I Z A Institute
of Labor Economics
Initiated by Deutsche Post Foundation

DISCUSSION PAPER SERIES

IZA DP No. 13763

A Subscription vs. Appropriation Framework for Natural Resource Conflicts

Dripto Bakshi
Indraneel Dasgupta

OCTOBER 2020

I Z A Institute of Labor Economics

Initiated by Deutsche Post Foundation

DISCUSSION PAPER SERIES

IZA DP No. 13763

A Subscription vs. Appropriation Framework for Natural Resource Conflicts

Dripto Bakshi
Indian Statistical Institute

Indraneel Dasgupta
Indian Statistical Institute and IZA

OCTOBER 2020

Any opinions expressed in this paper are those of the author(s) and not those of IZA. Research published in this series may include views on policy, but IZA takes no institutional policy positions. The IZA research network is committed to the IZA Guiding Principles of Research Integrity.

The IZA Institute of Labor Economics is an independent economic research institute that conducts research in labor economics and offers evidence-based policy advice on labor market issues. Supported by the Deutsche Post Foundation, IZA runs the world's largest network of economists, whose research aims to provide answers to the global labor market challenges of our time. Our key objective is to build bridges between academic research, policymakers and society.

IZA Discussion Papers often represent preliminary work and are circulated to encourage discussion. Citation of such a paper should account for its provisional character. A revised version may be available directly from the author.

ISSN: 2365-9793

IZA – Institute of Labor Economics

Schaumburg-Lippe-Straße 5–9
53113 Bonn, Germany

Phone: +49-228-3894-0
Email: publications@iza.org

www.iza.org

ABSTRACT

A Subscription vs. Appropriation Framework for Natural Resource Conflicts

We examine how cross-community cost or benefit spillovers, arising from the consumption of group-specific public goods, affect both inter-group conflicts over the appropriation of such goods and decentralized private provision for their production. Our model integrates production versus appropriation choices, vis-à-vis group-specific public goods, with their decentralized voluntary supply, against a backdrop of such cross-community consumption spillovers. Our flexible and general formulation of consumption spillovers incorporates earlier specifications as alternative special cases. We show that stronger negative (or weaker positive) consumption spillovers across communities may reduce inter-group conflict and increase aggregate income (and consumption) in society under certain conditions. Thus, stronger negative consumption spillovers may have socially beneficial consequences. We also identify conditions under which their impact will be both conflict-augmenting and income-compressing. Our general theoretical analysis offers a conceptual structure within which to organize investigation of feedback loops linking ethnic conflict and natural resource degradation in developing country contexts.

JEL Classification: D72, D74, O10, O20

Keywords: production versus appropriation, rent-seeking, public good contest, public bad, natural resource conflict

Corresponding author:
Indraneel Dasgupta
Economic Research Unit
Indian Statistical Institute
203 Barrackpore Trunk Road
Kolkata 700108
West Bengal
India

E-mail: indraneel@isical.ac.in

1. Introduction

Many forms of social conflicts are rooted in the fact that certain kinds of collectively consumed environment-related items have the characteristics of group-specific public goods. Their consumption, use or exploitation in some particular manner generates benefits that accrue in a non-rival manner to all members of one specific group. However, those benefits spill over to members of another group only to a lesser extent, or not at all. Indeed, in extreme cases, members of another group may all suffer some cost, so that what constitutes a public good within one community may constitute a public bad for another. In these situations, all members of either community benefit if a larger proportion of that item is reserved for exclusive use by their own community. Thus, there arises scope for inter-community conflict over environmental or natural resource policy and, more generally, modes of collective consumption.

To fix ideas, consider the case of cow protection in India. Orthodox Hindus consider the cow sacred. Hindu nationalist governments in many Indian states have drastically expanded the ambit of laws against cow slaughter and increased the penalty for their violation in recent years. At the same time, vigilante groups have engaged in violence against, and even murder of, cattle traders suspected of transporting cattle to other states and neighboring countries for slaughter. Consequently, farmers, unable to sell, have increasingly taken to abandoning their economically unproductive bullocks and older cows. This has led to a dramatic increase in the number of stray cattle, which in turn poses a serious threat to standing crops, imposes large fencing costs on farmers, degrades common grazing land and increases methane emissions. Thus, the crackdown on cow slaughter, and its attendant restrictions on inter-state and inter-country cattle trade, may be seen as generating non-rival and non-excludable benefits for Orthodox Hindus who are not farmers, but constituting a public bad for the farming community, with significant negative environmental consequences. Evidently, this creates the scope for political conflict over the content and implementation of laws against cow slaughter between the two groups.[1]

Inter-group conflicts over sharing of group-specific public goods in general, and such conflicts with environmental implications in particular, often acquire greater salience in developing countries due to traditional concentration of particular ethnic groups in specific economic locations. For example, resource conflicts between pastoralists and cultivators in West Africa often acquire broader ethno-linguistic and religious colors, because of strong correlations between such identities and economic

[1] For detailed discussions, see Alavi [2019] and Chari [2019]. Relatedly, in Sweden, while many nature lovers consider the Swedish wolf a public good, it constitutes a public bad for reindeer herders, whose livestock it preys on [Bostedt, 1999]. As noted by Buchholz *et al.* [2018], similar examples involving costly preservation of different animals can be provided from around the world.

interests.[2] In India, laws governing hunting and exploitation of forest produce, and conversion of forest land for commercial forestry, mining or industrial purposes, often bring traditionally forest-dwelling tribal communities into conflict with non-tribals [Aggarwal, 2020]. Thus, cross-community spillovers with environmental implications (whether positive or negative) in the group-specific use of natural resources carry important implications for both social conflict and overall economic wellbeing in many different, in particular developing country, contexts. But what exactly are those implications?

With a given stock of some natural resource, stronger negative spillovers, or greater exclusivity, across communities in its exploitation would imply stronger incentives for individuals to help appropriate that resource for exclusive use of their own community. This may be intuitively expected to increase conflict between competing communities and divert more resources overall to appropriation, rather than productive activities. But what happens then to individual incentives to contribute to public goods necessary for the maintenance or expansion of that resource stock itself?

For example, if degradation of common village grazing land leads to more destruction of standing crops by stray cattle, conflicts between farmers and 'cow protector' vigilante groups can be expected to increase. But what would happen to decentralized individual incentives to contribute to local veterinary clinics or NGOs that undertake immunization and treatment programs for all local cattle, thereby maintaining its stock? If reduced rainfall due to climatic change alters migration patterns of pastoralists and thereby causes greater damage to cultivators, conflict between the two groups over control of local waterbodies would rise. But what happens to individual incentives to contribute to the maintenance of local irrigation projects that replenish these contested waterbodies? If commercial afforestation involves planting trees that deplete groundwater more and thereby make both fruit trees and animal life less viable, conflict between hunter-gatherer communities and commercial planters, over what proportion of a given forest area should be allocated to commercial forestry, would rise. But what happens then to individual incentives to participate in forest protection associations meant to resist timber smuggling (or land grab attempts by third parties such as cultivator communities and mining companies)?

If individual incentives to contribute to its maintenance decline sufficiently overall, the stock of the resource being contested over would fall. The dampening effect of that contraction would counteract the conflict-expanding effect of stronger negative spillovers. What are the conditions, then, that determine whether the net effect would turn out to be conflict-expanding? Is it possible that stronger cross-community negative spillovers would actually increase the deployment of resources to production

[2] See Shettima and Tar [2008] for a discussion of the vast literature on such conflicts. Dasgupta and Kanbur [2005a, 2005b, 2007, 2011] develop the idea that intra-group sharing of group-specific public goods generates the cohesion of interest necessary to both make and maintain stable identity *communities* out of groups of individuals.

rather than appropriation, and thus increase aggregate social output, instead of reducing both, as one might perhaps intuitively expect? The purpose of this paper is to develop a parsimonious theoretical framework to address these issues.

We develop a model of a society populated by individuals who are partitioned into two equal-sized groups, or communities. All individuals are endowed with one unit of some resource ('money' or 'effort'). Individuals have to allocate their endowment among three alternative uses in decentralized manner. They can convert it to a privately consumed numeraire good, use it to subscribe to a society-wide common pool, or expend it on a Tullock [1980] style appropriation contest between the two communities. The common pool, or fund, generated through individual subscriptions produces, according to a strictly convex and iso-elastic production technology, the item whose inter-group division is contested over. This item has the characteristics of a group-specific public good, in that, ceteris paribus, all members of either group are strictly better off if their own group achieves a larger share. The benefit received by members of either group from a given production of the contested good falls monotonically as the share achieved by its antagonist increases, according to a strictly convex and iso-elastic *loss function*. A more elastic loss function implies a lower loss to a community from its antagonist achieving any given share, as does a fall in its scale parameter. Thus, the elasticity and scale parameters of the loss function capture the strength of consumption spillovers and externalities, whether positive or negative, impacting on either community in consequence of its antagonist exploiting any given share of the contested good accruing to it. All individuals allocate their respective endowments simultaneously.

Examining the properties of the Nash equilibrium, we find the following. Whether stronger cross-community negative consumption spillovers, modeled as either a fall in the elasticity of the loss function or an increase in its scale factor, will serve to increase group conflict depends critically on the production technology for the contested good. Strikingly, the total amount of resources wasted on appropriation will fall when the output elasticity of that production technology, which may vary over the open unit interval, is sufficiently close to unity. However, it will rise otherwise. A mean-preserving decrease in the spread of the scale parameters of the loss functions has the same effect. Given any elasticity of the production technology for the contested good, stronger cross-community negative consumption spillovers must reduce aggregate social income, measured in units of the numeraire good, when the numerical size of the communities exceeds a threshold value. In this very specific sense, stronger cross-community negative consumption spillovers may affect aggregate societal well-being adversely, as might be intuitively expected. Strikingly, however, given any arbitrary community size, and any arbitrary production elasticity for the contested good, there also exist parametric configurations under which the effect would be positive.

A large theoretical literature has developed, stemming from the seminal contributions by Hirshleifer [1991] and Skaperdas [1992], which investigates how, when property rights are not fully secure, appropriation opportunities impact on production incentives, and vice-versa.[3] In these (so-called 'production vs. appropriation') models, the magnitude of the item open to appropriation – the size of the prize – is endogenously determined. The item open to appropriation is typically one whose consumption is fully rival across individuals – a standard private good. Our analysis belongs to this tradition, in its focus on endogenous determination of the size of the prize, and, in consequence, mutual determination of production and appropriation. However, we extend this literature by highlighting inter-group appropriation conflicts over items which exhibit public good characteristics within groups; i.e., over group-specific public goods, whose size is endogenously determined through such interplay, via a process of society-wide voluntary subscriptions.

Parallel to the production vs. appropriation literature, and originating from Katz *et al.* [1990] and Ursprung [1990], a large theoretical literature has also developed to address inter-group contests over group-specific public goods. This literature originally developed to examine inter-community conflicts over the sharing of state investment in public goods of localized or jurisdiction-specific benefit like schools, roads, hospitals, security, public art and local antipollution measures when the communities exhibit locational segregation, but subsequently incorporated many other applications.[4] Most recently, it has expanded to include investigations of ethno-linguistic and religious conflicts over identity goods and social norms.[5] These models however almost all belong to the 'rent-seeking', rather than 'production and appropriation', tradition, in that the size of the public good prize being contested over between groups is exogenously given, rather than being determined as an endogenous consequence of the interplay of production and expropriation.[6] Our model expands this literature precisely through such endogenization.[7]

3 See, for example, Murphy *et al.* [1993], Grossman and Kim [1995], Anderton *et al.* [1999], Noh [2002], Hausken [2005], Caruso [2010], Dal Bo and Dal Bo [2011, 2012], Mitra and Ray [2014], Cornes *et al.* [2019] and Bakshi and Dasgupta [2020].

