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ABSTRACT

On the Optimality of Search Matching Equilibrium When Workers Are Risk Averse*

This paper revisits the normative properties of search-matching economies when homogeneous workers have concave utility functions and wages are bargained over. The optimal allocation of resources is characterized first when information is perfect and second when search effort is not observable. To decentralize these optima, employees should be unable to extract a rent when information is perfect. An appropriate positive rent is however needed in the second case. To implement these optima, non-linear income taxation is a key complement to unemployment insurance. According to the level of the workers' bargaining power, taxation has to be progressive or regressive. These properties are also studied through numerical simulations.

JEL Classification: J64, J65, J68, H21, D82

Keywords: unemployment, non-linear taxation, unemployment benefits, moral hazard, search, matching

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I Introduction

The optimality of search-matching equilibria (Pissarides 2000) is a non-trivial normative issue that has up to now only been studied under the assumption of risk neutrality. The so-called 'Hosios condition' (Hosios 1990) is then sufficient to guarantee that a laissez faire equilibrium is socially optimal. This condition requires that workers' bargaining power be equal to the elasticity of the matching function with respect to unemployment. Employed workers should receive a certain share of the rent generated by a match in order to compensate them for the cost inherent to job search activities and to prevent the creation of too many vacancies in equilibrium. When the bargaining power does not fulfill the Hosios condition, Boone and Bovenberg (2002) show how non lump-sum income taxation can be used to decentralize the optimum. Taxation can restore efficiency because a positive marginal tax rate (resp. a negative one) decreases (resp. increases) the share of the surplus that accrues to the workers. In the policy debate however, unemployment is typically not only seen as a waste of resources but also as a major source of risk for workers' income. This motivates our interest for revisiting the Hosios problem when workers are risk averse.

The social planner integrates endogenous job-search intensity, wage bargaining and job creation. Two informational settings are contrasted. In the first-best case, search intensity is observable while in the second-best it is not. We show that, compared to the first-best optimum, the second-best one is characterized by: *i*) lower search intensity and a tighter labor market to compensate for the decrease in job search intensity; *ii*) higher (resp., typically lower) income for employed (resp., unemployed) workers; *iii*) lower average and marginal income-tax rates. In both settings the optimum requires an appropriate combination of non-linear income taxation and unemployment benefits. Furthermore, the Hosios condition is neither sufficient nor necessary to reach an optimum.

This paper also contributes to the literature about the desirability of progressive labor taxes. Malcomson and Sator (1987), Lockwood and Manning (1993), Holmlund and Kolm (1995), Sorensen (1999) and Pissarides (1998 and 2000) among others emphasize that, for a given level of taxes, the negotiated wage is a decreasing function of the marginal tax rate. Accordingly, a more progressive labor tax schedule should reduce unemployment. However, the desirability of progressive labor income taxes has been recently questioned by papers that introduce in-work effort (Hansen 1999, Fuest and Huber 2000) or training decisions (Boone and de Mooij 2003). A more progressive tax schedule can reduce productivity *per capita* so that the total effect on output becomes ambiguous. We put forward another unfavorable effect of tax progressivity. Through a reduction in the rent extracted by employees, tax progressivity decreases the incentives unemployed people have to search.

We also contribute to the literature on optimal unemployment insurance (see Holmlund 1998). The seminal articles of Baily (1977) formulates the search for optimal unemployment insurance as a moral hazard problem. We extend this partial equilibrium view by including firms behavior and the negotiation of wages. We highlight that non linear taxation and unemployment insurance are complementary instruments. The literature about optimal unemployment insurance has already been extended in many directions that we do not consider. On the one hand, Shavell and Weiss (1979) or Hopenhayn and Nicolini (1997) show that unemployment benefits should decrease over the unemployment spell. This result was confirmed by Fredriksson and Holmlund (2001) and was toned down by Cahuc and Lehmann (2000) in general equilibrium search-matching models that endogeneize firms behavior and wage formation. On the other hand, sanctions (i.e. withdrawal of unemployment benefits if search effort is judged insufficient) are an alternative that allows to improve risk-sharing for those who comply with the rules (Boadway and Cuff, 1999, Boone and van Ours, 2000, Boone et al, 2001). This property is expected to hold as long as search effort is observable at a reasonable cost without (too frequent) errors.

The paper is organized as follows. Section II describes the structure of the economy. Section III is devoted to the equilibrium, Section IV to the first-best optimum and its decentralization, Section V to the second best optimum and its implementation. Section VI presents simulation results. Section VII concludes the paper.

II Assumptions and Notations

We look at a segment of the labor market which is made of a continuum of homogenous risk-averse workers. There are no financial markets. Workers can either be employed or unemployed. Jobs can either be filled or vacant. We assume infinitely-lived agents.

The model is based on the assumption that the matching between unemployed workers and vacant jobs is a time-consuming and costly process due to various frictions on the labor market. Assume a continuous-time setting. The flow of hires M is a function $M(S, v)$ of the number of job-seekers measured in efficiency units S and of the number of vacancies v. It is standard to assume that this function is increasing and concave in both arguments (with $M(0, v) = M(S, 0) = 0$) and that returns to scale are constant (see e.g. Petrongolo and Pissarides, 2001). Denoting by e the average search intensity and by u the mass of unemployed workers, one has $S = e \cdot u$. Let $\theta \equiv v/S$ be tightness on the labor market (measured in efficiency units). The rate at which a vacant job is filled is $m(\theta)$ with $m(\theta) = \frac{M(S,v)}{v} = M(1/\theta, 1)$, and $m'(.) < 0$. An unemployed with search intensity $e_i \geq 0$ flows out of unemployment at a rate $e_i \cdot \alpha(\theta) = \frac{e_i}{e} \cdot \frac{M(e \cdot u, v)}{u}$, with $\alpha(\theta) \equiv M(1, \theta) = \theta \cdot m(\theta)$ and $\alpha'(\theta) > 0$, $\alpha''(.) < 0$. Job matches end at the exogenous rate q.

We normalize the size of the labor force to 1. In steady state, equality between entries and exits yields the "Beveridge curve" equation:

$$e \cdot \alpha(\theta) \cdot u = q(1-u) \qquad \Leftrightarrow \qquad u = \frac{q}{q + e \cdot \alpha(\theta)} \tag{1}$$

that negatively links the unemployment rate to tightness θ.

Let r be the discount rate common to workers and firms. An employed worker has an instantaneous utility function $v(\omega)$, where ω denotes her after-tax income. An unemployed worker has an instantaneous utility $v(z - d(e))$ where z denotes her untaxed unemployment benefits. We assume $v'(.) > 0$, $d(.) \geq 0, d'(.) > 0$ and $d''(.) \geq 0$ (with $\lim_{e \to 0} d'(e) = 0$ and $\lim_{e \to \infty} d'(e) = +\infty$). The risk aversion assumption implies $v''(.) < 0$. Function $d(e)$ denotes the monetary cost of job-search activities. It also includes the money value of home production or of informal activities for which less time is available as e increases. Then, $z - d(e)$ stands for the net level of consumption of the unemployed.

The model is developed in steady state. Let V and V^u denote the expected lifetime utility of respectively an employed and an unemployed worker. V solves:

$$r \cdot V = v(\omega) + q(V^u - V) \tag{2}$$

Two cases will be considered. The one where search intensity is observable will be introduced later. When search cannot be observed, an unemployed worker has to choose her search intensity at any point in time. With a search intensity e_i, her instantaneous utility is $v(z - d(e_i))$ and her expected "capital gain" is $e_i \cdot \alpha(\theta)(V - V^u)$. Hence, the effort level is the solution of:

$$r \cdot V^u = \max_{e_i} \{v(z - d(e_i)) + e_i \cdot \alpha(\theta)(V - V^u)\} \tag{3}$$

Each firm is made of a unique filled or vacant job. Each filled job produces a flow of y units of output, whereas each vacant job costs c per unit of time. w denotes the gross wage (or equivalently the wage cost). Let J denote the intertemporal expected value of a filled vacancy and J^v the expected value of an open vacancy. J and J^v solve:

$$r \cdot J = y - w + q\left(J^v - J\right) \tag{4}$$

$$r \cdot J^v = -c + m\left(\theta\right)\left(J - J^v\right) \tag{5}$$

A tax T is levied on each filled job: $T = w - \omega$. According to the institutional setting, T could also denote social security contributions. These taxes are used to finance unemployed benefits z. Other public and social security expenses are here left aside. At any point in time, the budgetary surplus of the unemployment insurance system is defined as:

$$\chi = T\left(1 - u\right) - u \cdot z \tag{6}$$

Considering a given segment of the labor market, we will not impose that $\chi = 0$. One could for instance imagine that there is a deficit of the unemployment insurance system on some segments (say, those of the less-skilled workers) and a surplus on others. The State is assumed to be indifferent between giving an additional unit of expected utility and increasing the budget surplus by $1/\eta$ Euros. η stands for the social value of budget surplus expressed in terms of workers' utility units. η is positive, exogenous and segment-specific.

