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# ABSTRACT

## Modeling Life-Cycle Earnings Risk with Positive and Negative Shocks*

We study workers' idiosyncratic earnings risk over the life-cycle using a German administrative data set. Positive and negative earnings shocks both contain a highly persistent component. The variance and average size of positive persistent shocks is decreasing over the life-cycle. The (absolute) size of negative persistent shocks is increasing. The probability to experience either of these shocks is U-shaped in age; during prime-age it is around 35 percent. Negative transitory shocks are relatively larger and more dispersed than positive transitory shocks. Their size and variance are increasing over the life-cycle. Large persistent positive shocks early in life generate large wealth holdings for the top one percent of workers in an incomplete markets model. Moreover, age-varying risk implies a linear increase in consumption inequality late in working life.

**JEL Classification:** E21, E24, J31

**Keywords:** life-cycle, earnings risk, wealth dispersion

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# 1 Introduction

Workers face considerable earnings fluctuations during their working life. Observables, such as age, time effects or regional specific developments explain some of those fluctuations. However, most earnings variations cannot be traced back to such predictable patterns and are specific to an individual worker: Some workers become promoted, other demoted, health shocks limit the type of available work, unemployment reduces earnings, and a worker may find a firm paying a better wage than his current employer.

Early empirical studies such as Lillard and Weiss (1979), MaCurdy (1982), and Abowd and Card (1989) propose time series models where idiosyncratic log earnings changes result from a mean zero combination of persistent and transitory shocks that are independent of age. Yet, a large body of work on the labor market shows that the size and the nature of risk is age-varying. Topel and Ward (1992) show that workers' early careers are characterized by a large amount of job-to-job transitions and periods of non-employment. Jung and Kuhn (2015) find that prime-aged workers' careers are relatively stable, until, late in their working life, the risk of losing a high tenured job increases.

We use a German administrative data set for males[1] to highlight stylized facts of age-varying non-predicable earnings fluctuations. Positive residual earnings growth is relatively rare early in life but growth rates are large in magnitude and highly dispersed. Contrary, negative residual earnings growth is rare late in life but changes are large in magnitude[2]. As workers age, positive residual earnings innovations become more likely, but the size and dispersion of these innovations shrinks. Moreover, the probability of experiencing a small -or no earnings change whatsoever- becomes initially larger as workers age, which leads to an increase in the kurtosis of the distribution of residual earnings growth over the life-cycle.

Motivated by these age-varying patterns, we estimate a time series model of earnings dynamics that allows us to identify idiosyncratic earnings risk. In the model, workers' log earnings have a positive and a negative component. With age-varying probabilities, workers draw either an innovation to their positive component, an innovation to their negative component, or a purely transitory shock. An innovation to the positive (negative) component is a mixture of a transitory and a persistent log-normally distributed shock. We allow the means and variances of these shocks to vary by age.

[1] The moments for females are available upon request.

[2] Earnings rise on average when young and decline when old. We study deviations from this predictable age pattern.

The model provides a rich framework to study age-varying earnings risk. It allows the probability to receive large positive shocks (i.e., promotions) to vary over the life-cycle. Analogously, it allows for age-varying probabilities in large negative shocks (i.e., health shocks). Modeling shocks as mixtures of persistent and transitory shocks captures a wide range of labor market phenomena. Consider for example the earnings pattern Jacobson et al. (1993) document for displaced workers. Earnings, resulting from non-employment, are lowest in the year of displacement, recuperate somewhat afterwards, but stay permanently lower than before the displacement. Our model would identify this as an innovation to the negative component of log earnings. The initial reduction in hours would be identified as the transitory shock. The longer lasting earnings losses would be identified as the persistent shock. The model estimation yields the following set of results:

1. Both persistent positive and negative shocks are close to permanent.
2. The variance of persistent positive shocks is decreasing over the life-cycle. It is small and close to constant for negative shocks.
3. The average size of positive (negative) shocks is decreasing (increasing) over the life-cycle.
4. The probability to draw a negative shock is initially larger but decreases over the life-cycle. The probability to draw a positive shock is initially smaller but increases over the life-cycle.
5. At prime age, only 35 percent of workers receive any persistent shock.
6. Transitory shocks are quantitatively more important to explain negative earnings growth and their variance increases with age.