4 Contributions include Katz *et al.* [1990], Ursprung [1990], Gradstein [1993], Riaz *et al.* [1995]. Baik [2008, 2016], Epstein and Mealem [2009], Nitzan and Ueda [2009], Esteban and Ray [2001, 2011a, 2011b], Lee [2012], Kolmar and Rommeswinkel [2013], Chowdhury *et al.* [2013], Dasgupta [2017], Bakshi and Dasgupta [2018, 2020], Cheikbossian and Fayat [2018], Dasgupta and Guha Neogi [2018] and Baik and Jung [2020].

5 Esteban and Ray [2011a], Dasgupta [2017], Bakshi and Dasgupta [2018, 2020] and Dasgupta and Guha Neogi [2018] are examples of such recent application.

6 One partial exception is Gradstein [1993]. See footnote 7 below.

7 Gradstein [1993] examines endogenous determination of the size of a local public good prize in the context of inter-jurisdiction contests over location of that local public good. The local public good, whose size is determined according to the preferences of members of the winning jurisdiction, is however generated through taxation of the

Thus, like Bakshi and Dasgupta [2020], the present paper builds a bridge between the production and appropriation literature and that on inter-group contests over group-specific public. However, we depart fundamentally from that analysis by endogenizing the size of the public good prize being contested over between groups.[8]

The third dimension along which we expand the literature is by incorporating a quite general and flexible specification of cross-community consumption spillovers and externality effects with regard to the contested and group-specific public good. Ihori [2000] and Buchholz *et al.* [2018] formalize the idea that an item may be a public good for members of one group, but a public bad for members of another. They develop models where the benefits members of one group receive from an item generated by voluntary contributions on their part may be reduced by another item similarly generated by members of another group. We expand this idea to the domain of public good contests. In our model, the benefits received by all members of one group from its share of an item, acquired in consequence of contestation with another group (over a stock produced through voluntary contributions by members of *both* groups), may be reduced (or augmented) by the use its antagonist group makes of its own share. These inter-group spillover or externality effects may be asymmetric – the strength of the spillovers from, say, community A to community B may be different from that from B to A. This permitted asymmetry extends the formulation introduced by Dasgupta and Guha Neogi [2018], incorporating it as a special case. Indeed, we even permit the association of positive spillovers in one direction with negative spillovers in another. Furthermore, the strength of these spillovers varies in a non-linear fashion with the level of consumption. This last feature of our model extends it beyond the linear aggregative structure adopted by both Ihori [2000] and Buchholz *et al.* [2018], which constitutes a special, limiting case of our analysis.

Lastly, our contribution has relevance for the literature on voluntary contribution to public goods. This literature typically asks the following question: how would an exogenously supplied redistribution of

---

entire society in his model, not through society-wide voluntary subscriptions (as in ours). Another related contribution is Cheikbossian [2008], who focuses on the size of (uniform lump-sum) tax-funded government spending on public good provision, when two competing groups have differing preferences over that size and can lobby the government in Tullock fashion to reflect their respective preferences more closely. Neither voluntary private supply of public goods, nor group-specificity in their consumption, appears in that analysis. These however constitute critical elements of our model.

[8] The present paper also ignores both within-group conflict over sharing of private consumption and cross-territorial conflict spillovers – key features of the model in Bakshi and Dasgupta [2020].

income/wealth affect private supply of public goods?[9] The question we address instead is: how would inter-group conflict over their distribution affect private supply of public goods?

Section 2 sets up the model. The key equilibrium outcomes are characterized in section 3. Our main comparative static conclusions are presented in section 4. Section 5 concludes. Detailed proofs of propositions are provided in an appendix.

2. The Model

Consider a society partitioned into two mutually exclusive equal-sized groups or communities, H and M. Each community has n members, $n \geq 1$. Given any community $k \in \{H, M\}$, we shall use $-k$ to denote the other. Individuals are indexed by a pair $\langle i, k \rangle$, where $i \in \{1, 2, \dots, n\}$ and $k \in \{H, M\}$. Every individual in this society is endowed with one unit of a numeraire good, C (intuitively, 'money' or 'effort') which she can directly convert to fully rival and non-contestable consumption (c_{ik}), contribute to a common fund for the production of some contestable good Y (y_{ik}), or allocate to appropriation, i.e., inter-group conflict over division of that contestable good so produced (x_{ik}). Thus, each individual's budget constraint is:

$$c_{ik} + y_{ik} + x_{ik} = 1; \tag{1}$$

with $c_{ik}, y_{ik}, x_{ik} \geq 0$. The size of the common fund for production of Y, B, is given by the sum of individual contributions, so that:

$$B = \sum_{i=1}^{n} y_{iH} + \sum_{i=1}^{n} y_{iM}; \tag{2}$$

and that good is produced according to a strictly concave and iso-elastic production function:

$$\tilde{Y} = B^{\alpha}, \tag{3}$$

where $\alpha \in (0,1)$. The parameter α measures the output elasticity of Y (i.e., $\frac{Bd\tilde{Y}}{\tilde{Y}dB}$). Intuitively, the common fund B may be identified with either a pool of voluntary labor, or a sum of money raised through decentralized and voluntary individual subscriptions, that may be deployed to produce or maintain some collective good. The assumed strict concavity of the production function ascribed to Y may be interpreted

[9] This literature originates from Bergstrom *et al.* [1986]. See, for example, Andreoni [1990], Cornes [1993], Cornes and Sandler [1994, 1996, 2000] and Buchholz *et al.* [2018] for subsequent developments. Dasgupta and Kanbur [2005a, 2005b, 2007, 2011] analyze how private supply of public goods affects anti-poverty transfer policy, demand for income or wealth redistribution, and welfare inequality.

in standard fashion as a technological given. Evidently, the case of inter-group conflict over division of the common fund itself constitutes one limiting case of our model ($\alpha = 1$). We discuss a possible alternative formulation of the production technology for Y in Remark 2 below.

The good Y is contestable at a community level - its division between competing user groups is determined by a political process involving group-specific resource investment in lobbying, bribery, and possibly violence. More formally, the proportion of any produced amount of that good unilaterally controlled, and exploited or utilized, by community k is given by the symmetric Tullock (1980) contest success function:

$$p_k = \frac{X_k}{X} \text{ if } X > 0, \text{ and } \frac{1}{2} \text{ otherwise;} \tag{4}$$

where the community conflict allocations are simply the sum of individual members' allocations, so that $X_k = \sum_{i=1}^{n} x_{ik}$, and total (society-wide) conflict allocation is defined as $X = X_H + X_M$.[10]

The consumption of Y may, in effect, be less than or more than fully rival across communities due to cross-community consumption spillovers, which may be either positive or negative. This key feature of our model is captured via a distinction between a community's *control* share of that good, p_k, arrived at through a process of Tullock contestation, and the proportion that it would need to be able to use/exploit, in order to achieve the same benefit, in case its antagonist *did not* exploit or utilize its own control share, p_{-k}. We term this the community's *effective* share:

$$g_k = 1 - a_k \left(\frac{{p_{-k}}^{\rho}}{\rho} \right); \tag{5}$$

where $a_k \in (0,2]$ and $\rho > 1$. Lastly, consumption of Y is at least partly non-rivalrous within each community, so that the each member of community k has, effectively, consumption access to $\frac{\tilde{Y} g_k}{n^{\theta}}$ amount of the good Y, with $\theta \in [0,1)$. Y is a pure public good within either community when $\theta = 0$, and a pure intra-community private good in the limiting case of $\theta = 1$. For any individual i in community k, the payoff function is given by:

$$u_{ik} = c_{ik} + T\tilde{Y}\left(\frac{g_k}{n^{\theta}}\right); \tag{6}$$

[10] One can generalize the model to permit inter-community differences in conflict efficiency, by replacing (4) with the following condition: $p_H = \frac{\vartheta X_H}{X_M + \vartheta X_H}$ if $[X_M + \vartheta X_H] > 0$, and $\frac{\vartheta}{1+\vartheta}$ otherwise; where $\vartheta > 0$. Then ϑ captures the relative productivity of investment in appropriation by H. Evidently, (4) is simply the symmetric special case of this general formulation, where $\vartheta = 1$. This generalization would considerably increase the algebraic burden, but not add anything of substance to our comparative static conclusions.

where $T \in (0,1]$. All individuals simultaneously choose their private consumption c_{ik}, production common fund subscription y_{ik} and expropriation investment x_{ik} so as to maximize (6), subject to the constraints (1) – (5) above.

At this stage, it is useful to lay out in detail the key features of the effective share function introduced in (5). Since $\lim_{a_k \to 0} g_k = \lim_{\rho \to \infty} g_k = 1$, fully non-rival consumption of Y across communities constitutes one limiting case of the effective share function g_k. In this case, the benefits that can be derived by a community, say *H*, from *Y* does not depend on how much of that good is unilaterally exploited by *M*, instead of *H*. As $\lim_{\rho \to 1} g_k = p_k$ at $a_k = 1$, fully rival consumption constitutes another limiting case of the effective share function g_k. In this case, each community can only benefit from the public good to the extent it comes to own or control it through the process of political contestation, so that control and access shares become identical. In other words, the actual unilateral use of its control share by, say, M does not impose any additional cost or benefit on H - only possession matters.[11] When $g_k < p_k$, not only does the use of its control share p_{-k} of the public good by its opponent imply a loss of access for community k by the same proportion, such use also imposes an additional cost on k via negative externalities and spill-over effects. The magnitude of this cost, measured in units of the contested good, is $\tilde{Y}(p_k - g_k)$. Thus, if its antagonist were to merely cease exploiting its own share, community *k* would receive this additional benefit, even with the same control share p_k. Intuitively, this may happen through an across-the-board decline in the productivity of *k*'s stock of the good *Y*, because of negative externalities and spillovers stemming from the actions its antagonist takes while unilaterally exploiting its own stock.[12] Conversely, $g_k > p_k$ captures the case of positive externalities and spillovers.[13] Then, the use of its control share p_{-k} of the public good by its opponent augments the

---

[11] This particular limiting special case of our model is the standard formulation adopted in the literature on contests over group-specific public goods. When the contest success function is interpreted as providing the success probabilities, instead of the group *shares* as in our model, the expected utility of an individual must take the form: $\left[Eu_{ik} = c_{ik} + \left(\frac{T\tilde{Y}}{n^{\theta}}\right)\left[p_k + (1-p_k)\left(1-\frac{a_k}{\rho}\right)\right] = \left(1-\frac{a_k}{\rho}\right)\left(\frac{T\tilde{Y}}{n^{\theta}}\right) + c_{ik} + \left(\frac{a_k}{\rho}\right)\left(\frac{T\tilde{Y}}{n^{\theta}}\right)(1-p_{-k})\right]$. Hence, the individual pay-off function must always reduce to this special limiting form, regardless of the specification of the effective share function in (5). Thus, for it to have substantive consequences, the flexibility provided by our general specification of the effective share function in (5) requires that the contest outcomes be interpreted exclusively as group shares, *not* as success probabilities.

