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ABSTRACT

Job and Wage Mobility in a Search Model with Non-Compliance (Exemptions) with the Minimum Wage*

How well does a simple search on-the-job model fit the eighteen years of job and wage mobility of high school graduates? To answer this question we are confronted from the data with a prevalent non-compliance and exemptions from the minimum wage. We incorporate this observation in a job search model with three main ingredients: (i) search on-the-job; (ii) minimum wages, with potentially imperfect compliance or exemptions; and, (iii) exogenous wage growth on-the-job. We use panel data drawn from the NLSY79, US youth panel starting in 1979, to estimate the parameters of our simple job search model and, in particular, the extent of non-compliance/exemptions to the minimum wage. The model is solved numerically and we use simulated moments to estimate the parameters. The estimated parameters are consistent with the model and they provide a good fit for the observed levels and trends of the main job and wage mobility data. Furthermore, the estimated model indicates that the non-compliance and exemption rate with the federal minimum wage translates into a roughly 10% of jobs paying less than the minimum wage. Counterfactual experiment of increase of the compliance/non-exemption rate or the minimum wage shows a small effect on mean accepted wages but a significant negative effect on the non-employment rate.

JEL Classification: J42, J63, J64

Keywords: minimum wages, compliance, exemptions, job search, wage growth

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1 Introduction

How well does the standard Burdett (1978) and Lucas and Prescott (1974) search on-the-job models fit the first 18 years labor market history data of high school graduates? Wage and employment data from National Longitudinal Survey of the Youth of 1979 (NLSY79) indicate that, during the first ten years in the labor market the employment rate of white males increases, while their job-to-job mobility and work-to-nonemployment mobility decrease, remaining roughly stable afterwards. Both mean wages and their variance increase with labor market experience, and more so soon after labor market entry. However, during the first six months after graduatiing from high school, about 25 percent of white males earn an hourly wage below the US federal legal minimum wage. This figure falls to 15 percent a year after graduation, to 9 percent three years after, and to 3 percent from the tenth year onwards (see Figure 14).[1]

Despite the findings of Ashenfelter and Smith (1979) on the importance of imperfect compliance to minimum wage laws, most of the recent literature on labor market effects of minimum wages typically ignored compliance issues. Ashenfelter and Smith measure compliance as the fraction of workers earning less than the minimum wage before the enactment of the law, who earned exactly the minimum wage after enactment. They conclude that "for the country as a whole the point estimate of the compliance rate is 69%, although a conventional confidence interval would include the range in the 63-75%". More recent work by Cortes (2004) studies whether immigrants are more likely to be paid less than the minimum wage than natives and, overall, she finds no systematic pattern of noncompliance between immigrants and natives. Finally, Weil (2004) uses data on apparel contractors in the Los Angeles area, and find that 54% of employers in 2000 did not comply with minimum wage laws, and that 27% of employees were paid below the minimum wage.

This paper uses a search model to estimate the extent of noncompliance (and/or exemptions) to the US federal minimum wage. We will not distinguish between noncompliance with the law and exemptions to the law, and throughout the paper the noncompliance term will also refer to exemptions[2]. Observed wages below the minimum wage can result from both noncompliance and measurement error. But while measurement error should apply throughout the wage distribution, noncompliance by definition only applies below the minimum wage. This distinction will be the basis of our identification strategy of the extent of compliance with the US federal minimum wage.

We construct a continuous-time search model in a stationary labor market environment with the following ingredients: (i) search on the job; (ii) a minimum wage with imperfect compliance by firms; (iii) endogenous search effort; (iv) exogenous wage growth on-the-job.[3]

[1] In Figure 14 we also use the extensive data on wages by age for high school graduates from the Census of Population Surveys (CPS, 1979-97). The CPS data is fully consistent with the facts from the NLSY79.

[2] Meyer and Wise (1983a,b) were first to estimate the employment and wage impacts of the minimum wage. They used a statistical model and individual data from CPS. They identified the minimum wage effects under the assumption that the observed wage distribution (under the minimum wage) is a distortion from the potential wage distribution. The model is static it imposes the restriction that employment decreases in response to an increase in the minimum wage.

[3] Bowlus and Neuman (2004) use a search equilibrium model to empirically analyze the wage growth using NLSY data. Wolpin (1992) uses the first eight years in quarterly format to fit a finite horizon discrete time search model with similar components.

The model is solved numerically and we use simulated moments to estimate the parameters of interest. The estimated parameters provide a good fit for the level, the trend and the fluctuations of several moments that are used for estimation.

The literature on the minimum wage policy is large and we do not attempt to cover it all here. However, it is well recognized that the analysis of this policy requires an equilibrium model where firms and workers respond to the change in policy. This basic claim provides the reason why minimum wage policies were analyzed by Eckstein and Wolpin (1990), van den Berg and Ridder (1998), van den Berg (2003) and Flinn (2002, 2005), among others, in estimable search equilibrium models.[4]

Here we use a simple search model with an exogenous wage offer distribution, where equilibrium can be interpreted as in Lucas and Prescott (1974) islands' model. The main reason for our modelling choice is that it is not clear that one can empirically distinguish between different search models on NLSY79 data (see e.g., Eckstein and van den Berg, 2005). Our simple search model is a benchmark specification where the observed wage dispersion can be explained by productivity differentials across firms and worker mobility following dynamic search decisions.

The model is estimated using the simulated method of moments. We use the monthly work history moments from NLSY79 for white males who graduated from the high school and did not attend college. We follow their employment status and wages for eighteen years after graduation. The model is estimated using two separate set of moments, namely the sequences of monthly moments of labor market states (nonemployment and employment), transitions and wages; and moments computed on employment cycles, delimited by subsequent nonemployment spells. Unobserved heterogeneity is controlled for non-parametrically by allowing for two types of workers. The model fits reasonably well both sets of moments and captures the levels and trends in nonemployment and work transitions. Furthermore, the model with a stationary wage offer distribution and an estimated constant annual wage growth of 2.4% on-the-job fits very well the eighteen years of wage growth from 8 to 16 dollars per hour, as well as the trend and level of the wage variance.

The parameter estimates have plausible magnitudes and are in line with previous estimates of the parameters of a search model with search on-the-job. The arrival rate of job offers **is** higher for nonemployed than for employed workers, and these rates differ between the two types of individuals, such that the mean hazard rate of leaving nonemployment is decreasing. The estimated parameters from the two different sets of moments provide very similar sets of estimated parameters. One type of workers is less employable, with relatively low arrival rates and mean wage offer, and a reservation wage of about three dollars per hour, which is much below the federal minimum wage. The other type has much higher arrival rates of job offers and mean wage offer, and a reservation wage of about seven dollars per hour. This second type of invidual is thus not affected from minimum wage regulations. The estimated noncompliance rate is of about 25 percent. That is, the arrival rate of job offers below the minimum wage is one fourth of the arrival rate above the minimum wage. This result is robust with respect to the moments that we use for identification and it is not

[4] Eckstein and Wolpin (1990) use a wage-posting model **with unemployed job search only**. van den Berg and Ridder (1998) and van den Berg (2003) use an extended version of Burdett and Mortensen (1998) wage-posting model with **both employed and unemployed job search**. Flinn (2002, 2005) uses a search-bargaining model **with unemployed job search** only.

much affected by changes is measurement error estimates. This estimate in turn translates into a steady state proportion of jobs paying less than the minimum wage equal to roughly 10%.

Counterfactual analysis of changes in the minimum wage and the noncompliance/exemption rate imply that both have a significant effect on nonemployment and on the proportion of individuals working below the minimum wage. For example, a 20 percent increase in minimum wage increases nonemployment by about 2 percent. These effects are in line with the reported time series estimates for the US reported in Kennan(1995, page 1954). However, we find that the impact of the minimum wage and noncompliance/exemption changes on the mean and variance of wages is very small.

The rest of the paper is organized as follows. Section 2 presents our job search model. Section 3 describes our data set. Section 4 gives the estimation method, section 5 presents results and section 6 concludes.

2 The Search Model

We construct a continuous time search model in a stationary labor market environment with the following ingredients: (i) search on the job; (ii) minimum wages with imperfect compliance by firms; (iii) endogenous search effort; (iv) exogenous wage growth on-the-job.

Agents are infinitely lived, and at each moment in time they can be either nonemployed (a state denoted by n) or employed (a state denoted by e). When they are nonemployed, they enjoy some real return b (typically including the value of leisure and unemployment insurance benefits), and receive job offers at a Poisson rate λ_n. Generating job offers at rate λ_n requires some search effort, with related search costs $c_n(\lambda_n)$, with $c_n'(\lambda_n) > 0$ and $c_n''(\lambda_n) > 0$. When employed, they enjoy a real wage w, which is growing at an exogenous rate g, receive job offers at a Poisson rate λ_e, and bear search costs $c_e(\lambda_e)$, with $c_e'(\lambda_e) > 0$ and $c_n''(\lambda_e) > 0$. Existing jobs are hit by idiosyncratic shocks, which occur at a Poisson rate δ. The instantaneous discount rate is r. New wage offers for employed and unemployed are randomly drawn from some known, fixed distribution $F(w)$. Once an individual accepts a wage w, his wage on the same job grows with tenure, τ, such that $w_\tau = we^{g\tau}$.

Our wage offer distribution is motivated by underlying productivity differences across firms. This modelling choice closely resembles Lucas and Prescott (1974) islands' model, where wage dispersion stems from productivity differentials across different islands. As productivity in each island is subject to idiosyncratic shocks, workers need to spend some effort in order to locate better matching opportunities and eventually relocate across islands in pursuit of wage gains. In our model, each firm's productivity is given, but better matching opportunities arise to workers through search on-the-job.

There is an exogenously set federal minimum wage in the economy, denoted by w_M. However, there is imperfect compliance of firms to the minimum wage.[5] Let's normalize the

[5]Legal exemptions from the minimum wage level are treated here the same as non-compliance. We avoid the complexity of explicitly modelling non-compliance behavior, as it is not the focus of our work. Moreover, with exogenous compliance we keep the wage offer distribution continuous and differentiable. A simple way to model compliance is to let firms choose whether to comply with minimum wage regulations, based on the probabilty of being caught and the resulting sanction (Ashenfelter and Smith, 1979 and Lott and Roberts

number of existing firms to 1, and assume that they make zero profits. A minimum wage w_M would drive $F(w_M)$ firms out of business if they are forced to comply with minimum wage regulations, as their productivity falls short of the minimum wage. Under imperfect compliance, only a proportion $1-\alpha$ of these firms leave the market, and workers still face some positive probability to receive a wage offer below the minimum wage. The number of operationg firms is reduced to $1-(1-\alpha)F(w_M)$. The resulting wage density would be $\frac{\alpha f(w)}{1-(1-\alpha)F(w_M)}$ for all $w < w_M$ and $\frac{f(w)}{1-(1-\alpha)F(w_M)}$ for all $w \geq w_M$. Given the reduction in the number of operating firms, we also adjust arrival rates of job offers while employed and nonemployed to $\lambda_e(1-(1-\alpha)F(w_M))$ and $\lambda_n(1-(1-\alpha)F(w_M))$, respectively.[6]

To summarize, in this model the minimum wage policy has two parameters, the level of the minimum wage, w_M, and the level of noncompliance, α. The limiting case $\alpha = 0$ represents full compliance, with wage density zero for all $w < w_M$ and $\frac{f(w)}{1-F(W_M)}$ for all $w \geq w_M$. When $\alpha = 1$ there is no effective minimum wage regulation in the economy and the wage density is simply $f(w)$.

We assume that each individual starts job search as nonemployed, in the month he leaves school. To solve for the optimal search strategy, we compute the lifetime utilities for the employed and the nonemployed. The value of employment with tenure τ is denoted by $V_e(w_\tau)$ and the value of nonemployment does not depend on specific job attributes and is denoted by V_n.

A worker who is currently nonemployed enjoys a net flow of income $b-c_n(\lambda_n)$, receives job offers above or below the minimum wage at rate $\lambda_n(1-(1-\alpha)F(w_M))$, which are accepted if the value attached to them exceeds the value of nonemployment:

$$\begin{aligned} rV_n &= b - c_n(\lambda_n) + \lambda_n(1-(1-\alpha)F(w_M))\{E_{w \geq w_M} \max[0, V_e(w) - V_n]\} \\ &\quad + \lambda_n(1-(1-\alpha)F(w_M))\{E_{w < w_M} \max[0, V_e(w) - V_n]\}. \end{aligned} \tag{1}$$

A worker currently employed in a job with starting wage w and tenure τ receives net income $w_\tau - c_e(\lambda_e)$, enjoys wage growth at rate g, is forcibly separated from his employer at rate δ, and receives job offers above or below the minimum wage at rates $(1-(1-\alpha)F(w_M))\lambda_e$, which are accepted if the value attached to them exceeds the lifetime utility in the current job:

$$\begin{aligned} rV_e(w_\tau) &= w_\tau - c_e(\lambda_e) + \delta[V_n - V_e(w_\tau)] \\ &\quad + (1-(1-\alpha)F(w_M))\lambda_e E_{w \geq w_M} \max[0, V_e(w) - V_e(w_\tau)] \\ &\quad + (1-(1-\alpha)F(w_M))\lambda_e E_{w < w_M} \max[0, V_e(w) - V_e(w_\tau)] \\ &\quad + g w_\tau V_e'(w_\tau) \text{ for } w_\tau < w_M, \end{aligned}$$

and

1995). A model with this feature would generate a spike at the minimum wage and a discontinuity in the wage distribution close to the minimum wage.

[6] This adjustment is similar to that used by Eckstein and Wolpin (1990) in an equilibrium search model. If arrival rates of job offers were not adjusted, the effect of the introduction of the minimum wage would be equivalent to a rightwards shift of the wage offer distribution.

$$
\begin{aligned}
rV_e(w_\tau) &= w_\tau - c_e(\lambda_e) + \delta\left[V_n - V_e(w_\tau)\right] \\
&\quad + (1-(1-\alpha)F(w_M))\lambda_e E_w \max\left[0, V_e(w) - V_e(w_\tau)\right] + g w_\tau V_e'(w_\tau) \text{ for } w_\tau \geq w_M,
\end{aligned}
$$

where the last term in each case represents the change in value on the job, $\frac{\partial V_e(w_\tau)}{\partial w_\tau}$.