Our findings contribute to the existing literature on age-varying idiosyncratic earnings risk. Blundell et al. (2015), using Norwegian data, and Karahan and Ozkan (2013) and Lopez-Daneri (2016), using US data, find that the variance of shocks declines over the worker's life-cycle. However, they do not differentiate between positive and negative innovations. Guvenen et al. (2016) allow the probability to sample from different type of shocks to depend on age, but assume the mean and variance of these shocks to be constant over the life-cycle. Our results imply that the decreasing variance is driven by a decreasing variance of positive shocks that become more likely as workers age. Negative shocks, particularly transitory shocks, become more dispersed with age. This shift from rare but large positive shocks towards rare but large negative shocks over the life-cycle also implies that the skewness of earnings growth becomes more negative as workers age, a phenomenon also documented by Guvenen et al. (2016).

Workers that experience neither an innovation to their positive nor negative component of earnings receive a transitory shock that we estimate to have little dispersion and to be small in magnitude. We think of such a shock to arise for example from temporary fixed wages in the presence of inflation, or small temporary changes in hours. This type of shock represents the only source of contemporaneous earnings risk for 65 percent of workers at age 45.

To understand the implications of age-varying risk for consumption and savings decisions, we introduce the estimated earnings uncertainty into a model of incomplete insurance markets. We contrast the results to the literature that studies the consequences of normally distributed transitory and persistent shocks for consumption inequality (Deaton and Paxson (1994), Blundell et al. (2008), Kaplan and Violante (2010)), wealth inequality (Aiyagari (1994)), and the welfare costs of incomplete insurance markets (Heathcote et al. (2010b)). Regarding the latter, we find that the welfare losses are of similar magnitude across the different risk processes. In our age-varying risk process, large negative shocks are unlikely early in life when precautionary savings are lowest. Moreover, a large fraction of workers do not receive any persistent shock in a given year, particularly at prime-age. Both factors decrease the welfare costs of incomplete markets. Working in the opposite direction, shocks have heavy tails which increases the welfare costs. These opposing effects almost cancel each other.

At the same time, allowing jointly for age-varying shocks and sampling probabilities helps our model to reconcile two stylized patterns of inequality from the data. First, large but rare positive persistent shocks early in life lead to a small fraction of workers who become extremely rich relative to the median worker. These workers have incentives to accumulate large life-cycle wealth. As a consequence, our model features substantially more wealth inequality at the top of the distribution relative to a standard risk process. Second, as pointed out by Guvenen (2007), an increasing amount of precautionary savings over the life-cycle together with an age-invariant earnings risk implies a counterfactual concave consumption inequality profile in the standard model. Guvenen (2007) proposes learning over deterministic differences in earnings growth as one possible solution. In our model, a shift towards more persistent and large negative shocks offsets the increase in self-insurance late in life.

The rest of the paper is organized as follows. Section 2 describes the German data set. Section 3 presents a set of moments of residual earnings growth over the life-cycle and Section 4 the econometric model. Finally, Section 5 introduces our earnings process to a life-cycle savings model.

# 2 Data and Sample Construction

## 2.1 Data Description

We use the German labor income and demographic data from the *Sample of Integrated Labour Market Biographies* for the years 1975-2010 (*SIAB R 7510*). The data originates from the German notification procedure for social security. This requires employers to inform social security agencies about any of their employees' working spells. The data covers the population of German employment liable to social security, which excludes civil servants, self employment and regular students (about 20% of the employment population). From this population, the German employment agency draws a 2% random sample of individuals' careers. In total, the data has information on 1,594,466 individuals and 41,390,318 unique person-year records. It includes demographic information (such as gender, year of birth and region), socioeconomic information (such as education level and daily wage) and industrial information (such as occupation, economic activity and firm identifiers). Table A1 in the Appendix provides some selected summary statistics of the data.

The *SIAB R 7510* data set has several advantages over common survey data. First, it does not suffer from attrition. Second, it provides a large number of career-long earnings profiles for different cohorts. And third, administrative data entail less measurement error compared to survey data sets. The main disadvantage of the data arises from earnings (daily wages) that are top-coded by the limit liable to social security. This affects on average around 11% of observations by year. We follow Daly et al. (2016) and impute daily wages from an extrapolated Pareto density fitted to the non top-coded upper end of the observed distribution for each year. By doing so, we assume earnings growth behaves similar for the top decile of the German earnings distribution. Alternatively, we could drop workers affected by top-coding. The moments of residual earnings growth are almost identical for the two approaches. We opt for the former approach because it allows us to infer the entire cross-sectional earnings distribution of the German employment population.

## 2.2 Sample Construction

Our analysis focuses on earnings risk for the subset of workers with a high attachment to the labor force. Put differently, we omit any selection resulting from earnings shocks[3]. We drop all observations on apprenticeships, partial retirement, and part-time workers not eligible for unemployment benefits. Moreover, we consider only German male workers to avoid female decisions over maternity leave. We consider a worker employed within a year when he is contracted for at least 90 days of that year. Thus, our analysis abstracts from earnings shocks arising from long-term unemployment.