As it is standard in the literature (see e.g. Fredriksson and Holmlund (2001)), we ignore the transitional dynamics and henceforth assume that $r \to 0$. The social planner therefore maximizes

$$\Omega = \left(1 - u\right) v\left(\omega\right) + u \cdot v\left(z - d\left(e\right)\right) + \eta \cdot \chi \tag{7}$$

III The Market equilibrium

III.1 Free entry and efficiency

Assuming free entry of vacancies, a steady-state equilibrium should be characterized by $J^v = 0$. Hence, in such an equilibrium:

$$J = \frac{c}{m(\theta)} = \frac{y-w}{q} \qquad \Rightarrow \qquad w = \phi(\theta) \equiv y - \frac{c \cdot q}{m(\theta)} \tag{8}$$

This relationship between the gross wage w and tightness θ is downward-sloping. As w increases, the value of a filled job J declines and so do the number of vacancies and tightness θ. Since θ is measured in efficiency units, one should note that this relation does not depend on search intensity e.

The free entry condition (8) combined with the flow equality $q(1-u) = m(\theta) \cdot v$ imply that aggregate profits $\Pi = (1-u)(y-w) - c \cdot v$ equal zero. Equation (6) can then be rewritten as:

$$(1-u)\,\omega + u\,(z - d(e)) + \chi = Y \tag{9}$$

where $Y \equiv (1-u)\,y - u \cdot d(e) - c \cdot v$ stands for total output net of search and vacancy costs. As it is often done in the equilibrium search-matching literature, “efficiency” means here the maximization of Y.

III.2 Search Behavior

The search intensity solves (3) where V, V^u and θ are taken as given. The first-order condition of this problem is:

$$0 = \alpha(\theta)(V - V^u) - d'(e) \cdot v'(z - d(e)) \tag{10}$$

Together with equations (2) and (3), equation (10) implicitly defines the optimal search level e according to $0 = S(\theta, w, e)$ with:

$$S(\theta, w, e) \equiv \alpha(\theta)\left(v(w-T) - v(z - d(e))\right) - d'(e) \cdot v'(z - d(e))\,(q + e \cdot \alpha(\theta)) \tag{11}$$

Appendix A.1 proves that [1]: $S'_e < 0$, $S'_w > 0$, $S'_\theta > 0$. Therefore, the optimal search intensity increases with w and θ. It can be checked that an increase in T lowers search intensity (since $S'_T = -S'_w < 0$) while a rise in the level of unemployment benefits has an ambiguous effect on e. With the chosen instantaneous utility function, an increase in z reduces the marginal disutility of search effort. It also decreases the marginal gain of search. Hence the ambiguous net effect on e (see Mortensen 1977). Microeconometric estimations generally lead to the conclusion that the individual exit rate out of unemployment is negatively affected by the level of unemployment benefits. From this evidence, the case where:

$$S'_z < 0 \tag{12}$$

is the most plausible one (see Layard *et al* , 1991, and Holmlund, 1998, among others).

III.3 The Wage Bargain

A match generates a surplus that is shared between the worker and the firm-owner. Let γ be the exogenous bargaining power of the worker, with $0 < \gamma < 1$. The gross wage rate maximizes the following Nash product:

$$\max_w \quad (V - V^u)^\gamma (J - J^v)^{1-\gamma}$$

The level of taxes T is a function of the gross wage w. The wage setters realize that a marginal rise of the gross wage of an amount Δw changes the level of taxes by $T_m \cdot \Delta w$, where T_m denotes the marginal tax rate. Taking this relationship and θ as given, the first-order condition of the previous maximization can be written as :

$$V - V^u = \frac{\gamma (1 - T_m)}{1 - \gamma} \cdot v' (w - T) \cdot (J - J^v) \tag{13}$$

[1] For any function $f(., ..., .)$, f'_x denotes the partial derivatives of f with respect to x.

Let $\hat{\gamma}$ be such that:

$$\frac{\hat{\gamma}}{1-\hat{\gamma}} = \frac{\gamma(1-T_m)}{1-\gamma} \tag{14}$$

$\hat{\gamma}$ denotes the employees' *actual* bargaining power taking into account the negative effect of the marginal tax rate on their effective bargaining strength. For given tightness θ, search intensity e, bargaining power γ and level of taxes T, a higher marginal tax rate lowers the change in the after tax wage resulting from a given increase in the negotiated gross wage. This lowers the employees' rent $V - V^u$ and eventually leads to wage moderation (see e.g. Malcomson and Sator (1987), Lockwood and Manning (1993)).[2]

Combining (13) with (2) and (3) and the free entry condition (8) yields at a steady state $WS(\theta, w, e) = 0$ with :

$$WS(\theta, w, e) \equiv v(w-T) - v(z - d(e)) - \frac{\gamma(1-T_m)}{1-\gamma} \cdot \frac{q + e \cdot \alpha(\theta)}{m(\theta)} \cdot c \cdot v'(w-T) \tag{15}$$

This equation defines the wage-setting curve. From Appendix A.1, one has: $WS'_\theta < 0$, $WS'_w > 0$, $WS'_e = -\frac{S(\theta,w,e)}{r+q+e\cdot\alpha(\theta)}$, $WS'_T < 0$, $WS'_{T_m} > 0$ and $WS'_z < 0$. Conditional on e, the wage-setting curve is therefore upward-sloping in a (θ, w) space. If the marginal tax rate is fixed and θ and e are given, increasing the level of taxes T raises the net wage rate. On the contrary, for given levels of taxes T, tightness θ and search intensity e, a more progressive tax schedule will put a downward pressure on the negotiated wage. More generous unemployment benefits have the usual positive effect on wages.

[2] Proposition 6 of Manning (2004) also concludes that "a revenue neutral increase in tax progressivity reduces the average wage" in his wage-posting framework.

III.4 Equilibrium

Conditional on z, T, T_m, γ, a steady-state equilibrium (θ, w, e) is a solution of the system:

$$w = \phi(\theta) \qquad S(\theta, w, e) = 0 \qquad WS(\theta, w, e) = 0 \tag{16}$$

Equation (1) then gives the unemployment rate u and consequently the rate of vacant jobs v. Finally, equation (6) sets the level of the budget surplus χ.

Appendix A.2 proves the uniqueness of equilibrium conditional on the policy parameters (z, T, T_m, γ). This property will be useful to decentralize social optima. Furthermore, it is shown that $d\theta/dz < 0$, $d\theta/dT < 0$ and $d\theta/dT_m > 0$. These properties are standard when workers are risk neutral (see e.g. Pissarides, 1998, 2000, Fredriksson and Holmlund, 2001 or Cahuc and Zylberberg, 2004) but not under risk aversion. The direction in which the equilibrium values of tightness and wage vary with the policy parameters is entirely determined by the sign of their partial effects through the free-entry condition (8) and the wage-setting curve (15). Finally, Appendix A.2 explains why the marginal effect of T and T_m on e can only be signed if $\hat{\gamma}$ is equal to the elasticity of the matching function with respect to unemployment. General and partial equilibrium effects on e then coincide. Otherwise, in general, we cannot conclude about the net effects of these policy parameters on search intensity. In sum,

Proposition 1 *There is (at most) a single steady-state equilibrium in this economy. At the equilibrium, tightness θ (respectively, the gross wage w) decreases (resp. increases) with the levels of unemployment benefits z and tax T and increases (resp. decreases) with the marginal tax rate T_m.*

IV The First-Best optimum

In this section, we first look at the optimal allocation of resources that a benevolent social planner would implement if he could perfectly control search intensity. This section finally looks at the decentralization of the optimum.

IV.1 The central planner problem

In this subsection, the central planner controls tightness, the level of effort, the unemployment rate, net income and the unemployment benefit. He maximizes Ω subject to the resource constraint (9) and the flow equilibrium equation (1). Remembering that $v = e \cdot \theta \cdot u$, the planner's program then consists in [3]:

$$\max_{\theta,\omega,u,z,e} (1-u)\, v(\omega) + u \cdot v(z - d(e)) + \eta \left[(1-u)(y-\omega) - (z + c \cdot e \cdot \theta)\, u\right] \tag{17}$$
$$0 = e \cdot \alpha(\theta) \cdot u - q(1-u)$$

Introducing subscript 1 to denote the first-best optimum, Appendix A.3 implies that:

$$v'(\omega_1) = v'(z_1 - d(e_1)) = \eta \qquad \Leftrightarrow \qquad \omega_1 = z_1 - d(e_1) = \left(v'\right)^{-1}(\eta) \tag{18}$$

Under perfect information, the social planner perfectly insures workers against the unemployment risk. Appendix A.3 then shows that the first-best values of θ_1 and e_1 solves $G(\theta_1, e_1) = H(\theta_1, e_1) = 0$, where

$$G(\theta, e) \equiv \alpha(\theta)(y + d(e) + c \cdot e \cdot \theta) - \left(c \cdot \theta + d'(e)\right)(e \cdot \alpha(\theta) + q)$$
$$H(\theta, e) \equiv \alpha'(\theta)(y + d(e) + c \cdot e \cdot \theta) - c(e \cdot \alpha(\theta) + q)$$

Function $G(\theta, e)$ defines the optimal level of search intensity as a function of tightness. Function $H(\theta, e)$ defines the optimal level of tightness as a function

[3] Formally, one should maximize Ω with $r > 0$ under the dynamical constraint $\dot{u} = q(1-u) - e \cdot \alpha(\theta) \cdot u$, derive the first-order and envelope conditions and take the limits of those conditions for $r \to 0$. It can be verified that this method and the maximization of the following problem give the same results for $r \to 0$.

of search intensity. Notice that these functions, and hence θ_1, e_1 and u_1, are independent of the social value of budget surplus η. Appendix A.3 also shows that $G'_e < 0$, $H'_\theta < 0$ and that $G(.,.)$ and $H(.,.)$ can be represented as shown in Figure 1.