In either labor market state, agents set an acceptance rule for job offers and the optimal level of search effort. As job switching involves no cost, the optimal acceptance rule for the employed consists in accepting any job that pays more than their current wage, w_τ. For the nonemployed, the optimal acceptance rule consists in accepting the first job offer that pays at least some reservation wage w^*, such that $V_n = V_e(w^*)$. Note that such reservation wage exists and is unique because, while the value of search is constant, the value of employment is monotonically increasing in the wage paid. If $w_M \leq w^*$, minimum wages have no impact on agents' decisions or equilibrium outcomes. Therefore, we assume that the minimum wage is binding , i.e. $w_M > w^*$, such that the value functions in this model can be rewritten as:

$$
rV_n = b - c_n(\lambda_n) + \lambda_n \int_{w_M} \left[V_e(w) - V_n\right] f(w)dw + \alpha\lambda_n \int_{w^*}^{w_M} \left[V_e(w) - V_n\right] f(w)dw, \quad (2)
$$

and

$$
\begin{aligned}
rV_e(w_\tau) &= w_\tau - c_e(\lambda_e) + \delta\left[V_n - V_e(w_\tau)\right] \\
&\quad + \lambda_e \int_{w_\tau} \left[V_e(w) - V_e(w_\tau)\right] f(w)dw + g w_\tau V_e'(w_\tau) \text{ for } w_\tau \geq w_M, \quad (3) \\
rV_e(w_\tau) &= w_\tau - c_e(\lambda_e) + \delta\left[V_n - V_e(w_\tau)\right] \\
&\quad + \lambda_e \int_{w_M} \left[V_e(w) - V_e(w_\tau)\right] f(w)dw \\
&\quad + \alpha\lambda_e \int_{w_\tau}^{w_M} \left[V_e(w) - V_e(w_\tau)\right] f(w)dw + g w_\tau V_e'(w_\tau) \text{ for } w_\tau < w_M, \quad (4)
\end{aligned}
$$

Note that in (2), (3) and (4) the terms $(1-(1-\alpha)F(w_M))$ cancel out with the correction to the density function.

A nonemployed worker will choose λ_n in order to maximize (2). The resulting first-order condition is given by

$$
c_n'(\lambda_n) = \int_{w_M} \left[V_e(w) - V_n\right] dF(w) + \alpha \int_{w^*}^{w_M} \left[V_e(w) - V_n\right] dF(w), \quad (5)
$$

thus equating the marginal cost of an extra job offer to its marginal benefit.

Similarly, the first order condition for the choice of search intensity for the employed is given by

$$
\begin{aligned}
c_e'(\lambda_e) &= \int_{w_\tau} \left[V_e(w) - V_e(w_\tau)\right] dF(w), \qquad \text{if } w_\tau \geq w_M, \quad (6) \\
c_e'(\lambda_e) &= \int_{w_M} \left[V_e(w) - V_e(w_\tau)\right] dF(w) + \alpha \int_{w_\tau}^{w_M} \left[V_e(w) - V_e(w_\tau)\right] dF(w), \quad (7) \\
\text{if } w_\tau &< w_M.
\end{aligned}
$$

By convexity of the search cost functions, the unemployed will have a higher incentive to search for jobs, and, all else being equal, raise their arrival rate of job offers above that of the employed. Among the employed, search effort decreases with the current wage: in particular, those employed below the minimum wage will search more intensively than those employed above.

Given the acceptance rule $rV_n = rV_e(w^*)$, we can solve for the value of the reservation wage by setting equation (2) equal to equation (4) evaluated at $w_\tau = w^*$ and $\tau = 0$ (exploiting the continuity of $V_e(w_\tau)$ at w_M, a property that only holds if the wage offer distribution is the same above or below the minimum wage):

$$\begin{aligned} w^* &= b - c_n(\lambda_n) + c_e(\lambda_e) + (\lambda_n - \lambda_e)\int_{w^*} [V_e(w) - V_n]\, dF(w) \\ &\quad - (1-\alpha)(\lambda_n - \lambda_e)\int_{w^*}^{w_M} [V_e(w) - V_n]\, dF(w) - gw^* V_e'(w^*) \qquad (8) \\ &= b - c_n(\lambda_n) + c_e(\lambda_e) + (\lambda_n - \lambda_e)\int_{w^*} [1 - F(w)] V_e'(w)\, dw \\ &\quad - (1-\alpha)(\lambda_n - \lambda_e)\int_{w^*}^{w_M} [F(w_M) - F(w)] V_e'(w)\, dw - gw^* V_e'(w^*). \qquad (9) \end{aligned}$$

To solve equation (9) one needs to know the function $V_e'()$. In Appendix A we find the explicit solution for $V_e'(w_\tau)$ where $w_\tau > w_M$ (16) and where $w_\tau < w_M$ (17).

The reservation wage can be numerically computed substituting (16) and (17) into (9). If the job offer arrival rates are set exogenously, then the model is fully solved by calculating the reservation wage w^* using the solution to (9). Otherwise, the joint solution of w^*, λ_n and λ_e is found by solving jointly (9), (5), (6) and (7), and this solution enables us to simulate the dynamic decision sequence of the worker. This solution provides a joint dynamic distribution of labor market mobility from nonemployment to work, from job-to-job and back to nonemployment. In the empirical work that follows we set search effort exogenously.

We now have a framework for calculating the impact of changes in the minimum wage and the compliance parameter on nonemployment. As standard in search models we get that the reservation wage is increasing in search efforts (i.e. in the arrival rate of job offers).[7] Furthermore, when the reservation wage is lower than the minimum wage, an increase in the minimum wage lowers both the reservation wage and the arrival rate of job offers. Also note that the effect of increasing compliance, i.e. lower α, on reservation wages and arrival rates is the same as that of an increase in the minimum wage. Even in this simple model an increase in w_M or a reduction in α have an ambiguous impact on nonemployment, as both policy tools cause changes on opposite sign in the job offer arrival rate, λ_n, and in the acceptance rate, $1 - (1-\alpha)F(w_M) - \alpha F(w^*)$. The net impact of minimum wage policies on nonemployment is thus an empirical issue.[8]

[7] The proof of the comparative statics is in Appendix B, where we obviously only analyze the case where the reservation wage is lower than the minimum wage.

[8] Card and Krueger (1995) emphasize this as an important feature of an empirical model for the analysis of the minimum wage. This is the case in most equilibrium search models (see Eckstein and van den Berg (2005) and Flinn (2005)).

The solution to the model enables us to simulate the probability of all labor market states, distinguishing between employment above or below the minimum wage. If $\alpha = 0$, wage observations below the minimum wage can only be explained by measurement error in reported wages. When $0 < \alpha \leq 1$, they can be explained by both imperfect compliance and measurement error. But while measurement error should apply throughout the wage distribution, noncompliance only applies below the minimum wage. This distinction will be the basis of our identification strategy of α.

3 Data

We use data drawn from the National Longitudinal Survey of Youths, which contains information on a sample of 12,686 respondents who were between 14 and 21 years of age in January 1979 (NLSY79). We attempt to obtain a sample from a fairly homogenous population, which is relatively likely to participate in the labor force and receive wage offers below the minimum wage. Hence, we restrict our sample to white males who are high school graduates, and never returned to school.[9] Specifically, we select non-black, non-hispanic, males who have completed at most 12 years of schooling and declare to hold a high school degree. We exclude from our sample those who (i) ever went to the army; (ii) ever declared to be in college; (iii) ever declared to have a college or professional degree. We further restrict our sample to those who completed high school between age 17 and 19. These restrictions leave us with a sample of 577 individuals, with almost 12(months)x18(years) work history observations per-individual.

Information on selected respondents is available since January 1978. We construct individual monthly work histories using answers to retrospective questions. We assume that market entry coincides with the month an individual completed high school. Individuals in our sample completed high school between 1974 and 1984. More than 95% of them graduated in either May or June. We follow individuals for 18 years after high school graduation and the data is organized to be consistent with the model's definitions and assumptions.

Labor Market States From the NLSY79 work history file, we obtain the monthly employment and nonemployment status from January 1978 to December 1998. We define an individual as employed in a month if he works at least 10 hours per week and at least three weeks per month, or during the last two weeks in the month. Otherwise, an individual is classified as nonemployed, and we do not further distinguish between unemployed and out of the labor force. Figure 1 shows the monthly proportion of employed and nonemployed by time since high school graduation. The data shows clear patterns of seasonality.[10]

Among those who were employed upon finishing high school, 55% of individuals started work in the year before graduation. This may happen because job search starts while in school or, more likely, because high school students may take up temporary and part time jobs while in school. The latter explanation seems also supported by the clear seasonal pattern

[9] Flinn (2005) uses a CPS sample of individuals aged 16 to 24.

[10] We focus here on the growth of employment on the extensive margin. It should be noted that the average number of hours per-employed worker also shows a positive trend during the first eight years (Table 1). This intensive margin is not part of this paper but could be added to the search framework discussed here.

of employment rates during the last year before graduation. We assume that individuals employed before graduation enter the "official" labor market upon graduation, but we will treat the proportion of individuals employed at labor market entry as an initial condition in our analysis.

The employment history information is employer-based. All references to a "job" should be understood as references to an employer. Multiple jobs held contemporaneously are treated as new jobs altogether: the associated wage is the average of the two hourly wages and the associated hours are the sum of the hours worked on the different jobs. Duration of a given job is considered as completed when a new job is recorded or the work is terminated and the individual is back into nonemployment.

Table 1 gives employment statistics by years of labor market experience after graduation. Both the average number of months worked and the average annual hours increase with experience. As expected, the average yearly number of jobs decreases with experience from 1.52 to 1.14, while the total cumulative number of jobs per worker is on average 6.5 after 18 years.[11]

Table 2 reports the duration of nonemployment spells leading to the first 10 jobs in individual careers, which seems to fall roughly monotonically with the job rank. 103 individuals had more than 10 jobs, 12 of whom had more than 20 jobs. The maximum number of jobs held is 27. As we have several censored spells in our sample, the sample mean duration is downward biased. In columns 4 and 5 we therefore also present the Kaplan-Meier nonparametric durations estimates.[12]

Nonemployment duration is on average about nine months, and the correction for censoring adds one month. If the observation with the largest associated duration is censored, the Kaplan-Meier survivor function does not go to zero as duration goes to infinity. Consequently, the area under the cumulative duration distribution still underestimates the mean duration. We thus extrapolate the survivor function using an exponential density function to compute the area under the entire curve, and from this we obtain the Kaplan-Meier extended mean duration. This is reported in column 5, and the mean duration for each job is estimated to be about one to six months longer using the extended (column 5) rather than the restricted (column 4) Kaplan-Meier estimates. Job duration increases from the first to the second job, but from the third job onwards duration falls. Obviously, selection and sample attrition are important factors for these observations.

Figure 2 plots the job separation hazard by job tenure. We do not distinguish among job ranks, due to the insufficient number of observations for each rank. Duration is truncated at 10 years (and consequently 83 out of 2539 job spells are dropped). The monthly job hazard

[11] In the Bureau of Labor Statistics Report on "Number of jobs held, labor market activity, and earnings growth among younger baby boomers: results from more than two decades of a longitudinal survey" (BLS 2002, Table 1), the average number of jobs held by white high school graduates is 9.2, which is higher than our figure. Such discrepancy stems from the different definitions of jobs. BLS (2002) define a job as an uninterrupted period of work with a particular employer, excluding recalls from temporary layoffs. In our definition using job identifiers, individuals recalled by old employers after a nonemployment spell are considered as staying in the same job.

[12] Let n_t be the population alive at time t and d_t the number of failures. The nonparametric maximum likelihood estimate of the survivor function is: $\widehat{S}(t) = \Pi_{j|t_j \leqslant t}(\frac{n_j - d_j}{n_j})$. The Kaplan-Meier restricted mean duration is computed as the area under the Kaplan-Meier survivor function. And the associated standard error is given by the Greenwood formula: $\widehat{Var}\{\widehat{S}(t)\} = \widehat{S}^2(t) \sum_{j|t_j \leqslant t} \frac{d_j}{n_j(n_j - d_j)}$.

Table 1: Employment statistics by labor market experience

Year after graduation	Avg. months worked*	Avg. annual hours**	Avg. no. of jobs per year	Avg. cumulative no. of jobs
1	8.74 (3.96)	2031 (458)	1.52 (0.74)	1.52 (0.74)
2	9.54 (3.42)	2082 (440)	1.40 (0.64)	1.99 (1.14)
3	9.79 (3.51)	2130 (489)	1.36 (0.68)	2.41 (1.52)
4	10.10 (3.23)	2153 (443)	1.33 (0.65)	2.88 (1.84)
5	10.32 (3.07)	2200 (506)	1.31 (0.64)	3.28 (2.17)
6	10.48 (2.86)	2212 (469)	1.33 (0.65)	3.72 (2.53)
7	10.55 (2.90)	2195 (490)	1.29 (0.62)	4.07 (2.79)
8	10.86 (2.58)	2206 (492)	1.24 (0.55)	4.39 (3.03)
9	10.96 (2.49)	2226 (510)	1.29 (0.68)	4.72 (3.34)
10	10.86 (2.66)	2256 (569)	1.29 (0.64)	5.06 (3.61)
11	11.06 (2.34)	2292 (555)	1.21 (0.50)	5.29 (3.81)
12	11.16 (2.27)	2309 (548)	1.23 (0.56)	5.52 (4.00)
13	10.98 (2.44)	2337 (552)	1.21 (0.49)	5.74 (4.17)
14	10.93 (2.70)	2288 (549)	1.21 (0.56)	5.95 (4.32)
15	11.15 (2.32)	2352 (677)	1.21 (0.48)	6.15 (4.53)
16	11.10 (2.48)	2342 (616)	1.19 (0.47)	6.31 (4.68)
17	10.96 (2.84)	2360 (621)	1.19 (0.49)	6.45 (4.77)
18	11.00(2.79)	2356 (615)	1.14 (0.44)	6.55 (4.87)

Standard errors are in parentheses.