Following the past literature that focuses on workers with high attachment to the labor market, we keep for each individual the longest consecutive spell of earnings with at least 7 years of observations (see Meghir and Pistaferri (2004), Guvenen (2009) and Hryshko (2012)). The age range under consideration is of some importance because we want to avoid misinterpreting predictable earnings changes as shocks. For the time period of our sample in Germany, a high school degree takes from 9 to 13 years of schooling and male workers are obliged to perform military service (1 year), afterwards. Most workers enter professional training (2-3 years), thereafter. As a result, we can expect that workers have made a full transition to the labor market by the age of 24. Also, the intended retirement age in Germany used to be 65. Arnds and Bonin (2002) show that the average retirement age is around the age of 60 in Germany. They find that early retirement benefits targeted to those of age 60 and older lead to substantial early retirement. Moreover, generous unemployment benefits for high tenured workers often lead to an effective retirement age of 55. To avoid these endogenous decisions, we restrict the panel to workers aged 24 to 55.

We adjust earnings for inflation using the German consumer price index of 2010 (i.e. divide annual earnings by the consumer price index)[4]. Following Dustmann et al. (2009), we drop real daily earnings that are below 5 euros. Finally, we restrict the data to individuals working in West-Germany, as East-German observations are available only after 1991. We assign each individual to a birth cohort, defined as being born in a 7 year interval starting in 1923. On average, we have 27,928 individuals per birth cohort. Our final sample contains information for 251,352 individuals with a total of 3,566,212 person-year observations.

For each year, we aggregate an individual's earnings across all job spells. Therefore, changes in earnings may arise from a change in working hours, a change of employer, an unemployment spell, bonuses, promotions, etc. Workers entering the sample for the first time are statistically expected to enter in the middle of the

[3]See Low et al. (2010) for an analysis that allows employment selection upon earnings shocks.

[4]We obtain the consumer price index from OECD data; https://data.oecd.org/price/inflation-cpi.htm.

year. Daly et al. (2016) show that this may lead to a bias in the estimates of permanent shocks. To avoid the bias, we assume that earnings in the months the individual is not observed are the same as for those observed.

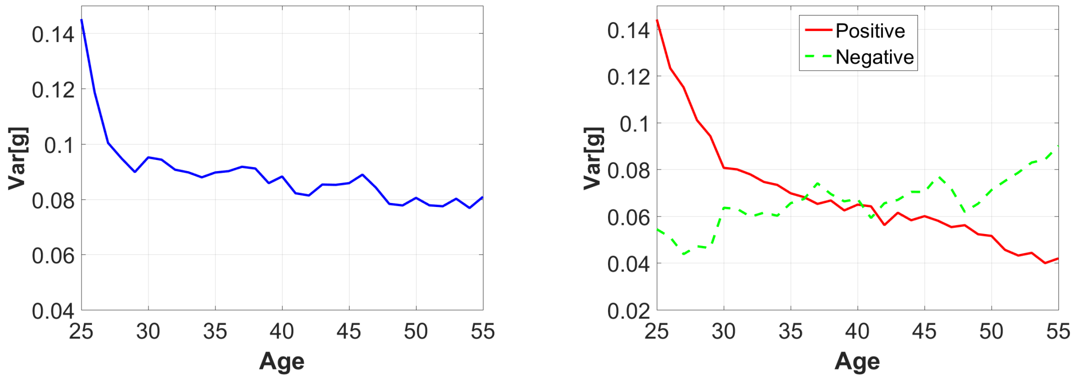
Most studies of earnings dynamics focus on US data. Figure A2 in the Appendix shows that key life-cycle moments of earnings growth are remarkably similar for the two economies. Yet, some institutional differences are worth highlighting. OECD (2011) shows that employment protection legislation plays an important role in shaping cross-country earnings uncertainty. Germany has a strong employment protection for high tenured workers that leads to a lower probability of becoming unemployed but also to a lower probability to find a new job for the non-employed. Moreover, for many sectors, wage floors are centrally bargained in Germany implying downward nominal wage rigidity and more concentrated earnings variations for workers. The relatively tighter institutional framework in Germany leads to a considerable lower labor turnover in comparison to the US. Bachmann et al. (2013) show that both the German accession and separation rate of workers within establishments are only 60% of the US level. Topel and Ward (1992) show that switching firms is a major source of earnings volatility. Resulting from these institutions, the variance of earnings growth is two to three times larger in US data.