[12] Ihori [2001] and Buchholz *et al.* [2018] use a linear aggregative structure for net benefits to capture the idea that an item may constitute a public good for one community, but a public bad for another. Since $\left[\lim_{\rho \to 1} g_k = 1 - a_k p_{-k}\right]$, translated to our framework, their formulation, in essence, falls out as this limiting case of our formulation when $a_k > 1$ (so that $\lim_{\rho \to 1} g_k < p_k$).

[13] Note that the specification introduced by Dasgupta and Guha Neogi [2018], viz., $[g_k = 1 - {p_{-k}}^{\rho}]$, with $\rho > 1$, falls in this class.

benefit k can receive from unilateral exploitation of its own control share, p_k, to the extent of $\tilde{Y}(g_k - p_k)$. If its antagonist were to stop exploiting its control share, community k's control share would need to be augmented by $(g_k - p_k)$, for it to be able to achieve the same benefit from *Y* as before.

Notice that the effective share function in (5) is monotonically increasing in own control share. Thus, the inter-community contest over control shares remains salient, despite the effective share diverging from the control share. Recalling (5), let $L_k \equiv a_k\left(\frac{p_{-k}{}^{\rho}}{\rho}\right)$. The term L_k can be thought of as a *loss* function, in that it specifies the loss (measured in terms of effective share) to *k*, that is generated in consequence of its antagonist exploiting any given control share. As noted above, $[L_k > p_{-k}]$ implies negative spillovers from its antagonist to community *k*, while $[L_k < p_{-k}]$ implies positive spillovers. Notice that spillovers must necessarily be positive if $\left[\left(\frac{a_k}{\rho}\right) < 1\right]$. However, if $\left[\left(\frac{a_k}{\rho}\right) > 1\right]$, spillovers will be negative at high values of p_{-k}, but positive at low values, reflecting non-monotone effects of its antagonists actions on community *k*.

The effective share g_k is monotonically increasing in ρ and monotonically decreasing in a_k. The parameter ρ measures the *elasticity* of the loss function of either community with respect to its antagonist's control share. A higher loss elasticity implies a lower loss to either community from any given control share accruing to its antagonist. Thus, intuitively, a more elastic loss function implies, in effect, less inter-group consumption rivalry with respect to *Y*. Since $\lim_{\rho\to 1}\frac{dL_k}{dp_{-k}} = \lim_{p_{-k}\to 1}\frac{dL_k}{dp_{-k}} = a_k$, the scale parameter a_k provides the upper bound for the loss to *k* due to a marginal increase in its antagonist's control share. We shall accordingly call a_k the *marginal degradation* rate for community *k*. Notice that the marginal degradation rates may vary across communities - a_H need not be equal to a_M. We thus permit asymmetric spillovers - the spillover effect on *H* due to *M*'s exploitation of its control share may differ from that flowing in the reverse direction even under an equal division of control. It follows that equal control shares do not, in general, imply equal effective shares in our model.

3. Equilibrium

We proceed now to characterize the equilibria of our model. In light of (1)-(6), remembering that $[p_k = 1 - p_{-k}]$, and assuming an interior solution, the first order conditions yield:

$$\text{for every } k \in \{H, M\}\text{: } X_{-k}T\tilde{Y}\left(\frac{dg_k}{dp_k}\right) = n^{\theta}X^2. \tag{7}$$

From (7), denoting equilibrium values of all variables by the superscript *E*, the relative marginal degradation rate by A_k ($A_k \equiv \frac{a_k}{a_{-k}}$), noting (5), and that, by (4), $\frac{X_{-k}}{X_k} = \frac{p_{-k}}{p_k}$, we have the equilibrium control share ratio:

$$\left(\frac{p_{-k}{}^E}{p_k{}^E}\right)^\rho = A_{-k}. \tag{8}$$

Equation (8) implies:

$$p_k^E = \left(A_{-k}{}^{\frac{1}{\rho}} + 1\right)^{-1}. \tag{9}$$

Together, (5) and (9) yield:

$$g_k^E = g_{-k}^E = 1 - \frac{a_H a_M}{\rho\left(a_H{}^{\frac{1}{\rho}} + a_M{}^{\frac{1}{\rho}}\right)^\rho}. \tag{10}$$

Thus, the equilibrium control shares differ across communities when their marginal degradation rates differ. The community with the higher marginal degradation rate stands to lose more from its antagonist achieving any given control share. It consequently allocates more resources to appropriation – to the contest over sharing of Y, and therefore receives the higher control share. In marked contrast, the equilibrium effective shares are always identical across communities. Thus, control shares provide a misleading picture of cross-community benefit disparity – any control advantage is completely compensated by a higher degradation rate – indeed, such control advantage comes about only as a compensatory response to higher negative spillovers, as captured by a higher degradation rate. Recall that $a_k \in (0,2]$ and $\rho > 1$ by assumption, which implies $[\frac{1}{\rho\left(a_H{}^{\frac{-1}{\rho}} + a_M{}^{\frac{-1}{\rho}}\right)^\rho} < 1]$. It then follows immediately from (10) that equilibrium effective shares must be positive, less than 1, increasing in the spillover elasticity and decreasing in the marginal degradation rates. These properties are stated more formally in Observation 1.

Observation 1. *For all* $k \in \{H, M\}$,

(i) $lim_{\rho\to\infty}\, g_k^E = 1$ *and* $\left[1 > lim_{\rho\to 1}\, g_k^E \geq 0\right]$;

(ii) $\frac{dg_k^E}{d\rho} > 0$;

and

(iii) $\frac{dg_k^E}{da_H}, \frac{dg_k^E}{da_M} < 0.$

Now notice that, in light of (10), the first order conditions also imply that, in an interior equilibrium:

$$T\left(\frac{\alpha}{B^{1-\alpha}}\right)\left(\frac{g_H}{n^\theta}\right) = T\left(\frac{\alpha}{B^{1-\alpha}}\right)\left(\frac{g_M}{n^\theta}\right) = 1. \tag{11}$$

Together, (10) and (11) imply:

$$B^E = \left(\left(\frac{T\alpha}{n^\theta}\right)\left(1 - \frac{a_H a_M}{\rho\left(a_H^{\frac{1}{\rho}} + a_M^{\frac{1}{\rho}}\right)^\rho}\right)\right)^{\frac{1}{1-\alpha}} . \tag{12}$$

The following conclusions may be then deduced about the equilibrium size of the subscription fund, *B*, dedicated to the production of the contested good *Y*.

Observation 2.

(i) $B^E \in (0,1)$*;*

(ii) $\frac{dB^E}{d\rho} > 0$ *and* $\frac{dB^E}{da_M}, \frac{dB^E}{da_H} < 0$*;*

(iii) *there exists* $\hat{\alpha} \in (0,1)$ *such that* $\frac{dB^E}{d\alpha} > 0$ *(resp.* < 0*) at every* $\alpha < \hat{\alpha}$ *(resp.*$> \hat{\alpha}$ *).*

Proof. See the appendix.

Total subscription for production of *Y* increases as the loss elasticity rises, reducing inter-community rivalry in its consumption and thereby increasing the effective shares of both communities. It decreases if either marginal degradation rate rises, reducing both effective shares. The effect of a marginal increase in the output elasticity of the contested good is however non-monotone. At low levels of such elasticity, total subscription rises as the elasticity increases, but it falls at output elasticities close to 1.

Since the equilibrium subscription fund expands as the loss function becomes more elastic, or as either marginal degradation rate declines, it is obvious from (3) that the actual amount of the contested good produced, $\tilde{Y}$, must correspondingly expand as well. More interestingly, it turns out that the latter necessarily declines as its production technology becomes more elastic, even when more input is forthcoming in response to such an increase (recall Observation 2(iii)).

Observation 3.

(i) $\tilde{Y}^E \in (0,1)$*;*

(ii) $\frac{d\tilde{Y}^E}{d\rho} > 0$ *and* $\frac{d\tilde{Y}^E}{da_M}, \frac{d\tilde{Y}^E}{da_H} < 0$*;*

(iii) $\frac{d\tilde{Y}^E}{d\alpha} < 0.$

Proof. See the appendix.

We now turn to characterizing aggregate conflict, measured in standard fashion by the total resource expended on appropriation, in equilibrium. Using (3) and (7), we get:

$$X^E = \left(\frac{T}{n^\theta}\right)(B^E)^\alpha\left(\left(\frac{dg_H(p_H^E)}{dp_H}\right)^{-1} + \left(\frac{dg_M(p_M^E)}{dp_M}\right)^{-1}\right)^{-1}; \tag{13}$$

where $\frac{dg_H(p_H^E)}{dp_H}$ and $\left(\frac{dg_M(p_M^E)}{dp_M}\right)$ are the derivatives of the effective share functions in (5) with respect to the corresponding own control shares, evaluated at the equilibrium values of the latter. Combining (4), (9) and (13), we get the expressions for community conflict allocations:

$$\text{for every } k \in \{H.M\}: X_k^E = \left(\frac{T}{n^\theta}\right)(B^E)^\alpha\left(\left(\frac{dg_H(p_H^E)}{dp_H}\right)^{-1} + \left(\frac{dg_M(p_M^E)}{dp_M}\right)^{-1}\right)^{-1}\left(A_{-k}{}^{\frac{1}{\rho}} + 1\right)^{-1}. \tag{14}$$

By (4) and (5), recalling that $a_k \in (0,2]$ by assumption,

$$0 < \frac{dg_k}{dp_k} = a_k(p_{-k})^{\rho-1} \leq 2. \tag{15}$$

An explicit solution for X^E can be derived by combining (9), (12), (13) and (15), while those for X_H^E and X_M^E are derived by combining (9), (12), (14) and (15). Note that, since, by assumption, $T \in (0,1], \alpha \in (0,1)$ and $n^\theta \geq 1$, together, Observation 2(i), (13) and (15) imply that:

$$X^E \in (0,1). \tag{16}$$

Remark 1. Since each community must have resource endowment of at least 1 (as $n \geq 1$), Observation 2(i) and (16) imply that an interior Nash equilibrium must be feasible. Any interior equilibrium must satisfy (9) and (12)-(15), and satisfaction of these conditions guarantees the interiority of the equilibrium. It can checked that, given our assumption $a_H, a_M \in (0,2]$, every Nash equilibrium must satisfy equations (9) and (12)-(15). Hence, our model uniquely characterizes total common fund subscription and group conflict allocations in equilibrium. However, individual consumption of the private good *C*, individual common fund subscription and individual conflict contribution are all indeterminate, as are group private consumption and group common fund subscription levels. Thus, the model produces multiple Nash equilibria – essentially a consequence of the quasi-linearity of the reduced form of our preference specification.