* The value is conditional on observations where all states are available in all months.

** Average hours are conditional on working in all months.

Table 2: Duration (months) of non-employment spells and job spells since high school gradaution

Job No.	No. of obs.	Sample Mean duration (s.d.)	Kaplan-Meier restricted Mean duration (s.d.)	Kaplan-Meier extended Mean duration
NE*	148	8.86 (23.90)	9.93 (2.37)	10.74
1	574	32.74 (53.94)	38.69 (2.76)	44.85
2	508	33.77 (49.05)	41.03 (2.85)	47.66
3	457	27.82 (38.35)	39.37 (3.10)	46.98
4	387	24.71 (33.28)	34.06 (2.77)	37.73
5	337	22.22 (30.91)	31.23 (2.73)	35.05
6	276	20.38 (28.84)	32.33 (3.83)	37.95
7	236	23.23 (31.30)	34.59 (3.40)	40.31
8	182	17.92 (25.78)	24.82 (3.07)	29.51
9	150	17.58 (19.22)	21.51 (2.12)	22.12
10	131	16.96 (22.31)	24.19 (3.40)	28.10

* Mean duration of nonemployment conditional on nonemployment in $\tau = 1$.

rate decreases significantly with duration, consistent with the above search model with wage growth on-the-job and/or endogenous search effort.

Figures 3 and 4 show labor market transition rates by potential experience. All transitions display some trend during the first ten years and then they stay constant for the additional eight years. Furthermore, there is strong evidence of seasonality, with large monthly fluctuations. The probability of staying on the same job increases from 65 percent to 90 percent (Figure 3). The probability of remaining out of work decreases from 20 percent to about 8 percent, and that of moving from employment to nonemployment decreases from about five percent to about two percent. The probabilities of moving from nonemployment to employment and from job to job (Figure 4) fall from about 5 percent to less than 2 percent and have relatively large fluctuations. An important goal of the search model described above is to fit the trends and levels of these transition rates, although it is not meant to fit the monthly seasonal fluctuations.

Wages and Employment Cycles We next define employment cycles, in order to set the data in a way that is consistent with our search model (see Wolpin, 1992). Each cycle starts with nonemployment and terminates with the last job before a subsequent nonemployment spell. Since 55% of individuals in the sample started working before graduation, their first cycle started with their first job instead of nonemployment. For an individual i, the sequence of cycles is denoted by

$$\{c_i^1(ne_i^1, J1_i^1, J2_i^1, \cdots), c_i^2(ne_i^2, J1_i^2, J2_i^2, \cdots), \cdots\},$$

where c_i^j denotes the cycle j for individual i, ne_i^j denotes nonemployment spells, and $J1_i^j, J2_i^j, ...$ denote job spells within each cycle. We also record wages in each job spell.

The NLSY collects data on respondents' usual earnings (inclusive of tips, overtime, and bonuses, before deductions) during every survey year for each employer for whom the respondent worked since the last interview date. The amount of earnings, reported in dollars and cents, is combined with information on the applicable unit of time, e.g., per day, per hour, per week, per year, etc. Combining earnings and time unit data, the variable "hourly rate of pay job #1-5" in the work history file provides the hourly wage rate for each job. We use coded real hourly wage in 2000 dollars. Nominal wage data are deflated by monthly CPI from BLS CPI-U. We top code and bottom code the hourly wage at 150$ and 1.0$ before 1990, respectively, and at 200$ and 1.5$ afterwards. Note that, given the way in which the NLSY constructs wage information, we do not exactly have hourly wages by month. In particular, an individual's hourly wage is constant within a year unless he moves job. Clearly, when we convert nominal wage in real terms, real wages may decrease with inflation, but this may or may not be the correct actual pay for each month.

We focus on real hourly wages. Table 3 reports the mean wage on the first five jobs in the first three cycles. As expected, mean wages increase with job moves within cycles. When a new cycle starts, the mean wage on the first job is lower than the mean wage on late jobs of previous cycles.

Mean hourly wages increase from 8$ upon high school graduation to 16$ after eighteen years (Figure 10). The wage variance during this time period increases from 4$ to 8$.

Table 3: Mean wages in the first three cycles of labor market careers

	Mean wage	Mean wage above w_M	Mean wage below w_M
First Cycle			
Job 1	8.22 (306)	9.16 (239)	4.89 (67)
Job 2	10.37 (192)	10.75 (179)	5.17 (13)
Job 3	11.85 (132)	11.96 (130)	4.99 (2)
Job 4	12.43 (77)	13.24 (70)	4.26 (7)
Job 5	12.64 (42)	12.83 (41)	4.87 (1)
Second Cycle			
Job 1	10.22 (311)	11.15 (264)	4.99 (47)
Job 2	11.02 (178)	11.50 (165)	4.96 (13)
Job 3	11.49 (84)	12.21 (77)	3.47 (7)
Job 4	13.13 (56)	13.27 (55)	5.46 (1)
Job 5	15.27 (29)	15.27 (29)	-
Third Cycle			
Job 1	9.95 (242)	10.61 (217)	4.29 (25)
Job 2	11.29 (125)	11.86 (115)	4.72 (10)
Job 3	11.81 (71)	12.22 (67)	5.02 (4)
Job 4	10.98 (39)	11.16 (38)	4.38 (1)
Job 5	14.20 (24)	14.20 (24)	-

Number of observations in brackets.

Minimum wage The federal minimum wage for covered nonexempt employees is currently at \$5.15 an hour.[13] Figure 5 describes the nominal increase in the federal minimum wage 1978 and 2002, from \$2.65 to \$5.15. However, the real minimum wage, deflated by monthly CPI-U and expressed in 2000 dollars, has been decreasing during this sample period. Several states also have state-level minimum wage laws. When an employee is subject to both the state and the federal minimum wage laws, he is entitled to the higher of the two. Seven states have no minimum wage law, namely Alabama, Arizona, Florida, Louisiana, Mississippi, South Carolina and Tennessee. Four states have minimum wage rates lower than the Federal level, namely Kansas, New Mexico, Ohio and Virgin Islands. All other states have minimum wage rates that are equal or higher than the Federal level. In our estimates we will only take into account the time path of the federal minimum wage.

Various minimum wage exemptions apply under specific circumstances to workers with disabilities, full-time students, youths under 20 in their first 90 consecutive calendar days of employment, tipped employees and student learners. A minimum wage of \$4.25 per hour applies to young workers under the age of 20 during their first 90 consecutive calendar days of employment with an employer. After 90 days or when the employee reaches age 20, he or she must receive a minimum wage of \$5.15. Full-time students can be paid not less than 85% of the minimum wage before they graduate or leave school for good. Student learners aged 16 or more can be paid not less than 75% of the minium wage for as long as they are enrolled in the vocational education program. Exemptions are well documented by the US Department of Labor, Employment Standards Aministration Wage and Hour Division at www.dol.gov/esa/. We view exemptions as part of noncompliance with minimum wage regulations.

Tables 4 and 5 show statistics on pay below the minimum wage. 47 percent of the individuals are observed to work for a wage below the minimum wage for at least one month. For these workers the average number of months worked below the minimum wage is 13.5, and the average number of jobs held below the minimum wage is 1.5. The mean job duration below the minimum wage is 8.9 months. These facts indicate that if wages are reported without error the noncompliance or exemptions with the minimum wage law is substantial among young high school graduates.

Table 6 presents the mean durations of nonemployment and the first five jobs in the first three cycles, conditional on wages above or below the minimum wage. The mean duration from nonemployment to the first job is lower for jobs paying at least the minimum wage. Also, mean duration on jobs paying at least the minimum wage is always longer than mean duration on jobs paying less than the minimum wage.

Table 7 gives the number of individuals making transitions from nonemployment to jobs, and between jobs, again conditional on wages above or below the minimum wage. Most transitions to jobs paying less than the minimum wage originate in nonemployment, and most workers earn wages above the minimum wage once they switch job. Very few workers are observed to move from a job paying more than the minimum wage to one paying less than the minimum wage, and in our identification strategy we assume that this results from

[13]The federal minimum wage provisions are contained in the Fair Labor Standards Act (FLSA).

Table 4: Number of months working below the minimum wage

No. of months	No. of obs.	%
0	309	53.55
1-6	125	21.67
7-12	62	10.74
13-24	43	7.45
25-36	21	3.64
36+	17	2.95
Total	577	100.00

Table 5: Number of jobs paying below the minimum wage

No. of jobs	No. of obs.	%
0	309	53.55
1	172	29.81
2	66	11.44
3	22	3.81
4	6	1.04
6	2	0.35
Total	577	100.00

Table 6: Mean duration of nonemployment and jobs in months

	First Cycle	Second Cycle	Third Cycle
NE	7.3 (116)	5.8 (311)	5.8 (242)
To job 1 above w_M	7.2 (87)	5.3 (264)	5.0 (217)
To job 1 below w_M	7.7 (29)	8.7 (47)	12.6 (25)
Job 1	29.6 (306)	27.5 (311)	23.9 (242)
Above w_M	31.0 (239)	29.4 (264)	25.0 (217)
Below w_M	24.5 (67)	17.3 (47)	14.4 (25)
Job 2	40.3 (192)	27.5 (178)	27.8 (125)
Above w_M	42.3 (179)	27 (165)	29.8 (115)
Below w_M	12.9 (13)	34.4 (13)	5.2 (10)
Job 3	39.4 (132)	25.9 (84)	21.1 (71)
Above w_M	39.8 (130)	26.9 (77)	22.1 (67)
Below w_M	13.5 (2)	14.9 (7)	3.5 (4)
Job 4	31.2 (77)	21.5 (56)	19.6 (39)
Above w_M	31.0 (70)	21.7 (55)	18.9 (38)
Below w_M	32.7 (7)	12 (1)	45 (1)
Job 5	25.2 (42)	18.5 (29)	20.6 (24)
Above w_M	25.7 (41)	18.5 (29)	20.6 (24)
Below w_M	5 (1)	-	-

Number of observations is in parentheses. All statistics are conditional on wage being observed and this is why there is little discrepancy between moments in table 6 and 9.

Table 7: Transitions to employment and from job-to-job (no. of obs.)

	First Cycle	Second Cycle	Third Cycle
Unemployed	(116)	(311)	(242)
UE to J1 above w_M	87	264	217
UE to J1 below w_M	29	47	25
First Job above w_M	(239)	(264)	(217)
Move to J2 above w_M	93	123	88
Move to J2 below w_M	4	6	6
First Job below w_M	(67)	(47)	(25)
Move to J2 above w_M	22	14	11
Move to J2 below w_M	4	2	2
Second Job above w_M	(179)	(165)	(115)
Move to J3 above w_M	86	67	50
Move to J3 below w_M	2	5	2
Second Job below w_M	(13)	(13)	(10)
Move to J3 above w_M	4	5	5
Move to J3 below w_M	0	2	1
Third Job above w_M	(130)	(77)	(67)
Move to J4 above w_M	59	48	33
Move to J4 below w_M	6	0	1
Third Job below w_M	(2)	(7)	(4)
Move to J4 above w_M	2	2	1
Move to J4 below w_M	0	0	0
Fourth Job above w_M	(70)	(55)	(38)
Move to J5 above w_M	31	28	18
Move to J5 below w_M	1	0	0
Fourth Job below w_M	(7)	(1)	(1)
Move to J5 above w_M	6	0	1
Move to J5 below w_M	0	0	0

measurement error in wages. In our estimates we separately identify noncompliance and measurement error.

On-the-Job Wage Growth We assume a constant wage growth rate g in all jobs, which can be interpreted as the return to both general and job-specific experience. In order to estimate g, we estimate a wage growth equation:

$$\ln w_{i\tau} = \ln w_{i0} + \tau \ln(1+g)$$

where w_{i0} is the first wage observation for individual i and τ is job tenure. The OLS estimate of g is 0.002 (with standard error of 2.24e-5), which should be interpreted as the average wage growth along worker careers. The corresponding annual growth rate is $(1+g)^{12}-1 = 2.43\%$. However, the data also show evidence of huge variation in wage growth across individuals, including cases of negative wage growth.

4 Estimation

Specification We estimate the model using simulated moments. We exogenously set arrival rates of job offers (without choice of search effort) and we allow for unobserved heterogeneity in arrival rates and the parameters of the wage offer distribution by assuming that there are two types of individuals in the population, with π denoting the proportion of type one (see Heckman and Singer, 1984). The wage density function is assumed to be log normal, $\ln w \sim N(\mu, \sigma_w^2)$. The time preference parameter r is known to be 4% annually, which is 0.3% monthly. We allow for measurement error in observed wages, such that $\ln w^o = \ln w + u$, where w^o is the observed wage, w is the true wage and the error term is normally distributed: $u \sim N(0, \sigma_u^2)$.

As 55% of individuals in our sample worked before graduation, we assume that a separate labor market exists before graduation, and we characterize this labor market by an initial (period 0) reservation wage w_0^*.[14] We estimate the reservation wage directly using equation (9).[15] The parameters of the model to be estimated are in the vector $\theta = [\lambda_{n1}, \lambda_{e1}, \lambda_{n2}, \lambda_{e2}, w_1^*, w_2^*, \mu_1, \mu_2, \sigma_{w1}, \sigma_{w2}, \delta_1, \delta_2, w_{01}^*, w_{02}^*, \pi, \sigma_u^2, \alpha]'$.

Data As we have described above, we have a sample of white male high school graduates indexed by $i = 1, ..., 577$. We observe their employment status and wage if employed every month after high school graduation. The data do not allow to differentiate between unemployment and out of labor force, thus employment and nonemployment are the only labor market states we consider. Let $d_{it_i} = 1$ if the individual is working and $d_{it_i} = 0$ if the individual is not working, where t_i is the month after graduation or, equivalently, the month since entry in the labor market. We observe the following data: $[d_{it_i}^D, w_{it_i}^D]$ for $i = 1, \cdots, 577$ and $t_i = 1, \cdots, T_i$, where the superscript D denotes the data.