Our interest is in annual earnings changes that are idiosyncratic to the individual. To this end, we remove predictable changes in earnings growth by running cross-sectional regressions on a dummy of workers' education, age, interaction of the two[5], year, region of residence, and 14 major industries[6]. Figure A1 in the Appendix, shows that residual earnings growth features both a calendar time and cohort effect. Following Blundell et al. (2015), we average all moments of residual earnings growth across cohorts to eliminate these types of time effects, assigning equal weight to all cohorts[7]. Therefore, our results can be interpreted as the risk a typical cohort is facing.

[5]In this way, we allow for an arbitrary amount of non-linearities.

[6]Quite likely, it is impossible for the worker to predict wage changes conditional on all these observables; therefore, we may underestimate earnings shocks. However, our moments are almost unchanged when excluding some of the observables.

[7]Averaging moments across cohorts, also partially controls for the fact that before 1984 one time payments were not reported in the data set.




(a) $Var[g_{i,h}]$ (b) $Var[g_{i,h}|g_{i,h} \gtrless 0]$

Notes: Figure Ia displays the variance of residual earnings growth by age. Figure Ib displays the variance of residual earnings growth by age conditional on residual earnings growth being positive (negative).

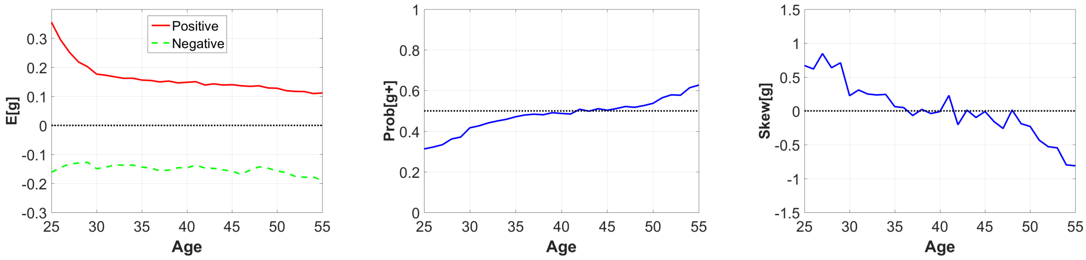
Figure I: Variance of Residual Earnings Growth

# 3 Moments of Residual Earnings Growth

This section highlights the salient features of residual earnings over the life-cycle that we want our econometric model to match. Our focus is on the dynamics of individual earnings; thus, we mostly concentrate on residual earnings growth (we use interchangeably the terms residual earnings growth/innovations/changes). We show that early in workers' working-life, positive residual earnings growth is rare but large and highly dispersed. Positive changes become more likely as workers age, and the size and variance of these changes become smaller. Contrary, negative changes are most likely early in life. Their variance increase throughout the life-cycle. Earnings growth is auto-correlated up to lag two, but the magnitude is relatively larger for negative changes. The kurtosis of earnings growth is well above a normal distribution and increases concavely over the life-cycle. Finally, cross-sectional earnings inequality increases close to linear over the life-cycle. We now discuss each of these points in more detail.

A common way to identify earnings shocks is studying the covariance structure of residual earnings growth, $g_{i,h}$, where $i$ denotes individual and $h$ denotes age. Figure Ia plots its cross-sectional variance over age. Residual earnings growth becomes significantly less dispersed over the life-cycle; its variance declines by almost 50% between the age of 25 and 55. Guvenen et al. (2016) demonstrates a similar pattern for the US.

In a standard decomposition into a mean zero persistent and transitory component, as in Blundell et al. (2015), one would conclude that the size of earnings




(a) $\mathbb{E}[g_{i,h}|g_{i,h} \gtrless 0]$ (b) $Prob\,(g_{i,h} > 0)$ (c) $Skew[g_{i,h}]$

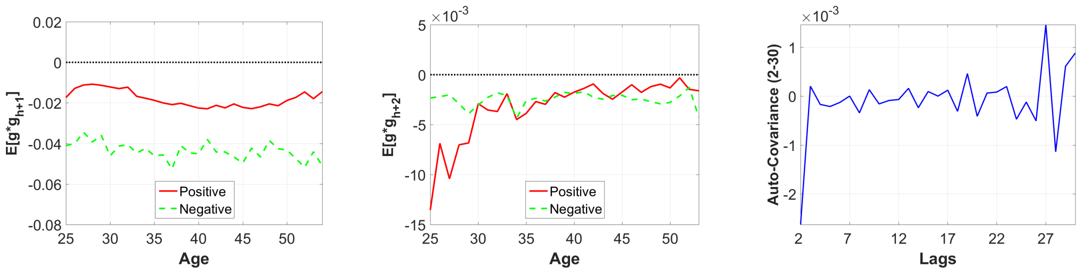
Notes: Figure IIa displays the average of residual earnings growth conditional on residual earnings growth being positive (negative) at each age. Figure IIb displays the fraction of residual earnings growth that is positive by age. Figure IIc displays the skewness of residual earnings growth by age.

