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Rasmus Lentz
Dale T. Mortensen

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Rasmus Lentz
University of Wisconsin-Madison,
Boston University and CAM

Dale T. Mortensen
Northwestern University,
NBER and IZA Bonn

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IZA

P.O. Box 7240
53072 Bonn
Germany

Phone: +49-228-3894-0
Fax: +49-228-3894-180
Email: iza@iza.org

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ABSTRACT

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Productivity dispersion across firms is large and persistent, and worker reallocation among firms is an important source of productivity growth. The purpose of the paper is to estimate the structure of an equilibrium model of growth through innovation that explains these facts. The model is a modified version of the Schumpeterian theory of firm evolution and growth developed by Klette and Kortum (2004). The data set is a panel of Danish firms than includes information on value added, employment, and wages. The model's fit is good and the structural parameter estimates have interesting implications for the aggregate growth rate and the contribution of worker reallocation to it.

JEL Classification: E22, E24, J23, J24, L11, L25, O3, O4

Keywords: labor productivity growth, worker reallocation, firm dynamics, firm panel data estimation

Corresponding author:

Dale T. Mortensen
Dept. of Economics
Northwestern University
Evanston, IL 60208
USA
Email: d-mortensen@northwestern.edu

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1 Introduction

In their review article, Bartelsman and Doms (2000) draw three lessons from empirical productivity studies based on longitudinal plant and firm data: First, the extent of dispersion in productivity across production units, firms or establishments, is large. Second, the productivity rank of any unit in the distribution is highly persistent. Third, a large fraction of aggregate productivity growth is the consequence of worker reallocation.

Although the explanations for productive firm heterogeneity are not fully understood, economic principles suggest that its presence should induce worker reallocation from less to more productive firms as well as from exiting to entering firms. There is ample evidence that workers do flow from one firm to another frequently. As Davis, Haltiwanger, and Schuh (1996) and others document, job and worker flows are large, persistent, and essentially idiosyncratic in the U.S. Recently, Fallick and Fleischman (2001) and Stewart (2002) find that job to job flows without a spell of unemployment in the U.S. represent at least half of the separations and is growing. In their analysis of Danish matched employer-employee IDA data, Frederiksen and Westergaard-Nielsen (2002) report that the average establishment separation rate over the 1980-95 period was 26%. About two thirds of the outflow represents the movement of workers from one firm to another.

In a companion paper, Lentz and Mortensen (2005) develop a stochastic general equilibrium model in which more profitable firms grow faster and contribute more to the aggregate growth rate through product innovation. The model is a variation on that proposed by Klette and Kortum (2004), which itself builds on the endogenous growth model of Grossman and Helpman (1991). By design, their model is consistent with stylized facts about product innovation and its relationship to the dynamics of firm size evolution. We adopt the approach because it provides an explanation for the fact that there is no correlation between labor force size and labor productivity but a strong positive association between value added and labor productivity in Danish firm data. Furthermore, the model provides a direct link between worker reallocation and productivity growth.

In the model, firms are monopoly suppliers of differentiated intermediate products that serve as inputs in the production of a final consumption good. Better quality products are introduced from time to time as the outcome of R&D investment by both existing firms and new entrants. As new products displace old, the process of creative destruction induces the need to reallocate workers across productive activities. In the version of the model estimated here, product quality differs across firms. In our earlier paper, we established the existence of a general equilibrium solution to the model. In this one, we use the equilibrium relationships implied by the model and information drawn from a Danish panel of firms to estimate the model's parameters.

Providing a good fit to data, the model is estimated on among other moments the relationship between firm size and firm growth which is slightly negative in the data. The model satisfies a theoretical version of Gibrat's law, but nevertheless replicates the negative relationship between size and growth found in data. The model is also estimated to fit a standard growth decomposition which suggests a large growth contribution from reallocation and while the model does in fact imply a large reallocation contribution, the reduced form decomposition is largely explained through measurement error and the fundamental sources of productivity growth are only loosely reflected in the reduced form decomposition.

Given the parameter estimates obtained, we explore the model's quantitative implications for productivity growth and its sources. The model implies an annual rate of overall productivity growth equal to 2.3%. We find that the reallocation of workers from less to more productive surviving firms accounts for 70% of productivity growth in equilibrium.

2 Danish Firm Data

Danish firm data provide information on productivity dispersion and the relationships among productivity, employment, and sales. The available data set is an annual panel of privately owned firms for the years 1992-1997 drawn from the Danish Business Statistics Register. The sample of approximately 4,900 firms is restricted to those with 20 or more employees. The sample does not include entrants.[1] The variables observed in each year include value added (Y), the total wage bill (W), and full-time equivalent employment (N). In this paper we use these relationships to motivate the theoretical model studied. Both Y and W are measured in Danish Kroner (DKK) while N is a body count.

[1] The full panel of roughly 6,700 firms contains some entry, but due to the sampling procedure, the entrant population suffers from significant selection bias. We have chosen not to rely on the entrant population for identification of the model.

Non-parametric estimates of the distributions of two alternative measures of a firm's labor productivity are illustrated in Figure 1. The first measure of firm productivity is value added per worker (Y/N) while the second is valued added per unit of quality adjusted employment (Y/N^*). Standard labor productivity misrepresents cross firm productivity differences to the extent that labor quality differs across firms. However, if more productive workers are compensated with higher pay, as would be true in a competitive labor market, one can use a wage weighted index of employment to correct for this source of cross firm differences in productive efficiency. Formally, the constructed quality adjusted employment of firm j is defined as $N_j^* = W_j/w$ where

$$w = \frac{\sum_j W_j}{\sum_j N_j} \tag{1}$$

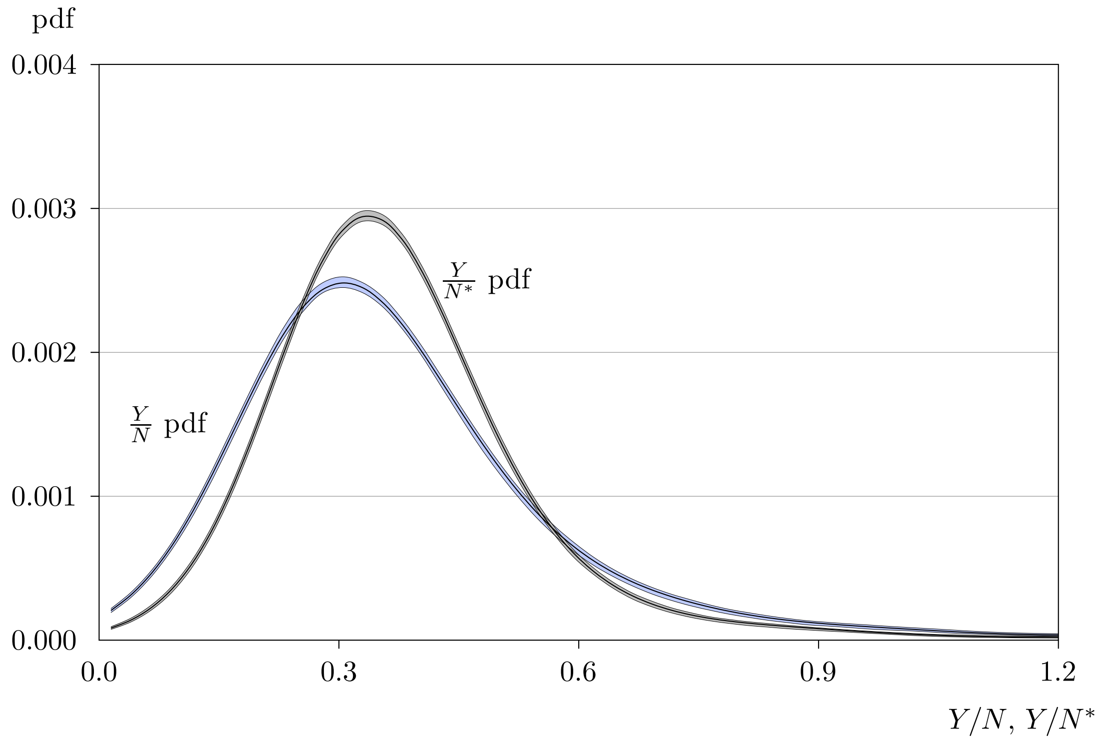
is the average wage paid per worker in the market.[2] Although correcting for wage differences across firms in this manner does reduce the spread and skew of the implied productivity distribution somewhat, both distributions have high variance and skew and are essentially the same general shape.

Both distributions are consistent with those found in other data sets. For example, productivity distributions are significantly dispersed and skewed to the right. In the case of the adjusted measure of productivity, the 5$^{\text{th}}$ percentile is roughly half the mode while the 95$^{\text{th}}$ percentile is approximately twice as large are the mode. The range between the two represents a four fold difference in value added per worker across firms. These facts are similar to those reported by Bartelsman and Doms (2000) for the U.S.

There are many potential explanations for cross firm productivity differentials. A comparison of the two distributions represented in Figure 1 suggests that differences in the quality of labor inputs does not seem to be the essential one. The process of technology diffusion is a well documented. Total factor productivity differences across firms can be expected as a consequence of slow diffusion of new techniques. If technical improvements are either factor neutral or capital augmenting, then one would expect that more productive firms would acquire more labor and capital. The implied consequence would seem to be a positive relationship between labor force size and labor

[2] In the case, where a firm is observed over several periods, the implicit identification of the firm's labor force quality is taken as an average over the time dimension to address issues of measurement error. The alternative approach of identifying a quality measure for each year has no significant impact on the moments of the data set.

Figure 1: Productivity Distributions.



Note: The shaded areas represent 90% bootstrap confidence intervals. Value added (Y) measured in 1 million DKK. N is the firm's labor force head count and N^* is the quality adjusted labor force size.

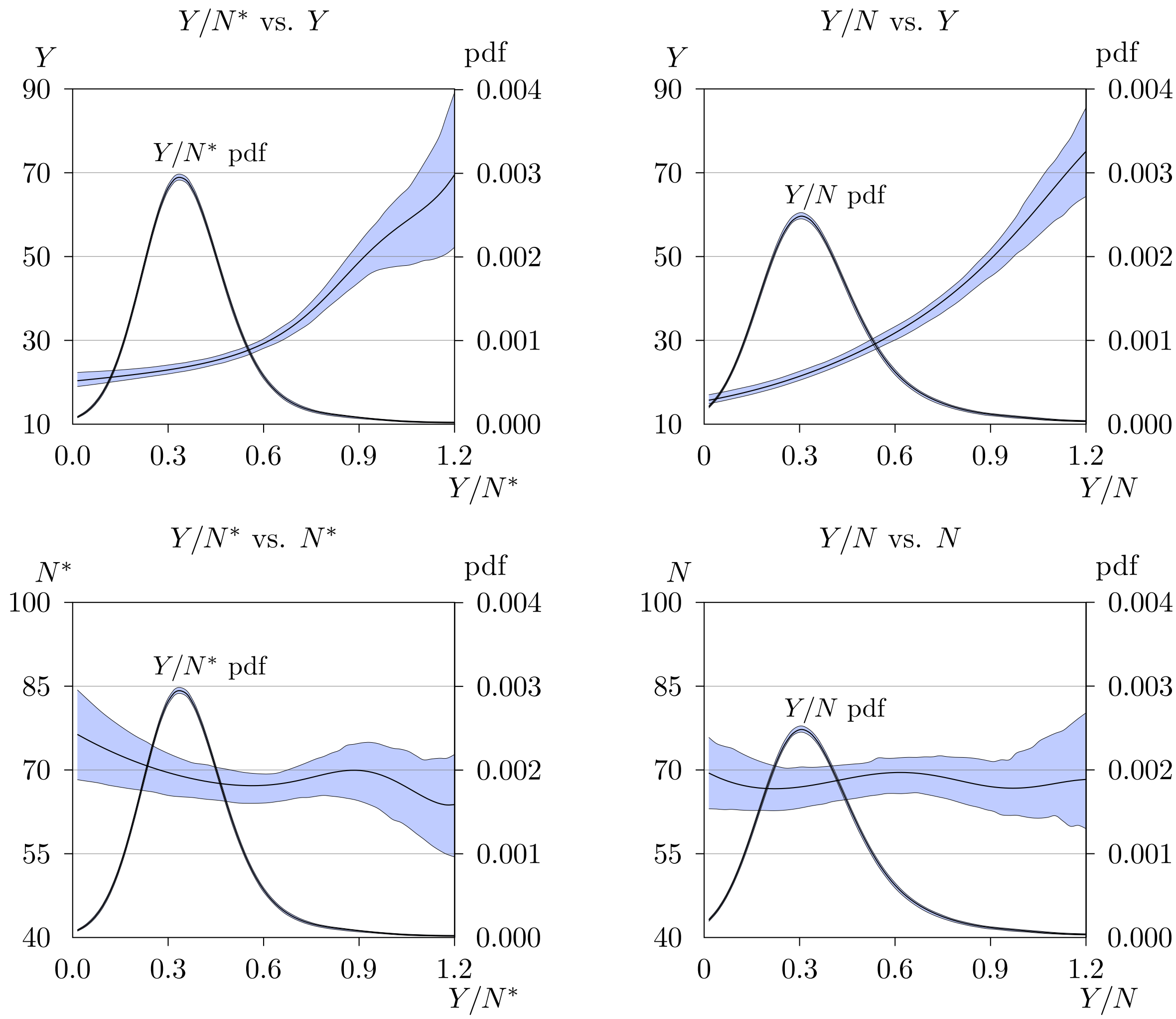
Table 1: Productivity – Size Correlations

	Employment (N)	Adjusted Employment (N*)	Value Added (Y)
Y/N	0.0017	0.0911	0.3138
Y/N*	-0.0095	-0.0176	0.1981

productivity. Interestingly, there is no correlation between the two in Danish data.