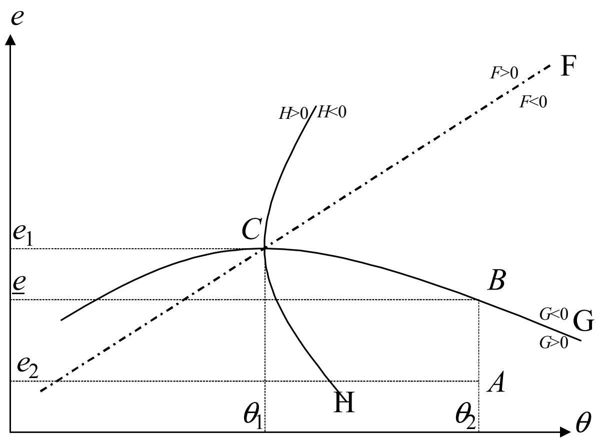

Figure 1: The first-best choice of (θ, e)

Finally, Appendix A.3 proves that (e_1, θ_1, u_1) are the values of (e, θ, u) that maximize output net of search cost Y subject to the flow equilibrium (1). Consequently, in the first best, allocative efficiency is reached independently of the social value of budget surplus and it is compatible with perfect insurance. In sum,

Proposition 2 *Under perfect information, the central planner is able to deal separately with allocative efficiency and with the risk-sharing. The first-best levels of search effort and tightness maximize total output net of costs. Given these optimal levels, the first-best income levels guarantee a constant level of utility whether workers are employed or not. This level is higher the lower the social value of budget surplus.*

IV.2 Decentralization of the First-Best optimum

The first-best setting with perfect monitoring of job search intensity is clearly a highly idealized case. However, looking briefly at the decentralization of this optimum highlights the complementarity between non-linear taxation and unemployment insurance. The State has to decentralize an equilibrium in which workers are perfectly insured against the unemployment risk. According to (15), this requires $\hat{\gamma}_1 = 0$. Such a low *actual* bargaining power is unavoidable to prevent insiders from extracting a rent $V - V^u > 0$ through wage bargaining. Whenever the workers' bargaining power γ is positive, this can only be achieved with a marginal tax rate $T_{m,1} = 100\%$ (see equation (14)). So, the decentralization of the first-best optimum is impossible without an "extremely" progressive income tax schedule:

Proposition 3 *The first best is decentralized with either* $\gamma = 0$ *or* $T_{m,1} = 100\%$

One may wonder why the decentralization with risk averse workers differ so much from the one under "linear" preferences (i.e. with $v''(.) = 0$ and $v'(.) \equiv \eta$). In the latter case, the social planner is only concerned with total output net of search costs Y, independently of the way this output is shared between the employed, the unemployed and the budget surplus. There is therefore a multiplicity of first-best optima. Any combination of ω, z and χ leading to the same total output Y_1 is actually a first-best optimum in this case. The *laissez faire* economy under the Hosios (1990) condition only corresponds to one of these optima. When preferences tends to the "linear" case, our decentralization (with $\hat{\gamma}_1 = 0$) leads to another efficient optimum with perfect unemployment insurance.

V The second-Best optimum

V.1 The central planner problem

In this section, we consider the polar case where search intensity is not observed by the State. As in the first best, the tax system and the level of unemployment benefits are the instruments used to promote efficiency and to insure workers. Since search effort is now chosen by the unemployed, the State faces a moral hazard problem. The incentive constraint $S(.,.,.) = 0$ (see equation 11) has therefore to be included. The second-best problem is:

$$\max_{\theta,\omega,u,z,e} (1-u)\cdot v(\omega) + u\cdot v(z-d(e)) + \eta\left[(1-u)(y-\omega) - z\cdot u - c\cdot e\cdot\theta\cdot u\right] \tag{19}$$

$$0 = e\cdot\alpha(\theta)\cdot u - q(1-u)$$

$$0 = \alpha(\theta)(v(\omega) - v(z-d(e))) - d'(e)\cdot v'(z-d(e))(q+e\cdot\alpha(\theta)) \tag{20}$$

Let subscript 2 denote the second-best optimum. Under risk aversion [4], the incentive constraint (20) implies that $\omega_2 > z_2 - d(e_2)$. Appendix A.4 shows that $G(\theta_2, e_2) > 0$ and $H(\theta_2, e_2) < 0$. Remembering the properties of functions G and H, these properties imply:

$$e_2 < e_1 \qquad \text{and} \qquad \theta_2 > \theta_1 \tag{21}$$

To induce search effort, employed workers necessarily enjoy higher utility levels than unemployed ones. Keeping search effort at its first-best level would however require a difference in utilities between employed and unemployed workers that would be too detrimental to the objective of insurance, hence $e_2 < e_1$. This property is less standard than often believed (see e.g. Mass-Colell *et al*, 1995, p 487, who provide a counter-example).

[4] Boone and Bovenberg (2002) show that first and second-best outcomes coincide when workers are risk neutral.

Compared to the first-best optimum, the social planner integrates the beneficial effect of a tighter labor market on search effort. An increase in tightness allows to relax the incentive constraint. This intuitively explains why $\theta_2 > \theta_1$. This property echoes similar results in Rosen (1985) where labor demand is above its efficient level when unemployment insurance is imperfect. Properties (21) have no clear implications on the comparison between V-U ratios (v_2/u_2 *vs* v_1/u_1) and between unemployment rates (u_2 *vs* u_1).

Appendix A.4 also proves that $\omega_2 > \omega_1$. In the most plausible case where higher unemployment benefits have a negative impact on search effort, i.e. when $S'_z < 0$, we also get $z_2 - d(e_2) < z_1 - d(e_1)$. Since, $e_2 < e_1$, these results imply $z_2 < z_1$. The intuition behind $z_2 - d(e_2) < z_1 - d(e_1)$ and $\omega_2 > \omega_1$ is similar to the one underlying $\theta_2 > \theta_1$. Compared to the first best, the social planner integrates the beneficial effect of a higher (respectively, a lower) utility level for employed (respectively, unemployed) workers on incentives to search.

One can compare total net output Y at the first-best and at the second-best optima. We have shown that the first-best levels of tightness and search intensity θ_1, e_1 maximize Y. Search intensity and tightness differ at the second-best compared to their first-best optima. Hence one has $Y_2 < Y_1$.

The following proposition summarizes our main results.

Proposition 4 *When search effort in unobservable, there is a trade-off between maximizing net output Y and insuring workers against the unemployment risk. Compared to the first-best optimum, search effort e and total net output Y are lower. Tightness θ and net income in employment ω are higher.*

To end this section, let us briefly emphasize how this analysis enriches studies such as Baily (1977), where both θ and w are exogenous. Let us assume that they are fixed at their second best optimal value [5]. The loss in efficiency when search effort is unobservable can then be captured by the distance between the second-best optimum e_2 and the solution $\underline{e}$ to equation $G(\theta_2, e) = 0$ (see Figure 1 where this distance is $A - B$). In our framework, the loss due to the unobservability of search effort is larger because now the second-best outcome, A in Figure 1, has to be compared to the first-best C.

V.2 Decentralization of the second best

To decentralize the second-best optimum, the policy parameters should necessarily solve $z = z_2$, $T_2 = \phi(\theta_2) - \omega_2$ and $WS(\theta_2, \phi(\theta_2), e_2) = 0$. This obviously gives a single vector of policy parameters $(z_2, T_2, T_{m,2})$. According to Proposition 1, we know that a single equilibrium exists for any vector of policy parameters. Hence for the policy parameters $(z_2, T_2, T_{m,2})$, we know that the second-best optimum is decentralized. To give unemployed workers an incentive to search, employees should extract some rent from a match. Therefore, the levels of the *actual* bargaining power can be ranked:

$$\hat{\gamma}_2 > \hat{\gamma}_1 = 0.$$

According to Equation (14), for any positive value of the bargaining power γ, implementing such an equilibrium requires:

$$T_{m,2} < 1 = T_{m,1}.$$

However, the position of $T_{m,2}$ with respect to 0 and to the average tax rate T_2/w_2 is ambiguous. Finally, since $\theta_2 > \theta_1$ and $\omega_2 > \omega_1$, one has

$$T_2 < T_1.$$

The level of tax required to decentralize the second-best optimum is lower than the first-best one. This follows from two effects. First, employed workers'

[5] The second-best value of the wage rate will be made precise in the following sub-section.

income has to be higher at the second best. Second, the inequality $\theta_2 > \theta_1$ implies that the gross wage is lower at the second best. From this, we can conclude that $T_2/\omega_2 < T_1/\omega_1$, so $T_2/w_2 < T_1/w_1$.