Figure II: Conditional Means, Probabilities, and Skewness

shocks gradually decrease over the life-cycle. To get a richer understanding of the decreasing variance, consider conditional changes (positive, $g^+_{i,h}$, and negative, $g^-_{i,h}$, residual earnings growth, with $g_{i,h} \neq 0 = \mathbb{1}\{g_{i,h,} > 0\}g^+_{i,h} + \mathbb{1}\{g_{i,h,} < 0\}g^-_{i,h}$). Figure Ib displays the conditional variances of these innovations, $Var[g_{i,h}|g_{i,h,} > 0]$ and $Var[g_{i,h}|g_{i,h,} < 0]$. The figure unveils an opposed behavior of the variance of positive and negative changes. The variance of positive growth decreases over the life-cycle, most pronounced early in life. The variance of negative growth is linearly increasing in age, reaching its peak at the end of the working life.

Figure IIa shows that the average size of conditional earnings innovations closely tracks their variances. Positive residual earnings growth is on average large early in life, and it becomes smaller throughout the life-cycle. Mean negative residual earnings growth is close to constant until the age of 50 and becomes larger in absolute size, afterwards. Figure IIb plots the probability of observing a positive innovation at each age, $Prob\,(g_{i,h} > 0)$. There is large heterogeneity over the life-cycle that reconciles the different means of conditional shocks. Early in life, close to 70% of innovations are negative, but the probability of a positive change is increasing throughout the working life, reaching 63% at the age of 55.

Both the age-varying sampling probabilities and the average size of innovations naturally map onto an age-varying skewness of these innovations. Figure IIc shows that the cross-sectional distribution of residual earnings growth is initially positively skewed, and skewness turns negative around the age of 40. The fall in skewness accelerates after the age of 48, representing the increased occurrence of large negative innovations. Guvenen et al. (2016) show a similar declining pattern of skewness in US earnings growth data, although they find that skewness




(a) $\mathbb{E}[g_{i,h+1}g_{i,h}|g_{i,h} \gtrless 0]$ (b) $\mathbb{E}[g_{i,h+2}g_{i,h}|g_{i,h} \gtrless 0]$ (c) $\mathbb{E}[g_{i,h+k}g_{i,h}]$

Notes: Figure IIIa and IIIb display the first and second order autocovariance of residual earnings growth by age conditional on residual earnings growth today being positive (negative). Figure IIIc displays the unconditional autocovariance of residual earnings growth for leads 2 and beyond.

Figure III: Autocovariances of Residual Earnings Growth

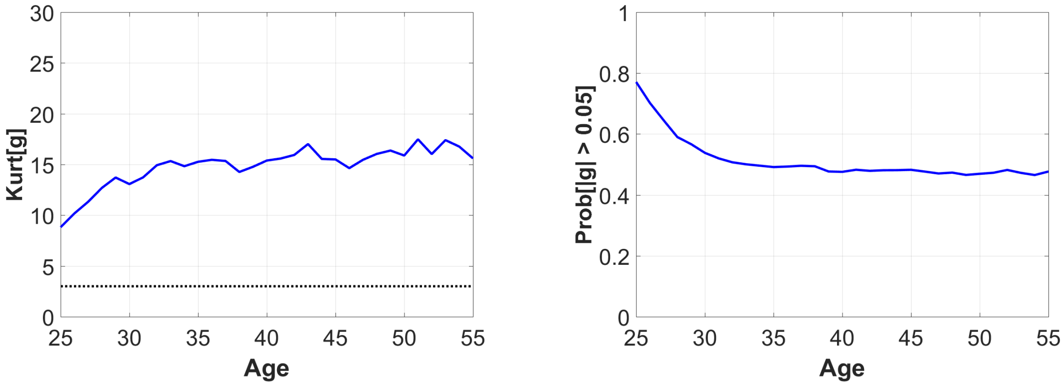
is more negative throughout the life-cycle[8]. Negative skewness is mostly driven by hours changes. We employ a more stringent employment definition than they do; a worker needs to be contracted at least 90 days per year. As a result, hours variation in our sample is smaller and skewness less negative.