The correlations between the two measures of labor productivity with the two employment measures and sales as reflected in value added are reported in Table 1. As documented in the table, the correlation between labor force size and productivity using either the raw employment measure or the adjusted one is zero. However, note the strong positive associate between value added and both measures of labor productivity. Non-parametric regressions of value added and employment on the two productivity measures are illustrated in Figure 2. The top and bottom curves in the figures represent a 90% confidence interval for the relationship. The positive relationships between value added and both measure of labor productivity are highly significant.

Figure 2: Firm Size–Productivity Relationships.



Note: The shaded areas represent 90% bootstrap confidence intervals. Value added (Y) measured in 1 million DKK. N is the firm's labor force head count and N^* is the quality adjusted labor force size.

The theory developed in this paper is in part motivated by these observations. Specifically, it is a theory that postulates labor saving technical progress of a specific form. Hence, the apparent fact that more productive firms produce more with roughly the same labor input per unit of value added is consistent with the model.

3 An Equilibrium Model of Creative Destruction

As is well known, firms come is an amazing range of shapes and sizes. This fact cannot be ignored in any analysis of the relationship between firm size and productivity. Furthermore, an adequate theory must account for entry, exit and firm evolution in order to explain the size distributions observed. Klette and Kortum (2004) construct a stochastic model of firm product innovation and growth that is consistent with stylized facts regarding the firm size evolution and distribution. The model also has the property that technical progress is labor saving. For these reasons, we pursue their approach in this paper.

Although Klette and Kortum allow for productive heterogeneity, firm productivity and growth are unrelated because costs and benefits of growth are both proportional to firm productivity in their model. Allowing for a positive relationship between firm growth and productivity is necessary for consistency with the relationships found in the Danish firm data studied in this paper.

3.1 Preferences and Technology

Intertemporal utility of the representative household at time t is given by

$$U_t = \int_t^{\infty} \ln C_s e^{-\rho(s-t)} ds \tag{2}$$

where $\ln C_t$ denotes the instantaneous utility of the single consumption good at date t and ρ represents the pure rate of time discount. Each household is free to borrow or lend at interest rate r_t. Nominal household expenditure at date t is $E_t = P_t C_t$. Optimal consumption expenditure must solve the differential equation $\dot{E}/E = r_t - \rho$. Following Grossman and Helpman (1991), we choose the numeraire so that $E_t = 1$ for all t without loss of generality, which implies $r_t = r = \rho$ for all t. Note that this choice of the numeraire also implies that the price of the consumption good expressed in terms of the numeraire, P_t, falls over time at a rate equal to the rate of growth in consumption.

The quantity of the consumption produced is determined by the quantity and quality of the economy's intermediate inputs. Specifically, there is a unit continuum of inputs and consumption is determined by the production function

$$\ln C_t = \int_0^1 \ln(A_t(j)x_t(j))dj = \ln A_t + \int_0^1 \ln x_t(j)dj \tag{3}$$

where $x_t(j)$ is the quantity of input $j \in [0, 1]$ at time t, $A_t(j)$ is the quality or productivity of input j at time t, and A_t represent aggregate productivity. The level of productivity of each input and aggregate productivity are determined by the number of technical improvements made in the past. Specifically,

$$A_t(j) = \prod_{i=1}^{J_t(j)} q_i(j) \text{ and } \ln A_t \equiv \int_0^1 \ln A_t(j)dj. \tag{4}$$

where $J_t(j)$ is the number of innovations made in input j up to date t and $q_i(j) > 1$ denotes the quantitative improvement (step size) in productivity attributable to the i^{th} innovation in product j. Innovations arrive at rate δ which is endogenous but the same for all intermediate products.

The model is constructed so that a steady state growth path exists with the following properties: Consumption output grows at a constant rate while the quantities of intermediate products and the endogenous innovation frequency are stationary and identical across all intermediate goods. As a consequence of the law of large numbers, the assumption that the number of innovations to date is Poisson with arrival frequency δ for all intermediate goods implies

$$\begin{aligned} \ln C_t &= \ln A_t + \int_0^1 \ln x(j)dj = \int_0^1 \sum_{i=1}^{J_t(j)} \ln q_i(j)dj + \int_0^1 \ln x(j)dj \\ &= E\ln(q)\delta t + \int_0^1 \ln x(j)dj. \end{aligned} \tag{5}$$

where $EJ_t(j) = \delta t$ for all j is the expected number of innovations per intermediate product and $E\ln(q) \equiv \int_0^1 \frac{1}{J_t(j)} \sum_{i=1}^{J_t(j)} \ln q_i(j)dj$ is the expected quality step size. In other words, consumption grows at the rate of growth in productivity which is the product of the creative-destruction rate and the expected log of the size of an improvement in productivity induced by each new innovation.

3.2 The Value of a Firm

Each individual firm is the monopoly supplier of the products it created in the past that have survived to the present. The price charged for each is limited by the ability of suppliers of previous versions to provide a substitute. In Nash-Bertrand equilibrium, any innovator takes over the market for its good type by setting the price just below that at which consumers are indifferent between the higher quality product supplied by the innovator and an alternative supplied by the last provider. The price charged is the product of relative quality and the previous producer's marginal cost of production. Given the symmetry of demands for the different good types and the assumption that future quality improvements are independent of the type of good, one can drop the good subscript without confusion. Given stationary of quantities along the equilibrium growth path, the time subscript can be dropped as well.

Labor is the only factor in the production of intermediate inputs. Labor productivity is the same across all inputs and is set equal to unity. Hence, $p = qw$ is the price in terms of the numeraire of every intermediate good as well as the value of labor productivity where w, the wage, represents the marginal cost of production of the previous supplier and $q > 1$ is the step up in quality of the innovation. As total expenditure is normalized at unity and there is a unit measure of product types, it follows that total revenue per product type is also unity given the specification of preferences and technology, i.e., $px = 1$. Hence, product output and employment are both equal to

$$x = \frac{1}{p} = \frac{1}{wq}. \tag{6}$$

and the gross profit associated with supplying the good is

$$1 > \pi = px - wx = 1 - \frac{1}{q} > 0. \tag{7}$$

The labor saving nature of improvements in intermediate input quality is implicit in the fact that labor demand is decreasing in q.

The model of quality improvements can equally well be viewed as a model of efficiency improvements, that is, a reduction of the amount of labor that is required to produce a unit of output. This is easily seen by re-interpreting the argument above in terms of quality units of output. Given that one unit of labor produces of unit of output, an increase in product quality of a unit of output is analogous to a reduction of the amount of labor that is required to produce a quality unit. In terms of quality units, the price is ever decreasing, demand for quality units of a product is ever increasing, and the amount of labor engaged in production in a given industry remains stable. In the short run, labor demand does fluctuate depending on the exact realization of the current lead that the industry leader has to the nearest follower - the greater the lead, the lower the demand.

Following Klette and Kortum (2004), the discrete number of products supplied by a firm, denoted as k, is defined on the integers and its value evolves over time as a birth-death process reflecting product creation and destruction. In their interpretation, k reflects the firm's past successes in the product innovation process as well as current firm size. New products are generated by R&D investment. The firm's R&D investment flow generates new product arrivals at frequency γk. The total R&D investment cost is $wc(\gamma)k$ where $c(\gamma)k$ represents the labor input required in the research and development process. The function $c(\gamma)$ is assumed to be strictly increasing and convex. According to the authors, the implied assumption that the total cost of R&D investment is linearly homogenous in the new product arrival rate and the number of existing product, "captures the idea that a firm's knowledge capital facilitates innovation." In any case, this cost structure is needed to obtain firm growth rates that are independent of size as typically observed in the data.

The market for any current product supplied by the firm is destroyed by the creation of a new version by some other firm, which occurs at the rate δ. Below we refer to γ as the firm's creation rate and to δ as the common destruction rate faced by all firms.[3]As product gross profit and product quality are one-to-one, the profits earned on each product reflects a firm's current labor productivity. The firm chooses the creation rate γ to maximize the expected present value of its future net profit flow.

Firms differ with the respect to the quality of their products. Hence, each type is characterized by profitability, π, as defined in equation (7). The value of the firm of type π that currently markets k products is

$$rV_k(\pi) = \max_{\gamma \geq 0} \left\{ \left[\pi - wc(\gamma)\right] k + \gamma k [V_{k+1}(\pi) - V_k(\pi)] + \delta k [V_{k-1}(\pi) - V_k(\pi)] \right\}. \tag{8}$$

The first term on the right side is current gross profit flow accruing to the firms product portfolio less current expenditure on R&D. The second term is the expected capital gain associated with the arrival of a new product line. Finally, the last term represents the expected capital loss associated with the possibility that one among the existing product lines will be destroyed.

The unique solution to (8) is proportional to the number of product lines. Formally,

$$V_k(\pi) = k \max_{\gamma \geq 0} \left\{ \frac{\pi - wc(\gamma)}{r + \delta - \gamma} \right\} \tag{9}$$

as one can verify by substitution. Consequently, any positive optimal choice of the product creation rate for a type π firm must satisfy the first order condition

$$wc'(\gamma(\pi)) = V_{k+1} - V_k = \max_{\gamma \geq 0} \left\{ \frac{\pi - wc(\gamma)}{r + \delta - \gamma} \right\}. \tag{10}$$

[3]These are in fact the continuous time job creation and job destruction rates respectively as defined in Davis, Haltiwanger, and Schuh (1996).

Hence, the second order condition, $c''(\gamma) > 0$, and the fact that the marginal value of a product line is increasing in π imply that the a firm's creation rate increases with profitability.

3.3 Firm Entry and Labor Market Clearing

The entry of a new firm requires innovation. Suppose that there are a constant measure m of potential entrants, identical ex ante. The rate at which any one of them generates a new product is γ_0 and the total cost is $wc(\gamma_0)$ where the cost function is the same as that faced by an incumbent. The firm's type is unknown ex ante but is realized immediately after entry. Since the expected return to innovation is $E_\pi\{V_1\}$ and the aggregate entry rate is $\eta = m\gamma_0$, the entry rate satisfies the following free entry condition

$$wc'\left(\frac{\eta}{m}\right) = \int_\pi V_1(\pi)\phi(\pi)d\pi = \int_\pi \max_{\gamma\geq 0}\left\{\frac{\pi - wc(\gamma)}{r+\delta-\gamma}\right\}\phi(\pi)d\pi \tag{11}$$

where $\phi(\pi)$ is the density entrant types. Of course, the second equality follows from equation (9).