Remark 2. We have assumed, via (2) and (3), that the subscriptions of the two communities combine in summative fashion to generate the total input allocation for the contestable good. An alternative modeling strategy might be to assume that each community separately produces the contestable good using the total subscription of its own community members as the input pool: $[B_k = \sum_{i=1}^{n} y_{iH}]$. Letting $\tilde{Y}_k$ denote the amount of the contestable good produced by community k (i.e., $\tilde{Y}_k \equiv (B_k)^{\alpha}$), the total stock of Y is then given as: $[\tilde{Y} = \tilde{Y}_H + \tilde{Y}_M]$, instead of (3). Notice that the equations derived from the first order conditions, (7) and (11), remain unchanged under this formulation (except that B is replaced, sequentially, by B_H and B_H in (11)). Since (7) remains unchanged, equations (8)-(10) remains unchanged as well, as does Observation 1. The expression for total common pool subscription in (12) now comes to stand for that within either community, $B_H^E = B_M^E$, so that we have $B^E = 2B_H^E$, with B_H^E given by (12). It follows that we now get $B_k^E, \tilde{Y}_k \in (0,1)$, so that $B^E, \tilde{Y}^E \in (0,2)$, while the comparative static properties stated in Observation 2 ((ii) and (iii)) and Observation 3 ((ii) and (iii)) remain unchanged. Equation (13) now comes to take the form:

$$X^E = 2\left(\frac{T}{n^{\theta}}\right)(B_H^E)^{\alpha}\left(\left(\frac{dg_H(p_H^E)}{dp_H}\right)^{-1} + \left(\frac{dg_M(p_M^E)}{dp_M}\right)^{-1}\right)^{-1},$$

and (14) is modified correspondingly. The claim in (15) remains unchanged, while that in (16) changes to: $X^E \in (0,2)$. The claims made in Remark 1 now come to hold under the assumption that $n \geq 3$, with the added proviso that each group's total contribution to the production of *Y*, i.e., B_k^E, is now uniquely defined by (12). Thus, in this alternative formulation, for each group, total private consumption, total subscription to the production of the contestable good and total expenditure on appropriation all come to be uniquely defined in equilibrium. However, individual private consumption, individual conflict allocation and individual contribution to the production of *Y* all remain indeterminate, as in our benchmark model (recall Remark 1). Since the expressions, respectively, for total spending on *Y* in (12) and total conflict allocation in (13) change only by a multiplicative factor of 2, the comparative static results presented in Propositions 1-3 below remain unchanged. Thus, in sum, very little of substance changes if we replace the specification of the contestable good's production technology adopted in our benchmark formulation by the alternative specification discussed above. A further generalization, which permits the elasticity of that production technology to vary across communities (i.e., $\tilde{Y}_k = (B_k)^{\alpha_k}$, where α_H need not equal α_M), drastically complicates the algebra, but does not yield much additional insight.

4. Conflict, Aggregate Income and Cross-community Spillovers

We are now ready to address our central questions. What happens to aggregate conflict as cross-community negative consumption spillover effects become stronger, due to either a decline in the loss elasticity or an increase in the marginal degradation rates? Furthermore, how do such changes affect aggregate social income (or consumption), measured in units of the numeraire good, *C*?

Consider first the case of aggregate conflict. Ceteris paribus, stronger negative spillovers from its antagonist increases the benefit, in terms of a gain in its effective share, to a community from a marginal increase in its control share (recall (15)). By itself, this effect will, clearly, incentivize greater conflict over control shares. However, stronger cross-community negative spillovers, by effectively degrading more the stock of *Y* controlled by either community) will also reduce the equilibrium effective shares (note (10)). Since this reduces the benefit to either community from contributing to the common pool for production of the contestable good, the common fund will shrink, correspondingly reducing the production of *Y* (recall (11)). This fall in the size of the prize being contested over will dampen individual incentives to engage in appropriation. Thus, two contradictory effects obtain, which makes the net conflict consequence of stronger negative spillovers a priori ambiguous, and therefore non-trivial.

Proposition 1. *Let* $\eta_b \equiv \left(\frac{b dX^E}{X^E db}\right)$.

(i) $\eta_\alpha < 0$.

(ii) *For every* $k \in \{H, M\}$*: (a)* $\frac{d\left(\eta_{a_k}\right)}{d\alpha} < 0$ *and (b) there exists* $\breve{\gamma} \in (0,1)$ *such that:* $\eta_{a_k} > 0$ *(resp.* < 0*) if* $\alpha < \breve{\gamma}$ *(resp.* $> \breve{\gamma}$*).*

(iii) *Let* $a_H = a + \Delta$*, let* $a_M = a - \Delta$*, and suppose* $\Delta > 0$*. Then* $\frac{d(\eta_\Delta)}{d\alpha} > 0$*; furthermore, there exists* $\delta \in (0,1)$ *such that:* $\eta_\Delta < 0$ *(resp.* > 0*) if* $\alpha < \delta$ *(resp.* $> \delta$*).*

(iv) $\frac{d(\eta_\rho)}{d\alpha} > 0$*; furthermore, there exists* $\gamma \in (0,1)$ *such that:* $\eta_\rho < 0$ *(resp.* > 0*) if* $\alpha < \gamma$ *(resp.* $> \gamma$*).*

Proof. See the appendix.

Proposition 1 specifies the sign of the elasticity of aggregate conflict with respect to the different parameters of the model. This elasticity with respect to the production technology parameter (α) is negative, implying that conflict falls as the production technology becomes more elastic (Proposition 1(i)). This happens because an increase in the latter reduces the production of the contested good (Observation 3(iii)), but does not affect effective shares (recall (10)). The impact of changes in the spillover parameters is however more complicated. The conflict elasticity with respect to either marginal degradation rate declines monotonically as the output elasticity rises – it is positive at low output elasticities, but negative at high ones (Proposition 1(ii)). Thus, greater negative spillover from, say, *M* to *H*, due to a rise in the marginal degradation rate for *H*, will reduce aggregate conflict when the output elasticity is close to 1, but increase it when the latter is close to 0. A mean-preserving decrease in the spread of the marginal degradation rates has the same effect (Proposition 1(iii)), as does a reduction in the elasticity of the loss function (Proposition 1(iv)). Thus, in sum, stronger negative spill-overs across communities, instead of increasing conflict, may indeed dampen it when the output elasticity is close to 1.

Proposition 1(parts (ii) and (iv)) immediately raises the intriguing possibility that greater negative consumption spillovers across communities may prove socially beneficial in terms of their role in expanding the extent of social peace, and thus aggregate consumption, under certain conditions. However, as already noted, they may reduce voluntary contributions to a common fund for production of a good whose consumption is at least partly non-rivalrous within a community. This effect may exacerbate the standard inefficiency associated with decentralized provision of any at-least-partly-public good, as well as that due to less than full marginal benefit of its subscription accruing to either community. The question then naturally arises – are there conditions under which the second effect necessarily dominates, so that stronger negative spillovers end up reducing aggregate consumption (or, equivalently, production or income) in society, measured in units of the numeraire good? We now turn to this issue.

Noting (1)-(6), define the total consumption of community $k \in \{H.M\}$, measured in terms of units of the numeraire – the private consumption good *C* – as follows:

$$W_k^E = \sum_{i=1}^{n}(1 - x_{ik} - y_{ik}) + nT\tilde{Y}^E(\frac{g_k^E}{n^\theta}). \tag{17}$$

Total equilibrium consumption in society, measured in units of *C*, is therefore:

$$W^E \equiv W_H^E + W_M^E = \left(2n - X^E - \left(\tilde{Y}^E\right)^{\frac{1}{\alpha}}\right) + \left(\frac{T}{n^{\theta-1}}\right)\tilde{Y}^E(g_H^E + g_M^E). \tag{18}$$

From (18), we get the following conclusions.

Proposition 2.

(i) *There must exist* $\dot{n} > 1$ *such that, if* $n > \dot{n}$*, then: (a)* $\frac{dW^E}{da_H}, \frac{dW^E}{da_M} < 0$*, and (b) given* $(a_H + a_M)$*,* $\frac{dW^E}{d\Delta} > 0$*, whenever* $\Delta \equiv \frac{\|a_H - a_M\|}{2} > 0$.

(ii) *There must exist* $\ddot{\hat{n}} > 1$ *such that, if* $n > \ddot{\hat{n}}$*, then* $\frac{dW^E}{d\rho} > 0$.

(iii) *There must exist* $\hat{n} > 1$ *such that, if* $n > \hat{n}$*, then* $\frac{dW^E}{d\alpha} < 0$.

Proof. See the appendix.

Proposition 2 essentially implies that, given the other parameters of the model, stronger negative cross-community spillovers unambiguously reduce aggregate social consumption when the communities are sufficiently numerous (parts (i) and (ii)). More rigorously, given the other parameters, one can always find a threshold community size with the following property: stronger negative cross-community spillovers, whether in the form of higher marginal degradation rates or a lower loss elasticity, must reduce aggregate social consumption whenever the communities are larger than this threshold size. A mean-preserving contraction in the spread of the marginal degradation rates and a rise in the output elasticity will both have the same effect when the communities are above this size threshold. The exact threshold size of the communities will of course depend on the parameters being held constant. All these conclusions essentially arise from the fact that negative effect of any fall in the subscription fund must dominate any consumption gains from consequent lower expenditure on appropriation when the communities are sufficiently large.

However, given any arbitrary community size, there do exist parametric configurations under which, at the margin, greater negative spillovers will increase aggregate consumption in society. This holds, for example, under a combination of high marginal degradation rates and low loss elasticity. This somewhat paradoxical possibility is presented formally in Proposition 3 below.

Proposition 3. *Suppose* $a_H, a_M = 2$. *Then, given any* $n \geq 1$, *there exists* $\breve{\rho} > 1$ *such that, for every* $\rho \in (1, \breve{\rho})$:

$$[\frac{dW^E}{da_H}, \frac{dW^E}{da_M}, \frac{dW^E}{d\alpha} > 0 \text{ and } \frac{dW^E}{d\rho} < 0].$$

Proof. *See the appendix.*

Proposition 3 implies the following. Given any community size, and any elasticity of the production technology for the contested good, one can find a threshold level of the loss elasticity, say $\boldsymbol{\breve{\rho}}$, with the following property. Provided the loss elasticity is lower than $\boldsymbol{\breve{\rho}}$, a marginal decline of either degradation rate from an initial situation of $\boldsymbol{a_H, a_M = 2}$ must reduce aggregate social consumption (or equivalently, production or income) measured in units of the numeraire good. Given $\boldsymbol{a_H, a_M = 2}$, and any loss elasticity below $\boldsymbol{\breve{\rho}}$, the same holds for a marginal decline in the output elasticity. Given $\boldsymbol{a_H, a_M = 2}$, any decline in the loss elasticity from some initial value below $\boldsymbol{\breve{\rho}}$ must increase aggregate social consumption. Thus, in sum, greater negative spillovers across communities may be aggregate consumption augmenting, and, in this specific sense, socially beneficial, under certain conditions, irrespective of the size of the communities.

5. Concluding Remarks

This paper has developed a parsimonious theoretical framework to examine how cross-community cost or benefit spillovers, arising from the consumption or exploitation of group-specific public goods, affects both inter-group conflicts over the appropriation of such goods and decentralized private provision for their production. We have offered a model which integrates production versus appropriation choices, vis-à-vis group-specific public goods, with their decentralized voluntary supply, against a backdrop of such cross-community consumption spillovers. Our flexible and general formulation of consumption spillovers incorporates earlier specifications as alternative special cases. We have shown that, somewhat counter-intuitively, stronger negative (or weaker positive) consumption spillovers across communities may serve to reduce inter-group conflict and increase aggregate income (and consumption) in society under certain conditions, which we have identified. Thus, stronger negative consumption spillovers, under certain conditions, may have socially beneficial consequences. Of course, their impact will be both conflict-augmenting and income-compressing, as may be intuitively expected, under other conditions. We have identified these latter conditions as well.

In many different developing country contexts, climate change and environmental degradation (as well as state policy, population growth or market pressures) increase the costs imposed on one ethnic group due to another group's exploitation of some natural resource. This increases inter-group competition over natural resources and often triggers persistent and widespread social conflict. Such conflict in turn affects decentralized community-level mechanisms for the maintenance and augmentation of the contested natural resource, which feeds back into the original conflict. Our general theoretical analysis offers a broad conceptual structure within which to organize case studies of such feedback loops, linking ethnic conflict and natural resource degradation, in specific developing country contexts.