Knowing how the second-best optimum can be decentralized, it is now possible to characterize the tax schedule in a more precise way. First, from $WS(\theta_2, \phi(\theta_2), e_2) = 0$, it is immediately seen that T_m increases with γ. Second, from the definition of $\hat{\gamma}$,

$$T_{m,2} \lesseqgtr 0 \Longleftrightarrow \gamma \lesseqgtr \hat{\gamma}_2$$

Third, combining (13) successively with (8), (2), (3) and again (8) it can be checked that:

$$\frac{1 - T_m}{1 - (T/w)} = \frac{1-\gamma}{\gamma} \frac{v(\omega) - v(z - d(e))}{\omega \cdot v'(\omega)} \frac{m(\theta) \cdot y - c \cdot q}{c(q + e \cdot \alpha(\theta))} \tag{22}$$

where $(1 - T_m)/(1 - (T/w))$ is the so-called coefficient of residual income progression CRIP [6]. Consequently, for $(\theta, w, e) = (\theta_2, \phi(\theta_2), e_2))$, the CRIP would be equal to 1 if $\gamma = \widetilde{\gamma}$, with:

$$\frac{\widetilde{\gamma}}{1 - \widetilde{\gamma}} = \frac{v(\omega_2) - v(z_2 - d(e_2))}{\omega_2 \cdot v'(\omega_2)} \frac{m(\theta_2) \cdot y - c \cdot q}{c(q + e_2 \cdot \alpha(\theta_2))} \quad \Rightarrow \quad \widetilde{\gamma} \in (0, 1) \tag{23}$$

For this particular value of γ, the tax schedule needed to decentralize the second-best optimum would actually be linear. Otherwise, a non linear tax schedule is required.

Proposition 5 *The decentralization of the second best requires a labor tax schedule which is typically neither linear nor lump sum. The optimal marginal tax rate is an increasing function of workers' bargaining power, ranging from negative to first-best values (100%). The average tax rate is lower than at the first best.*

[6] The latter measures the elasticity of after-tax wage with respect to pre-tax wage. This is a standard indicator of the progressivity of the tax schedule.

VI Numerical simulations

These simulations have three motives. First, we would like to quantify to what extent imperfect information affects optimal policies. Second, the analytically ambiguous effects of imperfect information on unemployment rates, V-U ratios and expected lifetime utilities should be signed numerically. Finally, from a policy point of view, it is essential to measure to what extent changes in structural parameters affect optimal unemployment insurance and labor income taxation.

In the following subsections, three parameters will be allowed to vary, namely risk aversion σ, the separation rate q and the workers' bargaining power γ. For the latter, we contrast the second-best outcomes with third-best ones accessible only through a linear tax schedule. Most of the simulation results will be summarized by way of a panel showing twelve figures. Each panel will have the same structure and will use the same conventions. Solid lines will correspond to the first best, dotted lines to the second best and dashed lines to the third best. Each panel will display tightness θ, search intensity e, the v/u ratio, the unemployment rate u, the net output Y, the budget surplus χ, the net income of employed workers ω, the unemployment benefits z, the after-tax replacement ratio $\rho = z/\omega$, the utility flows V, the average tax rate T/w and the marginal tax rate T_m. The figure related to the V's will show three dotted curves for the second best. The upper (respectively, the lower, the intermediate) curve will represent the employed workers' (respectively the unemployed workers', the average) instantaneous utility expressed in certainty equivalent.

VI.1 Calibration

The year is the unit of time. We consider a representative labor market segment of the French economy. The matching function is $m_0 \left(e \cdot u\right)^{0.5} v^{0.5}$. The utility function is $v(x) = \frac{x^{1-\sigma}}{1-\sigma}$ [7]. The disutility function of search effort is $d(e) = e^{\beta}$. The separation rate q equals 0.15, the net wage amounts to 18 000 Euros, and the after-tax replacement ratio is 0.7 (see Martin, 1996). For our benchmark economy, the relative risk aversion σ is put to 1 and the bargaining power $\gamma = 0.5$. To reproduce a microeconomic elasticity of unemployment duration with respect to the level of unemployment benefits of 0.5 (consistent with estimations surveyed by Layard *et al* (1991) and Holmlund (1998)), β is set to 2.8. Then, to match an unemployment rate of 0.1 and an average duration of vacancies of 0.1 year (see Maillard, 1997), m_0 is fixed to 0.96. The remaining parameters are such that the observed equilibrium is a second-best optimum. This leads to $y = 22\ 959$, $\eta = \eta_0 = 0.00006$ and $c = 171\ 876$.[8]

VI.2 General results

The following properties were found in all (reported and unreported) simulations. First, the v/u ratio is higher and the unemployment rate u is lower at the first-best than at the second-best optimum. Recall that unemployed workers search less in the second best but that the labor market is then also tighter. The increase in tightness is generally less important than the decrease in search intensity [9].

Second, compared to their first-best values, the budget surplus χ is lower and the workers' expected utility $(1-u)\,v(\omega) + u \cdot v(z - d(e))$ is higher. To understand these results, consider the dual problems of maximizing workers'

[7] If $\sigma = 1$, we take $v(x) = \ln(x)$.

[8] Following e.g. Romer (1996), c can also be interpreted as the cost of maintaining a job (either filled or vacant). Under this interpretation, y stands for gross output per worker minus the cost of maintaining a job.

[9] In the binary case where search intensity can equal zero or a fixed positive value, it can be shown that search intensity is the same at the first and the second best, whereas tightness remains higher at the second best. Hence, at the second best, unemployment is lower and the v/u ratio is higher.

expected utility given a budget surplus $\chi \geq \bar{\chi}$. The first-best problem is then:

$$\max_{\theta,\omega,u,z,e} (1-u)\, v(\omega) + u \cdot v(z - d(e)) + \eta\left[(1-u)(y-\omega) - (z + c \cdot e \cdot \theta)\, u\right]$$

$$e \cdot \alpha(\theta) \cdot u - q(1-u) = 0$$

$$\chi \geq \bar{\chi} \tag{24}$$

The second-best problem consists in adding (20) to the previous one. Denoting $\eta_i(\bar{\chi})$ the Lagrange multiplier of constraint (24) in the i^{th}-best program, one has the same first-order conditions as the previous ones except that now, the budget surplus χ's are exogenous whereas the multipliers η_i's are endogenous. It is then intuitive that: 1) a higher requirement for the budget surplus raises both first-best and second-best values of $\eta_i(\bar{\chi})$; 2) it is harder to fulfill a given budget requirement $\chi \geq \bar{\chi}$ at the second best, so $\eta_2(\bar{\chi}) > \eta_1(\bar{\chi})$. Putting these two features together imply that $\chi_2(\eta) < \chi_1(\eta)$ when we allow the budget surplus to vary but keep η exogenous (as we did). It turns out that this favorable effect on workers' expected utility outweighs two others. First, total net output is lower at the second best. So, fewer resources are available to insure workers against the unemployment risk. Second, net income levels in unemployment and in employment are different at the second best. So, risk-sharing is less efficient.

VI.3 Risk aversion σ

Figure 2 displays first- and second-best optima and their implementation in a decentralized economy for different values of the relative risk aversion parameter ($\sigma \in [0.05, 3]$). As σ increases, the weight given to insurance increases too. We normalize the value of η so that first-best values of incomes ω and z remain unchanged when σ varies [10]. So doing, changes in σ do not affect the relative weight attributed to the budget surplus at the first best.

[10] That is we impose $\eta = (\eta_0)^{\sigma}$.