There is a fixed measure of available workers, denoted by L, seeking employment at any positive wage. In equilibrium, these are allocated across production and R&D activities, those performed by both incumbent firms and potential entrants. Since the number of workers employed for production purposes per product of quality q is $x = 1/wq = (1-\pi)/w$ from equations (6) and (7), the total number demanded for production activity by firms of type π with k products is $L_x(k,\pi) = k(1-\pi)/w > 0$. The number of R&D workers employed by incumbent firms of type π with k products is $L_R(k,\pi) = kc(\gamma(\pi))$. Because each potential entrant innovates at frequency η/m, the aggregate number of worker engaged by all m in R&D is $L_E = mc(\eta/m)$. Hence, the equilibrium wage satisfies the labor market clearing condition

$$\begin{aligned} L &= \int_\pi \sum_{k=1}^{\infty}\left[L_x(k,\pi) + L_R(k,\pi)\right]M_k(\pi)d\pi + L_E \\ &= \int_\pi \left(\frac{1-\pi}{w} + c(\gamma(\pi))\right)\sum_{k=1}^{\infty} kM_k(\pi)d\pi + mc\left(\frac{\eta}{m}\right) \end{aligned} \tag{12}$$

where $M_k(\pi)$ represents the mass of firms of type π that supply k products.

3.4 The Steady State Distribution of Firm Size

Once a firm enters, its size as reflected in the number of product lines supplied evolves as a birth-death process. As the set of firms with k products at a point in time must either have had k

products already and neither lost nor gained another, have had $k-1$ and innovated, or have had $k+1$ and lost one to destruction over any sufficiently short time period, the equality of the flows into and out of the set of firms of type π with $k > 1$ products requires

$$\gamma(\pi)(k-1)M_{k-1}(\pi) + \delta(k+1)M_{k+1}(\pi) = (\gamma(\pi)+\delta)kM_k(\pi)$$

for every π where $M_k(\pi)$ is the steady state mass of firms of type π that supply k products. Because an incumbent dies when its last product is destroyed by assumption but entrants flow into the set of firms with a single product at rate η,

$$\phi(\pi)\eta + 2\delta M_2(\pi) = (\gamma(\pi)+\delta)M_1(\pi)$$

where $\phi(\pi)$ is the fraction of the new entrants that realize profit π. Births must equal deaths in steady state and only firms with one product are subject to death risk. Therefore, $\phi(\pi)\eta = \delta M_1(\pi)$ and

$$M_k(\pi) = \frac{k-1}{k}\gamma(\pi)M_{k-1} = \frac{\eta\phi(\pi)}{\delta k}\left(\frac{\gamma(\pi)}{\delta}\right)^{k-1} \tag{13}$$

by induction.

The size distribution of firms conditional on type can be derived using equation (13). Specifically, the total firm mass of type π is

$$\begin{aligned} M(\pi) &= \sum_{k=1}^{\infty} M_k(\pi) = \frac{\phi(\pi)\eta}{\delta}\sum_{k=1}^{\infty}\frac{1}{k}\left(\frac{\gamma(\pi)}{\delta}\right)^{k-1} \\ &= \frac{\eta}{\delta}\ln\left(\frac{\delta}{\delta-\gamma(\pi)}\right)\frac{\delta\phi(\pi)}{\gamma(\pi)}. \end{aligned} \tag{14}$$

where convergence requires that the aggregate rate of creative destruction exceed the creation rate of every incumbent type, i.e., $\delta > \gamma(\pi)\ \forall\pi$. Hence, the fraction of type π firm with k product is

$$\frac{M_k(\pi)}{M(\pi)} = \frac{\frac{1}{k}\left(\frac{\gamma(\pi)}{\delta}\right)^k}{\ln\left(\frac{\delta}{\delta-\gamma(\pi)}\right)}. \tag{15}$$

This is the logarithmic distribution with parameter $\gamma(\pi)/\delta$.[4] Consistent with the observations on firm size distributions, that implied by the model is highly skewed to the right.

[4] This result is in Klette and Kortum (2004). We include the derivation here simply for completeness.

By equation (15), the mean of the firm size distribution conditional on product profitability is

$$E\left[k|\pi\right] = \sum_{k=1}^{\infty} \frac{kM_k(\pi)}{M(\pi)} = \frac{\frac{\gamma(\pi)}{\delta-\gamma(\pi)}}{\ln\left(\frac{\delta}{\delta-\gamma(\pi)}\right)}, \tag{16}$$

As the product creation rate increases with profitability, expected size does also. Formally, because $(1+a)\ln(1+a) > a > 0$ for all positive values of a, the expected number of products is increasing in firm profitability,

$$\frac{\partial E[k|\pi]}{\partial \pi} = \left(\frac{(1+a(\pi))\ln(1+a(\pi)) - a(\pi)}{(1+a(\pi))\ln^2(1+a(\pi))}\right)\frac{\delta\gamma'(\pi)}{(\delta-\gamma(\pi))^2} > 0 \tag{17}$$

where $a(\pi) = \frac{\gamma(\pi)}{\delta-\gamma(\pi)}$.

Although more profitable firms supply more products, total expected employment, $nE[k]$ where $n = (1-\pi)/w + c(\gamma(\pi))$, need not increase with π in general and decreases with π if innovation is not related to profitability because innovation is labor saving. Hence, the hypothesis that firms with the ability to create products of better quality grow faster is consistent with dispersion in labor productivity and the correlations between value added, labor force size, and labor productivity observed in Danish data reported above.

Finally, the rate of creative-destruction is the sum of the entry rate and the aggregate creation rates of all the incumbents given that the total mass of products is fixed. Because the new product arrival rate of a firm of type π with k products is $\gamma(\pi)k$ and the measure of such firms is $M_k(\pi)$,

$$\delta = \eta + \int_{\pi} \gamma(\pi) \sum_{k=1}^{\infty} kM_k(\pi)d\pi. \tag{18}$$

3.5 Equilibrium

Definition A *steady state market equilibrium* is a triple composed of a labor market clearing wage w, entry rate η, and creative destruction rate δ together with an optimal creation rate $\gamma(\pi)$ and a steady state size distribution $M_k(\pi)$ for each type that satisfy equations (11), (12), (10), (13), and (18) provided that $\gamma(\pi) < \delta$, for every π in the support of the entry distribution.

Proposition If the cost of innovation, $c(\gamma)$, is strictly convex and $c'(0) = c(0) = 0$, then a steady state market equilibrium with positive entry exists. In the case of a single firm type, there is only one.

Proof. See Lentz and Mortensen (2005).[5] ■

4 Estimation

If product quality is a permanent firm characteristic, then differences in firm profitability are associated with differences in the product creation rates chosen by firms. Specifically, more profitable firms grow faster, are more likely to survive in the future, and supply a larger number of products on average. Hence, a positive cross firm correlation between current gross profit per product and sales volume should exist. Furthermore, worker reallocation from slow growing firms that supply products of lesser quality to more profitable fast growing firms will be an important sources of aggregate productivity growth. On the other hand, if product quality were iid across innovations and firms, all firms grow at the same rate even though persistent differences in profitability exist as a consequence of different realizations of product quality histories.

In this section, we demonstrate that firm specific differences in profitability are required to explain Danish interfirm relationships between value added, employment, and wages paid. In the process of fitting the model to the data, we also obtain estimates of the investment cost of innovation function that all firms face as well as the sampling distribution of firm productivity at entry.

4.1 Danish Firm Data

If more productive firm's grow faster in the sense that $\gamma'(\pi) > 0$, then (17) implies that more productive firms also supply more products and sell more on average. However, because production employment per product decreases with productivity, total expected employment, $nE[k]$ where $n = (1 - \pi)/w + c(\gamma(\pi))$, need not increase with π in general and decreases with π when growth is independent of a firm's past product quality realizations. These implications of the theory can be tested directly.

The model is estimated on an unbalanced panel of 4,872 firms drawn from the Danish firm panel described in Section 2. The panel is constructed by selecting all existing firms in 1992 with more than 20 workers and following them through time, while all firms that enter the sample in the

[5] Although the cost of entry is linear in the paper cited while the cost is convex here, the principal argument holds in this case as well.

subsequent years are excluded. In the estimation, the observed 1992 cross-section will be interpreted to reflect steady state whereas the following years generally do not reflect steady state since survival probabilities vary across firm types. Specifically, due to selection the observed cross-sections from 1993 to 1997 will have an increasing over-representation of high creation rate firm types relative to steady state. Entry in the original data set suffers from strong selection bias and the sampling choice to leave out entry altogether is consequently partly driven by data limitations but is also useful in identifying dynamic features of the model. Table 2 presents a number of data moments with standard deviations in parenthesis. The standard deviations are obtained by bootstrapping. Unless otherwise stated, nominal amounts are in 1,000 DKK.

The dynamic moments relating to firm growth rates ($\Delta Y/Y$) include firm death, so specifically an exiting firm will contribute to the statistic with a -1 observation. Should one exclude firm deaths from the growth statistic, one will obtain a more negative correlation between firm size and growth due to the strong negative correlation between firm size and the firm exit hazard rate.

In addition to the moments in Table 2, the model will also be estimated against a standard reduced form labor productivity growth decomposition. We use the preferred formulation in Foster, Haltiwanger, and Krizan (2001) which is taken from Baily, Bartelsman, and Haltiwanger (1996). The decomposition takes the form,

$$\begin{aligned}\Delta P_t = &\sum_{e\in C} s_{et-1}\Delta p_{et} + \sum_{e\in C}\left(p_{et-1} - P_{t-1}\right)\Delta s_{et} + \sum_{e\in C}\Delta p_{et}\Delta s_{et} + \sum_{e\in N}\left(p_{et} - P_{t-1}\right)s_{et} - \\ &\sum_{e\in X}\left(p_{et-1} - P_{t-1}\right)s_{et-1}, \qquad (19)\end{aligned}$$

where $P_t = \sum_e s_{et}p_{et}$, $p_{et} = Y_{et}/N_{et}$, and $s_{et} = N_{et}/N_t$. Thus, (19) will be used to decompose growth in value added per worker into 5 components in the order stated on the right hand side; within, between, a cross component, and entry and exit. The within component is interpreted to capture growth in the productivity measure due to productivity improvements by incumbents, the between component is designed to capture productivity growth from reallocation of labor from less to more productive firms. The cross component captures a covariance between share of input and productivity growth and the last two terms capture the growth contribution from entrants and exits. The decomposition shares in the data are shown in Table 3. As mentioned, the sample in this paper does not include entry, so there is no entry share in the decomposition and the

Table 2: Data Moments (std dev in parenthesis)

	1992	1997		1992	1997
Survivors	4,872.00 -	3,628.00 (32.13)	$Cor\left[\frac{Y}{N^*}, \frac{Y_{+1}}{N^*_{+1}}\right]$	0.476 (0.088)	0.550 (0.091)
$E[Y]$	26,277.26 (747.00)	31,860.85 (1,031.25)	$Cor\left[\frac{Y}{N^*}, \Delta\frac{Y}{N^*}\right]$	-0.227 (0.103)	-0.193 (0.057)
$Med[Y]$	13,471.00 (211.35)	16,432.10 (329.77)	$Cor[Y, W]$	0.852 (0.035)	0.857 (0.045)
$E[W]$	13,294.48 (457.47)	15,705.09 (609.60)	$Cor\left[\frac{Y}{N^*}, Y\right]$	0.198 (0.036)	0.143 (0.038)
$Med[W]$	7,229.70 (92.75)	8,670.28 (154.90)	$Cor\left[\frac{Y}{N^*}, N^*\right]$	-0.018 (0.013)	-0.026 (0.011)
$Std[Y]$	52,798.52 (5,663.63)	64,129.07 (7,742.51)	$E[\Delta Y/Y]$	-0.029 (0.008)	
$Std[W]$	30,616.94 (6,751.09)	35,560.60 (8,138.66)	$Std[\Delta Y/Y]$	0.550 (0.067)	
$E\left[\frac{Y}{N^*}\right]$	384.40 (2.91)	432.12 (5.10)	$Cor[\Delta Y/Y, Y]$	-0.061 (0.012)	
$Std\left[\frac{Y}{N^*}\right]$	205.09 (19.63)	305.35 (42.50)			

decomposition shares in Table 3. Consequently, the decomposition cannot be directly related to the results in Foster, Haltiwanger, and Krizan (2001), although a full decomposition is performed on the estimated model in section 4.5.2.