Dasgupta and Guha Neogi [2018] have examined how within-group fragmentation affects inter-group contests over group-specific public goods, while Esteban and Ray [2001], generalizing Olson [1965], have investigated how group size affects such conflicts under quite broad preference specifications. Analogous incorporation of within-group fragmentation and group size effects in suitably augmented versions of our model may generate useful insights. How income/wealth inequality and polarization affect social conflict [Esteban and Ray, 2011b] within our framework remains an open question. One may use alternatives to our perfect-substitutes summative specification for each community's aggregate group conflict effort, such as a constant elasticity of substitution aggregation [Kolmar and Rommeswinkel, 2013; Cheikbossian and Fayat, 2018], the best-shot specification [Chowdhury *et al.*, 2013] or the weakest-link formulation [Lee, 2012]. Similar alternatives have also been applied to specify the public good's production technology in voluntary subscription models [Cornes, 1993]. The consequences of cross-community spillovers with endogenous provisioning, in public good contests under such alternative specifications, whether of the conflict technology or the public good's production technology, constitute another promising avenue of future enquiry. We look forward to these and other extensions in future work.

Appendix

Proof of Observation 2. From Observation 1 ((i) and (ii)), for all $k \in \{H, M\}$, $g_k^E \in (0,1)$. By assumption, $T \in (0,1]$ and $\alpha \in (0.1)$, and $n \geq 1$. Then part (i) of Observation 1 follows immediately from (12) in light of (10). Now, from (12), we get:

$$B^E = \left(\left(\frac{T\alpha}{n^\theta}\right)\left(1 - \frac{1}{\rho\left(a_H^{\frac{-1}{\rho}} + a_M^{\frac{-1}{\rho}}\right)^\rho}\right)\right)^{\frac{1}{1-\alpha}} = \left(\left(\frac{T\alpha}{n^\theta}\right)\left(1 - \frac{a_M}{\rho\left(1 + \left(\frac{a_M}{a_H}\right)^{\frac{1}{\rho}}\right)^\rho}\right)\right)^{\frac{1}{1-\alpha}}. \qquad \text{(A.1)}$$

Without loss of generality, suppose $\left(\frac{a_M}{a_H}\right) \leq 1$. Then part (ii) of Observation 2 follows immediately from (A.1). Now, from (A.1),

$$\ln B^E = \left(\frac{1}{1-\alpha}\right)\left[\ln \alpha + \ln\left(\left(\frac{T}{n^\theta}\right)\left(1 - \frac{a_H a_M}{\rho\left(a_H^{\frac{1}{\rho}} + a_M^{\frac{1}{\rho}}\right)^\rho}\right)\right)\right];$$

implying:

$$\left(\frac{(1-\alpha)^2\alpha}{B^E}\right)\frac{dB^E}{d\alpha} = \left[(1-\alpha) + \ln\left(\left(\frac{T\alpha}{n^\theta}\right)\left(1 - \frac{a_H a_M}{\rho\left(a_H^{\frac{1}{\rho}} + a_M^{\frac{1}{\rho}}\right)^\rho}\right)\right)^\alpha\right]. \qquad \text{(A.2)}$$

Let $Z \equiv \left[(1-\alpha) + \ln\left(\left(\frac{T\alpha}{n^\theta}\right)\left(1 - \frac{a_H a_M}{\rho\left(a_H^{\frac{1}{\rho}} + a_M^{\frac{1}{\rho}}\right)^\rho}\right)\right)^\alpha\right]$. Then $\lim_{\alpha\to 0} Z = 1$, and $\lim_{\alpha\to 1} Z = \ln\left(\left(\frac{T}{n^\theta}\right)\left(1 - \frac{a_H a_M}{\rho\left(a_H^{\frac{1}{\rho}} + a_M^{\frac{1}{\rho}}\right)^\rho}\right)\right) < 0$. Now let $\tilde{Z} \equiv \alpha \ln\left(\left(\frac{T\alpha}{n^\theta}\right)\left(1 - \frac{a_H a_M}{\rho\left(a_H^{\frac{1}{\rho}} + a_M^{\frac{1}{\rho}}\right)^\rho}\right)\right)$. Then $\frac{d\tilde{Z}}{d\alpha} =$

$\ln\left(\left(\frac{T\alpha}{n^{\theta}}\right)\left(1-\frac{a_H a_M}{\rho\left(a_H^{\frac{1}{\rho}}+a_M^{\frac{1}{\rho}}\right)^{\rho}}\right)\right)+1$; implying $\frac{dZ}{d\alpha}=\ln\left(\left(\frac{T\alpha}{n^{\theta}}\right)\left(1-\frac{a_H a_M}{\rho\left(a_H^{\frac{1}{\rho}}+a_M^{\frac{1}{\rho}}\right)^{\rho}}\right)\right)<0$. Part (iii) of Observation 2 follows. □

Proof of Observation 3. Parts (i) and (ii) of Observation 3 follow immediately from parts (i) and (ii), respectively, of Observation 2, in light of (3). Now, using (3) and (12),

$$\tilde{Y}^E=\left(\left(\frac{T\alpha}{n^{\theta}}\right)\left(1-\frac{a_H a_M}{\rho\left(a_H^{\frac{1}{\rho}}+a_M^{\frac{1}{\rho}}\right)^{\rho}}\right)\right)^{\frac{\alpha}{1-\alpha}}. \tag{A.3}$$

Hence

$$\ln\tilde{Y}^E=\left(\frac{\alpha}{1-\alpha}\right)\left[\ln\alpha+\ln\left(\left(\frac{T}{n^{\theta}}\right)\left(1-\frac{a_H a_M}{\rho\left(a_H^{\frac{1}{\rho}}+a_M^{\frac{1}{\rho}}\right)^{\rho}}\right)\right)\right];$$

implying:

$$\left(\frac{(1-\alpha)^2}{\tilde{Y}}\right)\frac{d\tilde{Y}^E}{d\alpha}=\left[(1-\alpha)+\ln\left(\left(\frac{T\alpha}{n^{\theta}}\right)\left(1-\frac{a_H a_M}{\rho\left(a_H^{\frac{1}{\rho}}+a_M^{\frac{1}{\rho}}\right)^{\rho}}\right)\right)\right]. \tag{A.4}$$

Let $Z\equiv\left[(1-\alpha)+\ln\left(\left(\frac{T\alpha}{n^{\theta}}\right)\left(1-\frac{a_H a_M}{\rho\left(a_H^{\frac{1}{\rho}}+a_M^{\frac{1}{\rho}}\right)^{\rho}}\right)\right)\right]$. Then $\lim_{\alpha\to0}Z=-\infty$, and $\lim_{\alpha\to1}Z=\ln\left(\left(\frac{T}{n^{\theta}}\right)\left(1-\frac{a_H a_M}{\rho\left(a_H^{\frac{1}{\rho}}+a_M^{\frac{1}{\rho}}\right)^{\rho}}\right)\right)<0$. Furthermore, $\frac{dZ}{d\alpha}=-1+\frac{1}{\alpha}>0$. Part (iii) of Observation 3 follows. □

Proof of Proposition 1. By (5) and (9), $\left(\left(\frac{dg_H(p_H^E)}{dp_H}\right)^{-1}+\left(\frac{dg_M(p_M^E)}{dp_M}\right)^{-1}\right)^{-1}$ is independent of α. Part (i) of Proposition 1 then follows directly from (13) in light of (3) and Observation 3(iii).

We shall prove parts (ii), (iii) and (iv) of Proposition 1 via the following lemma.

Lemma 1.

(i) *For every* $k \in \{H, M\}$, $lim_{\alpha\to 0}\left(\frac{a_H dX^E}{X^E da_k}\right)=p_{-k}^E, lim_{\alpha\to 1}\left(\frac{a_H dX^E}{X^E da_k}\right)=-\infty$, *and* $\frac{d\left(\frac{a_H dX^E}{X^E da_k}\right)}{d\alpha}<0.$

(ii) *Let* $a_H=a+\Delta$, *let* $a_M=a-\Delta$, *and suppose* $\Delta>0$. *Then,* $lim_{\alpha\to 0}\left(\frac{\Delta dX^E}{X^E d\Delta}\right)=\Delta\left[\frac{p_M^E}{a_H}-\frac{p_H^E}{a_M}\right]<0, lim_{\alpha\to 1}\left(\frac{\Delta dX^E}{X^E d\Delta}\right)=\infty$, *and* $\frac{d\left(\frac{\Delta dX^E}{X^E d\Delta}\right)}{d\alpha}>0.$

(iii) $lim_{\alpha\to 0}\left(\frac{\rho dX^E}{X^E d\rho}\right)=K\,, lim_{\alpha\to 1}\left(\frac{\rho dX^E}{X^E d\rho}\right)=\infty,$ *and* $\frac{d\left(\frac{\rho dX^E}{X^E d\rho}\right)}{d\alpha}>0$*;* *where* $K=\rho\left(\frac{ln\,p_M^E+\left(\frac{p_H^E}{p_M^E}\right)ln\,p_H^E}{1+\left(\frac{p_H^E}{p_M^E}\right)}\right)<0.$

Proof of Lemma 1.

(i) Using (13),

$$\left(\frac{n^\theta}{T}\right)X^E=\tilde{Y}^E\left(\left(\frac{dg_H(p_H^E)}{dp_H}\right)^{-1}+\left(\frac{dg_M(p_M^E)}{dp_M}\right)^{-1}\right)^{-1}. \tag{A.5}$$

Hence,

$$\left(\frac{n^\theta}{T}\right)\left(\left(\frac{dg_H(p_H^E)}{dp_H}\right)^{-1}+\left(\frac{dg_M(p_M^E)}{dp_M}\right)^{-1}\right)\left(\frac{dX^E}{da_k}\right)=$$

$\frac{d\tilde{Y}^E}{da_k}-$

$$\tilde{Y}^E\left(\left(\frac{dg_H(p_H^E)}{dp_H}\right)^{-1}+\left(\frac{dg_M(p_M^E)}{dp_M}\right)^{-1}\right)^{-1}\left[\left(\frac{\partial\left(\left(\frac{dg_H(p_H^E)}{dp_H}\right)^{-1}+\left(\frac{dg_M(p_M^E)}{dp_M}\right)^{-1}\right)}{\partial p_H}\right)\left(\frac{dp_H}{da_k}\right)+\right.$$

$$\left.\left(\frac{\partial\left(\left(\frac{dg_H(p_H^E)}{dp_H}\right)^{-1}+\left(\frac{dg_M(p_M^E)}{dp_M}\right)^{-1}\right)}{\partial a_k}\right)\right]. \tag{A.6}$$

Define $S\equiv\left(\frac{\partial\left(\left(\frac{dg_H}{dp_H}\right)^{-1}+\left(\frac{dg_M}{dp_M}\right)^{-1}\right)}{\partial p_H}\right)$. Then, noting that $p_H=1-p_M$, we get:

$$S=-\left(\left(\frac{dg_H}{dp_H}\right)^{-2}\frac{d^2g_H}{dp_H{}^2}-\left(\frac{dg_M}{dp_M}\right)^{-2}\frac{d^2g_M}{dp_M{}^2}\right). \tag{A.7}$$