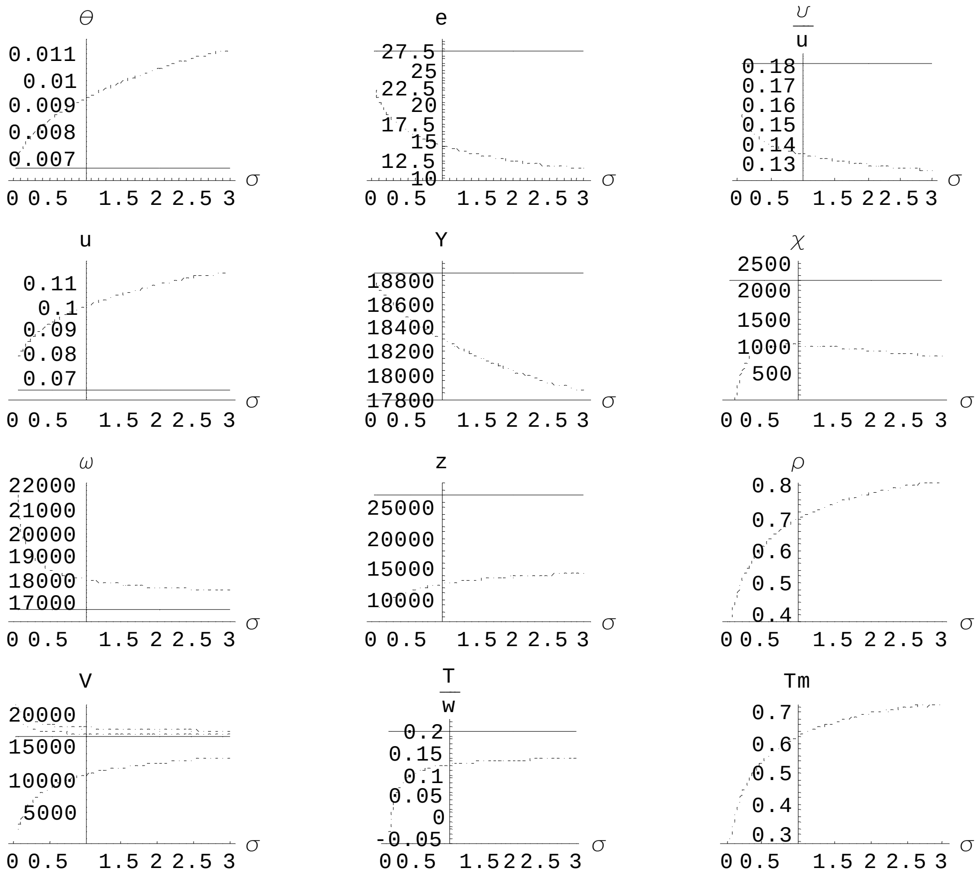

Figure 2: Simulations with respect to risk aversion

When σ increases the central planner put less incentive on search effort. Search intensity e, the υ/u ratio and net output Y decrease while the unemployment rate u and tightness θ increase with σ. While the previous variables move away from their first-best counterpart as σ increases, employed and unemployed workers' net incomes and utility levels converge to their first best values. This was expected as insurance matters more and more. Starting from quite high values for extremely low risk averseness, the after-tax replacement ratio and the marginal tax rate increase a lot with σ, respectively from 0.38 to around 0.8 and from 0.26 to 0.7. From a policy viewpoint, these results highlight the complementarity between tax progressivity and unemployment insurance. The order of magnitude for the after-tax replacement ratio observed in Continental Europe (namely, between 0.7 and 0.8) is obtained for a wide range of relative risk aversion values (namely for $\sigma \in [1, 3]$). As expenditures for unemployed workers $z \cdot u$ rises, the level of taxes increases and so does the average tax rate T/w.

VI.4 Separation rate q

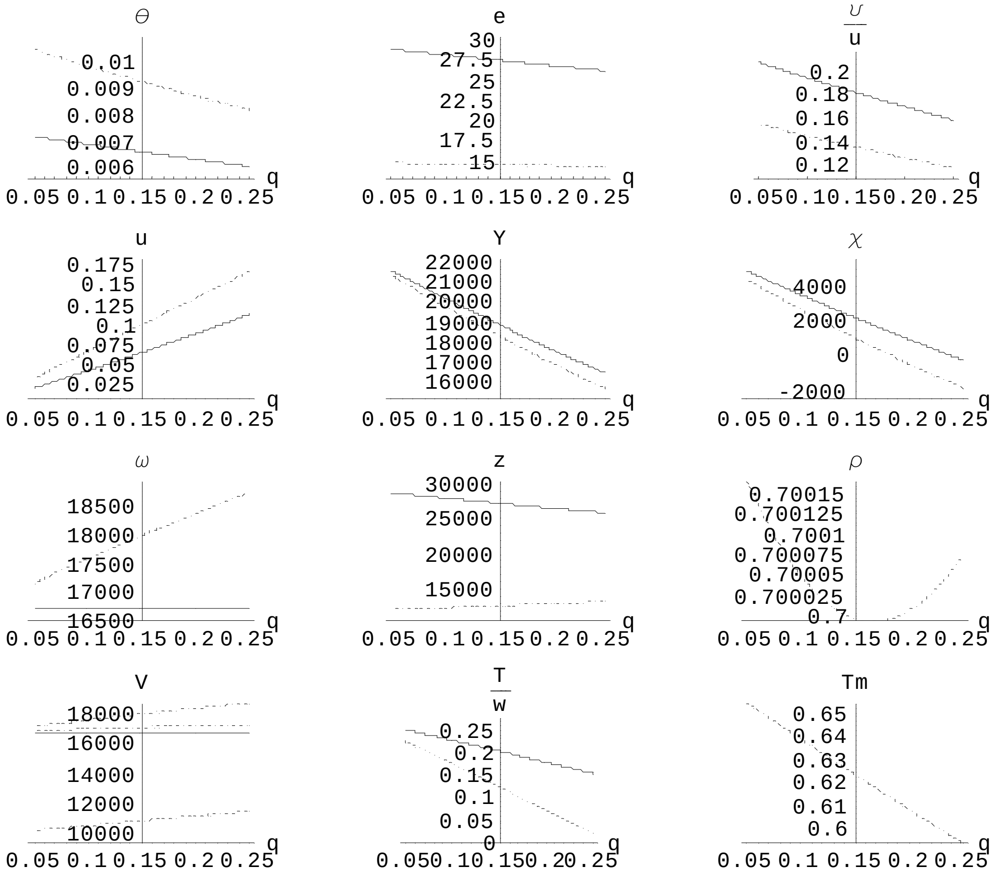

Figure 3: Simulations with respect to the separation rate.

A rise in the separation rate q has similar effects as a decrease in the scale parameter of the matching function m_0. Both capture a rise in the frictional determinants of unemployment. First, when q increases, both first and second-best optima become somehow less "efficient". This means that total net output, tightness, search intensity, the $V-U$ ratio decrease and unemployment increases. Second, it turns out that employed and unemployment utility levels increase. Since total output decreases, this can be only achieved through a dramatic decrease in the budget surplus and in average tax rates. Third, the marginal tax rate decreases very slightly, whereas the after tax replacement ratio remains unchanged. In sum, increases in the separation rate have large effects on the unemployment rate. Conversely, the generosity of the unemployment insurance system and the marginal tax rate are roughly constant.

VI.5 Non-linear versus proportional taxation

This section illustrates to what extent non-linear taxation is a key ingredient to decentralize the second-best optimum. For our benchmark economy, it turns out that second-best tax schedule is proportional when workers' bargaining power equals to $\widetilde{\gamma}=0.3$. Two cases will now be contrasted when $\gamma \neq \tilde{\gamma}$: first, the already considered case where non-linear taxation is allowed; second, the so-called third-best case where taxation has to be proportional to wages. The latter setting is the one typically considered in the literature.

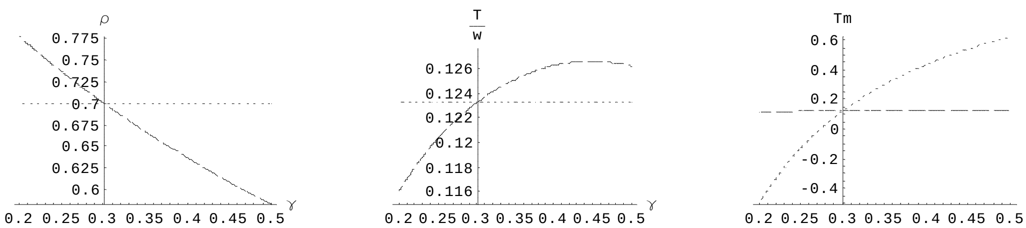

Figure 4: Proportional versus non-linear taxation

Figure 4 compares second- (dotted lines) and third-best policies (dashed lines) when γ varies. Obviously, policies are identical when $\gamma=\tilde{\gamma}$. When $\gamma>\tilde{\gamma}$ (respectively when $\gamma<\tilde{\gamma}$), the second-best requires a progressive (regressive) tax schedule that is no more available at the third best. Consequently, workers extract too much (too few) rent, so the after tax replacement ratio has to decrease (increase).

VII Conclusion

We have contrasted a first-best optimum where the State can perfectly monitor unemployed workers' search intensity and a second-best optimum with moral hazard. In the first-best setting, efficiency can be achieved independently of the redistributive issues and the State can perfectly insure workers against the unemployment risk. The implementation of the first best requires a 100% marginal tax rate. In the second-best case, search intensity and total net output are lower and tightness is higher. Marginal tax rate, average tax rates and typically unemployment benefits are lower. Whether, income taxation should be progressive or regressive heavily depends on the value of the workers' bargaining power. For sufficiently low values of the latter, the marginal tax rate could even become negative in order to provide appropriate incentives to search and to create vacancies. A non proportional tax schedule is a necessary complement to unemployment insurance in a second-best world. Numerical simulations confirm this conclusion. They also show that the optimal replacement ratio and marginal tax rate are very sensitive to changes in risk aversion but very inelastic to the separation rate.

In a more realistic framework, the State can imperfectly observe search behavior. So, reality lies somewhere between our first and second best. A continuity argument suggests that as search behavior becomes better observed, the social optimum would be characterized by higher search intensity and unemployment benefits and lower tightness. A less expected result is that marginal tax rates should also increase. International institutions such as the OECD and the academic profession (see Boadway and Cuff, 1999, Boone and van Ours 2000, or Boone et al 2001) work more and more on the effects of monitoring search effort. From our paper, it can be concluded that a better control of job-search effort should not only lead to better insurance against the unemployment risk (conditional on the income tax schedule). To be optimal, such reforms should also be accompanied by an increase in tax progressivity.

This paper could be extended in different ways. First, the introduction has cited several papers that have been concerned with the optimal profile of unemployment benefits over the unemployment spell rather than a single level of unemployment benefits. Second, we could consider heterogeneity in workers productivity as in Mirrlees (1971), Saez (2002), Hungerbühler *et alii* (2003) and Boone and Bovenberg (2004). Finally, it would be interesting to analyze the same issues from a political economy viewpoint instead of a normative one.