The decomposition provides additional information on dynamics in the data and is therefore valuable for identification purposes. But it is also a useful method of directly relating the model to a standard reduced form measure of sources of productivity growth. In section 5, we determine the labor productivity growth rate and the structural decomposition for the estimated model. Labor productivity growth in the model is only loosely related to growth in value added per worker and consequently, there is no reason to expect the decomposition in equation (19) to coincide with the structural decomposition in section 5.

Table 3: Y/N 1992 to 1997 Growth Decomposition. Std Dev in parentheses.

	Growth Shares
Within	1.015 (0.146)
Between	0.453 (0.112)
Cross	-0.551 (0.196)
Exit	0.084 (0.066)

4.2 Model Estimator

An observation in the panel is given by $\psi_{it} = (Y_{it}, W_{it}, N^*_{it})$, where Y_{it} is real value added, W_{it} the real wage sum, and N^*_{it} quality adjusted labor force size of firm i in year t. Let ψ_i be defined by, $\psi_i = \left(\psi_{i1,...,}\psi_{iT}\right)$ and finally, $\psi = (\psi_1, \ldots, \psi_I)$

Simulated minimum distance estimators, as described in for example Gourieroux, Monfort, and Renault (1993), Hall and Rust (2003), and Alvarez, Browning, and Ejrnæs (2001), are computed as follows: First, define a vector of auxiliary data parameters, $\Gamma(\psi)$. The vector consists of all the items in Table 2 except the number of survivors in 1992 and three of the moments in table 3. Thus, $\Gamma(\psi)$ has length 33. Second, $\psi^s(\omega)$ is simulated from the model for a given set of model parameters ω. The model simulation is initialized by assuming that the economy is in steady state in the first year and consequently that firm observations are distributed according to the ω-implied steady state distribution. Alternatively, one can initialize the simulation according to the observed data in the first year, $(\psi_{11}, \ldots, \psi_{1I})$. The assumption that the economy is initially in steady state provides additional identification in that $(\psi_{11}, \ldots, \psi_{1I})$ can be compared to the model-implied steady state distribution $(\psi^s_{11}(\omega), \ldots, \psi^s_{1I}(\omega))$. The simulated auxiliary parameters are then given by,

$$\Gamma^s(\omega) = \frac{1}{S}\sum_{s=1}^{S}\Gamma(\psi^s(\omega)),$$

where S is the number of simulation repetitions.

The estimator is then the choice of parameters that minimizes the weighted distance between the data and simulated auxiliary parameters,

$$\hat{\omega} = \arg\min_{\omega \in \Omega} \left(\Gamma^s(\omega) - \Gamma(\psi)\right)' A^{-1} \left(\Gamma^s(\omega) - \Gamma(\psi)\right), \tag{20}$$

where A is some positive definite matrix. If A is the identity matrix, $\hat{\omega}$ is the equally weighted minimum distance estimator (EWMD). If A is the covariance matrix of the data moments $\Gamma(\psi)$, $\hat{\omega}$ is the optimal minimum distance estimator (OMD). The OMD estimator is asymptotically more efficient than the EWMD estimator. However, Altonji and Segal (1996) show that the estimate of A as the second moment matrix of $\Gamma(\cdot)$ may suffer from serious small sample bias. Horowitz (1998) suggests a bootstrap estimator of A. The estimation in this paper adopts Horowitz's bootstrap estimator of the covariance matrix A.

In addition to the $\hat{\omega}$ estimator, the analysis also presents a bootstrap estimator as in Horowitz (1998). In each bootstrap repetition, a new set of data auxiliary parameters $\Gamma\left(\psi^b\right)$ is produced, where ψ^b is the bootstrap data in the b^{th} bootstrap repetition. ψ^b is found by randomly selecting observations ψ_i from the original data with replacement. Thus, the sampling is random across firms but is done by block over the time dimension (if a particular firm i is selected, the entire time series for this firm is included in the sample). For the b^{th} repetition, an estimator ω^b, is found by minimizing the weighted distance between the re-centered bootstrap data auxiliary parameters $\left[\Gamma\left(\psi^b\right) - \Gamma(\psi)\right]$ and the re-centered simulated auxiliary parameters $\left[\Gamma^s\left(\omega^b\right) - \Gamma^s(\hat{\omega})\right]$,

$$\omega^b = \arg\min_{\omega \in \Omega} \Big(\left[\Gamma^s(\omega) - \Gamma^s(\hat{\omega})\right] - \left[\Gamma(\psi^b) - \Gamma(\psi)\right] \Big)' A^{-1} \Big(\left[\Gamma^s(\omega) - \Gamma^s(\hat{\omega})\right] - \left[\Gamma(\psi^b) - \Gamma(\psi)\right] \Big).$$

In each bootstrap repetition, a different seed is used to generate random numbers for the determination of $\Gamma^s(\omega)$. Hence, the bootstrap estimator of $V(\hat{\omega})$ captures both data variation and variation from the model simulation.

The bootstrap estimator of the structural parameters is then the simple average of all the ω^b estimators,

$$\hat{\omega}^{bs} = \frac{1}{B} \sum_{b=1}^{B} \omega^b, \tag{21}$$

where B is the total number of bootstrap repetitions. In the estimation below, $B = 500$ and $S = 10$.

4.3 Model Simulation

To fit the data, the model simulation produces time paths for value added (Y), the wage sum (W), and labor force size (N) for I firms. The estimation introduces a stochastic demand realization for each of a firm's products, $\tilde{Z}$. Thus, the demand for product j is given by $x_j = \tilde{Z}/p_j$. The random variable, $\tilde{Z}$, is iid across products and is assumed to follow a log-normal distribution,

$$\tilde{Z} = \exp\left(\tilde{\xi}\sigma_z + \mu_z\right) \text{ where } \tilde{\xi} \sim N(0,1). \tag{22}$$

Denote the expected value of $\widetilde{Z}$ by $E\left[\tilde{Z}\right] = Z$. Given the formulation of the firm's problem, the innovation rate is affected by the $\tilde{Z}$ distribution only through the expectation of $\tilde{Z}$ and not by any of the higher order moments.

To properly capture the labor share in the data, a capital cost $\kappa \equiv K/Z$ is added to the model where K is the capital associated with the production of a given product and κ is the capital cost relative to average product expenditure. This modifies the pricing of the intermediary goods. Now, providing an intermediary good at price p yields expected operational profits, $Z(1 - w/p - \kappa)$. Thus, the price of intermediary good j is, $p = qw/(1-\kappa)$ since consumers are exactly indifferent between buying from the quality leader at this price and the from the immediate follower at price $p = w/(1-\kappa)$, which is as low as the follower is willing to go. The inclusion of a non-labor cost then modifies the definition of production profits, π, as defined in (7). The more general definition that allows for non-labor cost is given by,

$$\pi = (1-\kappa)(1-q^{-1}), \tag{23}$$

which is identical to (7) if $\kappa = 0$.

The quality of each new innovation (and thereby the profit associated with it) is a stochastic realization drawn from a distribution which is contingent on the firm's type. Specifically, the profit of any particular innovation is assumed to satisfy

$$\widetilde{\pi} = (1-\kappa)(1-\widetilde{q}^{-1}), \text{ where } \widetilde{q} = 1 + \exp\left(\xi\sigma_{\widetilde{\pi}} + \mu_{\widetilde{\pi}}(\pi)\right) \text{ and } \xi \sim N(0,1). \tag{24}$$

where the mean $E[\widetilde{\pi}|\pi] = \pi$ represents the firm's profitability type, the determinant of its creation rate. Each firm's type is itself a random variable realized after entry. We assume that the steady state distribution profit distribution, denoted as $p(\pi)$, is characterized by

$$\pi = (1-\kappa)\left(1-q^{-1}\right), \text{ where } q = 1+\exp\left(z\sigma_\pi + \mu_\pi\right) \text{ and } z \sim N(0,1) \tag{25}$$

where in both (24) and (25) $N(0,1)$ represents the standard normal distribution. For future reference,

$$p(\pi) \equiv \frac{M(\pi)}{\int_\pi M(\pi)d\pi} \tag{26}$$

where $M(\pi)$, the steady state mass of firms of type, is given by equation (14).

Denote by $\Pi^k = (\pi_1, \ldots, \pi_k)$ the quality realizations of a firm's k products. Similarly let $Z^k = (Z_1, \ldots, Z_k)$ be the demand realizations of the firm's k products. The value added of a type π firm with k products characterized by $\left(\Pi^k, Z^k\right)$ is given by,

$$Y_k\left(\Pi^k, Z^k, \pi\right) = \sum_{i=1}^{k} Z_i, \tag{27}$$

where each product demand realization Z_i is drawn according to (22). The wage bill is given by,

$$W_k\left(\Pi^k, Z^k, \pi\right) = \sum_{i=1}^{k} Z_i\left(1-\kappa-\pi_i\right) + kZw\tilde{c}\left(\gamma(\pi)\right), \tag{28}$$

where $\widehat{c}(\gamma) = c(\gamma)/Z$.

The estimation allows for measurement error in both value added and the wage bill. The measurement error is introduced as a simple log-additive process,

$$\begin{aligned} \ln \tilde{Y}_k\left(\Pi^k, Z^k, \pi\right) &= \ln Y_k\left(\Pi^k, Z^k, \pi\right) + \xi_Y \\ \ln \tilde{W}_k\left(\Pi^k, Z^k, \pi\right) &= \ln W_k\left(\Pi^k, Z^k, \pi\right) + \xi_W, \end{aligned}$$

where $\xi_Y \sim N\left(0, \sigma_Y^2\right)$ and $\xi_W \sim N\left(0, \sigma_W^2\right)$. The estimation is performed on the quality adjusted labor force size. Consequently, the wage bill measurement error is assumed to carry through to the labor force size, $\tilde{N}_k\left(\Pi^k, Z^k, \pi\right) = \tilde{W}_k\left(\Pi^k, Z^k, \pi\right)/w$ since by construction, $N_i^* w = W_i$ for all firms in the data.

Lentz and Mortensen (2005) analyze the firm's creation rate choice in the general case where product quality is a stochastic process. Because the value of the next product is linear in profit and the profit realizations across products are iid for each firm, the optimal choice of creation rate for a firm of type π solves,

$$\gamma(\pi) = \arg\max_{\gamma} \frac{E[\widetilde{\pi}] - w\widehat{c}(\gamma)}{r + \delta - \gamma} = \arg\max_{\gamma} \frac{\pi - w\widehat{c}(\gamma)}{r + \delta - \gamma} \tag{29}$$

as in the deterministic case sketch above. Specify the cost function as $\widehat{c}(\gamma) = c_0\gamma^{1+c_1}$. Then, the first order condition for the optimal creation rate choice is,

$$w(1 + c_1) c_0 \gamma^{c_1} (r + \delta - \gamma) = \pi - wc_0\gamma^{1+c_1}. \tag{30}$$

Equations (27) and (28) provide the foundation for the model simulation. It then remains to simulate product paths for all firms. The simulation is initialized by the assumption of steady state. By (15), the steady state product size distribution conditional on survival is given by,

$$\Pr(k^* = k|\pi) = \frac{M_k(\pi)}{M(\pi)} = \frac{\frac{1}{k}\left(\frac{\gamma(\pi)}{\delta}\right)^k}{\ln\left(\frac{\delta}{\delta - \gamma(\pi)}\right)}. \tag{31}$$

First, a firm's type, π, is determined according to (25). Then, the initial product size of a firm (k_1) is determined following (31).

