as follows:

T: treatment (0 for PAY and 1 for HYP)

SS: session number

SB: subject number

SX: subject's sex (0 for Female and 1 for Male)

Choices between plans are coded as follows: ABn indicates the choice between plans A and B of length n periods. A value of 0 indicates that A was chosen over B, whereas a value of 1 indicates that B was chosen over A. The only exception to this coding strategy is for plan SJSI, which was only available for three period sequences and where 0 indicates the choice of the SJ sequence over the SI sequence, while 1 denotes the opposite choice.

T	SS	SB	SX	KI3	ID3	DK3	IJ3	JK3	JD3	SJSI	KI2	ID2	DK2	IJ2	JK2	JD2
0	1	0	0	0	1	0	0	1	1	1	0	1	0	0	1	1
0	1	1	0	0	0	1	0	0	0	0	0	1	1	1	1	1
0	1	2	0	1	0	1	0	1	1	1	0	0	0	0	1	1
0	1	3	0	0	0	1	0	1	0	1	0	0	1	0	1	1
0	1	4	0	0	1	0	0	1	1	1	0	1	0	0	1	1
0	1	5	0	0	1	0	0	1	1	1	0	1	1	0	1	1
0	1	6	1	0	1	0	0	1	1	1	0	1	0	0	0	1
0	1	7	1	0	1	0	0	1	1	1	0	1	0	0	1	1
0	1	8	1	0	1	0	0	1	1	1	0	1	0	0	1	1
0	1	9	0	1	0	1	1	0	0	1	1	0	1	0	0	0
0	2	0	1	0	1	0	0	1	1	1	0	1	0	0	1	1
0	2	1	0	1	1	0	0	0	1	1	0	0	1	0	1	1
0	2	2	1	0	1	0	0	1	1	1	0	1	0	0	1	1
0	2	3	0	0	1	0	0	1	1	1	0	1	0	0	1	1
0	2	4	1	0	1	1	0	1	1	1	0	1	1	0	1	1
0	2	5	0	0	0	1	0	1	1	1	0	0	1	0	1	1
0	2	6	1	0	1	0	0	1	1	1	0	1	0	0	1	1
0	2	7	0	0	1	0	0	1	1	1	0	1	0	0	1	1
0	2	8	1	0	1	0	0	1	1	1	0	1	0	0	1	1
0	2	9	1	1	0	1	0	1	0	1	0	0	1	0	1	0
0	2	10	1	0	1	0	0	1	1	1	0	1	0	0	1	1
0	2	11	0	0	1	0	0	1	1	1	0	0	0	0	1	1
0	2	12	0	0	1	0	0	1	1	1	0	1	0	0	1	1
0	2	13	1	0	1	0	0	1	1	1	0	1	0	0	1	1
0	2	14	1	0	1	0	0	1	1	1	0	1	0	0	1	1
0	2	15	0	0	1	0	0	1	1	1	0	1	0	0	1	1
0	3	0	1	1	1	1	0	1	1	1	1	0	1	0	1	1
0	3	1	0	0	1	0	0	1	1	0	0	1	0	0	1	1
0	3	2	0	0	1	0	0	1	1	1	0	1	0	0	1	1
0	3	3	0	0	1	1	0	1	1	1	0	1	1	0	1	1
0	3	4	1	0	1	0	0	1	1	1	0	1	1	0	1	1
0	3	5	1	0	1	1	0	1	1	1	0	1	0	0	1	1
0	3	6	0	0	1	1	0	0	1	1	0	1	0	0	1	1
0	3	7	0	0	1	1	0	1	1	1	0	1	0	0	1	1

0	3	8	1	0	1	0	0	1	1	1	0	1	0	0	1	1
0	4	0	0	0	1	1	0	1	1	1	0	1	0	0	1	1
0	4	1	1	0	1	0	1	1	1	1	0	1	0	0	1	1
0	4	2	1	0	1	0	0	1	1	1	0	1	0	0	1	1
0	4	3	1	0	1	1	0	1	1	1	0	1	0	0	1	1
0	4	4	1	0	1	0	0	1	1	1	0	1	0	0	0	1
0	4	5	0	0	1	0	0	1	1	1	0	1	0	0	1	1
0	4	6	1	0	1	0	0	1	1	1	0	1	0	0	1	1
0	4	7	1	0	1	0	0	1	1	1	0	1	0	0	1	1
0	4	8	1	0	1	0	0	1	1	1	0	1	0	0	1	1
0	4	9	1	0	0	1	0	1	0	1	0	0	1	0	0	0
0	4	10	1	0	1	0	0	1	1	1	0	1	0	0	1	1
0	4	11	0	0	0	1	0	1	1	1	0	0	0	0	1	1
0	5	0	0	0	0	0	1	1	1	0	0	0	0	1	1	1
0	5	1	0	0	0	1	0	1	0	0	1	0	1	1	0	0
0	5	2	1	0	1	0	0	1	1	1	0	1	0	0	1	1
0	5	3	0	0	1	0	0	1	1	0	0	1	0	0	1	1
0	5	4	0	0	1	0	0	1	1	1	0	1	0	0	1	1
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0	5	6	1	0	1	0	0	1	1	1	0	1	0	0	1	1
0	5	7	0	0	0	1	0	1	0	1	1	0	1	0	1	0
0	6	0	0	0	1	0	0	1	1	0	0	1	0	1	1	1
0	6	1	1	0	1	0	1	0	0	0	1	1	0	1	0	1
0	6	2	0	0	1	0	0	1	1	0	0	1	1	0	1	1
0	6	3	0	0	1	0	0	1	1	1	0	1	0	0	1	1
0	6	4	1	0	0	1	0	1	0	1	0	0	1	1	1	1
0	6	5	1	0	1	0	0	1	1	1	0	1	0	0	1	1
0	6	6	1	0	0	0	0	0	0	0	0	0	1	0	0	1
0	6	7	1	0	1	0	0	1	1	1	0	1	0	0	1	1
0	6	8	1	0	1	0	0	1	1	1	0	1	0	0	1	1
0	6	9	0	0	1	0	0	1	1	1	0	1	0	0	1	1
0	6	10	0	0	1	0	0	1	1	1	0	1	1	0	1	1
0	6	11	1	0	1	0	0	1	1	1	0	1	0	0	1	1
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0	7	3	0	0	1	1	0	1	1	1	0	1	0	0	1	1
0	7	4	0	0	1	1	0	1	1	1	0	1	1	0	1	1
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0	8	0	1	0	0	1	0	0	0	1	1	0	1	0	0	0
0	8	1	1	0	1	0	0	1	1	1	0	1	0	0	1	1
0	8	2	1	0	1	0	1	1	1	0	0	1	0	1	0	0
0	8	3	0	0	1	0	0	1	1	1	0	1	0	0	1	1
0	8	4	1	0	1	0	0	1	1	1	0	1	0	0	1	1