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A Appendix

A.1 Partial Derivatives

From (11), one has:

$$\begin{aligned}
S'_e &= \left(-d''(e)\cdot v'(z-d(e)) + \left[d'(e)\right]^2 v''(z-d(e))\right)(q+e\cdot\alpha(\theta)) \\
S'_w &= \alpha(\theta)\cdot v'(w-T) > 0 \\
S'_\theta &= \alpha'(\theta)\left[v(w-T) - v(z-d(e)) - e\cdot d'(e)\cdot v'(z-d(e))\right]
\end{aligned}$$

Since, $v''(.) < 0$ and $d''(.) \geq 0$, it is easily checked that $S'_e < 0$. Equation $S(.,.,.) = 0$, can be rearranged to yield:

$$v(w-T) - v(z-d(e)) = \frac{q+e\cdot\alpha(\theta)}{\alpha(\theta)} d'(e)\cdot v'(z-d(e))$$

Therefore,

$$S'_\theta = \frac{\alpha'(\theta)}{\alpha(\theta)}\cdot q\cdot d'(e)\cdot v'(z-d(e)) > 0$$

Finally, one has:

$$\begin{aligned}
S'_{T_m} &= 0 \\
S'_T &= -\alpha(\theta)\cdot v'(w-T(w)) = -S'_w < 0 \\
S'_z &= -\alpha(\theta)\cdot v'(z-d(e)) - d'(e)(q+e\cdot\alpha(\theta))\, v''(z-d(e))
\end{aligned}$$

From (15), the following partial derivatives can be computed:

$$\begin{aligned}
WS'_\theta &= -\frac{c\cdot\gamma}{1-\gamma}\cdot\frac{1-T_m}{m(\theta)}\left(e\cdot\alpha'(\theta) - \frac{m'(\theta)(q+e\cdot\alpha(\theta))}{m(\theta)}\right) < 0 \\
WS'_w &= v'(w-T) - \frac{c\cdot\gamma}{1-\gamma}(1-T_m)\frac{q+e\cdot\alpha(\theta)}{m(\theta)}\cdot v''(w-T) > 0 \\
WS'_e &= d'(e)\cdot v'(z-d(e)) - \frac{c\cdot\gamma}{1-\gamma}(1-T_m)\frac{\alpha(\theta)}{m(\theta)}\cdot v'(w-T)
\end{aligned}$$

After some manipulation, $WS(.,.,.) = 0$ becomes:

$$\frac{c\cdot\gamma}{1-\gamma}\cdot\frac{(1-T_m)\cdot v'(w-T)}{m(\theta)} = \frac{v(w-T) - v(z-d(e))}{q+e\cdot\alpha(\theta)}$$

Taking this equality into account leads to:

$$WS'_e = d'(e) \cdot v'(z - d(e)) - \alpha(\theta) \frac{v(w-T) - v(z - d(e))}{q + e \cdot \alpha(\theta)} = -\frac{S(\theta, w, e)}{q + e \cdot \alpha(\theta)}$$

Hence, WS'_e is equal to zero in equilibrium. Finally, one has:

$$\begin{aligned}
WS'_T &= -v'(w-T) + \frac{c \cdot \gamma}{1-\gamma}(1 - T_m) \frac{q + e \cdot \alpha(\theta)}{m(\theta)} \cdot v''(w-T) < 0 \\
WS'_{T_m} &= \frac{c \cdot \gamma}{1-\gamma} \frac{q + e \cdot \alpha(\theta)}{m(\theta)} \cdot v'(w-T) > 0 \\
WS'_z &= -v'(z - d(e)) < 0
\end{aligned}$$

A.2 Uniqueness of the equilibrium and comparative statics

We define

$$\mathbb{W}(\theta, e) \equiv WS(\theta, \phi(\theta), e) \qquad \mathbb{S}(\theta, e) \equiv S(\theta, \phi(\theta), e)$$

and show that the system $\mathbb{S}(\theta, e) = \mathbb{W}(\theta, e) = 0$ admits at most one solution. Since equation (8) depends neither on search intensity e nor on policy parameters (z, T, T_m), one gets

$$\mathbb{S}'_e(\theta, e) = S'_e(\theta, \phi(\theta), e) < 0 \qquad \mathbb{S}'_x(\theta, e) = S'_x(\theta, \phi(\theta), e) \qquad \text{for} \qquad x = z, T, T_m$$

Similarly, one has

$$\begin{aligned}
\mathbb{W}'_\theta(\theta, e) &= WS'_\theta(\theta, \phi(\theta), e) + \phi'_\theta \cdot WS'_w(\theta, \phi(\theta), e) < 0 \\
\mathbb{W}'_e(\theta, e) &= WS'_e(\theta, \phi(\theta), e) = -\frac{\mathbb{S}(\theta, e)}{q + e \cdot \alpha(\theta)} \\
\mathbb{W}'_x(\theta, e) &= W'_x(\theta, \phi(\theta), e) \qquad \text{for} \qquad x = z, T, T_m
\end{aligned}$$

First, we prove uniqueness. Since $\mathbb{S}'_e(\theta, e) < 0$, for any θ, the equation $\mathbb{S}(\theta, e) = 0$ admits at most one solution. Call this solution $\mathbb{E}(\theta)$ if it exists. The implicit function theorem insures that function $\mathbb{E}(\theta)$ is continuous and differentiable wherever it is defined. Now, let $\mathcal{W}(\theta) \equiv \mathbb{W}(\theta, \mathbb{E}(\theta))$. An equilibrium necessarily solves $\mathcal{W}(\theta) = 0$. Differentiating function $\mathcal{W}(.)$ yields $\mathcal{W}'(\theta) = \mathbb{W}'_\theta(\theta, \mathbb{E}(\theta)) + \mathbb{E}'(\theta) \cdot \mathbb{W}'_e(\theta, \mathbb{E}(\theta))$. Since $\mathbb{E}(\theta)$ solves $\mathbb{S}(\theta, \mathbb{E}(\theta)) = 0$, one has $\mathbb{W}'_e(\theta, \mathbb{E}(\theta)) = 0$. Hence, $\mathcal{W}'(\theta) = \mathbb{W}'_\theta(\theta, \mathbb{E}(\theta)) < 0$. So, equation $\mathcal{W}(\theta) = 0$ admits at most one solution. The equilibrium, if any, is therefore unique.

Second, we look at the comparative statics of the equilibrium. Differentiating $\mathbb{S}(\theta, e) = \mathbb{W}(\theta, e) = 0$ and taking into account that $\mathbb{W}'_e = 0$ around the equilibrium, one has:

$$\begin{pmatrix} \mathbb{W}'_\theta & 0 \\ \mathbb{S}'_\theta & \mathbb{S}'_e \end{pmatrix} \begin{pmatrix} d\theta \\ de \end{pmatrix} = -\begin{pmatrix} WS'_z & WS'_T & WS'_{T_m} \\ S'_z & S'_T & S'_{T_m} \end{pmatrix} \begin{pmatrix} dz \\ dT \\ dT_m \end{pmatrix}$$

Hence:

$$d\theta = -\frac{WS'_z}{\mathbb{W}'_\theta} dz - \frac{WS'_T}{\mathbb{W}'_\theta} dT - \frac{WS'_{T_m}}{\mathbb{W}'_\theta} dT_m$$

Since $\mathbb{W}'_\theta < 0$, $WS'_z < 0$, $WS'_T < 0$, $WS'_{T_m} > 0$ one has $d\theta/dz < 0$, $d\theta/dT < 0$ and $d\theta/dT_m > 0$. Moreover,

$$de = \frac{\mathbb{S}'_\theta \cdot WS'_z - \mathbb{W}'_\theta \cdot S'_z}{\mathbb{S}'_e \cdot \mathbb{W}'_\theta} dz + \frac{\mathbb{S}'_\theta \cdot WS'_T - \mathbb{W}'_\theta \cdot S'_T}{\mathbb{S}'_e \cdot \mathbb{W}'_\theta} dT + \frac{\mathbb{S}'_\theta \cdot WS'_{T_m} - \mathbb{W}'_\theta \cdot S'_{T_m}}{\mathbb{S}'_e \cdot \mathbb{W}'_\theta} dT_m$$

$$\mathbb{S}'_\theta = S'_\theta + \phi'(\theta) \cdot S'_w = \frac{\alpha'(\theta)}{\alpha(\theta)} q \cdot d'(e) \cdot v'(z - d(e)) + \frac{c \cdot q}{m(\theta)} \cdot \frac{m'(\theta)}{m(\theta)} \alpha(\theta) v'(w - T)$$

By equations (10), (8) and (13), one has

$$\begin{aligned} \mathbb{S}'_\theta &= q \cdot \frac{v(w-T) - v(z - d(e))}{q + e\alpha(\theta)} \left\{ \alpha'(\theta) + \theta m'(\theta) \frac{1-\gamma}{\gamma(1-T_m)} \right\} \\ &= q \cdot m(\theta) \cdot \frac{v(w-T) - v(z - d(e))}{q + e\alpha(\theta)} \left\{ 1 + \frac{\theta m'(\theta)}{m(\theta)} \left(1 + \frac{1-\gamma}{\gamma(1-T_m)} \right) \right\} \end{aligned}$$

Recall that $\hat{\gamma}/(1-\hat{\gamma}) = \gamma(1-T_m)/(1-\gamma)$. Then:

$$\mathbb{S}'_\theta = q \cdot m(\theta) \cdot \frac{v(w-T) - v(z - d(e))}{q + e\alpha(\theta)} \left\{ 1 + \frac{\theta m'(\theta)}{m(\theta)} \cdot \frac{1}{\hat{\gamma}} \right\}$$

So, under the Hosios condition, $\hat{\gamma} = -\frac{\theta m'(\theta)}{m(\theta)}$, one gets $\mathbb{S}'_\theta = 0$ around the equilibrium, and:

$$de = -\frac{\mathbb{S}'_z}{\mathbb{W}'_\theta} dz - \frac{\mathbb{S}'_T}{\mathbb{W}'_\theta} dT - \frac{\mathbb{S}'_{T_m}}{\mathbb{S}'_e} dT_m$$

Hence, one has $\frac{de}{dT} < 0$, $\frac{de}{dT_m} = 0$ but $\frac{de}{dz}$ remains ambiguous. However, whenever $\hat{\gamma} \neq -\frac{\theta m'(\theta)}{m(\theta)}$, $\mathbb{S}'_\theta$ has an ambiguous sign, so the marginal effect of z, T, T_m on e cannot be signed.