With a given initial product size, simulation of the subsequent time path requires knowledge of the transition probability function $\Pr(k_2 = k|k_1, \pi)$. Denote by $p_{\pi,n}(t)$ the probability of a type π firm having product size n at time t. As shown in Klette and Kortum (2004), $p_{\pi,n}(t)$ evolves according to the ordinary differential equation system,

$$\begin{aligned} \dot{p}_{\pi,n}(t) &= (n-1)\gamma(\pi) p_{\pi,n-1}(t) + (n+1)\delta p_{\pi,n+1}(t) - (\delta + \gamma(\pi)) p_{\pi,n}(t), \ \forall n \geq 1 \\ \dot{p}_{\pi,0}(t) &= \delta p_{\pi,1}(t). \end{aligned} \tag{32}$$

Hence, with the initial condition,

$$p_{\pi,n}(0) = \begin{cases} 1 \text{ if } n = k_1 \\ 0 \text{ otherwise.} \end{cases} \tag{33}$$

one can determine $\Pr(k_2 = k|k_1, \pi)$ by solving the differential equation system in (32) for $p_{\pi,k}(1)$. Solving for $p_{\pi,k}(1)$ involves setting an upper reflective barrier to bound the differential equation system. It has been set sufficiently high so as to avoid biasing the transition probabilities. Based on the transition probabilities $\Pr(k_{t+1} = k|k_t, \pi)$ one can then iteratively simulate product size paths for each firm. The procedure correctly captures the evolution of k_t but it does not identify the exact evolution of $\left(\Pi^{k_t}, Z^{k_t}\right)$. The evolution of $\left(\Pi^{k_t}, Z^{k_t}\right)$ is assumed to follow the net change in products.[6]

Finally, the simulation allows for an exogenous growth factor in both value added and the wage bill, denoted as $\widehat{g}$, that is independent of the endogenous quality improvements produced by incumbents and entrants.

4.4 Identification

The set of model parameters to be identified (ω) is given by,

$$\omega = \left\{c_0, c_1, \delta, \kappa, Z, \sigma_z^2, \sigma_{\widetilde{\pi}}^2, \sigma_\pi^2, \mu_\pi, \widehat{g}, \sigma_Y^2, \sigma_W^2\right\} \in \Omega,$$

where Ω is the feasible set of model parameters choices. The interest rate will be set at $r = .05$. The wage w is immediately identified as the average worker wage in the sample $w = 190.24$. Experimentation with non-parametric identification of the firm type distribution has been performed with a distribution with 4 support points. Because the results showed little sensitivity in the remaining model parameters to this alternative specification, we report only those parameters obtained given the assumed parametric distribution of types.

4.5 Estimation Results

The model parameter estimates are given in Table 4. Table 5 produces a comparison of the data moments and the simulated moments associated with the model parameter estimates.

The estimated model does well in fitting the labor productivity distribution and the correlations between productivity and firm size. These relationships are also shown in Figure 3. Notice that the model has not been fitted to the higher order moments of these relationships but fits them quite well nonetheless.

The estimation implies a significant level of firm type heterogeneity. In Table 4, it is expressed via the distribution of the firm's expected quality improvement of an innovation. The type dis-

[6] Suppose firm i is simulated to lose one product in a given year. In this case, $\left(\Pi^{k_{it}}, Z^{k_{it}}\right)$ is updated by randomly eliminating one element from it. This assumes that the net loss of one product took place by the gross destruction of one product and zero gross creation. This is the most likely event by which the firm loses one product. However, the net loss could also come about by the gross destruction of two products and gross creation of one product during the year. In this case, $\left(\Pi^{k_{it}}, Z^{k_{it}}\right)$ should be updated by randomly eliminating two elements and adding one. There are in principle an infinite number of ways that the firm can loose one product over the year. The estimation consequently over-estimates the persistency of $\left(\Pi^{k_{it}}, Z^{k_{it}}\right)$. The bias will go to zero as the period length is reduced, though.

Table 4: Model Parameter Estimates

	Point Estimate	Bootstrap Estimator	Std Deviation
c_0	595.2774	598.6864	50.8455
c_1	4.4186	4.4224	0.0785
κ	0.4420	0.4403	0.0055
Z	17,024.5242	17,053.0482	428.3154
δ	0.0794	0.0791	0.0029
σ_z^2	0.9138	0.8612	0.0487
$\sigma_{\widetilde{\pi}}^2$	2.3317	2.0939	0.3692
μ_π	-4.6093	-4.7304	0.3004
σ_π^2	5.9086	6.8098	0.7993
$\widehat{g}$	0.0163	0.0166	0.0012
σ_Y^2	0.0114	0.0093	0.0047
σ_W^2	0.0283	0.0293	0.0039
Inferred Estimates			
η	0.0456	0.0448	0.0018
m	1.2370	1.2116	0.0973
M	0.7174	0.7069	0.0209
L	44.8899	44.6207	1.2804
$\bar{\gamma}$	0.0338	0.0344	0.0029
Entry q-distribution			
10^{th} percentile	1.0004	1.0003	0.0001
Median	1.0072	1.0064	0.0018
90^{th} percentile	1.1327	1.1402	0.0261
Steady state q-distribution			
10^{th} percentile	1.0004	1.0003	0.0001
Median	1.0099	1.0091	0.0025
90^{th} percentile	1.2206	1.2410	0.0462

tribution at entry is such that the median firm expects to produce a 0.72% quality improvement upon discovering an innovation. The 90^{th} percentile firm expects a 13.27% improvement. The heterogeneity in creation rates across types is reflected in the steady state distribution where the high type firms are over-represented relative to the entry distribution to the point where the 90^{th} percentile firm in steady state expects a 22.06% quality improvement when it innovates. We have experimented with more flexible choices of type distributions and have found the current choice to be non-restrictive.

Given the steady state equilibrium definition, one can infer the overall entry rate, η, and the measure of potential entrant, m.[7] The implied values of these parameters are also reported in Table 4. The average incumbent creation rate, $\bar{\gamma}$, is simply the difference between the entry rate

[7] The formulas used to make the calculations are presented in the appendix.

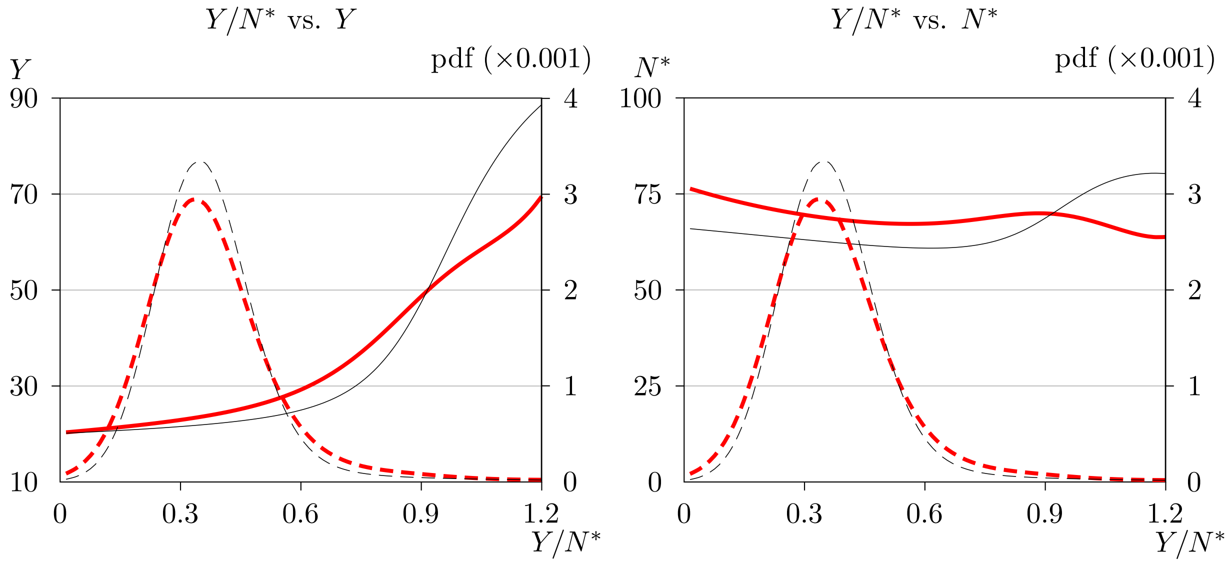
Table 5: Model Fit

	Data		Simulated Model	
	1992	1997	1992	1997
Survivors	4,872.000	3,628.000	4,872.000	3,594.300
$E[Y]$	26,277.262	31,860.851	23,832.346	28,088.419
$Med[Y]$	13,471.000	16,432.098	13,536.529	15,718.961
$E[W]$	13,294.479	15,705.087	11,976.439	13,868.172
$Med[W]$	7,229.704	8,670.279	7,146.571	8,234.080
$Std[Y]$	52,798.524	64,129.072	39,536.429	46,974.187
$Std[W]$	30,616.944	35,560.602	15,439.476	17,810.208
$E\left[\frac{Y}{N^*}\right]$	384.401	432.118	384.046	421.007
$Std\left[\frac{Y}{N^*}\right]$	205.095	305.348	199.185	216.350
$Cor\left[\frac{Y}{N^*}, \frac{Y_{+1}}{N^*_{+1}}\right]$	0.476	0.550	0.798	0.793
$Cor\left[\frac{Y}{N^*}, \Delta\frac{Y}{N^*}\right]$	-0.227	-0.193	-0.295	-0.312
$Cor[Y, W]$	0.852	0.857	0.855	0.853
$Cor\left[\frac{Y}{N^*}, Y\right]$	0.198	0.143	0.207	0.230
$Cor\left[\frac{Y}{N^*}, N^*\right]$	-0.018	-0.026	-0.021	-0.011
$E[\Delta Y/Y]$	-0.029	–	0.011	–
$Std[\Delta Y/Y]$	0.550	–	0.844	–
$Cor[\Delta Y/Y, Y]$	-0.061	–	-0.029	–
Growth decomp.				
– Within	1.015	–	0.939	–
– Between	0.453	–	0.350	–
– Cross	-0.551	–	-0.429	–
– Exit	0.084	–	0.140	–

and the destruction rate. It is seen that the estimates imply that more than half of all innovation comes from entrants. Given the estimated steady state distribution of firms, $p(\pi)$, and the other parameters of the model, one can also infer the ex ante type distribution, $\phi(\pi)$. The cdf's of the two distributions are shown in Figure 4 along with the incumbent creation rate choice conditional on firm type. It is clear from the figure that the higher quality type firms choose higher creation rates and consequently grow faster. Therefore, those with better products will make up a larger fraction of firms in steady state relative to their shares at entry. The consequences of this fact for aggregate growth are explored more fully below.

The estimation is performed given the assumption that the true firm population of interest coincides with the size censoring in the data. That is, the estimation does not correct for size censoring bias. While this is obviously a strong assumption, it reasonable assumption that the large number of very small firms in the economy are qualitatively different from those in this

Figure 3: Firm Productivity and Size, 1992 (Data and Simulation).



Note: Observed relationships drawn in bold pen and estimated relationships drawn in thin pen. Value added measured in 1 million DKK.

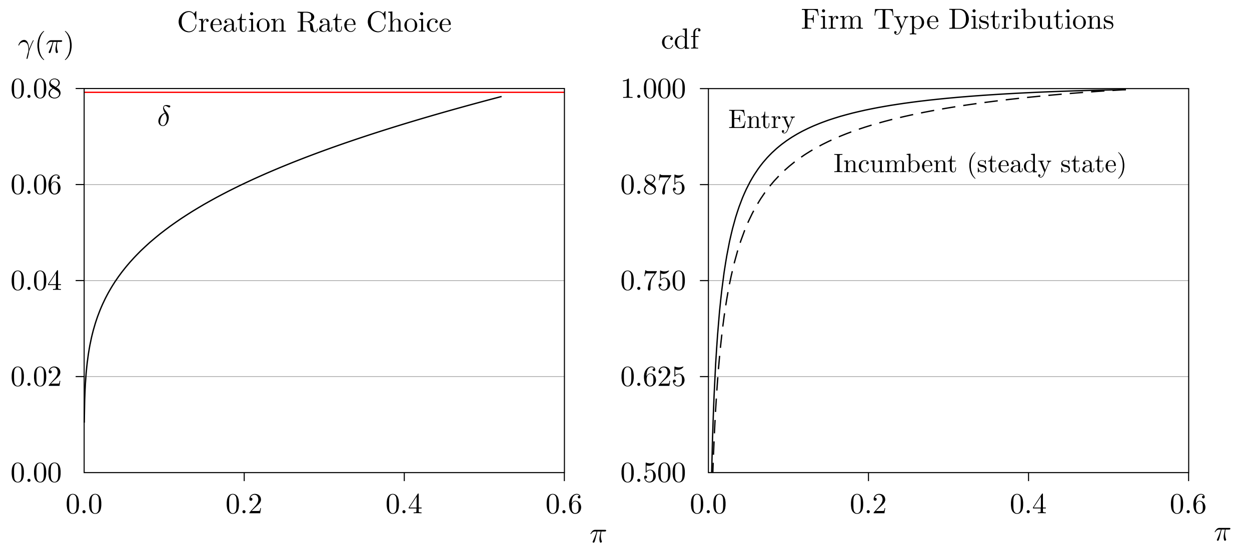
analysis and are not just firms with fewer products.