Simulations We simulate both conditional moments, i.e. predicted values of wages and employment, conditional on the observed (data) values in the previous month, and unconditional moments, which only depend on the simulated values for the previous month. We take the 2.43% annual wage growth rate as given, as resulting from the estimates of the previous section.

In the first month ($t = 1$), individual i has 0.45 probability being nonemployed since 45% of the individuals are nonemployed. If he is employed, we simulate a wage w_{i0} such that $w_{i0} \geq w_0^*$. If individual i is nonemployed, with probability λ_n, he receives a random wage draw from a truncated log normal distribution $\Phi(\cdot)$. The truncation is due to the minimum wage and the wage density function **is**

$$\begin{aligned} \frac{\phi(w)}{1-(1-\alpha)\Phi(w_M)} \quad & \text{if } w \geq w_M \\ \frac{\alpha\phi(w)}{1-(1-\alpha)\Phi(w_M)} \quad & \text{if } w < w_M \end{aligned}$$

[14] Wolpin (1987) documents similar **evidence** and argues that it is consistent with the notion that the search process begins prior to graduation.

[15] Assuming measurement error in reported wages enable us to estimate the reservation wage by the moments rather than by the lowest observed wage.

where $\phi(\cdot)$ is the log normal wage density function with mean μ and variance σ^2 **and** $\Phi(\cdot)$ is the corresponding c.d.f.. If the wage offer is above w^*, he moves from nonemployment to employment. When he is employed at wage w_{it}, with probability λ_e he receives a random wage draw w' from $\Phi(\cdot)$. If $w' > w_{it}$, he moves to the new job. Otherwise he stays on current job and his wage increases to $w_{it+1} = w_{it}(1+g)$. In any period he goes back to nonemployment with probability δ.

Let's first consider all individuals who have observations on $d^D_{it_i}, w^D_{it_i}$ from $t_i = 1, ...T_i$, i.e. that are not left-censored. In a *conditional* simulation s, the model predicts $d^s_{it_i}$ and $w^s_{it_i}$, conditional on $d^D_{it_i-1}$ and $w^{TD}_{it_i-1}$. If an individual is working and a wage is observed, we simulate the measurement error to obtain the "true" wage according to $w^{TD}_{it_i} = w^D_{it_i} - u$.[16] Now TD indicates a predicted "true" wage that should be related to the observed wage. Conditional on the "true" wage in $t_i = 1$, we simulate the outcome for $t_i = 2$, i.e. $[d^s_{it_i=2}, w^s_{it_i=2}]$, again for 25 simulations. We thus generate a sequence of 25 simulated observations $[d^s_{it_i}, w^s_{it_i}]$ for $t_i = 1, ..., T_i$, that follow the true sequence $[d^D_{it_i-1}, w^{TD}_{it_i-1}]$ for $t_i = 1, ..., T_i$. When a wage is not observed, the simulated wage is dropped from the simulated sample for all simulations. In an *unconditional* simulation, the prediction of $[d^s_{it_i}, w^s_{it_i}]$ is conditional on the last period simulations $[d^s_{it_i-1}, w^s_{it_i-1}]$. Also in this case we run 25 simulations.

For the left censored observations, suppose that the first observation for individual i is available at $t_i = 2$. The simulation for period 2 is based on the sequence of two simulations: we first simulate period 1 employment status and wages, and conditional on these we simulate period 2 employment status and wages, $[d^s_{it_i=2}, w^s_{it_i=2}]$.[17] Similarly if the first available observation is at time 3, and so on. Having said this, we have $N^S = 25$ simulations based on the parameter vector θ.

Monthly moments and identification We use two sets of moments: the first set is computed by months and the second set is computed by employment cycle.[18]. Among the monthly moments, the conditional ones include the nonemployment rate mne; the proportion of individuals that move from nonemployment to employment mtr_1; the proportion of individuals that move from job to job mtr_2; the proportion of individuals that move from employment to nonemployment mtr_3; the mean wage mw_1; its standard deviation mw_2; the mean wage below the minimum wage mw_3; and the standard deviation of the wage below the minimum wage mw_4. The unconditional moments include all previous 8 series of monthly moments plus the proportion of individuals that work below the minimum wage mp. All these moments are computed from the data and simulated 25 times, either conditionally or not. The simulated moments used in estimation are the averages across all simulations.[19]

Transition moments from nonemployment to employment are used to identify the offer arrival rate when nonemployed. Similarly, job-to-job transitions identify the offer arrival rate when employed and transitions from employment to nonemployment identify the job destruction rate. The reservation wage is identified by the nonemployment rate. Wage moments can identify the parameters of the wage distribution. In particular, the initial

[16] Note that the "true" data here, w^{TD}, has a simulated aspect the we do not specifically indicate.

[17] In this example the data starts at period 2, and, therefore, the initial period and period 1 data are simulated.

[18] We have 18 years' data. So each monthly moment is a vector of 216 elements.

[19] See Appendix C for the exact defenitions of the moments.

wage identifies the initial reservation wage. The mean and the variance of the wage are identified from the observed monthly mean and variance as well as from the transitions from job-to-job.

A key aspect of the paper is the identification of the noncompliance parameter α. The proportion of workers who earn below the minimum wage identifies α, but it should be noted that this moment is also affected by the measurement error. However, the measurement error in wages also affects the variance of the observed wage distribution without affecting the job-to-job transitions. Hence, conditional on the variance of the offered wage and job-to-job transitions, the proportion of workers earning less than the minimum wage and the variance of the monthly wage jointly identify the measurement error and α.

Cycle moments and identification The second set of moments are based on employment cycles, as described in the previous section. In particular, we use moments from the first three employment cycles. We first use duration moments, namely mean nonemployment and employment duration on the first three jobs in the first three employment cycles. Second, we use wage moments, namely mean and standard deviation of accepted wages (either global or below the minimum wage) on the first three jobs in the first three cycles. Third, we use transition moments, including the proportion of individuals who start the first three cycles from nonemployment, the proportion of individuals who move from the first to the second job, from the second to the third job in the first three cycles. Last, we also use the proportion of individuals who work below the minimum wage on the first three jobs in the first three cycles.

As with monthly moments, nonemployment duration identifies the offer arrival rate when nonemployed. Job-to-job transitions identify the offer arrival rate when employed. The reservation wage is identified by the wage on the first job. The mean and variance of the wage offer distribution are identified by the mean wage and its standard deviation. The job destruction rate is identified by the nonemployment rate when new cycles start. The initial reservation wage is identified by the initial nonemployment rate and the mean wage on the first job in the first cycle (being significantly lower than wage on the first job in the second and third cycles). The monthly proportion below the minimum wage and the variance of the wage identifies the noncompliance parameter α and the measurement error variance (see the discussion above).

Implementation. We implement the SGMM by using these two sets of moments and then compare results. Let mom_j^D be moment j in the data and $mom_j^S(\theta)$ be moment j from the model simulation, given the parameter vector θ. The moment vector is

$$g(\theta)' = [mom_1^D - mom_1^S(\theta), \cdots, mom_j^D - mom_j^S(\theta), \cdots, mom_J^D - mom_J^S(\theta)]$$

where J is the total number of moments. For first set of moments, $J = 3672$ and for the second set of moments, $J = 66$.[20] The objective function to be minimized with respect to θ

[20] The first set of moments include 8 series of conditional monthly moments and 9 series of unconditional monthly moments, so J=17*216=3672. The second set of moments consist of 12 duration moments, 36 wage moments, 9 transition moments and 9 moments on proportions below the minimum wage.

is

$$J(\theta) = g(\theta)' W g(\theta),$$

where the weighting matrix W is set to be diagonal.[21]

The Asymptotic Theory of the SGMM Let $y_{it} = [d^D_{it_i}, w^D_{it_i}]$ be a vector of observed data for individual i after t months of market experience. Given the observed state vector at t for individual i, z_{it}, the simulated values of the random events at simulation s, $\varepsilon_s(z_{it})$, and the value of the parameters θ^* the model implies that,

$$y^s_{it} = G(z_{it}, \varepsilon_s(z_{it}); \theta^*).$$

The function $G(z_{it}, \varepsilon_s(z_{it}); \theta^*)$ is given by the solution to the model. We assume that the data $\{y_{it}, z_{it}\}^I_{i=1}$ for all t are *i.i.d.* By the independence of the simulated random variables we have the orthogonality condition that $E[G(z_{it}, \varepsilon_s(z_{it}); \theta^*) - y_{it} \mid z_{it}] = 0$. Now for N^S simulations of $\varepsilon_s(z_{i\eta})$, we define $h(y_{i\tau})$ as the contribution of individual i for the vector of data moments at time t, and $h(y^s_{it})$ as the contribution of simulation s of individual i for the vector of simulated moments at time t.

$$\begin{aligned} g_{tI}(\theta) &= [\frac{1}{I_t}\sum_{i=1}^{I_\tau} h(y_{it}) - \frac{1}{N^S}\sum_{s=1}^{N^S}(\frac{1}{I_t}\sum_{i=1}^{I_\tau} h(y^s_{it};\theta)] \\ &\equiv \frac{1}{I_t}\sum_{i=1}^{I_\tau} h_i(\theta) \end{aligned}$$

and we have the result that $g_{tI}(\theta) \longrightarrow 0$ as $I \longrightarrow \infty$. And under the standard regularity conditions $\theta \longrightarrow \theta^*$. Note that for any function of z_{it} that multiply $y_{it} - \frac{1}{N^S}\sum_{s=1}^{N^S} G(z_{it}, \varepsilon_s(z_{it}); \theta)$ and the average of this product converges to zero as I converges to infinity.[22] The asymptotic variance is given by $(1 + \frac{1}{N^s})(A'WA)^{-1}A'W\Omega WA(A'WA)^{-1}/I$, where N^S is the number of simulations, $A \equiv E[\nabla_\theta h_i(\theta^*)]$ and $\Omega \equiv E[h_i(\theta^*)\, h_i(\theta^*)']$.

5 Results

Parameters The estimates of the parameters are presented in Table 8. The two unobserved types of individuals are allowed to differ in all parameters but in the level of compliance α and the measurement error variance σ_u.

Starting from monthly moments, the parameter estimates have plausible magnitudes and are in line with previous estimates of the parameters of a search model with search on-the-job. That is, arrival rates of job offers are higher for nonemployed than employed individuals and these rates are different across types of individuals, delivering a decreasing hazard rate. Type 1 individuals, representing about 43% of our sample, have lower job offer arrival rates

[21] In our estimates, the weight on each moment is set to be one over its sample mean for the monthly moments. We use the identity matrix as the weighting matrix for cycle moments.

[22] For a recent survey of the asymptotic distribution of the estimated parameters, tests and references see Carrasco and Florens (2002).

Table 8: Parameter estimates of search model with $g = 0.0243$

Parameters	Estimates (monthly moments)		Estimates (cycle moments)	
	coef.	(s.e.)	coef.	(s.e.)
λ_{n1}	0.428	(0.102)	0.451	(0.001)
λ_{n2}	0.958	(0.100)	0.958	(0.034)
λ_{e1}	0.097	(0.024)	0.129	(0.017)
λ_{e2}	0.237	(0.047)	0.264	(0.011)
π	0.432	(0.009)	0.473	(0.147)
w_1^*	2.696	(0.270)	2.695	(0.373)
w_2^*	8.520	(0.454)	10.309	(0.134)
μ_1	1.527	(0.057)	1.571	90.043)
μ_2	1.652	(0.047)	1.667	(0.017)
σ_{w1}	0.482	(0.067)	0.471	(0.027)
σ_{w2}	0.517	(0.022)	0.513	(0.010)
δ_1	0.061	(0.004)	0.040	(0.002)
δ_2	1.88e-3	(0.0003)	0.012	(0.0003)
w_{01}^*	2.843	(0.323)	3.128	(0.086)
w_{02}^*	6.791	(0.085)	6.610	(0.181)
σ_u	6.14e-3	(0.009)	0.012	(0.001)
α	0.249	(0.037)	0.254	(0.042)

Notes. The sample includes male high-school graduates from the NLSY. Number of observations: 577. Estimation methods: Simulated GMM.

while both nonemployed and employed, a higher job destruction rate, a lower mean wage offer and a lower reservation wage.[23] One would expect that reservation wages during the final year of high school are lower than after graduation, and this is indeed the case for type-2 individuals, while the two reservation wage values are not significantly different from each other for type-1 individuals.

Type 2 individuals have reservation wages above the minimum wage, which implies that for more than half of our sample the minimum wage policy is not binding. Hence, changes in the minimum wage and noncompliance do not affect 57% of high school graduates.

The types are identified from the panel dimension of the data. One way to illustrate identification is to look at the posterior probability of being type 1, conditional on a particular event. For example, let's look at transitions into nonemployment. We can compute the posterior probability of being type 1, conditional on observing a transition form employment to nonemployment. Given the model estimates, this is

$$\begin{aligned}\Pr\left(type\ 1|e\rightarrow ue\right) &= \frac{\Pr\left(e\rightarrow ue|type\ 1\right)\Pr(type\ 1)}{\Pr\left(e\rightarrow ue\right)}\\ &= \frac{\delta_1\pi_1}{\delta_1\pi_1+\delta_2\pi_2}=0.96,\end{aligned}$$

implying that type-1 individuals represent 96% of employment to nonemployment transitions.

The novelty of our results consists in providing an estimate for the extent of noncompliance of firms' job offers to minimum wage regulations, represented by the parameter α. We find that the arrival rate of job offers below the minimum wage is about a quarter of that above the minimum wage. Having estimated α, the steady state proportion of jobs that pay less than the minimum wage is given by

$$\frac{\pi_1\alpha[F_1\left(w_M\right)-F_1\left(w_1^*\right)]}{\pi_1[1-F_1\left(w_1^*\right)]+\pi_2[1-F_2\left(w_2^*\right)]}=0.105.$$

This number is clearly lower than α as only type 1 individuals are affected by the compliance level. Furthermore, it is only the density of offers between the reservation wage of type-1 individuals and the minimum wage that matters for the job count below w_M.