A.3 First Best and decentralization

Denoting δ_1 the Lagrange multiplier, the first-order conditions of program (17) with respect to ω, z, e, u, θ are respectively:

$$0 = (1 - u_1)\left[v'(\omega_1) - \eta\right] \tag{25}$$

$$0 = u_1\left[v'(z_1 - d(e_1)) - \eta\right] \tag{26}$$

$$0 = u_1\left[-v'(z_1 - d(e_1)) \cdot d'(e_1) - \eta \cdot c \cdot \theta_1 + \delta_1 \cdot \alpha(\theta_1)\right] \tag{27}$$

$$0 = v(z - d(e_1)) - v(\omega_1) + \eta(\omega_1 - y - z_1 - c \cdot e_1 \cdot \theta_1) + \delta_1(e_1 \cdot \alpha(\theta_1) + q) \tag{28}$$

$$0 = -c \cdot e_1 \cdot \eta \cdot u_1 + \delta_1 \cdot e_1 \cdot \alpha'(\theta_1) \cdot u_1 \tag{29}$$

Equations (25) and (26) lead to equalities (18). Conditions (27) (28) and (29) can therefore be respectively rewritten as:

$$\left(\frac{\delta_1}{\eta} =\right) \frac{d'(e_1) + c \cdot \theta_1}{\alpha(\theta_1)} = \frac{y + d(e_1) + e_1 \cdot c \cdot \theta_1}{e_1 \cdot \alpha(\theta_1) + q} = \frac{c}{\alpha'(\theta_1)} \tag{30}$$

From the equalities in (30), we get that the social optimum is determined by either $F(\theta_1, e_1) = G(\theta_1, e_1) = 0$, or $F(\theta_1, e_1) = H(\theta_1, e_1) = 0$ or $G(\theta_1, e_1) = H(\theta_1, e_1) = 0$, where:

$$F(\theta, e) \equiv \alpha'(\theta)\left(d'(e) + c \cdot \theta\right) - c \cdot \alpha(\theta)$$

$$G(\theta, e) \equiv \alpha(\theta)(y + d(e) + c \cdot \theta \cdot e) - \left(c \cdot \theta + d'(e)\right)(e \cdot \alpha(\theta) + q)$$

$$H(\theta, e) \equiv \alpha'(\theta)(y + d(e) + e \cdot c \cdot \theta) - c(e \cdot \alpha(\theta) + q)$$

These functions are independent of η. The partial derivatives of F have unambiguous signs:

$$F'_\theta = \alpha''(\theta)\left(d'(e)+c\cdot\theta\right)+c\cdot\alpha'(\theta)-c\cdot\alpha'(\theta)<0$$
$$F'_e = \alpha'(\theta)\cdot d''(e)>0$$

Consequently, function $F(.,.)$ is upward-sloping in the (θ,e) plane (see Figure 1). Second,

$$G'_e = -d''(e)\left(e\cdot\alpha(\theta)+q\right)<0$$
$$G'_\theta = \alpha'(\theta)\left(y+d(e)-e\cdot d'(e)\right)-c\cdot q$$

However, along $G(\theta,e)=0$, one has:

$$y+d(e)=\frac{c\cdot\theta\cdot q+d'(e)\left(e\cdot\alpha(\theta)+q\right)}{\alpha(\theta)}=\frac{q}{\alpha(\theta)}\cdot\left(c\cdot\theta+d'(e)\right)+e\cdot d'(e)$$

Therefore,

$$G'_\theta=\alpha'(\theta)\cdot\frac{q}{\alpha(\theta)}\cdot\left(c\cdot\theta+d'(e)\right)-c\cdot q=\frac{q}{\alpha(\theta)}F(\theta,e) \tag{31}$$

Consequently, in the (θ,e) plane function $G(.,.)$ is upward-sloping (respectively downward-sloping) at the left (respectively at the right) of function $F(.,.)$ and intersects function $F(.,.)$ horizontally (see Figure 1). Third,

$$H'_\theta = \alpha''(\theta)\left(y+d(e)+e\cdot c\cdot\theta\right)+e\cdot c\cdot\alpha'(\theta)-e\cdot c\cdot\alpha'(\theta)<0$$
$$H'_e = \alpha'(\theta)\left(d'(e)+c\cdot\theta\right)-c\cdot\alpha(\theta)=F(\theta,e)$$

Hence, in the (θ,e) space, function $H(.,.)$ is upward-sloping (respectively downward-sloping) above (respectively below) function $F(.,.)$ and intersects function $F(.,.)$ vertically (see Figure 1). This configuration guarantees the uniqueness of a solution to the system (30) [11].

[11] The proof is similar to the one of the uniqueness of equilibrium. One simply has to replace $\mathbb{S}$ by $-F$ and $\mathbb{W}$ by H.

We now show that the first-best solution (e_1, θ_1, u_1) maximizes total output net of search costs Y:

$$\max_{e,u,\theta} \quad Y = (1-u)\,y - u \cdot d\,(e) - c \cdot e \cdot \theta \cdot u \tag{32}$$
$$s.t. \quad : \quad e \cdot \alpha\,(\theta) \cdot u = q\,(1-u)$$

Denoting δ^Y the Lagrange multiplier, the first-order conditions are:

$$\left(\delta^Y =\right) \frac{d'\,(e) + c \cdot \theta}{\alpha\,(\theta)} = \frac{y + d\,(e) + e \cdot c \cdot \theta}{e \cdot \alpha\,(\theta) + q} = \frac{c}{\alpha'\,(\theta)}$$

Comparing these expressions with (30), it is obvious that the solution (e, θ) to problem (32) is exactly the first-best optimum $(e_{1,}\theta_1)$. Hence, the unemployment rate u is equal to u_1. At the first-best optimum, total output net of costs is therefore maximized. Consequently, efficiency and equity goals can be achieved separately.

To decentralize the first best, the State fixes the level of unemployment benefits to z_1. By assumption, it is also able to impose a search intensity e_1. Knowing the optimal value θ_1, let then the level of tax be given by $T_1 = \phi\,(\theta_1) - \omega_1$. Since, $\hat{\gamma}_1 = 0$, $z = z_1$ and $e = e_1$, the wage bargaining process implies $\omega_1 = z_1 - d\,(e_1)$. So, $w = \phi(\theta_1) = \omega_1 + T_1$ according to (15). Given this gross wage, the representative firm chooses its optimal level of vacancies so that $\theta = \theta_1$.

A.4 The second-best optimum

Let δ_2 (respectively ψ_2) denote the Lagrange multiplier associated with the flow equilibrium (respectively the incentive constraint). The first order conditions of problem (19) with respect to ω, z, e, u and θ are:

$$0 = (1-u_2)\left[v'(\omega_2)-\eta\right]+\psi_2\cdot\alpha(\theta_2)\cdot v'(\omega_2) \tag{33}$$

$$0 = u_2\left[v'(z_2-d(e_2))-\eta\right] \tag{34}$$
$$-\psi_2\left\{\alpha(\theta_2)\cdot v'(z_2-d(e_2))+v''(z_2-d(e_2))\cdot d'(e_2)\cdot(q+e_2\cdot\alpha(\theta_2))\right\}$$

$$0 = u_2\left[-d'(e_2)\cdot v'(z_2-d(e_2))-\eta\cdot c\cdot\theta_2+\delta_2\cdot\alpha(\theta_2)\right] \tag{35}$$
$$+\psi_2\left\{-d''(e_2)\cdot v'(z_2-d(e_2))+\left(d'(e_2)\right)^2\cdot v''(z_2-d(e_2))\right\}(q+e_2\cdot\alpha(\theta_2))$$

$$0 = v(z_2-d(e_2))-v(\omega_2)+\eta(\omega_2-y-z_2-c\cdot e_2\cdot\theta_2)+\delta_2(e_2\cdot\alpha(\theta_2)+q) \tag{36}$$

$$0 = -c\cdot e_2\cdot\eta\cdot u_2+\delta_2\cdot\alpha'(\theta_2)\cdot e_2\cdot u_2+ \tag{37}$$
$$\psi_2\cdot\alpha'(\theta_2)\cdot\left\{v(\omega_2)-v(z_2-d(e_2))-e_2\cdot d'(e_2)\cdot v'(z_2-d(e_2))\right\}$$