So far, we have not differentiated between different types of positive and negative changes. The literature on earnings dynamics commonly decomposes shocks into a persistent (promotions, large health shocks) and a transitory (bonuses, temporary sickness, temporary demand shifts) component. In order to delve into the different types of shocks, we study the first and second conditional autocovariance. A negative first autocovariance of residual earnings growth implies that part of the current residual earnings growth is offset next year, i.e., it provides information regarding the amount of mean reversion following a shock. The second autocovariance identifies whether this mean reversion lasts longer than one year.

Figures IIIa and IIIb display the conditional first and second autocovariances of residual earnings growth, respectively. The positive first autocovariance is small relatively to the negative first autocovariance. Put differently, mean reversion is more prevalent for negative earnings growth. The second autocovariance is negative suggesting mean reversion takes at least an extra year. Figure IIIc displays the (unconditional) autocovariance for longer lags. Quantitatively speaking, modeling persistence in transitory shocks beyond lags 1-2 is of little importance; all autocovariances are close to zero.

The distant lags of the unconditional autocovariance also provide information concerning the presence of heterogeneity in deterministic (but unobserved) earnings growth rates. As discussed in Hryshko (2012), the autocovariance function

[8]Figure A2b displays their data.




(a) $Kurt[g_{i,h}]$ (b) $Prob(|g_{i,h}| > 5\%)$

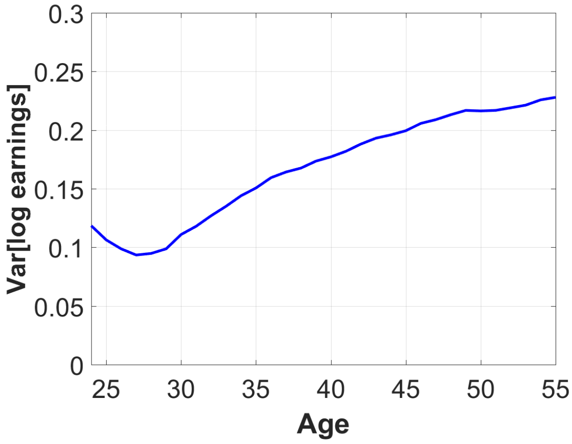
Notes: Figure IVa displays the kurtosis of residual earnings growth by age. The black dashed line displays the kurtosis of a normal distribution. Figure IVb displays the fraction of residual earnings growth that are larger than 5% in absolute value by age.

Figure IV: Kurtosis of Residual Earnings Growth

of earnings growth converges to the variance of these heterogeneous profiles. As indicated by Figure IIIc, the scope for deterministic profiles playing an important role in labor earnings dynamics is limited. The distant lags of the autocovariance oscillate around zero and we do not observe convergence to a positive constant.

Guvenen et al. (2016) highlight that earnings uncertainty features fat tail behavior. Figure IVa shows that the magnitude and life-cycle behavior of kurtosis in residual earnings growth is very similar in Germany. Kurtosis increases in a concave fashion throughout the life-cycle. At its peak, it is 5 times larger than what is suggested by a normally distributed shock. The large kurtosis implies that a substantial fraction of workers experience none, or very small residual earnings changes. Figure IVb displays the fraction of residual earnings growth by age that are above 5% in magnitude. The profile is an inverse image of kurtosis over the life-cycle. Strikingly, at prime-age, 50% of workers experience residual earnings changes of a magnitude smaller than 5 percent (in absolute value). In contrast, early in life, close to 80% of innovations are of a magnitude above 5%.

Finally, Figure V shows how these residual earnings dynamics map onto cross-sectional residual earnings inequality. Inequality is falling for the first three years and reaches a low of 0.1. Cross-sectional inequality is substantially lower than in the US. Guvenen et al. (2016) report a value of 0.47 at the same point of the life-cycle. Inequality accelerates up to age 40 when its growth slows down somewhat. In total, between the age of 27 and 55, similar to US data, residual earnings inequality more than doubles.




Notes: Figure V displays the variance of log residual earnings by age. The figure displays the age component obtained from regressing the variance of log earnings on a cohort and age dummies.

Figure V: Cross-sectional Residual Earnings Dispersion

## 3.1 Sources of Earnings Innovations

Taken together, the data suggests that earnings fluctuations are particularly large early and late in life. At prime-age, most workers experience no, or only small, changes in earnings. We finish this section by relating large earnings changes early (before age 30) and late (after age 50) in life to observable labor market outcomes.