The estimation explicitly includes a number of dynamic moments. In addition, it should be noted that since the estimation is performed on cross-section moments not just in 1992 but also in 1997 and because of the specific sampling procedure in the data, the estimation implicitly address dynamic features of the model. The trends in the moments over time are in part interpreted as a result of systematic selection bias due to creation rate heterogeneity across types.

Size Distributions The model captures the medians of the Y and W distributions, but underestimate the mean and the variance. Thus, the model is not quite capturing the heaviness of the right-tail of the size distributions. This can likely be remedied by a more flexible choice of demand and supply shock processes.

The dispersion estimate is a result of a combination of the stochastic nature of the birth-death process of products, the demand shock process, and to a lesser extend the measurement error processes. Model simulation without measurement error ($\sigma_Y^2 = \sigma_W^2 = 0$) yields a reduction in the 1992 value added standard deviation estimate from $39,536.43$ to $37,697.31$. A model simulation

Figure 4: Creation Rate Choice and Firm Type Distributions.



Note: Type distribution at entry drawn in solid pen. The steady state type distribution drawn in dashed line.

with no demand shocks ($\sigma_Z^2 = 0$) yields a reduction in the 1992 value added standard deviation estimate from $39,536.43$ to $28,838.20$.

Productivity–Size Correlations Type heterogeneity and supply side shocks, σ_π^2 and $\sigma_{\tilde{\pi}}^2$ respectively, play an important role in explaining the productivity – size correlations. Type heterogeneity provides the foundation for a positive correlation between productivity and output size through a greater product creation rate for higher productivity type firms. The overall heterogeneity in product quality realizations both through type heterogeneity and random quality realizations within types explains the difference between the productivity – input size correlation and the productivity – output size correlation. Together σ_π^2 and $\sigma_{\tilde{\pi}}^2$ are chosen to get the exact levels of the correlations right. Measurement error has the potential of explaining these correlations as well. The estimation allows for both input and output measurement error which are estimated at fairly moderate amounts. If the model is simulated without the measurement error ($\sigma_Y^2 = \sigma_W^2 = 0$), the 1992 size–productivity correlations change to $corr\,(Y/N, Y) = 0.210$ and $corr\,(Y/N, N) = 0.0190$. Thus, measurement error is estimated to have virtually no impact on these moments in the data. Rather, these moments are explained to be a result of the labor saving innovation process at the heart of the model.

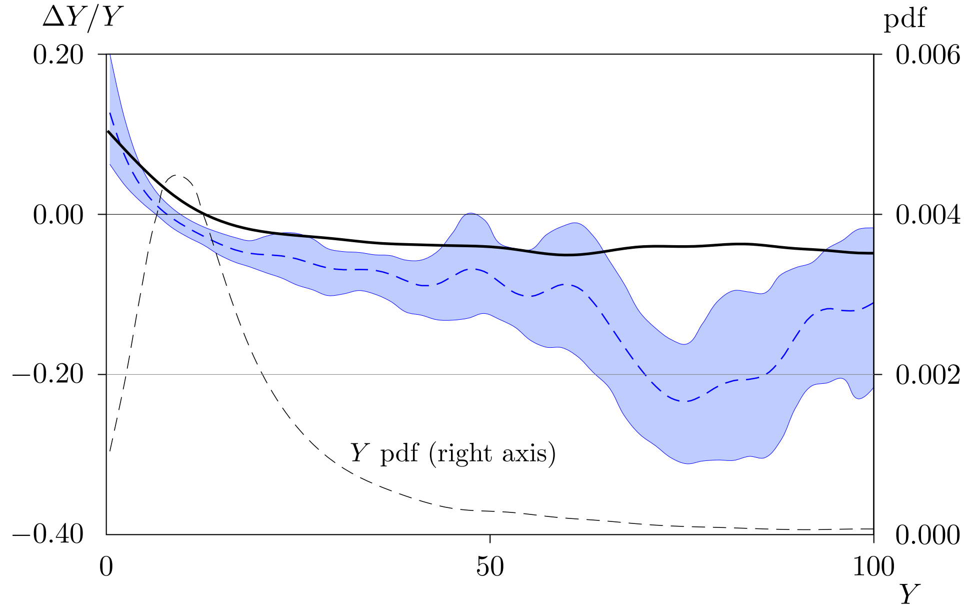
Right-Shift of Size Distributions Notice that the model successfully captures the right shift of the Y and W distributions of survivors from 1992 to 1997. There are three effects that contribute to the right shift: Generally, since the sampling eliminates the flow in of entrants, the model predicts a general decrease in mass of firms of all product sizes and types, $M_k(\pi)$, since all firms face an overall negative product growth rate. However, since entrants are assumed to flow in from the lower end of the size distribution, the reduction in mass is relatively stronger at the lower end and consequently the size distribution of survivors will begin to place relatively more weight on the upper end as time passes. Thus, the model predicts that the use of an unbalanced panel that excludes entry will itself produce a right shift of the distributions since entrants are assumed to enter as small firms. Second, the positive exogenous growth estimate directly predicts a right shift of the Y and W distributions. The third effect comes from type heterogeneity. In steady state, larger firms will over-represent high type firms with high creation rates and small firms will over-represent low type firms with low creation rates. Thus, smaller firms face greater net product destruction than large firms. In the absence of entry, the negative correlation between size and net product destruction rate will in isolation produce a right shift of the Y and W distributions over time. Hence, this effect is also a consequence of the use of an unbalanced panel that excludes entry, but is separate from the first explanation which is not a result of destruction rate heterogeneity.

Value Added per Worker Distribution The distribution of firm labor productivity Y/N is explained primarily through type heterogeneity, the capital share, the structural noise processes, and measurement error. The mean level of value added per worker is closely linked to the estimate of κ. The dispersion in Y/N across firms is explained primarily and in roughly equal parts through type heterogeneity and the positive estimate of $\sigma_{\tilde{\pi}}^2$ - supply side shocks. Measurement error adds to the dispersion measure, but to a smaller extend. Simulation without measurement error ($\sigma_Y^2 = \sigma_W^2 = 0$) yields a reduction in the 1992 Y/N standard deviation measure from 199.19 to 174.62. To an even lesser extend dispersion in Y/N is also affected by the positive estimate of σ_Z^2, that is, demand side shocks because the size of the R&D department is unaffected by particular demand realizations for a firm's products. In the absence of the R&D department, demand side shocks cannot affect labor productivity because an increase in Z realizations will increase value added and manufacturing labor demand by the same fraction. However, since the demand for R&D labor is unaffected by an increase in overall demand, a positive demand shock will result in an increase in the overall labor productivity measure, Y/N. Demand side shocks turn out to be a secondary source of labor productivity dispersion, though. Simulating the model with $\sigma_Z^2 = 0$ yields a reduction in the 1992 Y/N standard deviation measure from 199.19 to 194.03.

The right shift of the value added per worker distribution from 1992 to 1997 is explained as a combination of the exogenous growth estimate and the selection effect in that more productive firms have lower exit hazard rates. However, given the relatively low estimate of overall creative destruction, the primary effect is from the exogenous growth estimate.

Value Added per Worker Persistence and Mean Reversion The persistence in firm labor productivity $cor\left(\frac{Y}{N}, \frac{Y_{+1}}{N_{+1}}\right)$ can be explained directly through σ_π^2, σ_Z^2, the magnitudes of the creation and destruction rates $\gamma(\pi)$ and δ, and measurement error. The estimate of the relatively low level of overall creation and destruction implies that both the supply and the demand shock processes are fairly permanent and they turn out to contribute very little in the explanation of the persistence and mean reversion of value added per worker. Thus it is left to the transitory nature of the measurement error processes to explain the exact persistence and mean reversion of the value added per worker measures. Simulating the model without measurement error ($\sigma_Y^2 = \sigma_W^2 = 0$) results in 1992 persistence and mean reversion moments of $cor\left(\frac{Y}{N}, \frac{Y_{+1}}{N_{+1}}\right) \approx .97$ and $cor\left(\frac{Y}{N}, \Delta\frac{Y}{N}\right) \approx -.015$. So, without the measurement error, the model implies a high level of value added per worker persistence, which is ultimately reduced by the measurement error components. It is important to note that transitory demand shocks have much the same impact as the measurement error components along this dimension. One can speculate that the introduction of an additional demand noise component of a more transitory nature will result in a lower measurement error noise estimate.

Figure 5: Kernel Regression of Firm Growth Rate and Size (1992).



Note: Estimated model drawn in solid line. Data drawn in dashed line. Value added measured in 1 million DKK. Shaded area represents 90% confidence bounds. Value added distribution from data drawn on right axis.

4.5.1 Growth Rate and Size

Beginning with Gibrat (1931), much emphasis has been placed on the relationship between firm growth and firm size. Gibrat's law is interpreted to imply that a firm's growth rate is size independent and a large literature has followed testing the validity of this law. See Sutton (1997) for a survey of the literature. No real consensus seems to exist, but at least on the study of continuing establishments, a number of researchers have found a negative relationship between firm size and growth rate. For a recent example, see Rossi-Hansberg and Wright (2005). One can make the argument that Gibrat's law should not necessarily hold at the establishment level and that one must include firm death in order to correct for survivor bias. Certainly, if the underlying discussion is about issues of decreasing returns to scale in production, it is more likely to be relevant at the establishment level than at the firm level. However, as can be seen from Figure 5, in the current sample of firms where the growth rate – size regression includes firm exits, one still obtains a negative relationship.

Table 6: Firm Size and Growth Moments. Estimate and Counterfactuals

	Data	Point Estimate	$\sigma_Y^2 = 0$ $\sigma_W^2 = 0$	$\sigma_Z^2 = 0$ $\sigma_Y^2 = 0$ $\sigma_W^2 = 0$
$E[\Delta Y/Y]$	-0.029	0.011	-0.001	-0.034
$Std[\Delta Y/Y]$	0.550	0.844	0.083	0.300
$Cor[\Delta Y/Y, Y]$	-0.061	-0.029	-0.022	0.016

At a theoretical level, the model satisfies Gibrat's law; A firm's net innovation rate is size independent. But two opposing effects will impact the unconditional size–growth relationship: First, due to selection, larger firms will tend to over-represent higher creation rate types and in isolation the selection effect will make for a positive relationship between size and and the unconditional firm growth rate. Second, the mean reversion in demand shocks, measurement error, and to a smaller extend in supply shocks introduces an opposite effect: The group of small firms today will tend to over-represent firms with negative demand and measurement error shocks. Chances are that the demand realization of the next innovation will reverse the fortunes of these firms and they will experience relatively large growth rates. On a period-by-period basis, the same is true for the measurement error processes that are assumed to be iid over time. Large firms have many products and experience less overall demand variance. The demand shock and measurement error effects dominate in the estimated model as can be seen in Figure 5.[8] Note that the growth statistics include firm death. If firm deaths are excluded and the statistic is calculated only on survivors, the survival bias will steepen the negative relationship between firm size and firm growth both for the data and for the model since the model reproduces the higher exit hazard rate for small firms that is also found in data.