The estimates obtained on cycle moments are quite similar to those obtained on monthly moments, including the ranking of values for type 1 and type 2 individuals. The difference that is worthwhile mentioning is that under cycle moments we obtaine a higher estimate of the measurement error variance. But the estimate for α remains virtually unchanged. Based on our assumed model, the measurement error variance and the compliance parameter estimates should be linked by the fact that, whenever one comes across a wage observation below w_M, this should stem from either measurement error or noncompliance. However, in practice the estimated value of α does not seem to be too sensitive to variations in σ_u.

[23] It should be noted that the reservation wage depends on the level of the minmum wage, but in practice we find that w_M has a negligible impact on w^*. We back out the net value of nonemployment for both types: $b_1=-0.357$ and $b_2=-3.399$, which are interpreted here as very high search costs.

Model Fit and Interpretation Figures 6-13 show the fit of all the monthly moments that are used for estimation of the parameters of interest. It is quite remarkable to see how well the conditional simulated moments fit the data moments in terms of the eighteen-year trends, levels and seasonal fluctuations.

The model fits well the life cycle decrease in nonemployment (Figure 6), and the slight increase in the transition rate from nonemployment to employment (Figure 7) during the first 10 years in the labor market, but fails to fit the increase in the seasonal fluctuations in these transition rates. The model fits well the decrease in job-to-job transitions (Figure 8): but while conditional moments also fit well its level, the unconditional ones seem to underpredict mobility. Finally, the model fails to fit the decreasing trend in transitions from employment to nonemployment, but it does fit its level. Note that the only potential source of dynamics here is the unobserved heterogeneity in the job destruction rate: the implication is that the two-type heterogeneity does not seem to be sufficient to fit the job destruction decreasing trend.

The model with the *same* wage offer distribution for the entire eighteen years and a constant wage growth on the same job (tenure) of 2.43 annually, fits very well the eighteen years of mean hourly wage growth from about 8 to 16 dollars (Figures 10). The upward trend in the hourly wage variance is also well predicted by the model (Figure 11). Furthermore, the conditional moments of the model do well in predicting the mean and variance of wages below the minimum wage (Figures 12 and 13). The unconditional moments fit the trends in the data but do not fit the levels and fluctuations in wages.

The fit results are quite important in supporting the search model interpretation of wage growth upon high-school graduation. During the first 18 years in the labor market, 55% of wage growth is due to on-the-job wage growth and the remaining 45% is due to job mobility.[24] Topel and Ward (1992) find that wage gains attributable to job mobility explain about one third of total earnings growth during the first ten years of labor market experience. Their analysis mostly refer to the 1960s and their sample consist of young men of all schooling levels. A more comparable recent study by Omer (2005) uses the same NLSY79 white male high school graduate sample and finds that wage growth between jobs accounts for about 45% of the worker's entire wage growth.

Figure 14 presents model and data moments for the proportion of individuals working below the minimum wage. In estimation we have used the unconditional prediction of the model as the conditional moments have no information on the noncompliance parameter, α. The model fit of the level and the trend of this proportion is remarkable. This result provides support for using the model as a basis for explaining of the fact that a large proportion of workers receive wages below the minimum wage. Hence, the analysis of the implications of changing the compliance rate and the minimum wage on labor market outcome can be well trusted.

The cycle moments used in estimation are also reproduced well by our estimates (see Table 9). Mainly the mean duration, mean wage and standard deviation of the wage (above and below the minimum wage) are captured quite well by the model for cycle one and

[24] Over the entire sample mean hourly wage grows by \$8.1, from \$8.32 in the first month to \$16.42 in the last month. Consider an individual staying on the same job, on average his wage would increase by \$4.46 from \$8.32 to $8.32 * (1 + 0.2\%)^{215} = 12.78$.

somewhat less accurately for the other two cycles. The model fits nonemployment duration and duration of each job in the cycles, as well as the mean and standard deviation of wages by jobs and cycles, both overall and below the minimum wage. The fit for the proportion of workers below the minimum wage is good in the first cycle but not as good for the second and third jobs in the later two cycles. This maybe mostly due to the low number of observations in later jobs/cycles.

The fit of transition moments is less accurate as it can be noted from the last three rows in Table 9. In particular, the model predicts a much higher transition rate from nonemployment to work in all three cycles. Job-to-job transitions also deviate from the data but to a lesser extent.

Counterfactual Policies We use the model to get quantitative implications of changing the level of the minimum wage (Table 10) and the rate of noncompliance with the minimum wage (Table 11).

An increase in the minimum wage by $1.35, from $5.15 to $6.50, increases nonemployment by 1.8% to 2.1% and a decrease in the minimum wage by $1.65 decreases nonemployment by about 2.3%. The same increase (decrease) in minimum wage increases or decreases mean accepted hourly wages by 10-30 cents. These results indicate a significant impact of the minimum wage level on unemployment, consistently with the main findings reported by Kennan (1998). That is, time series correlations indicated that a 10 percent rise in the minimum wage is associated with one percentage point of unemployment. It should be noted that for the model we estimated it would have been possible to get lower unemployment due to an increase in the minimum wage (see section 2).

However, the impact of the change in minimum wage on mean wages is very small. Furthermore, the impact of the minimum wage on inequality, measured as the ratio of the 90th to the 10th percentiles and standard deviation of wages is very small. However, the impact of the same change on the proportion of workers that are employed for a wage below the minimum wage is large. These findings from the model provide a very simple and convincing explanation to facts that a-priori seem to be inconsistent with an economic model. That is, potential large changes in the proportion of individuals working below the minimum wage due to changes in real minimum wage could be consistent with the fact that the same changes in minimum wage have small effect on the wage distribution and unemployment.

The effects of changing the noncompliance rate from the estimated level of 25 percent to full compliance ($\alpha = 0$), to less compliance ($\alpha = 0.5$) and no compliance ($\alpha = 1.0$) are qualitatively similar to the results on the change in the level of the minimum wage. That is, lower (higher) compliance decreases (increases) the nonemployment rate by one to four percentage points depending on the change. Similarly, the hourly mean wage and the ratio of the 90th to 10th percentile in the wage distribution (opposite from mean wage!) decreases (increases) by few cents as compliance decreases (increases). However, the proportion of workers that are employed for a wage below the minimum wage changes dramatically (zero to 12 percent after ten years in the labor market) with the rate of the compliance policy.

Table 10: Counterfactual: Change the Level of the Minimum Wage

All individuals	Years in The Labor Market	Model $w_m = 5.15$	Counterfactuals $w_m = 3.5$	$w_m = 6.5$
Non-employment	1	21.4	18.8	23.5
rate	2	10.7	8.3	12.5
(percentage)	5-9	9.8	7.5	11.6
	10-18	9.9	7.6	11.9
Mean wage	1	9.2	9.0	9.2
(2000 dollars)	2	10.3	10.1	10.4
	5-9	12.8	12.5	13.0
	10-18	15.0	14.6	15.2
90p/10p wage	1	2.9	3.1	3.2
	2	2.8	3.2	3.1
	5-9	3.4	3.9	3.3
	10-18	4.1	4.7	4.0
Proportion below	1	12.5	2.4	20.8
the minimum wage	2	6.8	0.8	14.3
(percentage)	5-9	4.9	0.4	10.6
	10-18	4.9	0.4	10.9

Table 11: Counterfactual: Change the Level of Compliance

All individuals	Years in The Labor Market	Model $\alpha = 0.25$	Counterfactuals $\alpha = 0$	$\alpha = 0.5$	$\alpha = 1.0$
Non-employment	1	21.4	23.3	20.1	18.2
rate	2	10.7	12.7	9.4	7.9
(percentage)	5-9	9.8	11.7	8.6	7.1
	10-18	9.9	11.8	8.7	7.1
Mean wage	1	9.2	9.3	9.1	8.9
(2000 dollars)	2	10.3	10.6	10.1	10.0
	5-9	12.8	13.1	12.6	12.4
	10-18	15.0	15.3	14.8	14.5
90p/10p wage	1	2.9	2.7	3.1	3.2
	2	2.8	2.6	3.0	3.2
	5-9	3.4	3.1	3.6	4.0
	10-18	4.1	3.7	4.4	4.9
Proportion below	1	12.5	9.0	14.8	17.8
the minimum wage	2	6.8	2.2	9.8	12.7
(percentage)	5-9	4.9	0.0	7.9	11.2
	10-18	4.9	0.0	7.9	11.6

6 Conclusions

This paper interprets the individual monthly work history data of white male high school graduates using a continuous time search on-the-job model. The model is simple and the number of parameters is small relative to the empirical dynamic stochastic models that attempt to interpret the same data (for example, Keane and Wolpin, 1997). The estimated model fits well the labor market mobility from school to work, job-to-job transitions and the wage growth in the sample.

We extended the standard model to account for the fact that early in their careers a large fraction of individuals are observed to work for a wage below the official minimum wage. To do it, we design a simple extension to the standard model, where observed wages below the minimum wage are a result of noncompliance and exemptions from the federal level of the minimum wage and/or measurement error in wage data. We assume that a constant fraction of jobs disappears as the minimum wage is set, but workers may still receive job offers below the minimum wage due to firm noncompliance or exemptions. We also assume that the job arrival rate changes with noncompliance. Under these circumstances the effect of an increase in the minimum wage and/or the level of compliance on nonemployment may be ambiguous. Having estimated this model, we find that about ten percent of accepted offered wages are below the federal minimum wage. We also find that the estimated model predicts an increase in nonemployment that is consistent with aggregate data on the decrease in minimum wage and observed changes in the unemployment rates.

It is clear that there are alternative search models that fit well the observed sample of wages and job mobility due to a general known property of observational equivalence of alternative theories to a given sample. For example, we also estimated a model where the arrival rates of jobs *are not* affected by changes in the level of the minimum wage and the rate of compliance.[25] This model could fit the observed data almost as well as the one we report here. However, the predictions on nonemployment rates due to changes in the level of compliance are reversed since less noncompliance/exemptions only shift the offered wage to have more weight on higher wages and has no impact on the demand (offer rate) for labor.

The main task one needs to accomplish is to formulate and estimate a dynamic equilibrium search model where the demand for labor, the level of compliance and the job arrival rates are endogenously determined. This paper shows that the analysis of a model with productivity heterogeneity among firms could well fit the observed labor supply and wage data. The main question is whether it will fit also the firm level data and provide new predictions on the impact of a minimum wage policy where noncompliance and exemptions are included.

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Appendix A: Computation of $V_e'()$

Using integration by parts, (3) and (4) can be rewritten as:

$$\begin{aligned} rV_e(w_\tau) &= w_\tau - c_e(\lambda_e) + \delta\left[V_n - V_e(w_\tau)\right] \\ &\quad + \lambda_e \int_{w_\tau} [1 - F(w)] V_e'(w)\, dw + g w_\tau V_e'(w_\tau) \text{ for } w_\tau \geq w_M \end{aligned} \tag{10}$$

and

$$\begin{aligned} rV_e(w_\tau) &= w_\tau - c_e(\lambda_e) + \delta\left[V_n - V_e(w_\tau)\right] \\ &\quad + \lambda_e \int_{w_M} [1 - F(w)] V_e'(w)\, dw \\ &\quad + \lambda_e \int_{w_\tau}^{w_M} V_e'(w)\left[1 - \alpha F(w) - (1-\alpha) F(w_M)\right] dw \\ + g w_\tau V_e'(w_\tau) \text{ for } w_\tau &< w_M \end{aligned} \tag{11}$$

By differentiating (10) and (11):

$$g w_\tau V''(w_\tau) = \left\{r + \delta + \lambda_e\left[1 - F(w_\tau)\right] - g\right\} V'(w_\tau) - 1, \text{ for } w_\tau \geq w_M \tag{12}$$

$$\begin{aligned} g w_\tau V''(w_\tau) &= \left\{r + \delta + \lambda_e\left[1 - \alpha F(w_\tau) - (1-\alpha) F(w_M)\right] - g\right\} V'(w_\tau) \\ -1, \;\text{ for } w_\tau &< w_M. \end{aligned} \tag{13}$$

Consider the case $w_\tau \geq w_M$ first, and let $u(w_\tau) = -\frac{1}{g w_\tau}\left\{r + \delta - g + \lambda_e\left[1 - F(w_\tau)\right]\right\}$ and $q(w_\tau) = -\frac{1}{g w_\tau}$. Equation (12) implies

$$\begin{aligned} V_e'(w_\tau) &= e^{-\int u(w_\tau) dw_\tau}\left[A + \int q(w_\tau) e^{\int u(w_\tau) dw_\tau} dw_\tau\right] \\ &= e^{-\int_{w_0}^{w_\tau} u(s) ds}\left[A + \int_{w_0}^{w_\tau} q(s) e^{\int_{s_0}^{s_\tau} u(z) dz} ds\right] \end{aligned} \tag{14}$$

where A is an arbitrary constant. Now let

$$\begin{aligned} R(w_\tau; w_0) &= -\int_{w_0}^{w_\tau} u(s) ds \\ &= \int_{w_0}^{w_\tau} \frac{r + \delta - g + \lambda_e[1 - F(s)]}{gs} ds. \end{aligned} \tag{15}$$

This in turn implies

$$\int_{w_0}^{w_\tau} q(s) e^{\int_{s_0}^{s_\tau} u(z) dz} ds = \int_{w_0}^{w_\tau} q(s) e^{-R(s)} ds = -\int_{w_0}^{w_\tau} \frac{1}{gs} e^{-R(s)} ds.$$