First, we consider workers younger than age 30. We define a large positive innovation as a positive change in residual log earnings of 0.4 (or approximately 50%) or above. Starting with the work of Topel and Ward (1992), a large literature studies the role of job-ladder effects for early career building. Consistent with this idea, we find that in 30% of cases where we observe a large positive earnings change early in life, the individual changed his employer. Topel and Ward (1992) also show that young workers' careers are characterized by repeated non-employment spells between jobs. In this vein, we ask how many of the large positive innovations in the data can be explained by workers increasing the amount of days worked in a year[9]. We define a "substantial" increase in days worked as one where the amount of contracted days increases by more than 30 days from one year to the next. Around 33% of large positive earnings innovations early in life are associated with such an increase in working days.

Turning to workers older than age 50, we define a large negative innovation as a negative change of residual log earnings of -0.6931 (or approximately -50%) or below. Jacobson et al. (1993) show that in incidences where workers lose a highly

[9]Optimally we would like to study the change in work hours. However, our data has only information on the amount of days contracted.

tenured job, their wages at reemployment are substantially lower. To understand the importance of this effect for elderly workers in our data, we calculate the share of large negative earnings changes associated with the worker changing employers. We find that in only 8% of cases where we observe a large negative innovation the worker changed employers. Put differently, losing a high paying job and reentering with a lower paying job is not a common phenomenon of elderly German workers. Instead, large negative residual earnings changes are predominantly associated with a reduction in working days. In 72% of the cases where we observe a large negative earnings change workers reduce their amount of working days by at least 30 per year.

# 4 A Time Series Model of Earnings Dynamics

We now present our time series model of residual log earnings and discuss its implications in comparison to other models of idiosyncratic earnings uncertainty.

## 4.1 Model

We model residual log earnings as the sum of a deterministic and a stochastic component:

$$y_{i,h} = \underbrace{\alpha_i + \beta_i h}_{\text{profile heterogeneity}} + \underbrace{u_{i,h}}_{\text{stochastic component}} \tag{1}$$

where $\alpha_i \sim N(0, \sigma_\alpha^2)$ reflects individual initial heterogeneity and $\beta_i \sim N(0, \sigma_\beta^2)$ reflects deterministic (unobserved) heterogeneity in life-cycle earnings growth. For simplicity, we impose $Corr(\alpha, \beta) = 0$[10]. Motivated by the different dynamics that positive and negative residual earnings growth displays throughout the life-cycle, we allow the stochastic component $(u_{i,h})$ to consist of a positive and a negative component and an i.i.d. error:

$$u_{i,h} = \underbrace{W_{i,h}^{+}}_{\text{positive}} + \underbrace{W_{i,h}^{-}}_{\text{negative}} + \underbrace{\iota_{i,h}^{n}}_{\text{neutral}}, \tag{2}$$

where $\iota_{i,h}^n \sim N(\mu_{\iota^n,h}, \sigma^2_{\iota^n,h})$ is a transitory shock to earnings that realizes at each age. By differentiating between a positive and a negative component in residual

[10] As already hinted at by the autocovariance function of residual earnings growth, we find $\sigma_\beta$ close to zero. Hence, any possible correlation with individual initial heterogeneity is of little importance.

log earnings, we allow positive and negative shocks to have different time series properties. Specifically, the positive component, $W^+_{i,h}$, and the negative component, $W^-_{i,h}$, contain both a persistent and a transitory part:

$$W^+_{i,h} = \underbrace{w^+_{i,h}}_{\text{persistent}} + \underbrace{\tau^+_{i,h}}_{\text{transitory}} \tag{3}$$

$$W^-_{i,h} = \underbrace{w^-_{i,h}}_{\text{persistent}} + \underbrace{\tau^-_{i,h}}_{\text{transitory}} \tag{4}$$

$$w^j_{i,h} = \rho^j w^j_{i,h-1} + \xi^j_{i,h} \quad \text{for } \ j = -, + \tag{5}$$

$$\tau^j_{i,h} = \theta^j \iota^j_{i,h-1} + \iota^j_{i,h} \quad \text{for } \ j = -, +. \tag{6}$$

Thus, innovations to the positive and the negative component are a mixture of a persistent ($\xi^j_{i,h}$) and a transitory ($\iota^j_{i,h}$) shock. We model the transitory components as a $MA(1)$ processes to account for the first and second order conditional autocovariances that we observe in the data[11]. The mixture of persistent and transitory shocks allows us to capture a wide range of economic phenomena. For example, a bad health shock may reduce earnings persistently by restricting the type of performable work, and additionally reduces earnings transitionally by lowering the amount of days worked initially.