Thus, in our interpretation the model satisfies Gibrat's law by design, but it nevertheless exhibits a negative relationship between observed firm size and growth rate. As shown in Table 6, the model explains the negative relationship found in data through demand fluctuations and measurement error.[9] Gibrat's law may at one level simply be a statement about the observed

[8]Figure 5 uses value added as the firm size measure. Using labor force size as the size measure instead results in a very similar looking figure and no significant change in the correlation between size and growth.

[9]It is important to note that identification of the demand shock and measurement error processes comes from other aspects of the data as well such as dispersion in the size distribution and a number of the dynamic moments. If the Gibrat related moments are excluded from the estimation, the estimated model still exhibits a negative relationship between observed firm size and growth rate.

relationship between firm size and growth, and its validity is in this sense an issue that can be settled through observations such as the one in Figure 5. However, we have interpreted Gibrat's law to be a statement about a more fundamental proportionality between size and the firm's growth process, specifically innovation. In this case, the structural estimation shows that observation of the relationship between firm growth and firm size is not enough to falsify the statement.

4.5.2 Y/N Growth Decomposition

With the introduction of longitudinal micro-level data sets, a large literature has emerged with the focus on firm level determinants of aggregate productivity growth. See Bartelsman and Doms (2000) for a review of the literature. Given the observation of extensive firm level productivity dispersion, one particular area of interest has been the contribution to aggregate productivity growth from resource reallocation. The discussion has been quantified through decompositions such as (19), where productivity has been defined either as value added per worker or firm TFP. In the estimation in this paper, we have used the value added per worker measure. It should be immediately clear that value added per worker is only loosely related to actual productivity growth in our model, so we should at the outset expect some level of divergence between the reduced form decomposition in (19) and the structural decomposition that we present in the following section.

In the estimation and in the data sample, entry is excluded and the decomposition consequently has no value added per worker growth contribution from entry. The first two columns of Table 7, presents the decomposition results from the data and the simulated steady state that excludes entry. The remaining three columns in the table presents the simulated steady state with entry for the actual point estimate and for the two counterfactuals where measurement error noise and demand shocks have been eliminated.

The steady state with entry simulates not only the dynamic evolution of the sample of incumbents, which is the sample that the estimation is based on, it also simulates the entry process implied by the steady state general equilibrium. The entry process is described in section 3.3. For the estimated model, the size of the potential entrant pool is $4,872m/M = 8,400$. At any point in time, each of these entrants will enter according to entry rate $\gamma_0 = \eta/m = 0.0369$. The entry process is simulated to fit the one year observation frequency in the data. Thus, for each entrant who starts the year in the potential entrant pool, we calculate the transition probability that after 1 year the potential entrant has k products, $\Pr(k_e = k|\pi)$, where the type conditioning refers to the firm type realization at entry. The type realization is obviously unknown to the potential entrant prior to entry, but is subsequently of importance in terms of determining the birth-death process of product lines in the remainder of the year after entry. If $k_e > 0$ the firm is registered as an entrant with k_e products and the subsequent life of the entrant is simulated through the incumbent transition probability described in section 4.3.

The type π conditional potential entrant transition probability, $\Pr(k_e = k|\pi)$, is calculated in a similar fashion to the incumbent transition probability as described in section 4.3. However, in this case, the differential equation system that describes the probability that the potential entrant has product size n at time t, takes the form,

$$\begin{aligned}
\dot{p}_e(t) &= -\gamma_0 p_e(t) \\
\dot{p}_{\pi,1}(t) &= \gamma_0 p_e(t) - (\delta + \gamma(\pi)) p_{\pi,1}(t) \\
\dot{p}_{\pi,n}(t) &= (n-1)\gamma(\pi) p_{\pi,n-1}(t) + (n+1)\delta p_{\pi,n+1}(t) - (\delta + \gamma(\pi)) p_{\pi,n}(t), \ \forall n \geq 2 \\
\dot{p}_{\pi,0}(t) &= \delta p_{\pi,1}(t),
\end{aligned}$$

where the notation follows the notation in section 4.3 with the addition that $p_e(t)$ refers to the probability that the potential entrant is still a potential entrant at time t (and obviously has product size 0). Given the initial condition $p_e(0) = 1$, the potential entrant transition probability is found by solving the above differential equation system for $p_e(1)$ and $p_{\pi,k}(1)$. Thus, the probability that the potential entrant will not have entered after one year is $p_e(1) + p_{\pi,0}(1)$. The latter term reflects the event that a firm enters but exits again before the year's end, in which case the firm is not included in the pool of entrants. It is also seen that the discrete observation frequency implies that entry with more than one product is a positive likelihood event.

The decomposition results on the data suggest a significant contribution to productivity growth from reallocation, roughly 45%, which is a bit higher than results in Foster, Haltiwanger, and Krizan (2001), but still within the general range of their results. Part of this could have been interpreted to be a result of a missing entry component. The model does reasonably well in

Table 7: Y/N Growth Decomposition. Estimate and Counterfactuals

	Data	Point Estimate	Steady State with Entry Point Estimate	$\sigma_Y^2 = 0$ $\sigma_W^2 = 0$	$\sigma_Z^2 = 0$ $\sigma_Y^2 = 0$ $\sigma_W^2 = 0$
Within	1.015	0.939	1.108	0.796	0.820
Between	0.453	0.350	0.301	0.037	0.053
Cross	-0.551	-0.429	-0.612	-0.065	-0.104
Exit	0.084	0.140	0.160	0.161	0.160
Entry	–	–	0.069	0.072	0.072

capturing the decomposition. The third column introduces the model implied steady state entry to the decomposition and does confirm the idea that the somewhat high reallocation contribution could be a result of missing entry observations.

The fourth column in Table 7 shows the model decomposition results without the measurement error. Both the cross-term and reallocation contribution components drop to close to zero magnitude and measurement error is in this case shown to be a very important issue for the form in (19). Obviously, true productivity growth is unaffected by measurement error. We quantify true productivity growth and a structural decomposition in section 5.

Foster, Haltiwanger, and Krizan (2001) note the potential importance of measurement error and present alternative forms that may be less sensitive to measurement error. But it is doubtful that these alternative measures will be better reflections of productivity growth for the structure in this paper given the loose connection between value added per worker and TFP to the actual productivity contribution of a firm. This issue is related to points raised in Klette and Griliches (1996) where unobserved endogenous pricing at the firm level is discussed. It is an interesting issue whether one can obtain a simple reduced form that can approximate the true decomposition for this paper's model.

In terms of identification, the cross-term component turns out to be of particular importance for the input measurement error parameter. If the model is estimated subject to $\sigma_W^2 = 0$, the remaining model parameters change a little towards a bit more estimated type dispersion, but leaves the estimated cross term component close to zero. Allowing for input measurement error results in the fairly good fit of the cross-term component as shown in Table 7. In isolation, the input

Table 8: Data Moments by Industry

	Manufacturing		Wholesale and retail		Construction	
	1992	1997	1992	1997	1992	1997
Survivors	2,051.000	1,536.000	1,584.000	1,189.000	651.000	480.000
$E[Y]$	30,149.460	35,803.473	22,952.920	28,386.719	15,191.354	16,869.550
$Med[Y]$	15,117.552	18,855.682	12,740.250	15,288.949	8,688.501	10,691.434
$E[W]$	15,047.636	17,318.195	10,696.683	12,712.899	9,973.166	10,594.737
$Med[W]$	8,031.273	9,530.273	6,417.403	7,650.565	5,785.053	6,832.554
$Std[Y]$	56,095.672	69,597.651	33,410.862	41,426.484	31,311.623	22,478.083
$Std[W]$	24,673.900	27,168.284	15,365.073	16,809.785	24,545.298	14,195.942
$E\left[\frac{Y}{N^*}\right]$	379.047	422.471	410.234	466.591	305.075	342.273
$Std\left[\frac{Y}{N^*}\right]$	163.214	226.934	171.716	278.613	133.213	174.052
$Cor\left[\frac{Y}{N^*}, \frac{Y_{+1}}{N^*_{+1}}\right]$	0.650	0.728	0.325	0.674	0.428	0.345
$Cor\left[\frac{Y}{N^*}, \Delta\frac{Y}{N^*}\right]$	-0.024	-0.195	-0.195	-0.259	-0.327	-0.560
$Cor[Y, W]$	0.889	0.855	0.922	0.914	0.967	0.922
$Cor\left[\frac{Y}{N^*}, Y\right]$	0.236	0.200	0.252	0.188	0.131	0.174
$Cor\left[\frac{Y}{N^*}, N^*\right]$	0.011	-0.003	-0.028	-0.039	-0.040	-0.093
$E[\Delta Y/Y]$	-0.035	–	-0.042	–	-0.025	–
$Std[\Delta Y/Y]$	0.474	–	0.425	–	0.448	–
$Cor[\Delta Y/Y, Y]$	-0.073	–	-0.090	–	-0.122	–
Growth decomp.						
– Within	0.863	–	1.176	–	0.986	–
– Between	0.365	–	0.618	–	0.635	–
– Cross	-0.297	–	-0.826	–	-0.870	–
– Exit	0.068	–	0.032	–	0.249	–

measurement error implies some Y/N dispersion and the estimation responds by lowering the type dispersion estimate a little to fit the actual Y/N dispersion. It is interesting that the measurement error estimate is very moderate, and has little effect on the remaining model parameter estimates, but it has a very significant impact on the decomposition results.

4.6 Estimation by Industry

It is of course possible that the correlations and other data moments in Table 2 are a result of firm heterogeneity across industries and does not reflect the true picture within more homogenous subgroups of firms. This turns out not to be the case. Data moments by industry reveal the same qualitative picture as in Table 2 for each industry. Table 8 presents data moments for the 3 largest industries (by firm count). All industries show evidence of significant firm productivity dispersion, a roughly zero correlation between productivity and firm input size and a positive

Table 9: Point Estimate by Industry

	Manu-facturing	Wholesale and retail	Construction
c_0	821.1786	639.0215	93.0344
c_1	4.2496	3.7907	3.2079
κ	0.4515	0.5000	0.3163
Z	19,588.1611	17,962.8955	10,717.1519
δ	0.0687	0.0584	0.0704
σ_z^2	0.8291	0.7748	0.5938
$\sigma_{\tilde{\pi}}^2$	1.6388	0.2173	0.0012
μ_π	-6.1584	-7.3303	-5.2908
σ_π^2	8.1522	13.0962	6.1584
$\widehat{g}$	0.0194	0.0211	0.0131
σ_Y^2	0.0140	0.0151	0.0188
σ_W^2	0.0215	0.0194	0.0301
Inferred Estimates			
η	0.0483	0.0465	0.0556
m	1.8490	2.5405	2.8637
M	0.8139	0.8765	0.8777
L	53.0892	45.3600	36.3381
$\bar{\gamma}$	0.0205	0.0120	0.0149
Entry q-distribution			
10^{th} percentile	1.0000	1.0000	1.0002
Median	1.0016	1.0005	1.0042
90^{th} percentile	1.0500	1.0384	1.0869
Steady state q-distribution			
10^{th} percentile	1.0001	1.0000	1.0002
Median	1.0021	1.0007	1.0050
90^{th} percentile	1.0821	1.0677	1.1212

correlation between productivity and firm output size (roughly 0.2). All industries also display a fair amount of productivity persistence and mean reversion. Finally, both the value added and wage bill distributions are characterized by a strong right shift over time across industries.

The estimates by industry are reported in Table 9. The model estimates by industry are not qualitatively different from the full sample estimate but it is worth noting a consistent drop in the estimated type dispersion in the industry estimates. This is likely a result of effectively allowing for more heterogeneity in other model parameters.

5 Reallocation and Growth

If more profitable firms grow faster, then workers move from less to more profitable surviving firms as well as from exiting to entering firms. This selection effect can be demonstrated by noting that more profitable firms are over represented (relative to their fraction at entry) among those that produce more than one product and that this "selection bias" increases with the number of products produced. Namely, by equation (13), the difference between the relative fraction of a given firm type in the surviving population with k products and relative the fraction in its entry cohort,

$$\frac{M_k(\pi')}{M_k(\pi)} - \frac{\phi(\pi')}{\phi(\pi)} = \frac{\phi(\pi')}{\phi(\pi)} \left[\left(\frac{\gamma(\pi')}{\gamma(\pi)} \right)^{k-1} - 1 \right], \tag{34}$$

is positive and increasing in k when $\pi' > \pi$.