Let us first show that $\psi_2 > 0$. At the second best, the unemployed choose a search intensity e that solves:

$$0=\alpha(\theta_2)(v(\omega_2)-v(z_2-d(e)))-d'(e)\cdot v'(z_2-d(e))(q+e\cdot\alpha(\theta_2))$$

The incentive constraint (20) evaluated at the second best encompasses two constraints, namely

$$0\le\alpha(\theta_2)(v(\omega_2)-v(z_2-d(e_2)))-d'(e_2)\cdot v'(z_2-d(e_2))(q+e_2\cdot\alpha(\theta_2)) \tag{38}$$

and

$$0\ge\alpha(\theta_2)(v(\omega_2)-v(z_2-d(e_2)))-d'(e_2)\cdot v'(z_2-d(e_2))(q+e_2\cdot\alpha(\theta_2)) \tag{39}$$

Equation (38) (respectively (39)) requires that the chosen search intensity e be at least (at most) equal to e_2. According to Kuhn and Tucker conditions, the former (latter) constraint is associated with a Lagrange multiplier $\psi_2^+ \ge 0$ ($\psi_2^- \le 0$). Obviously, one has $\psi_2=\psi_2^+ + \psi_2^-$ and either $\psi_2^-=0$ or $\psi_2^+=0$. From an economic point of view, only the constraint $e \ge e_2$, namely (38), matters.

For the constraint $e \geq e_2$ ($e \leq e_2$) means that the net earnings ω_2 should be sufficiently (not too) higher than $z_2 - d(e_2)$ which is detrimental (beneficial) to the insurance objective. So $\psi_2^- = 0$ and $\psi_2 = \psi_2^+ \geq 0$. Finally, the incentive constraint (20) implies that $\omega_2 > z_2 - d(e_2)$, thereby $v'(\omega_2) < v'(z_2 - d(e_2))$. Therefore, $\psi_2 \neq 0$ according to (33) and (34). Consequently, $\psi_2 > 0$.

It will now be shown that one has $G(\theta_2, e_2) > 0$ at the second-best optimum. Dividing first-order condition (36) by η, adding $d(e_2)$ on both sides and rearranging yields:

$$y + d(e_2) + c \cdot e_2 \cdot \theta_2 = \frac{\delta_2}{\eta}(e_2 \cdot \alpha(\theta_2) + q) - \frac{v(\omega_2) - v(z_2 - d(e_2))}{\eta_2} + \omega_2 - z_2 + d(e_2) \tag{40}$$

Multiplying both sides by $\alpha(\theta_2)$ yields:

$$\begin{aligned} \alpha(\theta_2)(y + d(e_2) + c \cdot e_2 \cdot \theta_2) &= \frac{\delta_2 \cdot \alpha(\theta_2)}{\eta}(e_2 \cdot \alpha(\theta_2) + q) \\ -\frac{\alpha(\theta_2)[v(\omega_2) - v(z_2 - d(e_2))]}{\eta} &+ \alpha(\theta_2)(\omega_2 - z_2 + d(e_2)) \end{aligned}$$

Taking the incentive constraint (20) into account, the previous equality can be rewritten:

$$\begin{aligned} \alpha(\theta_2)(y + d(e_2) + c \cdot e_2 \cdot \theta_2) &= \frac{e_2 \cdot \alpha(\theta_2) + q}{\eta}\left[\delta_2 \cdot \alpha(\theta_2) - d'(e_2) \cdot v'(z_2 - d(e_2))\right] \\ &\quad + \alpha(\theta_2)(\omega_2 - z_2 + d(e_2)) \end{aligned}$$

The right-hand side of the last equality can be substituted in the definition of function G evaluated at (θ_2, e_2). After some manipulations, this yields:

$$\begin{aligned} G(\theta_2, e_2) &= \frac{e_2 \cdot \alpha(\theta_2) + q}{\eta}\left[\delta_2 \cdot \alpha(\theta_2) - d'(e_2) \cdot v'(z_2 - d(e_2)) - c \cdot \theta_2 \cdot \eta\right] \\ &\quad + \alpha(\theta_2)(\omega_2 - z_2 + d(e_2)) - (e_2 \cdot \alpha(\theta_2) + q)\, d'(e_2) \end{aligned}$$

Using once again the incentive constraint, $G(\theta_2, e_2)$ can be restated as:

$$\begin{aligned} G(\theta_2, e_2) &= \frac{e_2 \cdot \alpha(\theta_2) + q}{\eta}\left[\delta_2 \cdot \alpha(\theta_2) - d'(e_2) \cdot v'(z_2 - d(e_2)) - c \cdot \theta_2 \cdot \eta\right] \\ &\quad + \frac{\alpha(\theta_2)}{v'(z_2 - d(e_2))}\left[v'(z_2 - d(e_2)) \cdot (\omega_2 - z_2 + d(e_2)) - v(\omega_2) + v(z_2 - d(e_2))\right] \end{aligned} \tag{41}$$

However, the first-order condition (35) insures that:

$$\begin{aligned} & \delta_2 \cdot \alpha(\theta_2) - d'(e_2) \cdot v'(z_2 - d(e_2)) - c \cdot \theta_2 \cdot \eta \\ = & \frac{\psi_2}{u_2} \left[d''(e_2) \cdot v'(z_2 - d(e_2)) - \left(d'(e_2)\right)^2 \cdot v''(z_2 - d(e_2)) \right] (q + e_2 \cdot \alpha(\theta_2)) > 0 \end{aligned}$$

So, the first term on the right hand side of (41) is positive. In addition, the concavity of $v(.)$ implies that :

$$v(\omega_2) - v(z_2 - d(e_2)) < v'(z_2 - d(e_2)) \cdot (\omega_2 - z_2 + d(e_2))$$

by which the second term on the right hand side of (41) is positive too. Therefore, function G evaluated at the second-best optimum is positive while the same function was zero and reached a maximum evaluated at the first-best optimum. So, $e_2 < e_1$.

Next, it will be shown that $H(\theta, e) < 0$ at the second-best optimum. The first-order condition (37) together with the incentive constraint (20) gives:

$$c \cdot e_2 \cdot \eta \cdot u_2 = \delta_2 \cdot \alpha'(\theta_2) \cdot e_2 \cdot u_2 + \psi_2 \cdot \alpha'(\theta_2) \cdot \frac{q}{\alpha(\theta_2)} \cdot v'(z_2 - d(e_2)) \cdot d'(e_2)$$

Substituting the flow equilibrium (1) yields:

$$c = \alpha'(\theta_2) \left\{ \frac{\delta_2}{\eta} + \frac{\psi_2}{\eta} \cdot \frac{1}{1 - u_2} \cdot v'(z_2 - d(e_2)) \cdot d'(e_2) \right\}$$

Substituting this expression and equation (40) into $H(\theta_2, e_2)$, i.e. in $\alpha'(\theta_2)(y + d(e_2) + c \cdot e_2 \cdot \theta_2) - c(e_2 \cdot \alpha(\theta_2) + q)$ leads to

$$\begin{aligned} H(\theta_2, e_2) &= \alpha'(\theta_2) \left\{ \omega_2 - z_2 + d(e_2) - \frac{v(\omega_2) - v(z_2 - d(e_2))}{\eta} \right. \\ & \left. - \frac{\psi_2}{\eta(1 - u_2)} \cdot v'(z_2 - d(e_2)) \cdot d'(e_2) \cdot (e_2 \cdot \alpha(\theta_2) + q) \right\} \end{aligned}$$

Taking (20) into account, this expression can be rewritten as:

$$\begin{aligned} H(\theta_2, e_2) = \alpha'(\theta_2) \quad & \left\{ \omega_2 - z_2 + d(e_2) - \frac{v(\omega_2) - v(z_2 - d(e_2))}{\eta} \right. \\ & \left. - \frac{\psi_2 \cdot \alpha(\theta_2)}{\eta(1 - u_2)} [v(\omega_2) - v(z_2 - d(e_2))] \right\} \end{aligned}$$

From first-order condition (33)

$$\frac{\psi_2 \cdot \alpha(\theta_2)}{\eta(1-u_2)} = \frac{1}{v'(\omega_2)} - \frac{1}{\eta}$$

Consequently,

$$H(\theta_2, e_2) = \alpha'(\theta_2)\left\{\omega_2 - z_2 + d(e_2) - \frac{v(\omega_2) - v(z_2 - d(e_2))}{v'(\omega_2)}\right\}$$

Finally, concavity of function $v(.)$ implies that:

$$0 < v'(\omega_2) \cdot (\omega_2 - z_2 + d(e_2)) < v(\omega_2) - v(z_2 - d(e_2)) < v'(z_2 - d(e_2)) \cdot (\omega_2 - z_2 + d(e_2))$$

Therefore, function H evaluated at the second-best optimum (θ_2, e_2) is always negative. This implies that $\theta_2 > \theta_1$.