0	8	5	1	0	1	1	0	1	0	1	0	1	1	1	1	0
0	8	6	0	0	0	1	0	1	1	1	0	0	1	0	1	1
0	8	7	1	0	1	0	1	1	1	1	0	1	0	0	1	1
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0	9	10	0	0	1	1	0	1	1	1	0	1	1	0	1	1
0	9	11	1	0	0	1	1	1	0	0	0	0	1	0	1	0
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1	1	1	0	0	0	1	0	1	1	1	0	1	1	0	1	1
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1	1	10	0	0	0	0	0	1	1	1	0	1	0	0	0	1
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1	2	3	1	0	1	0	0	1	1	1	0	1	0	0	1	1
1	2	4	1	0	1	0	0	1	1	1	0	1	0	0	1	1
1	2	5	1	0	0	0	1	1	1	1	1	1	1	0	1	0
1	2	6	1	0	0	1	0	1	0	0	0	0	1	0	1	1
1	2	7	0	0	1	0	0	1	1	1	0	1	0	0	1	1
1	2	8	1	1	0	1	0	0	0	1	1	0	1	0	1	0
1	2	9	0	0	1	0	0	1	1	1	0	1	0	0	1	1
1	2	10	0	0	1	1	0	1	1	1	0	1	0	0	1	1
1	3	0	1	0	1	0	0	1	1	1	0	1	0	0	1	1
1	3	1	0	0	1	0	0	1	1	1	0	1	0	0	1	1
1	3	2	0	0	1	0	0	1	1	1	0	1	0	0	1	1
1	3	3	0	0	1	0	0	1	1	1	0	1	0	0	1	1
1	3	4	1	1	1	0	0	0	1	1	0	0	1	0	1	1
1	3	5	0	0	0	1	0	1	1	1	0	1	1	0	0	1

1	3	6	0	0	1	0	0	1	1	1	0	1	0	0	1	1
1	3	7	0	1	0	1	1	0	0	0	1	0	1	1	0	0
1	3	8	1	0	0	1	0	1	1	1	0	1	0	0	1	1
1	3	9	1	1	0	1	0	0	0	1	1	0	1	0	0	0
1	3	10	0	0	1	0	0	1	1	1	0	1	0	0	1	1
1	3	11	0	0	0	1	0	1	1	1	0	1	0	0	1	1
1	3	12	1	0	1	0	0	1	1	1	0	1	0	0	1	1
1	3	13	1	0	1	1	0	1	1	1	1	1	0	0	0	1
1	3	14	0	0	0	1	0	1	0	1	0	0	1	0	1	1
1	3	15	0	0	1	0	0	1	1	1	0	1	0	0	1	1
1	4	0	1	1	0	1	1	0	0	0	1	0	1	0	0	0
1	4	1	0	0	1	0	0	1	1	1	0	1	0	0	1	1
1	4	2	1	0	1	0	0	1	1	1	0	1	0	0	1	1
1	4	3	1	0	0	1	1	1	0	1	0	0	1	0	1	1
1	4	4	0	1	1	1	0	1	0	1	1	0	1	0	1	0
1	4	5	1	0	1	0	0	1	1	1	0	1	0	0	1	1
1	4	6	1	0	1	1	0	1	1	1	0	1	1	0	1	1
1	4	7	0	0	1	0	0	1	1	1	0	1	0	0	1	1
1	4	8	0	0	1	0	0	1	1	1	0	1	0	0	1	1
1	4	9	1	0	1	1	0	1	1	1	0	1	1	0	1	1
1	4	10	1	0	1	1	0	1	1	1	0	1	1	0	1	1
1	4	11	0	0	1	0	0	1	1	1	0	1	0	0	1	1
1	4	12	0	0	1	0	0	1	1	1	0	1	0	0	1	1
1	4	13	1	0	1	0	0	1	1	1	0	1	0	0	1	1