From equation (5), the equilibrium rate of growth in consumption is

$$\begin{aligned} \frac{\dot{C}}{C} &= g = \delta E \ln q \\ &= \delta \left(\int_\pi E[\ln q(\pi)] \frac{\eta\phi(\pi)d\pi}{\delta} + \int_\pi E[\ln q(\pi)] \frac{\gamma(\pi)\sum_{k=1}^\infty kM_k(\pi)d\pi}{\delta} \right) \\ &= \eta \int_\pi E[\ln q(\pi)]\,\phi(\pi)d\pi + \int_\pi \gamma(\pi)E[\ln q(\pi)] \sum_{k=1}^\infty kM_k(\pi)d\pi \end{aligned}$$

where $q(\pi) = (1-\kappa)/(1-\pi-\kappa)$ is the quality of the products of a type π firm and δ is the aggregate rate of creative destruction as defined in equation (18). The decomposition of the rate of productivity growth,

$$\begin{aligned} g &= \eta \int_\pi E[\ln q(\pi)]\,\phi(\pi)d\pi + \int_\pi \gamma(\pi)E[\ln q(\pi)]\,\phi(\pi)d\pi \\ &\quad + \int_\pi \gamma(\pi)E[\ln q(\pi)] \left[\frac{\eta}{\delta-\gamma(\pi)} - 1 \right] \phi(\pi)d\pi \end{aligned} \tag{35}$$

where $\frac{\eta\phi(\pi)}{\delta-\gamma(\pi)} = \sum_{k=1}^\infty kM_k(\pi)$ from equation (13), highlights the role of worker reallocation from exiting to entering firms as well as from less to more productive firms as sources of productivity growth. The first term $\eta\int_\pi E[\ln q(\pi)]\,\phi(\pi)d\pi$ is the net effect of entry and exit on productivity growth. The second term $\int_\pi \gamma(\pi)E[\ln q(\pi)]\,\phi(\pi)d\pi$ is the average contribution of continuing firms to growth were there no selection. Finally, the last term $\int_\pi \gamma(\pi)E[\ln q(\pi)]\left[\sum_{k=1}^\infty kM_k(\pi) - \phi(\pi)\right]d\pi$ represents the contribution of worker reallocation from firms with products of lesser quality to firms that produce higher quality products.

Table 10: Labor Productivity Growth Rate Estimates.

	Point Estimate	Bootstrap Estimate	Std Deviation
Growth rate g	0.0213	0.0232	0.0034
Decomposition shares:			
– Entry	0.1436	0.1301	0.0138
– Continuing	0.1765	0.1662	0.0250
– Reallocation	0.6800	0.7032	0.1120

Since the total measure of products is unity $\left(\int_\pi \sum_{k=1}^{\infty} kM_k(\pi)d\pi = 1\right)$ and $\phi(\pi)d\pi$ is the fraction of entrants of type π, $\left(\int_\pi \phi(\pi)d\pi = 1\right)$, it follows that

$$0 = \int_\pi \left[\sum_{k=1}^{\infty} kM_k(\pi) - \phi(\pi)\right] d\pi = \int_\pi \left(\frac{\eta}{\delta - \gamma(\pi)} - 1\right) \phi(\pi)d\pi.$$

Hence, the fact that $\gamma(\pi)$ is strictly increasing in π implies that the contribution to growth of the reallocation of workers among continuing firms, the last term in (35), is positive. Equivalently, it is positive because $\gamma(\pi)E\left[\ln q(\pi)\right]$ is strictly increasing in π and the steady state distribution of types stochastically dominates the distribution of types at entry as a consequence of the firm size selection process.

Given the parameter estimates reported in the previous section, the implied aggregate growth rate and its components are those reported in in table 10. These calculations raise several interesting issues. First, they imply an over all growth rate in productivity somewhat higher than the typical estimate. This fact provides indirect support for arguments that the measurement methodologies currently in use fail to fully separate quality improvements from price increases.[10] In addition, the estimates imply that worker reallocation from both exiting to entering firms and among surviving firms account for 13% and 70% respectively of the aggregate rate of growth. These numbers suggest a very important role to both forms of reallocation.

It is seen that the reduced form growth decomposition in (19) discussed in section 4.5.2 is not a useful reflection of the actual structural decomposition as it has been presented in this section. This is in large part because the empirical measure of labor productivity Y/N is not a direct reflection of the productivity contribution of a given firm in the model. This is partly because the product of the labor engaged in innovation is not measured in Y. Furthermore, while there

[10] In the U.S., this argument is fully articulated in Boskin, Dulberger, Gordon, Griliches, and Jorgenson (1996).

will be a monotonic relationship between value added per manufacturing labor and the quality improvement of the innovation, value added per manufacturing labor will not necessarily correctly reflect the exact labor productivity growth contribution. The problem is unfortunately not solved by looking at TFP rather than Y/N since exactly the same problems apply. Thus, the reduced form decomposition in (19) will not be very informative about sources of aggregate productivity growth for a structure like the one in this paper. It is an interesting question whether a simple reduced form measure on standard observable statistics exists that will provide a good approximation of the growth decomposition for the model in this paper.

6 Concluding Remarks

Large and persistent differences in firm productivity and firm size exist. Evidence suggests that the reallocation of workers across firms and establishments is an important source of aggregate economic growth. In a companion paper, Lentz and Mortensen (2005), we explore a variant of the equilibrium Schumpeterian model of firm size evolution developed by Klette and Kortum (2004) that provided insights into these and other empirical regularities. In our version of the model, firms that can develop products of higher quality grow larger at the expense of less profitable firms though a process of creative destruction. Worker reallocation from less to more profitable firms induced by the process contributes to aggregate productivity growth. Furthermore, the model is consistent with the observation that there is no correlation between employment size and labor productivity and a positive correlation between value added and labor productivity observed in Danish firm data.

In this paper, we take the model to the data. Namely, we fit its structure to Danish firm panel data for the 1992–1997 time period. We find that the parameter estimates are sensible and that the model provides a reasonable fit to many of the moments of the joint distribution of size as measured by value added and employment. The model also explains the evolution of the size distribution of firms in the panel over the observation period.

By design, the growth rate of a firm is size independent, but the model fits the negative unconditional firm size growth relationship in data. The model also captures the reduced form growth decomposition form that is standard to the literature, which suggests a strong contribution to growth from reallocation. But the model explains a large part of the fit with a moderate amount of measurement error and the actual determinants of productivity growth in the model are not reflected well by the reduced form decomposition.

Finally, the quantitative model has interesting aggregate implications for the growth process. First, the implied rate of productivity growth, 2.1% per year, is larger than estimates based on standard accounting methods. Second, reallocation of workers from less to more productive surviving firms is shown to account for more than 2/3 of aggregate productivity growth.

A Appendix

In this section, we present the algorithm used to compute the values of model parameters implied by the estimates and the equilibrium and optimal growth rates, all reported in the text. To do so, one must account for the two parameters not explicitly used in the initial presentation of the model, the average demand per product, Z, which was normalized to unity in the model, and the cost of capital per product line, denoted κZ. Hence, profit per product line can be represented as πZ for a firm of type π where

$$\pi = (1-\kappa)(1-q^{-1}) \tag{36}$$

is now profit express as a fraction of value average sales.

Since the parametric form of the steady state distribution of firms over profit, denoted $p(\pi)$ in the text, is specified in the model estimated, one needs to derive its relationship to the initial density of entering firms over profit, $\phi(\pi)$, by inverting the steady state relationship implied by the model. Specifically,

$$p\left(\pi\right) = M\left(\pi\right)/M$$

where $M(\pi)$ is the steady state mass of firms of type π and $M = \int_\pi M(\pi)d\pi$ is the total mass of firms. Since

$$M\left(\pi\right) = \sum_{k=1}^{\infty} M_k(\pi) = \ln\left(\frac{\delta}{\delta-\gamma(\pi)}\right)\frac{\eta\phi(\pi)}{\gamma(\pi)}$$

from equation (13), it follows that

$$\eta\phi(\pi) = \frac{\gamma(\pi)M\left(\pi\right)}{\ln\left(\frac{\delta}{\delta-\gamma(\pi)}\right)} = \frac{\gamma(\pi)p\left(\pi\right)M}{\ln\left(\frac{\delta}{\delta-\gamma(\pi)}\right)},$$

At this stage, the aggregate entry rate η and the total mass of firms M have yet to be separately identified. But by $\int_\pi \phi\left(\pi\right)d\pi = 1$, it follows that,

$$\eta = \eta\int_\pi \phi(\pi)d\pi = M\int_\pi \frac{\gamma(\pi)p\left(\pi\right)}{\ln\left(\frac{\delta}{\delta-\gamma(\pi)}\right)}d\pi. \tag{37}$$

Consequently, the profit density at entry is

$$\phi(\pi) = \frac{\frac{\gamma(\pi)p(\pi)}{\ln\left(\frac{\delta}{\delta-\gamma(\pi)}\right)}}{\int_x \frac{\gamma(x)p(x)}{\ln\left(\frac{\delta}{\delta-\gamma(x)}\right)}dx}. \tag{38}$$

Equation (15) and the assumption that the measure of products is unity, the steady state measure of continuing firms in the market solves

$$\begin{aligned} 1 &= \int_{\pi} \sum_{k=1}^{\infty} k M_k(\pi) d\pi = \int_{\pi} M(\pi) \sum_{k=1}^{\infty} \frac{k M_k(\pi)}{M(\pi)} d\pi \\ &= \int_{\pi} \frac{\gamma(\pi) M(\pi)}{(\delta - \gamma(\pi)) \ln\left(\frac{\delta}{\delta - \gamma(\pi)}\right)} d\pi = M \int_{\pi} \frac{\gamma(\pi) p(\pi)}{(\delta - \gamma(\pi)) \ln\left(\frac{\delta}{\delta - \gamma(\pi)}\right)} d\pi. \end{aligned} \tag{39}$$

Hence,

$$\eta = \frac{\int_{\pi} \frac{\gamma(\pi) p(\pi)}{\ln\left(\frac{\delta}{\delta - \gamma(\pi)}\right)} d\pi}{\int_{\pi} \frac{\gamma(\pi) p(\pi)}{(\delta - \gamma(\pi)) \ln\left(\frac{\delta}{\delta - \gamma(\pi)}\right)} d\pi}. \tag{40}$$

from by equations (37) and (39).

To solve the planner's problem, one also needs the size of the aggregate labor force, L, and the measure of potential entrants, m. Because one can show that the limit price charged by the current supplier of each product solves $p(1-\kappa) = wq$ when a capital cost exists, the demand for production workers is $Zx(\pi) = 1/p = Z(1-\kappa)/wq = Z(1-\kappa-\pi)/w$ from (36). Hence, equations (12) and (13) imply

$$L = Z \left[\int_{\pi} \left(\frac{1-\kappa-\pi}{w} + \widehat{c}(\gamma(\pi)) \right) \frac{\eta \phi(\pi) d\pi}{\delta - \gamma(\pi)} + m \widehat{c}(\eta/m) \right] \tag{41}$$

where, as specified in the text, $\widehat{c}(x) = c_0 x^{1+c_1}$. Finally, one can obtain the value of m by using the fact that the marginal cost of entry must equal the expected marginal cost of innovation by incumbents. Specifically, equations (11) and (10) imply require that m solves

$$\widehat{c}' \left(\frac{\eta}{m} \right) = \int_{\pi} \widehat{c}' \left(\gamma(\pi) \right) \phi(\pi) d\pi \tag{42}$$

Finally, the parametric specification of heterogeneity in product quality is

$$q(z) = 1 + e^{\mu_\pi + \sigma_\pi z} \tag{43}$$

where z is the standard normal random variable. Hence, one can use the fact that $f(z)dz = p(\pi(z))d\pi(z)$, where $f(z)$ is the standard normal pdf and $\pi(z) = (1-\kappa)(1-q(z)^{-1})$ by (36), to compute all the necessary integrals in the equations above and those that define the components of the growth rate found in the text.

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