Having set $\tau = 0$ in (14), one obtains $V_e'(w_0) = A = \int_{w_0}^{\infty} \frac{1}{gs} e^{-R(s)} ds$ and thus

$$\begin{aligned} V_e'(w_\tau) &= e^{R(w_\tau)} \left[\int_{w_0}^{\infty} \frac{1}{gs} e^{-R(s)} ds - \int_{w_0}^{w_\tau} \frac{1}{gs} e^{-R(s)} ds \right] \\ &= e^{R(w_\tau)} \int_{w_\tau}^{\infty} \frac{1}{gs} e^{-R(s)} ds \\ &= \int_{w_\tau}^{\infty} \frac{1}{gs} e^{R(w_\tau) - R(s)} ds \end{aligned}$$

where

$$R(w_\tau) - R(s) = \int_s^{w_\tau} \frac{r + \delta - g + \lambda_e [1 - F(z)]}{gz} dz.$$

Therefore:

$$V_e'(w_\tau) = \int_{w_\tau}^{\infty} \frac{1}{gs} \exp\left(\int_s^{w_\tau} \frac{r + \delta - g + \lambda_e [1 - F(z)]}{gz} dz \right) ds, \tag{16}$$

is the solution when $w_\tau > w_M$. Similarly, when $w_\tau < w_M$

$$V_e'(w_\tau) = \int_{w_\tau}^{\infty} \frac{1}{gs} \exp\left(\int_s^{w_\tau} \frac{r + \delta - g + \lambda_e [1 - \alpha F(z) - (1-\alpha) F(w_M)]}{gz} dz \right) ds. \tag{17}$$

Appendix B: Comparative Statics

When $w^* > w_M$, it is trivial that the reservation wage is independent of the minimum wage or the compliance rate. We consider the case when $w^* < w_M$. Optimal search efforts and reservation wage are jointly determined by equation (5) and equation (9). Combining these two equations together gives

$$w^* = b - c_n(\lambda_n) + c_e(\lambda_e) + (\lambda_n - \lambda_e) c_n'(\lambda_n) - g w^* V_e'(w^*).$$

Taking derivative with respect to λ_n, we have

$$\frac{\partial w^*}{\partial \lambda_n} = (\lambda_n - \lambda_e) c_n''(\lambda_n) - g V_e'(w^*) \frac{\partial w^*}{\partial \lambda_n} - g w^* V_e''(w^*) \frac{\partial w^*}{\partial \lambda_n}.$$

Thus

$$\begin{aligned} \frac{\partial w^*}{\partial \lambda_n} &= (\lambda_n - \lambda_e) c_n''(\lambda_n) [1 + g V_e'(w^*) + g w^* V_e''(w^*)]^{-1} \\ &= \frac{(\lambda_n - \lambda_e) c_n''(\lambda_n)}{\{r + \delta + \lambda_e [1 - \alpha F(w^*) - (1-\alpha) F(w_M)]\} V'(w^*)} > 0. \end{aligned}$$

The reservation wage is increasing in search efforts, or the job arrival rate.

Differentiating (5) with respect to w_M gives

$$\frac{\partial \lambda_n}{\partial w_M} = \frac{-(1-\alpha) \int_{w^*}^{w_M} F'(w_M) V_e'(w) dw}{c_n''(\lambda_n) + [1 - \alpha F(w^*) - (1-\alpha) F(w_M)] V_e'(w^*) \frac{\partial w^*}{\partial \lambda_n}} < 0.$$

Therefore, when the minimum wage increases, both search effort and reservation wage decrease. Similarly differentiating (5) with respect to α gives

$$\frac{\partial \lambda_n}{\partial \alpha} = \frac{\int_{w^*}^{w_M} [F(w_M) - F(w)] V_e'(w)\, dw}{\{c_n''(\lambda_n) + [1 - \alpha F(w^*) - (1-\alpha) F(w_M)] V_e'(w^*) \frac{\partial w^*}{\partial \lambda_n}\}} > 0.$$

Thus, decreasing a, that is, increasing the compliance rate has the same effect as increasing the minimum wage on the reservation wage and search efforts.

To analyze the effect of the level and compliance of the minimum wage on nonemployment rate, we consider the unemployment hazard rate. Recall that the wage density function after imposing w_M is given by

$$\phi(w) = \left\{ \begin{array}{ll} \frac{\alpha f(w)}{1-(1-\alpha)F(w_M)} & w < w_M \\ \frac{f(w)}{1-(1-\alpha)F(w_M)} & w \geq w_M \end{array} \right\}.$$

Thus, the c.d.f. is given by

$$\Phi(w) = \left\{ \begin{array}{ll} \frac{\alpha F(w)}{1-(1-\alpha)F(w_M)} & w < w_M \\ \frac{\alpha F(w_M)}{1-(1-\alpha)F(w_M)} + \frac{F(w)-F(w_M)}{1-(1-\alpha)F(w_M)} & w \geq w_M \end{array} \right\}.$$

Hazard rate is determined by

$$h = \lambda_n [1 - (1-\alpha) F(w_M)] [1 - \Phi(w^*)].$$

If $w^* > w_M$, then $h = \lambda_n [1 - F(w^*)]$. The hazard rate as well as the nonemployment rate is not affected by the minimum wage policy. If $w^* < w_M$, however,

$$h = \lambda_n [1 - (1-\alpha) F(w_M) - \alpha F(w^*)].$$

Increasing w_M (decreasing α) implies decreasing hazard rate and increasing nonemployment rate, when keeping the reservation wage fixed. However, the reservation wage decreases when increasing w_M (decreasing α) and this will increase the hazard rate. Hence, the net impact of the minimum wage policy on nonemployment is an empirical issue.

Appendix C: Moments

Monthly Moments To compute the moments in the data, we use following formulas. Note that all moments are calculated by each month in the labor market $\tau = 1, 2, \cdots, 216$. For example, mne^D is a column vector of 216 dimensions and each element $mne^D(\tau)$ is determined by

$$mne^D(\tau) = \frac{\sum_i I(d_{i\tau}^D = 0)}{\sum_i I(d_{i\tau}^D = 0) + \sum_i I(d_{i\tau}^D = 1)};$$

$I(\cdot)$ is an indicator function, which equals one if the condition is satisfied and equals zero otherwise. Similarly

$$mtr_1^D(\tau) = \frac{\sum_i I(d_{i\tau}^D = 0, d_{i\tau+1}^D = 1)}{\sum_i I(d_{i\tau}^D = 0)}, \tau = 1, 2, \cdots, 215,$$

$$mtr_2^D(\tau) = \frac{\sum_i I(d_{i\tau}^D = 1, d_{i\tau+1}^D = 1)}{\sum_i I(d_{i\tau}^D = 1)}, \tau = 1, 2, \cdots, 215,$$

where $d_{i\tau}^D = 1$ and $d_{i\tau+1}^D = 1$ refer to two different jobs one after another.

$$mtr_3^D(\tau) = \frac{\sum_i I(d_{i\tau}^D = 1, d_{i\tau+1}^D = 0)}{\sum_i I(d_{i\tau}^D = 1)}, \tau = 1, 2, \cdots, 215, ;$$

$$mw_1^D(\tau) = \frac{\sum_i (w_{i\tau}^D | w_{i\tau}^D > 0)}{\sum_i I(d_{i\tau}^D = 1 | w_{i\tau}^D > 0)};$$

$$mw_2^D(\tau) = \sqrt{\frac{\sum_i ((w_{i\tau}^D - mw_1^D(\tau))^2 | w_{i\tau}^D > 0)}{\sum_i I(d_{i\tau}^D = 1 | w_{i\tau}^D > 0) - 1}};$$

$$mw_3^D(\tau) = \frac{\sum_i (w_{i\tau}^D | 0 < w_{i\tau}^D < w_{M\tau})}{\sum_i I(d_{i\tau}^D = 1 | 0 < w_{i\tau}^D < w_{M\tau})};$$

$$mw_4^D(\tau) = \sqrt{\frac{\sum_i ((w_{i\tau}^D - mw_3^D(\tau))^2 | 0 < w_{i\tau}^D < w_{M\tau})}{\sum_i I(d_{i\tau}^D = 1 | 0 < w_{i\tau}^D < w_{M\tau}) - 1}}$$

$$mp^D(\tau) = \frac{\sum_i I(w_{i\tau}^D < w_{M\tau} | w_{i\tau}^D > 0)}{\sum_i I(d_{i\tau}^D = 1 | w_{i\tau}^D > 0)},$$

where $w_{M\tau}$ is the minimum wage.

One period ahead conditional simulated moments are defined as following:

$$mne^S(\tau) = \frac{1}{N^S} \sum_{s=1}^{N^S} \frac{\sum_i I(d_{i\tau}^s = 0)}{\sum_i I(d_{i\tau}^s = 0) + \sum_i I(d_{i\tau}^s = 1)};$$

$$mtr_1^S(\tau) = \frac{1}{N^S} \sum_{s=1}^{N^S} \frac{\sum_i I(d_{i\tau}^D = 0, d_{i\tau+1}^s = 1)}{\sum_i I(d_{i\tau}^D = 0)}, \tau = 1, 2, \cdots, 215;$$

$$mtr_2^S(\tau) = \frac{1}{N^S} \sum_{s=1}^{N^S} \frac{\sum_i I(d_{i\tau}^D = 1, d_{i\tau+1}^s = 1)}{\sum_i I(d_{i\tau}^D = 1)}, \tau = 1, 2, \cdots, 215,$$

where $d_{i\tau}^D = 1$ and $d_{i\tau+1}^s = 1$ refer to two different jobs;

$$mtr_3^S(\tau) = \frac{1}{N^S} \sum_{s=1}^{N^S} \frac{\sum_i I(d_{i\tau}^D = 1, d_{i\tau+1}^s = 0)}{\sum_i I(d_{i\tau}^D = 1)}, \tau = 1, 2, \cdots, 215;$$

$$mw_1^S(\tau) = \frac{1}{N^S} \sum_{s=1}^{N^S} \frac{\sum_i (w_{i\tau}^s | w_{i\tau}^s > 0)}{\sum_i I(d_{i\tau}^s = 1 | w_{i\tau}^s > 0)};$$

$$mw_2^S(\tau) = \frac{1}{N^S} \sum_{s=1}^{N^S} \sqrt{\frac{\sum_i ((w_{i\tau}^s - mw_1^s(\tau))^2 | w_{i\tau}^s > 0)}{\sum_i I(d_{i\tau}^s = 1 | w_{i\tau}^s > 0) - 1}};$$

$$mw_3^S(\tau) = \frac{1}{N^S}\sum_{s=1}^{N^S}\frac{\sum_i(w_{i\tau}^s|0 < w_{i\tau}^s < w_{M\tau})}{\sum_i I(d_{i\tau}^s = 1|0 < w_{i\tau}^s < w_{M\tau})};$$

$$mw_4^S(\tau) = \frac{1}{N^S}\sum_{s=1}^{N^S}\sqrt{\frac{\sum_i((w_{i\tau}^s - mw_3^s(\tau))^2|0 < w_{i\tau}^s < w_{M\tau})}{\sum_i I(d_{i\tau}^s = 1|0 < w_{i\tau}^s < w_{M\tau}) - 1}};$$

$$mp^S(\tau) = \frac{1}{N^S}\sum_{s=1}^{N^S}\frac{\sum_i I(w_{i\tau}^s < w_{M\tau}|w_{i\tau}^s > 0)}{\sum_i I(d_{i\tau}^s = 1|w_{i\tau}^s > 0)},$$

where $w_{M\tau}$ is the minimum wage and $N^S = 25$ is the total number of simulations.

All unconditional simulated moments are defined the same as the conditional simulated moments except for the transition moments:

$$mtr_1^S(\tau) = \frac{1}{N^S}\sum_{s=1}^{N^S}\frac{\sum_i I(d_{i\tau}^s = 0, d_{i\tau+1}^s = 1)}{\sum_i I(d_{i\tau}^s = 0)}, \tau = 1, 2, \cdots, 215;$$

$$mtr_2^S(\tau) = \frac{1}{N^S}\sum_{s=1}^{N^S}\frac{\sum_i I(d_{i\tau}^s = 1, d_{i\tau+1}^s = 1)}{\sum_i I(d_{i\tau}^s = 1)}, \tau = 1, 2, \cdots, 215,$$

where $d_{i\tau}^s = 1$ and $d_{i\tau+1}^s = 1$ refer to two different jobs;

$$mtr_3^S(\tau) = \frac{1}{N^S}\sum_{s=1}^{N^S}\frac{\sum_i I(d_{i\tau}^s = 1, d_{i\tau+1}^s = 0)}{\sum_i I(d_{i\tau}^s = 1)}, \tau = 1, 2, \cdots, 215.$$

Cycle Moments Recall the way we construct employment cycles. To calculate the empirical moments, we follow each individual i for the first three cycles and the first three jobs in each cycle, i.e. $\{c_i^1(ne_i^1, J1_i^1, J2_i^1, J3_i^1), c_i^2(ne_i^2, J1_i^2, J2_i^2, J3_i^2), c_i^3(ne_i^3, J1_i^3, J2_i^3, J3_i^3)\}$. We convert our monthly data $[d_{i\tau}^D, w_{i\tau}^D]$ into $[\overline{d}_{it}^{Dcj}, \overline{w}_i^{Dcj}]$ where i denotes individual i, $c = 1, 2, 3$ denotes the number of cycle, $j = 0, 1, 2, 3$ corresponds to nonemployment, first, second and third job, t is the tenure on each job (or nonemployment). For example $\overline{d}_{i10}^{12} = 1$ means individual i works (otherwise equals 0) in the 10th month on the second job of his first employment cycle and $\overline{d}_{i5}^{D20} = 1$ denotes fifth month nonemployment in the second cycle. $\overline{w}_i^{Dcj}$ presents the accepted wage for job j in cycle c, which is the first wage observation on the job.