Recall that, from (5): $\frac{dg_k}{dp_k}=a_k p_{-k}{}^{\rho-1}$, $\frac{d^2g_k}{dp_k{}^2}=-\left(\frac{\rho-1}{p_{-k}}\right)\frac{dg_k}{dp_k}$. Then, from (A.7), we have:

$$S=\left(\frac{dg_M}{dp_M}\right)^{-1}(\rho-1)\left(\left(\frac{dg_M}{dp_M}\right)\left(\frac{dg_H}{dp_H}\right)^{-1}\left(\frac{1}{p_M}\right)-\left(\frac{1}{p_H}\right)\right).$$

Since $\left(\frac{dg_M}{dp_M}\right)\left(\frac{dg_H}{dp_H}\right)^{-1}=\left(\frac{a_M}{a_H}\right)\left(\frac{p_H}{p_M}\right)^{\rho-1}$, and, from (8), $\left[1=\left(\frac{a_H}{a_M}\right)\left(\frac{p_M}{p_H}\right)^{\rho}\right]$ in equilibrium, we then have:

$$S^E=\left(\frac{dg_M}{dp_M}\right)^{-1}(\rho-1)\left(\left(\frac{p_H}{p_M}\right)^{-1}\left(\frac{1}{p_M}\right)-\left(\frac{1}{p_H}\right)\right)=0. \tag{A.8}$$

In light of (A.8), (A.6) reduces to:

$$\left(\frac{n^\theta}{T}\right)\left(\left(\frac{dg_H(p_H^E)}{dp_H}\right)^{-1}+\left(\frac{dg_M(p_M^E)}{dp_M}\right)^{-1}\right)\left(\frac{dX^E}{da_k}\right)=$$

$$\frac{d\tilde{Y}^E}{da_k}-\tilde{Y}^E\left(\left(\frac{dg_H(p_H^E)}{dp_H}\right)^{-1}+\left(\frac{dg_M(p_M^E)}{dp_M}\right)^{-1}\right)^{-1}\left(\frac{\partial\left(\left(\frac{dg_H(p_H^E)}{dp_H}\right)^{-1}+\left(\frac{dg_M(p_M^E)}{dp_M}\right)^{-1}\right)}{\partial a_k}\right). \tag{A.9}$$

Now recall that, by (A.5):

$$\left(\frac{dg_H}{dp_H}\right)^{-1}+\left(\frac{dg_M}{dp_M}\right)^{-1}=\left(\frac{1}{a_H p_M{}^{\rho-1}}+\frac{1}{a_M p_H{}^{\rho-1}}\right), \tag{A.10}$$

implying:

$$\left(\frac{\partial\left(\left(\frac{dg_H}{dp_H}\right)^{-1}+\left(\frac{dg_M}{dp_M}\right)^{-1}\right)}{\partial a_k}\right) = -{a_k}^{-1}\left(\frac{dg_k}{dp_k}\right)^{-1}. \tag{A.11}$$

Combining (A.10) and (A.11), and recalling that $A_k = \frac{a_k}{a_{-k}}$, we get:

$$\left(\left(\frac{dg_H}{dp_H}\right)^{-1} + \left(\frac{dg_M}{dp_M}\right)^{-1}\right)^{-1}\left[\left(\frac{\partial\left(\left(\frac{dg_H}{dp_H}\right)^{-1}+\left(\frac{dg_M}{dp_M}\right)^{-1}\right)}{\partial a_k}\right)\right] = -\frac{{a_k}^{-1}}{\left(1+A_k\left(\frac{p_{-k}}{p_k}\right)^{\rho-1}\right)}. \tag{A.12}$$

Hence, combing (A.9) and (A.12), we have:

$$\left(\frac{n^\theta}{T}\right)\left(\left(\frac{dg_H(p_H^E)}{dp_H}\right)^{-1} + \left(\frac{dg_M(p_M^E)}{dp_M}\right)^{-1}\right)\left(\frac{dX^E}{da_k}\right) = \frac{d\tilde{Y}^E}{da_k} + \frac{{a_k}^{-1}\tilde{Y}^E}{\left(1+A_k\left(\frac{p_{-k}}{p_k}\right)^{\rho-1}\right)}. \tag{A.13}$$

By (13),

$$\left(\frac{n^\theta}{T}\right)\left(\left(\frac{dg_H(p_H^E)}{dp_H}\right)^{-1} + \left(\frac{dg_M(p_M^E)}{dp_M}\right)^{-1}\right) = \tilde{Y}^E(X^E)^{-1}. \tag{A.14}$$

Since, by (3), $\left[\left(\frac{d\tilde{Y}^E}{da_k}\right) = \alpha\tilde{Y}^E(B^E)^{-1}\left(\frac{dB^E}{da_k}\right)\right]$, (A.13) and (A.14) together yield:

$$a_k(X^E)^{-1}\left(\frac{dX^E}{da_k}\right) = \alpha a_k(B^E)^{-1}\left(\frac{dB^E}{da_k}\right) + \frac{1}{\left(1+A_k\left(\frac{p_{-k}}{p_k}\right)^{\rho-1}\right)}. \tag{A.15}$$

Since, from (8), $\left[1 = \left(\frac{a_H}{a_M}\right)\left(\frac{p_M}{p_H}\right)^{\rho}\right]$ in equilibrium, (A.15) reduces to:

$$a_k(X^E)^{-1}\left(\frac{dX^E}{da_k}\right) = \alpha a_k(B^E)^{-1}\left(\frac{dB^E}{da_k}\right) + p_{-k}^E. \tag{A.16}$$

Now, from (11),

$$B^E = \left(\frac{\alpha T g_k^E}{n^\theta}\right)^{\frac{1}{1-\alpha}}, \tag{A.17}$$

so that:

$$\alpha\left(\frac{a_k}{B^E}\right)\frac{dB^E}{da_k} = a_k\left(\frac{\alpha}{1-\alpha}\right)\left(g_k^E\right)^{-1}\left(\frac{d{g^E}_k}{da_k}\right). \tag{A.18}$$

From (9) and (10),

$$\frac{dg_k^E}{da_k} = -\rho^{-1}\left({p^E}_{-k}\right)^{(\rho+1)}. \tag{A.19}$$

Combining (A.16), (A.18) and (A.19), we get:

$$a_k(X^E)^{-1}\left(\frac{dX^E}{da_k}\right) = -a_k\left(\frac{\alpha}{1-\alpha}\right)\left(g_k^E\right)^{-1}\rho^{-1}\left(p_{-k}^E\right)^{(\rho+1)} + p_{-k}^E. \tag{A.20}$$

Noting that, by (9) and (10), both p_{-k}^E and g_k^E are independent of α, part (i) of Lemma 1 follows.

(ii) From (A.20), recalling (10),

$$\Delta(X^E)^{-1}\left[\frac{dX^E}{d\Delta}\right] = \Delta(X^E)^{-1}\left[\frac{dX^E}{da_H} - \frac{dX^E}{da_M}\right]$$

$$= \left(\frac{\alpha}{1-\alpha}\right)\Delta(g_M^E)^{-1}\rho^{-1}\left[(p_H^E)^{(\rho+1)} - (p_M^E)^{(\rho+1)}\right] + \Delta[a_H{}^{-1}p_M^E - a_M{}^{-1}p_H^E].$$

Since $\Delta > 0$, $a_H > a_M$. Hence, by (8), $p_H^E > p_M^E$, so that $[a_H{}^{-1}p_M^E - a_M{}^{-1}p_H^E] < 0$ and $\left[(p_H^E)^{(\rho+1)} - (p_M^E)^{(\rho+1)}\right] > 0$. Part (ii) of Lemma 1 follows.

(iii) From (A.5),

$$\left(\frac{n^\theta}{T}\right)\left(\left(\frac{dg_H(p_H^E)}{dp_H}\right)^{-1} + \left(\frac{dg_M(p_M^E)}{dp_M}\right)^{-1}\right)\left(\frac{dX^E}{d\rho}\right) =$$

$$\frac{d\tilde{Y}^E}{d\rho} -$$

$$\tilde{Y}^E\left(\left(\frac{dg_H(p_H^E)}{dp_H}\right)^{-1} + \left(\frac{dg_M(p_M^E)}{dp_M}\right)^{-1}\right)^{-1}\left[\left(\frac{\partial\left(\left(\frac{dg_H(p_H^E)}{dp_H}\right)^{-1} + \left(\frac{dg_M(p_M^E)}{dp_M}\right)^{-1}\right)}{\partial p_H}\right)\left(\frac{dp_H}{d\rho}\right) +\right.$$

$$\left.\left(\frac{\partial\left(\left(\frac{dg_H(p_H^E)}{dp_H}\right)^{-1} + \left(\frac{dg_M(p_M^E)}{dp_M}\right)^{-1}\right)}{\partial \rho}\right)\right]; \tag{A.21}$$

which, in light of (A.7), (A.8), (A.10) and (A.14) reduces to:

$$(X^E)^{-1}\left(\frac{dX^E}{d\rho}\right) = \tilde{Y}^{E^{-1}}\left(\frac{d\tilde{Y}^E}{d\rho}\right) - \left(\frac{1}{a_H(p_M^E)^{\rho-1}} + \frac{1}{a_M(p_H^E)^{\rho-1}}\right)^{-1}\left(\frac{\partial\left(\left(\frac{dg_H}{dp_H}\right)^{-1} + \left(\frac{dg_M}{dp_M}\right)^{-1}\right)}{\partial\rho}\right). \tag{A.22}$$

From (A.10),

$$\left(\frac{\partial\left(\left(\frac{dg_H}{dp_H}\right)^{-1}+\left(\frac{dg_M}{dp_M}\right)^{-1}\right)}{\partial\rho}\right) = -\left(\left(\frac{dg_H}{dp_H}\right)^{-1}(\ln p_M) + \left(\frac{dg_M}{dp_M}\right)^{-1}(\ln p_H)\right). \tag{A.23}$$

Using (5), (A.22) and (A.23), we then get:

$$(X^E)^{-1}\left(\frac{dX^E}{d\rho}\right) = \tilde{Y}^{E^{-1}}\left(\frac{d\tilde{Y}^E}{d\rho}\right) + \left(\frac{1}{a_H(p_M^E)^{\rho-1}} + \frac{1}{a_M(p_H^E)^{\rho-1}}\right)^{-1}\left(\frac{\ln p_M^E}{a_H(p_M^E)^{\rho-1}} + \frac{\ln p_H^E}{a_M(p_H^E)^{\rho-1}}\right). \tag{A.24}$$

Since, by (3), $\left[\left(\frac{d\tilde{Y}^E}{d\rho}\right) = \alpha\tilde{Y}^E(B^E)^{-1}\left(\frac{dB^E}{d\rho}\right)\right]$, in light of (8), (A.24) reduces to:

$$\rho(X^E)^{-1}\left(\frac{dX^E}{d\rho}\right) = \rho\alpha(B^E)^{-1}\left(\frac{dB^E}{d\rho}\right) + \rho\left(\frac{\ln p_M^E + \left(\frac{p_H^E}{p_M^E}\right)\ln p_H^E}{1+\left(\frac{p_H^E}{p_M^E}\right)}\right). \tag{A.25}$$

Without loss of generality, suppose $A_M \geq 1$. From (A.17), we have:

$$\rho\alpha(B^E)^{-1}\frac{dB^E}{d\rho} = \rho\left(\frac{\alpha}{1-\alpha}\right)(g_H^E)^{-1}\left(\frac{dg_H^E}{d\rho}\right). \tag{A.26}$$

Then, (A.25) reduces to:

$$\rho(X^E)^{-1}\left(\frac{dX^E}{d\rho}\right) = \rho\left(\frac{\alpha}{1-\alpha}\right)(g_H^E)^{-1}\left(\frac{dg_H^E}{d\rho}\right) + \rho\left(\frac{\ln p_M^E + \left(\frac{p_H^E}{p_M^E}\right)\ln p_H^E}{1+\left(\frac{p_H^E}{p_M^E}\right)}\right). \tag{A.27}$$

Now, from (5),

$$\frac{dg^E{}_H}{d\rho} = -a_H\left(\frac{p_M^{E\,\rho}}{\rho}\right)\left[\ln p_M^E - \left(\frac{\rho}{p_M^E}\right)\frac{dp_H^E}{d\rho} - \frac{1}{\rho}\right]. \tag{A.28}$$

From (9), $\frac{dp_H^E}{d\rho} = \left(A_M{}^{\frac{1}{\rho}} + 1\right)^{-2}\left(\frac{A_M{}^{\frac{1}{\rho}}}{\rho^2}\right)\ln(A_M) \geq 0$, since $A_M \geq 1$ by assumption. Hence, from (A.28), $\frac{dg_H^E}{d\rho}$ is positive, finite and independent of α. The terms g_H^E and p_H^E are both finite and independent of α as well. Lemma 1(iii) then follows from (A.27). □

Parts (ii), (iii) and (iv) of Proposition 1 follow immediately from parts (i), (ii) and (iii) of Lemma 1, respectively. □

Proof of Proposition 2.