Data cycle moments are defined as following. Duration of cycle c job j for individual i is $\sum_t \overline{d}_{it}^{Dcj}$, thus mean duration

$$mdur^{Dcj} = \frac{\sum_i(\sum_t \overline{d}_{it}^{Dcj})}{\sum_i I(\sum_t \overline{d}_{it}^{Dcj} \geq 1)}, c = 1, 2, 3, j = 0, 1, 2, 3.$$

Mean accepted wage

$$mwage_1^{Dcj} = \frac{\sum_i(\overline{w}_i^{Dcj}|\overline{w}_i^{Dcj} > 0)}{\sum_i I(\overline{d}_{i1}^{Dcj} = 1|\overline{w}_i^{Dcj} > 0)}, c, j = 1, 2, 3.$$

Standard deviation of accepted wage

$$stdwage_1^{Dcj} = \sqrt{\frac{\sum_i((\overline{w}_i^{Dcj} - mwage_1^{Dcj})^2 | \overline{w}_i^{Dcj} > 0)}{\sum_i I(\overline{d}_{i1}^{Dcj} = 1 | \overline{w}_i^{Dcj} > 0) - 1}}, c, j = 1, 2, 3.$$

Mean accepted wage below the minimum wage

$$mwage_2^{Dcj} = \frac{\sum_i(\overline{w}_i^{Dcj} | 0 < \overline{w}_i^{Dcj} < w_{M\tau})}{\sum_i I(\overline{d}_{i1}^{Dcj} = 1 | 0 < \overline{w}_i^{Dcj} < w_{M\tau})}, c, j = 1, 2, 3.$$

Standard deviation of accepted wage below the minimum wage

$$stdwage_2^{Dcj} = \sqrt{\frac{\sum_i((\overline{w}_i^{Dcj} - mwage_2^{Dcj})^2 | 0 < \overline{w}_i^{Dcj} < w_{M\tau})}{\sum_i I(\overline{d}_{i1}^{Dcj} = 1 | 0 < \overline{w}_i^{Dcj} < w_{M\tau}) - 1}}, c, j = 1, 2, 3.$$

Proportion of workers paid below the minimum wage on job j in cycle c

$$prop^{Dcj} = \frac{\sum_i I(\overline{w}_i^{Dcj} < w_{M\tau} | \overline{w}_i^{Dcj} > 0)}{\sum_i I(\overline{d}_{i1}^{Dcj} = 1 | \overline{w}_i^{Dcj} > 0)}, c, j = 1, 2, 3.$$

Proportion of workers start cycle c as nonemployed

$$ne^{Dc} = \frac{\sum_i I(\overline{d}_{i1}^{Dc0} = 1)}{577}, c = 1, 2, 3.$$

Proportion of workers move from job 1 to job 2 in cycle c

$$tr_1^{Dc} = \frac{\sum_i I(\overline{d}_{i1}^{Dc1} = 1, \overline{d}_{i1}^{Dc2} = 1)}{\sum_i I(\overline{d}_{i1}^{Dc1} = 1)}, c = 1, 2, 3.$$

Proportion of workers move from job 2 to job3 in cycle c

$$tr_2^{Dc} = \frac{\sum_i I(\overline{d}_{i1}^{Dc2} = 1, \overline{d}_{i1}^{Dc3} = 1)}{\sum_i I(\overline{d}_{i1}^{Dc2} = 1)}, c = 1, 2, 3.$$

Simulated cycle moments are defined similarly for each simulation s and we take average over $N^s = 25$ simulations.

Table 9: Data and Predicted Moments

		Cycle One				Cycle Two				Cycle Three			
		NE	Job 1	Job 2	Job 3	NE	Job 1	Job 2	Job 3	NE	Job 1	Job 2	Job 3
Mean Duration	data	8.07	33.96	40.89	35.87	6.73	25.06	27.02	23.33	6.45	21.41	28.05	19.98
(months)	(model)	(8.14)	(26.09)	(32.06)	(37.47)	(7.65)	(27.21)	(30.53)	(26.31)	(6.78)	(22.00)	(22.97)	(21.37)
Mean Wage	data		8.22	10.37	11.85		10.22	11.02	11.49		9.95	11.29	11.81
(2000 dollars)	(model)		(9.32)	(11.96)	(13.25)		(10.41)	(12.42)	(13.06)		(9.54)	(11.03)	(11.72)
S.D. Wage	data		3.43	4.28	4.82		7.50	5.48	5.97		5.25	6.18	6.07
	(model)		(4.28)	(4.78)	(4.54)		(4.41)	(4.89)	(5.20)		(4.34)	(4.65)	(4.73)
Mean wage below w_M	data		4.89	5.17	4.99		4.99	4.96	3.47		4.29	4.72	5.02
(2000 dollars)	(model)		(4.57)	(4.94)	(5.12)		(4.36)	(4.80)	(4.88)		(4.13)	(4.48)	(4.53)
S.D. Wage Below w_M	data		1.38	1.21	1.35		1.20	1.17	1.10		1.56	0.85	1.00
	(model)		(1.00)	(0.91)	(0.94)		(0.97)	(0.81)	(1.10)		(0.87)	(0.73)	(1.00)
Proportion (%)	data		21.90	6.77	1.52		15.11	7.30	8.33		10.33	8.00	5.63
below w_M	(model)		(23.64)	(4.87)	(1.50)		(14.74)	(3.29)	(0.90)		(15.62)	(3.42)	(0.78)
	data	20.10*				53.90**				41.94**			
	(model)	(36.23)				(91.14)				(75.15)			
% moving from	data		40.20				46.62				44.21		
job 1 to job 2	(model)		(56.04)				(36.01)				(35.85)		
% moving from	data			47.92				44.38				46.40	
job 2 to job 3	(model)			(38.51)				(24.30)				(23.61)	

* The proportion of non-employed people in cycle one.

** Proportions of people moving to work from non-employment.

Figure 1: Monthly Employment and Nonemployment Rates

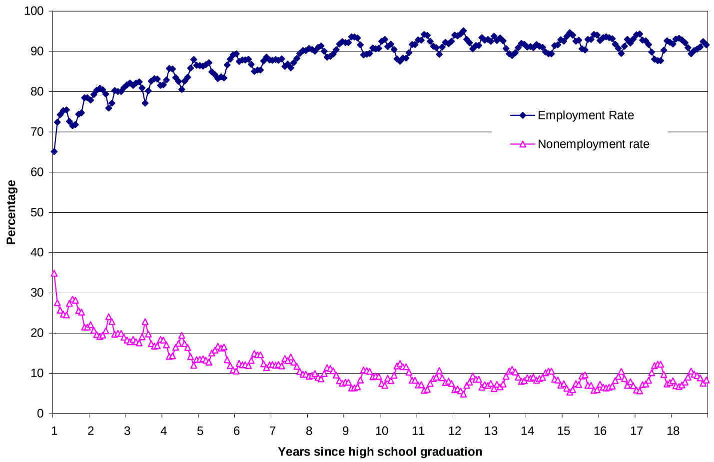


Figure 2: Job Separation Rate by Job Tenure (Months)

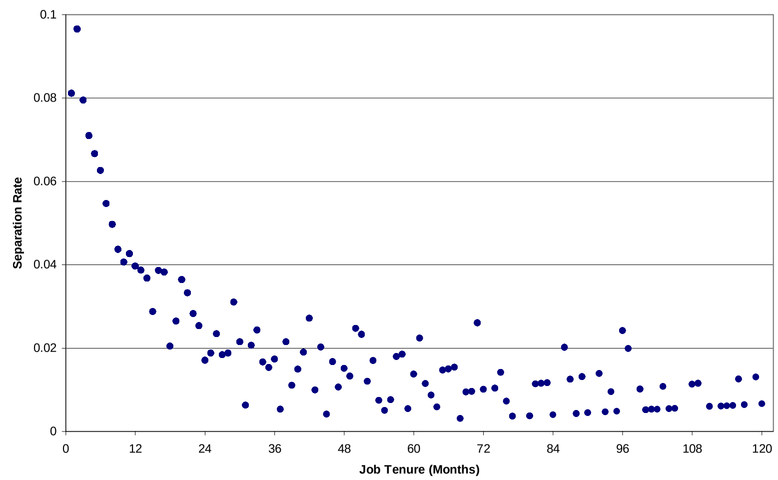


Figure 3: Transition Probabilities From and To Employment and Nonemployment

Figure 4: Transition Probabilities from Job to Job and from Nonemployment to Employment

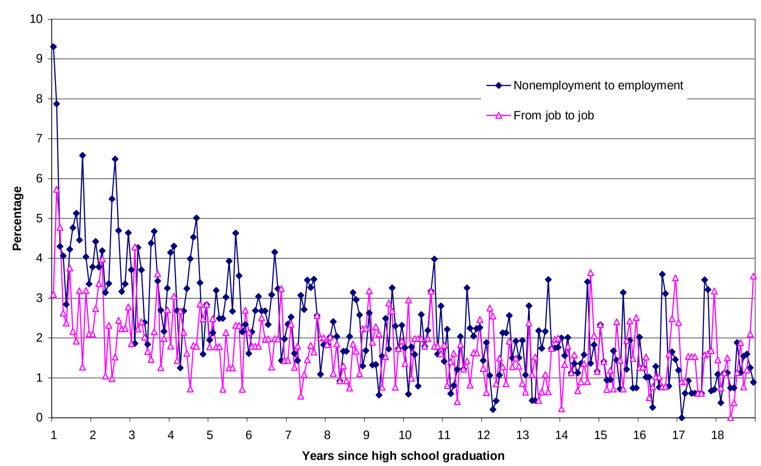


Figure 5: Federal Minimum Wage Under the Fair Labor Standards Act

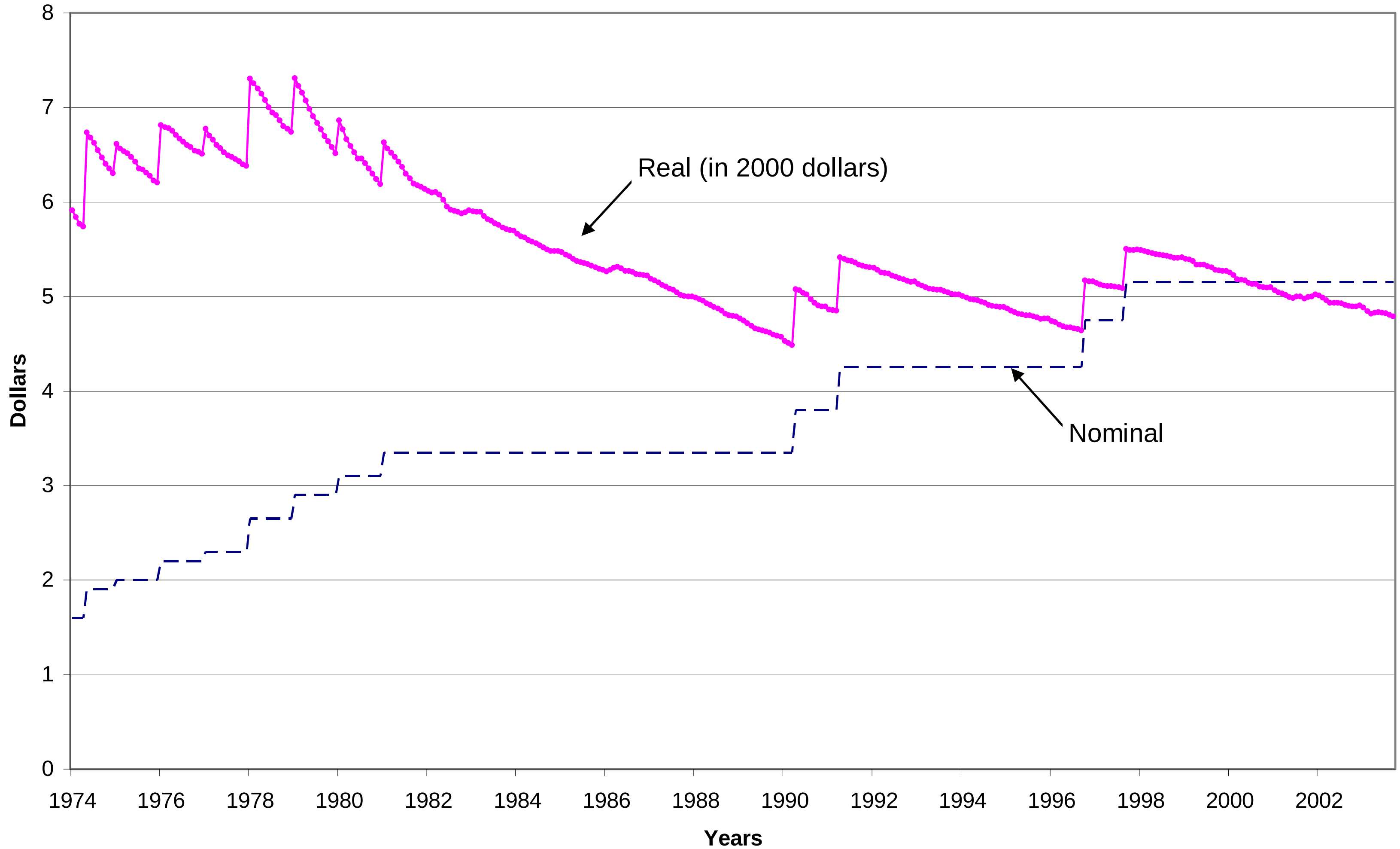


Figure 6: Actual and Predicted Monthly Nonemployment Rate

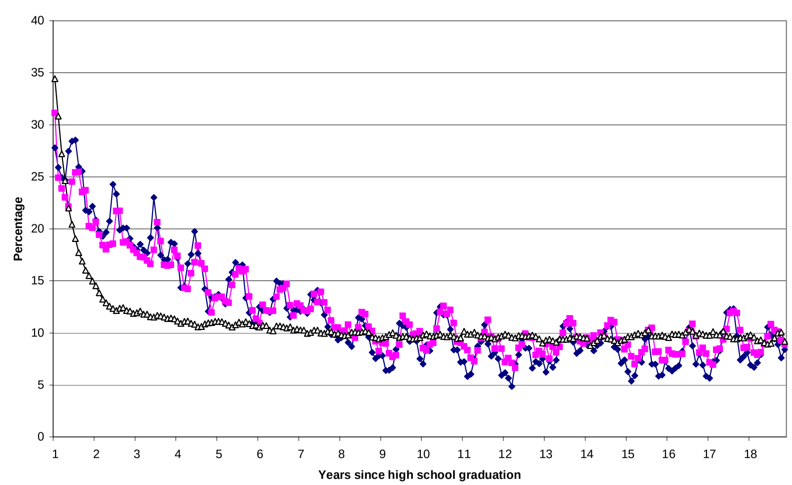


Figure 7: Actual and Predicted Monthly Transition Rate from Nonemployment to Work

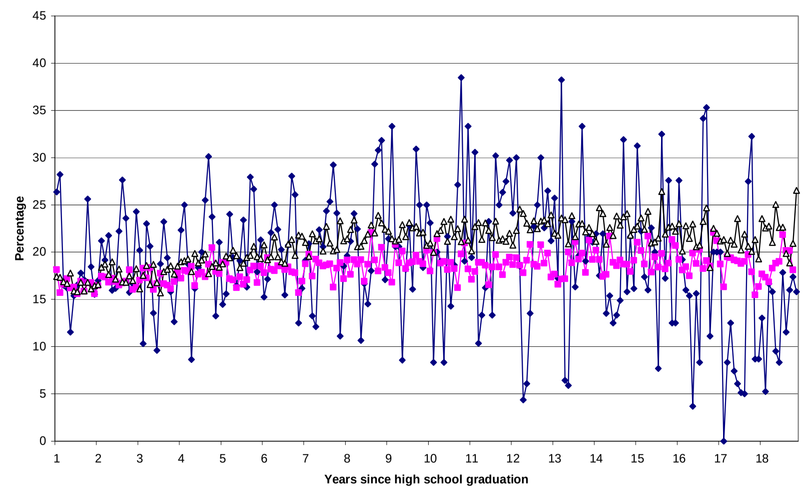


Figure 8: Actual and Predicted Monthly Transition Rate from Job-to-job

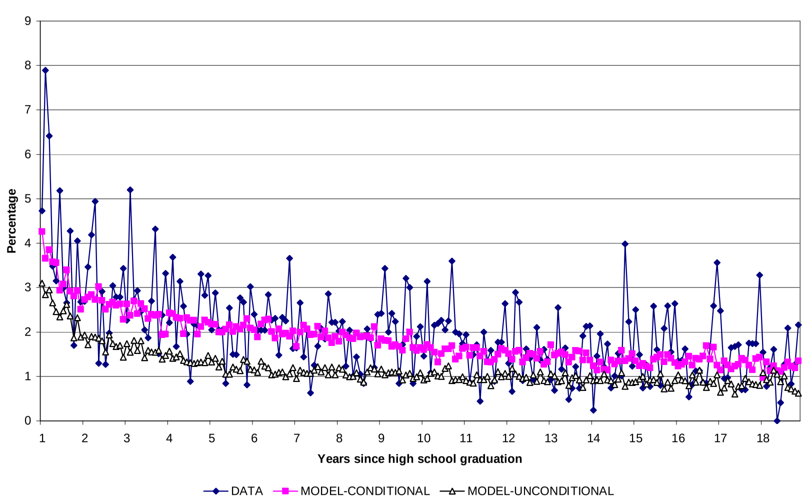


Figure 9: Actual and Predicted Monthly Transition Rate from Employment to Nonemployment

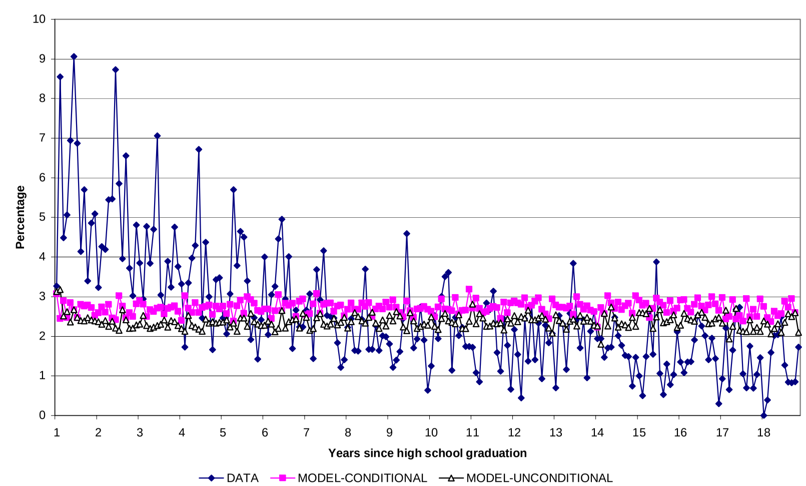


Figure 10: Actual and Predicted Hourly Mean Wage

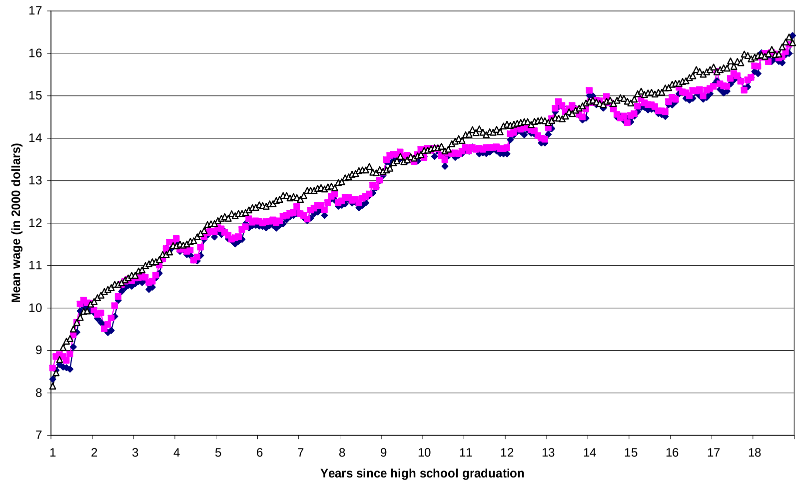


Figure 11: Actual and Predicted Standard Deviation of Hourly Wage

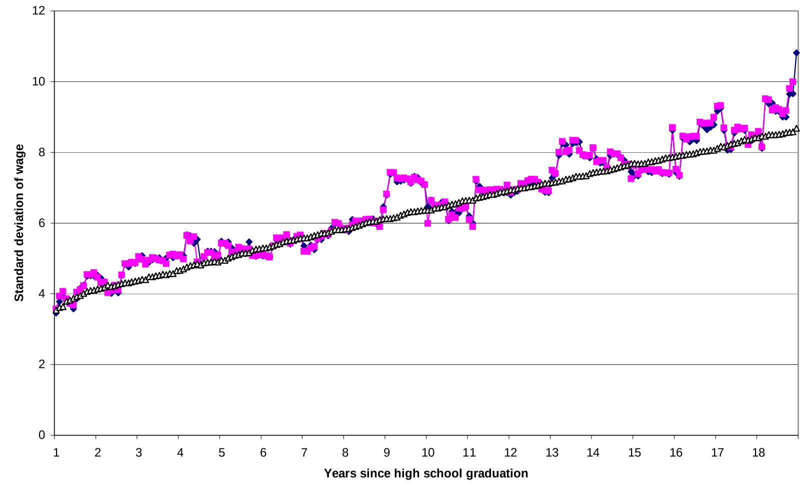


Figure 12: Actual and Predicted Hourly Mean Wage Below the Minimum Wage

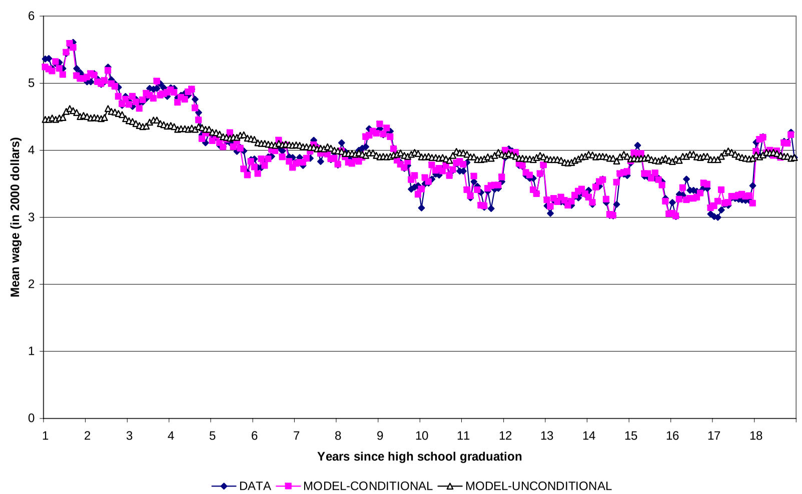


Figure 13: Actual and Predicted Standard Deviation of Hourly Wage Below the Minimum Wage

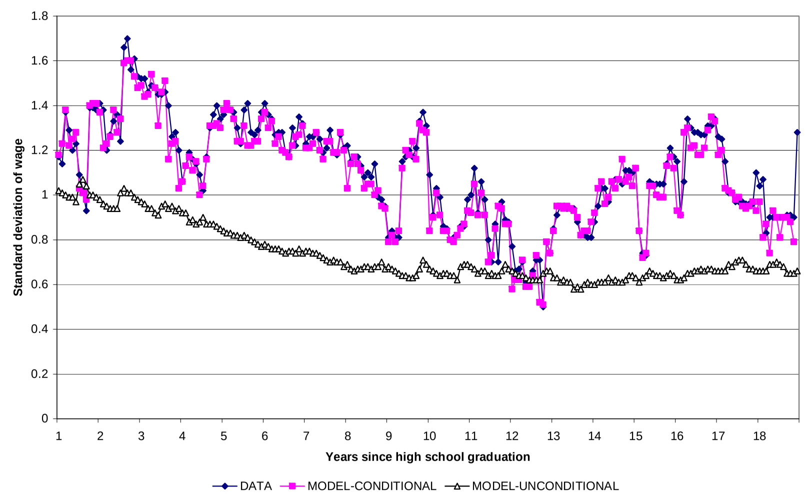
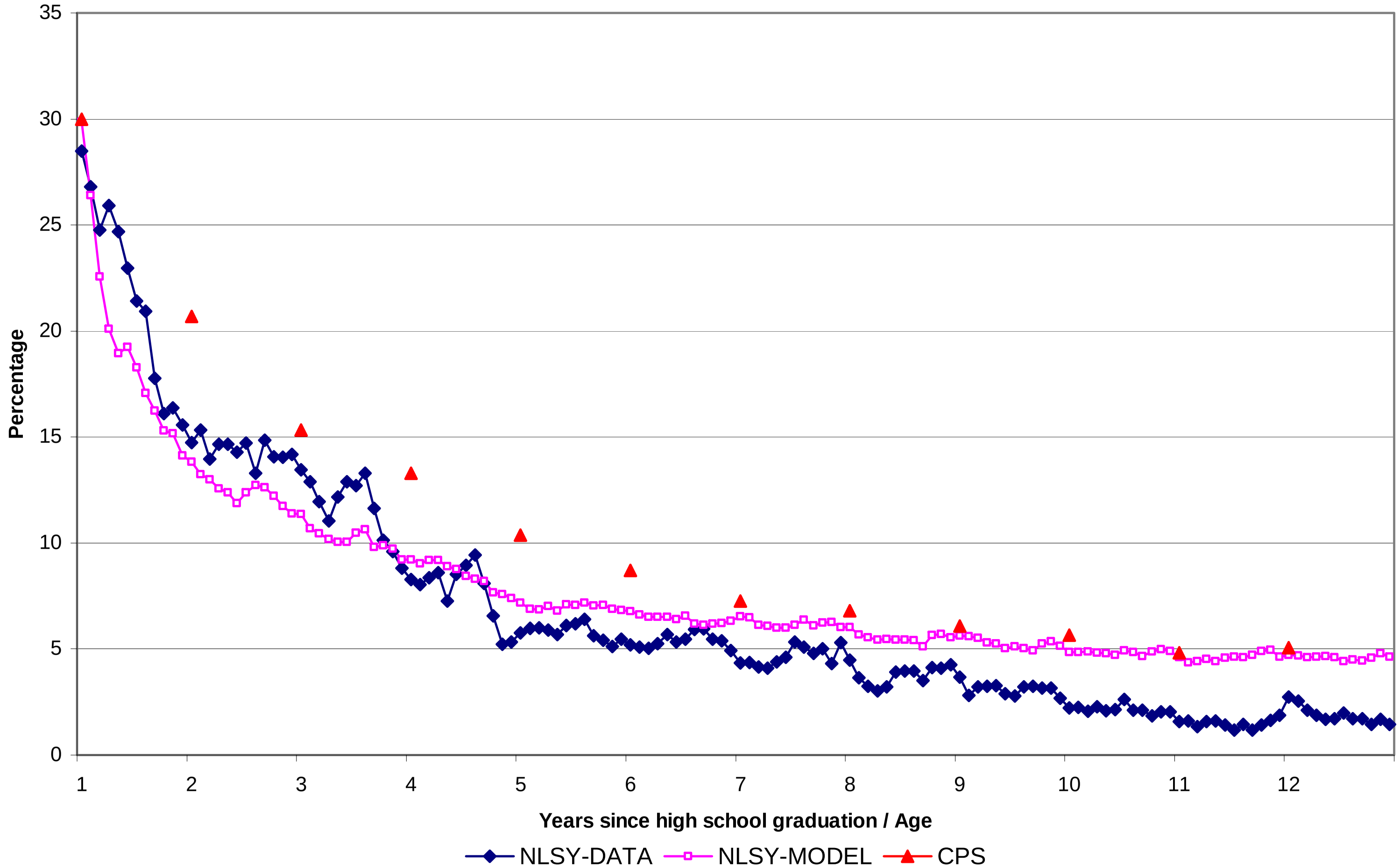


Figure 14: Percentage of workers paid below the minimum wage



Note: CPS Wage data has the same restrictions as for NLSY wage data. We calculate the mean of the proportions of workers paid below the minimum wage between 1979-1997 for youth aged between 19 to 36.