(i) From (18),

$$\frac{dW^E}{da_k} = \left(-\frac{dX^E}{da_k} - \frac{dB^E}{da_k}\right) + \alpha\left(\frac{T}{n^{\theta-1}}\right)(B^E)^{\alpha-1}\left(\frac{dB^E}{da_k}\right)(g_H^E + g_M^E) + \left(\frac{T}{n^{\theta-1}}\right)\tilde{Y}^E\left(\frac{d(g_H^E+g_M^E)}{da_k}\right). \quad \text{(A.29)}$$

Together, (A.29) and (11) yield:

$$:\frac{dW^E}{da_k} = \left((2n-1)\left(\frac{dB^E}{da_k}\right) - \frac{dX^E}{da_k}\right) + \left(\frac{T}{n^{\theta-1}}\right)\tilde{Y}^E\left(\frac{d(g_H^E+g_M^E)}{da_k}\right). \quad \text{(A.30)}$$

In light of (A.16), (A.30) reduces to:

$$:\frac{dW^E}{da_k} = [(2n-1) - \alpha(B^E)^{-1}X^E]\left(\frac{dB^E}{da_k}\right) - \frac{X^E p_{-k}^E}{a_k} + \left(\frac{T}{n^{\theta-1}}\right)\tilde{Y}^E\left(\frac{d(g_H^E+g_M^E)}{da_k}\right). \quad \text{(A.31)}$$

From (11) and (13),

$$\alpha\left(\frac{X^E}{B^E}\right) = g_H{}^{-1}\left(\left(\frac{dg_H(p_H^E)}{dp_H}\right)^{-1} + \left(\frac{dg_M(p_M^E)}{dp_M}\right)^{-1}\right)^{-1}. \quad \text{(A.32)}$$

From (A.8) and (A.10),

$$g_H{}^{-1}\left(\left(\frac{dg_H}{dp_H}\right)^{-1} + \left(\frac{dg_M}{dp_M}\right)^{-1}\right)^{-1} = \frac{\rho a_H p_M{}^{\rho}}{\rho - a_H p_M{}^{\rho}}, \quad \text{(A.33)}$$

In light of (A.32) and (A.33), (A.31) yields:

$$\frac{dW^E}{da_k} = \left[\frac{(2n-1)\left(\rho - a_H(p_M^E)^{\rho}\right) - \rho a_H(p_M^E)^{\rho}}{\rho - a_H(p_M^E)^{\rho}}\right]\left(\frac{dB^E}{da_k}\right) - \frac{X^E p_{-k}^E}{a_k} + \left(\frac{T}{n^{\theta-1}}\right)\tilde{Y}^E\left(\frac{d(g_H^E+g_M^E)}{da_k}\right). \quad \text{(A.34)}$$

Now notice that, by (11), (A.18) and (A.19),

$$\frac{dB^E}{da_k} = -\left(\frac{\alpha}{1-\alpha}\right)\left(\tilde{Y}^E\right)\left(\frac{T}{n^{\theta}}\right)\rho^{-1}\left(p_{-k}^E\right)^{(\rho+1)}; \quad \text{(A.35)}$$

Now, from (10), $\left(\frac{dg^E{}_H}{da_k}\right) = \left(\frac{dg^E{}_M}{da_k}\right)$. Then (A.19), (A.34) and (A.35) yield:

$$-\left(\tilde{Y}^E\right)^{-1}\frac{dW^E}{da_k} = \left(\left[\frac{(2n-1)\left(\rho - a_H(p_M^E)^{\rho}\right) - \rho a_H(p_M^E)^{\rho}}{\rho - a_H(p_M^E)^{\rho}}\right]\left(\frac{\alpha}{1-\alpha}\right) + 2n\right)\left(\frac{T}{n^{\theta}}\right)\rho^{-1}\left(p_{-k}^E\right)^{(\rho+1)} + \frac{X^E p_{-k}^E}{\tilde{Y}^E a_k}. \quad \text{(A.36)}$$

Together, (A.5) and (A.36) yield:

$$-\left(\tilde{Y}^E\right)^{-1}\left(\frac{n^{\theta}}{T}\right)\frac{dW^E}{da_k} = \left(\left[\frac{(2n-1)(\rho - a_H(p_M^E)^{\rho}) - \rho a_H(p_M^E)^{\rho}}{\rho - a_H(p_M^E)^{\rho}}\right]\left(\frac{\alpha}{1-\alpha}\right) + 2n\right)\rho^{-1}\left(p_{-k}^E\right)^{(\rho+1)}$$

$$+\left(\frac{p_{-k}^E}{a_k}\right)\left(\left(\frac{dg_H(p_H^E)}{dp_H}\right)^{-1}+\left(\frac{dg_M(p_M^E)}{dp_M}\right)^{-1}\right)^{-1}. \quad \text{(A.37)}$$

Consider the term $Z \equiv (2n-1)(\rho - a_H(p_M^E)^\rho) - \rho a_H(p_M^E)^\rho$. By Observation 1, $(\rho - a_H(p_M^E)^\rho) > 0$. Since $\rho > 1$, there must exist $\dot{n} = \dot{n}(\rho) > 1$, with $\dot{n}'(\rho) < 0$, such that, if $n > \dot{n}$, then $Z > 0$. Part (a) of Proposition 2(i) follows. Now, using (A.37), and noting (10), we have:

$$-\left(\tilde{Y}^E\right)^{-1}\left(\frac{n^\theta}{T}\right)\left(\frac{dW^E}{da_H}-\frac{dW^E}{da_M}\right)=$$

$$\left(\left[\frac{(2n-1)\left(\rho-a_H(p_M^E)^\rho\right)-\rho a_H(p_M^E)^\rho}{\rho-a_H(p_M^E)^\rho}\right]\left(\frac{\alpha}{1-\alpha}\right)+2n\right)\rho^{-1}\left[(p_M^E)^{(\rho+1)}-(p_H^E)^{(\rho+1)}\right]+$$

$$\left[\left(\frac{p_M^E}{a_H}\right)-\left(\frac{p_H^E}{a_M}\right)\right]\left(\left(\frac{dg_H(p_H^E)}{dp_H}\right)^{-1}+\left(\frac{dg_M(p_M^E)}{dp_M}\right)^{-1}\right)^{-1}. \quad \text{(A.38)}$$

Without loss of generality, suppose $a_H > a_M$. Then, from (8), $p_H^E > p_M^E$. Part (b) of Proposition 2(i) then follows from (A.38).

(ii) From (18),

$$\frac{dW^E}{d\rho}=\left(-\frac{dX^E}{d\rho}-\frac{dB^E}{d\rho}\right)+\alpha\left(\frac{T}{n^{\theta-1}}\right)(B^E)^{\alpha-1}\left(\frac{dB^E}{d\rho}\right)(g_H^E+g_M^E)+\left(\frac{T}{n^{\theta-1}}\right)\tilde{Y}^E\left(\frac{d(g_H^E+g_M^E)}{d\rho}\right). \quad \text{(A.39)}$$

Together, (A.39) and (11) yield:

$$:\frac{dW^E}{d\rho}=\left((2n-1)\left(\frac{dB^E}{d\rho}\right)-\frac{dX^E}{d\rho}\right)+n\left(\frac{B^E}{\alpha}\right)(g_H^E)^{-1}\left(\frac{d(g_H^E+g_M^E)}{d\rho}\right). \quad \text{(A.40)}$$

In light of (A.25), (A.40) reduces to:

$$\frac{dW^E}{d\rho}=\left[(2n-1)-\alpha(B^E)^{-1}X^E\right]\left(\frac{dB^E}{d\rho}\right)-X^E\left(\frac{\ln p_M^E+\left(\frac{p_H^E}{p_M^E}\right)\ln p_H^E}{1+\left(\frac{p_H^E}{p_M^E}\right)}\right)+n\left(\frac{B^E}{\alpha}\right)(g_H^E)^{-1}\left(\frac{d(g_H^E+g_M^E)}{d\rho}\right). \quad \text{(A.41)}$$

Noting (A.32) and (A.33), (A.41) further reduces to:

$$\frac{dW^E}{d\rho}=\left[\frac{(2n-1)\left(\rho-a_H(p_M^E)^\rho\right)-\rho a_H(p_M^E)^\rho}{\rho-a_H(p_M^E)^\rho}\right]\left(\frac{dB^E}{d\rho}\right)$$

$$-X^E\left(\frac{\ln p_M^E+\left(\frac{p_H^E}{p_M^E}\right)\ln p_H^E}{1+\left(\frac{p_H^E}{p_M^E}\right)}\right)+n\left(\frac{B^E}{\alpha}\right)(g_H^E)^{-1}\left(\frac{d(g_H^E+g_M^E)}{d\rho}\right). \quad \text{(A.42)}$$

Together, (A.42), (A.26) and (10) yield:

$$\left(\frac{\alpha g_H^E}{B^E}\right)\left(\frac{dW^E}{d\rho}\right) = \left(\left[\frac{(2n-1)\left(\rho - a_H(p_M^E)^\rho\right) - \rho a_H(p_M^E)^\rho}{\rho - a_H(p_M^E)^\rho}\right]\left(\frac{\alpha}{1-\alpha}\right) + 2n\right)\left(\frac{dg_H^E}{d\rho}\right) - \frac{\alpha g_H^E X^E}{B^E}\left(\frac{\ln p_M^E + \left(\frac{p_H^E}{p_M^E}\right)\ln p_H^E}{1+\left(\frac{p_H^E}{p_M^E}\right)}\right). \quad \text{(A.43)}$$

By (A.28), $\frac{dg_H^E}{d\rho}$ is positive. By Observation 1, $(\rho - a_H(p_M^E)^\rho) > 0$. Note that, by (12) and (13), $\frac{X^E}{B^E}$ is positive and finite, as is $\lim_{\alpha\to 1}\frac{X^E}{B^E}$. Since $\rho > 1$, there must exist $\hat{\hat{n}} = \hat{\hat{n}}(a_H, a_M) > 1$ such that, if $n > \hat{\hat{n}}$, then $Z > 0$. Proposition 2(ii) follows.

(iii) Combining (13) and (18) and using (10), we have:

$$\frac{dW^E}{d\alpha} = g_H^E\left(\frac{T}{n^\theta}\right)\left[2n - \left(g_H^E\left(\frac{dg_H(p_H^E)}{dp_H}\right)^{-1} + \left(\frac{dg_M(p_M^E)}{dp_M}\right)^{-1}\right)^{-1}\right]\frac{d\tilde{Y}^E}{d\alpha} - \frac{dB^E}{d\alpha}. \quad \text{(A.44)}$$

Together, (A.33) and (A.44) yield:

$$\frac{dW^E}{d\alpha} = g_H^E\left(\frac{T}{n^\theta}\right)\left[2n - \frac{\rho a_H(p_M^E)^\rho}{\rho - a_H(p_M^E)^\rho}\right]\frac{d\tilde{Y}^E}{d\alpha} - \frac{dB^E}{d\alpha}. \quad \text{(A.45)}$$

Since: $\left[\frac{dB^E}{d\alpha} = \left(\frac{B^E}{\alpha\tilde{Y}^E}\right)\frac{d\tilde{Y}^E}{d\alpha} - \left(\frac{B^E}{\alpha}\right)\ln B^E\right]$, (A.45) yields:

$$\frac{dW^E}{d\alpha} = \left[g_H^E T\left[2n^{1-\theta} - \frac{\rho a_H(p_M^E)^\rho}{n^\theta\left(\rho - a_H(p_M^E)^\rho\right)}\right] - \left(\frac{B^{E^{1-\alpha}}}{\alpha}\right)\right]\frac{d\tilde{Y}^E}{d\alpha} + \left(\frac{B^E}{\alpha}\right)\ln B^E. \quad \text{(A.46)}$$

Noting that $B^E \in (0,1)$ by Observation 2(i), so that $[\ln B^E < 0]$, and recalling that $\frac{d\tilde{Y}^E}{d\alpha} < 0$ by Observation 3(iii), part (iii) of Proposition 2 follows from (A.46). □

Proof of Proposition 3.

Let $Z = \frac{\rho a_H(p_M^E)^\rho}{\rho - a_H(p_M^E)^\rho}$. By (9), $\left[a_H(p_M^E)^\rho = \frac{1}{\left(a_H^{\frac{-1}{\rho}} + a_M^{\frac{-1}{\rho}}\right)^\rho}\right]$. Hence, when $a_H = a_M = 2$, $\left[a_H(p_M^E)^\rho = \frac{1}{2^{\rho-1}}\right]$. It follows that, when $a_H = a_M = 2$, $\lim_{\rho\to 1} Z = \infty$. Proposition 3 then follows, by continuity, from (A.37), (A.43) and (A.46). □

References

Aggarwal, M. (2020). India is missing a clear forest policy and its jungle dwellers are the worst off, *Quartz India*, January 13,

https://qz.com/india/1783965/indias-missing-forest-policy-is-hurting-tribals-the-most/

Alavi, F. (2019). Stray cattle issue – a reality check, *The Indian Express*, June 29,

https://indianexpress.com/article/opinion/columns/stray-cattle-issue-a-reality-check-cow-slaughter-india-livestock-5805684/.

Anderton, C., Anderton, R. and Carter, J. R. (1999). Economic activity in the shadow of conflict, *Economic Inquiry*, 37 (1), 166-79.

Andreoni, J. (1990). Impure altruism and donations to public goods: a theory of warm-glow giving, *Economic Journal*, 100, 464–477.

Baik, K. H. (2008). Contests with group-specific public-good prizes, *Social Choice and Welfare*, 30, 103–117.

Baik, K. H. (2016). Contests with alternative public-good prizes, *Journal of Public Economic Theory*, 18 (4), 545-559.

Baik, K. H. and Jung, H. M. (2020). Contests with multiple alternative prizes – public good/bad prizes and externalities, *Journal of Mathematical Economics*, forthcoming, https://doi.org/10.1016/j.jmateco.2020.06.004 .

Bakshi, D. and Dasgupta, I. (2018). A model of dynamic conflict in ethnocracies, *Defence and Peace Economics*, 29 (2), 147-170.

Bakshi, D. and Dasgupta, I. (2020). Identity conflict with cross-border spillovers, *Defence and Peace Economics*, forthcoming, https://doi.org/10.1080/10242694.2019.1614279 .

Bergstrom, T.C., Blume, L. and Varian, H. (1986). On the private provision of public goods, *Journal of Public Economics*, 29, 25–49.

Bostedt, K.G. (1999). Threatened species as public goods and public bads: An application to wild predators in Sweden, *Environmental and Resource Economics*, 13, 59–73.

Buchholz, W., Cornes, R. and Rübbelke, D. (2018). Public goods and public bads, *Journal of Public Economic Theory*, 20 (4), 525-540.

Caruso, R. (2010). Butter, guns and ice-cream: theory and evidence from sub-Saharan Africa, *Defence and Peace Economics*, 21 (3), 269-283.

Chari, M. (2019). Hindutva paranoia about cow slaughter has given India a stray cattle problem that's here to stay, *Scroll.in*, January 13,

https://scroll.in/article/908702/the-cow-and-bull-story-as-farmers-vent-anger-indias-stray-cattle-problem-is-here-to-stay.

Cheikbossian, G. (2008). Heterogeneous groups and rent-seeking for public goods, *European Journal of Political Economy*, 24, 133–150.

Cheikbossian, G. and Fayat, R. (2018). Group size, collective action and complementarities in efforts, *Economics Letters*, 168, 77-81.

Chowdhury, S. M., Lee, D. and Sheremeta, R, M. (2013). Top guns may not fire: best-shot group contests with group-specific public good prizes, *Journal of Economic Behavior & Organization*, 92, 94–103.

Cornes, R. (1993). Dyke maintenance and other stories: some neglected types of public goods, *Quarterly Journal of Economics*, 108, 259–271.

Cornes, R. and Sandler, T. (1994). The comparative static properties of the impure public good model, *Journal of Public Economics*, 54, 403–421.

Cornes, R. and Sandler, T. (1996) *The Theory of Externalities, Public Goods and Club Goods*, 2nd Ed. (Cambridge University Press, Cambridge UK).

Cornes, R. and Sandler, T. (2000). Pareto-improving redistribution and pure public goods, *German Economic Review*, 1, 169–186.

Cornes, R., Hartley, R. and Tamura, Y. (2019). Two-aggregate games: demonstration using a production–appropriation model, *The Scandinavian Journal of Economics*, 121 (1), 353-378.

Dal Bo, E. and Dal Bo, P. (2011). Workers, warriors, and criminals: social conflict in general equilibrium, *Journal of the European Economic Association*, 9 (4), 646-77.

Dal Bo, E. and Dal Bo, P. (2012). Conflict and policy in general equilibrium: Insights from a standard trade model, in M. Garfinkel and S. Skaperdas (eds.), *The Oxford Handbook of The Economics of Peace and Conflict*, (Oxford University Press, New York) 611-632.

Dasgupta, I. and Guha Neogi, R. (2018). Between-group contests over group-specific public goods with within-group fragmentation, *Public Choice*, 174 (3-4), 315-334.

Dasgupta, I. (2017). Linguistic assimilation and ethno-religious conflict, in W. Buchholtz and D. Ruebbelke (eds.), *The Theory of Externalities and Public Goods: Essays in Memory of Richard C. Cornes*, (Springer, Berlin) 219-242.

Dasgupta, I. and Kanbur, R. (2005a). Community and anti-poverty targeting, *Journal of Economic Inequality* 3 (3), 281-302.

Dasgupta, I. and Kanbur, R. (2005b). Bridging communal divides: separation, patronage, integration, in C. Barrett (ed.) *The Social Economics of Poverty: On Identities, Groups, Communities and Networks*, (Routledge, London) 146-170.

Dasgupta, I. and Kanbur, R. (2007). Community and class antagonism, *Journal of Public Economics*, 91 (9), 1816-1842.

Dasgupta, I. and Kanbur, R. (2011). Does philanthropy reduce inequality? *Journal of Economic Inequality*, 9, 1–21.

Epstein, G. S. and Mealem, Y. (2009). Group specific public goods, orchestration of interest groups with free riding. *Public Choice*, 139, 357–369.

Esteban, J. and Ray, D. (2001). Collective action and the group size paradox, *American Political Science Review*, 95, 663–672.

Esteban, J. and Ray, D. (2011a). A model of ethnic conflict, *Journal of the European Economic Association*, 9, 496–521.

Esteban, J. and Ray, D., (2011b). Linking conflict to inequality and polarization, *American Economic Review*, 101 (4), 1345–1374.

Gradstein, M. (1993). Rent-seeking and the provision of public goods, *Economic Journal*, 103, 1236–1243.

Hausken, K. (2005). Production and conflict models versus rent-seeking models, *Public Choice*, 123, 59–93.

Grossman, H. I. and Kim, M. (1996). Predation and accumulation, *Journal of Economic Growth*, 1, 333-350.

Hirshleifer, J. (1991). The paradox of power, *Economics &* Politics, 3 (3), 177–200.

Ihori, T. (2000). Defense expenditures and allied cooperation, *Journal of Conflict Resolution*, 44, 854–867.

Katz, E., Nitzan, S. and Rosenberg, J. (1990). Rent-seeking for pure public goods, *Public Choice*, 65, 49-60.

Kolmar, M. and Rommeswinkel, H. (2013). Contests with group-specific public goods and complementarities in efforts, *Journal of Economic Behavior and Organization*, 89, 9–22.

Lee, D. (2012). Weakest-link contests with group-specific public good prizes, *European Journal of Political Economy*, 28 (2), 238–248.

Mitra, A. and Ray, D. (2014). Implications of an economic theory of conflict: Hindu-Muslim violence in India, *Journal of Political Economy*, 122 (4), 719-765.

Murphy, K.M., Schleifer, A. and Vishny, R. W. (1993). Why is rent-seeking so costly to growth?, *American Economic Review*, 83 (2), 409-14.

Nitzan, S. and Ueda, K. (2009). Collective contests for commons and club goods, *Journal of Public Economics*, 93, 48-55.

Noh, S. J. (2002). Production, appropriation and income transfer, *Economic Inquiry*, 40 (2), 279–287.

Olson, M. (1965) *The Logic of Collective Action: Public Goods and the Theory of Groups* (Harvard University Press, Cambridge MA). .

Riaz, K., Shogren, J. F. and Johnson, S. R. (1995). A general model of rent-seeking for pure public goods, *Public Choice*, 82, 243–259.

Shettima, A. G. and Tar, U. A. (2008). Farmer-pastoralist conflict in West Africa: exploring the causes and consequences, *Information, Society and Justice*, 1.2, 163-184.

Skaperdas, S. (1992). Cooperation, conflict, and power in the absence of property rights, *American Economic Review*, 82, 720–739.

Tullock, G. (1980). Efficient rent seeking, in J.M. Buchanan, R.D. Tollison and G. Tullock (eds.) *Toward a Theory of the Rent-seeking Society*, (Texas A and M University Press, College Station) 97-112.

Ursprung, H. W. (1990). Public goods, rent dissipation, and candidate competition, *Economics and Politics*, 2, 115–132.