We allow the probability to receive innovations from the positive and negative components to vary with age. This allows us to capture the age variation in the fraction of positive and negative earnings changes observed in Figure IIb. Mutually exclusive, and at each age, an individual draws with probability $p^-_{i,h}$ an innovation to his negative component, (both $\xi^-_{i,h}, \iota^-_{i,h}$), and with probability $p^+_{i,h}$ an innovation to his positive component, (both $\xi^+_{i,h}, \iota^+_{i,h}$). With probability $1-p^+_{i,h}-p^-_{i,h}$ he draws neither[12]. We specify second order polynomials in age for these probabilities to capture possible non-linearities in the data[13]:

$$p^j_{i,h} = \delta^j_I + \delta^j_{II} h + \delta^j_{III} h^2 \quad \text{for } \ j = -, +. \tag{7}$$

The persistent and the transitory shocks to the positive and negative components

[11] We assume that shocks are independent across time. Meghir and Pistaferri (2004) allow the variance of shocks to follow an ARCH process. Arellano et al. (2015) allow the properties of past shocks to be altered by today's shocks

[12] In particular, we obtain a draw from a uniform distribution, $s_{i,h} \sim U(0,1)$, for each worker at each age, and assign the innovation to the negative component of that worker when $s_{i,h} \in [0, p^-]$. Similarly, we assign an innovation to the positive component of that worker when $s_{i,h} \in (p^-, p^+ + p^-]$. Finally, we assign no innovation to these components when $s_{i,h} \in (p^+ + p^-, 1]$.

[13] We found that moving to a third order polynomial provides little improvement in the model.

follow age-varying log-normal distributions[14]:

$$\xi^{+}_{i,h} \sim exp(N(\mu^{+}_{h}, \sigma^{2}_{\xi^{+},h})), \quad \xi^{-}_{i,h} \sim -exp(N(\mu^{-}_{h}, \sigma^{2}_{\xi^{-},h})) \tag{8}$$

$$\iota^{+}_{i,h} \sim exp(N(\mu^{+}_{h}, \sigma^{2}_{\iota^{+},h})), \quad \iota^{-}_{i,h} \sim -exp(N(\mu^{-}_{h}, \sigma^{2}_{\iota^{-},h})) \tag{9}$$

We opt for a log-normal distribution partly for convenience. With the log-normal specification, the tail of the positive (negative) shock distribution does not cross into the negative (positive) domain, providing stability in the implied moments of the process, particularly the autocovariances. What is more, the log-normal assumption implies that our model is able to match the shape of the distribution of residual earnings growth. The large kurtosis and the fraction of shocks smaller than 5% (Figures IVa and IVb) imply that residual earnings growth is centered around zero and exhibits fat tails. Figure A3 in the Appendix displays the resulting density function of residual earnings growth at age 36. The figure also shows that the tail behavior of the two log-normally distributed transitory shocks together with the normally distributed shock, $\iota^{n}_{i,h}$, imply a similar density function.

Figure Ib shows that the dispersion of positive and negative residual earnings growth varies significantly by age. To accommodate for these age-varying conditional variances, we allow the dispersion parameters in equations (8) and (9) to vary with age in linear fashions:

$$\sigma^{j}_{k,h} = \gamma^{j}_{a,k} + \gamma^{j}_{b,k} h \;\; \text{for} \;\; j = -,+ \;\; \text{and} \;\; k = \xi, \iota. \tag{10}$$

Also, to allow for an age-varying conditional mean (Figure IIa), we model the location parameters of these shocks to be age-varying:

$$\mu^{j}_{h} = \lambda^{j}_{a} + \lambda^{j}_{b} h \;\; \text{for} \;\; j = -,+. \tag{11}$$

Age-varying location parameters are also crucial for us to match the age variation in the kurtosis we observe in the data. Simply speaking, increasing either location parameter stretches out the distribution of earnings growth; thereby, increases the kurtosis. We restrict the autocorrelation parameters of persistent shocks to be age-invariant. However, the age-varying probabilities to sample different shocks from age-varying distributions imply that the persistence of residual earnings growth is varying with age, a fact documented by Karahan and Ozkan (2013).

We also allow the purely transitory shock, $\iota^{n}_{i,h}$, to have an age-varying mean and

[14] To keep the number of parameters manageable, we impose the same means for transitory and persistent shocks.

variance:

$$\sigma_{\iota^n,h} = \gamma_{a,\iota^n} + \gamma_{b,\iota^n} h \tag{12}$$

$$\mu_{\iota^n,h} = \lambda_{a,\iota^n} + \lambda_{b,\iota^n} h. \tag{13}$$

We do not impose it in the estimation, but it is natural to think of these shocks as mostly representing small changes in real earnings that are close to zero (nominal contracts, small changes in hours, etc...). Thus, the probability of only drawing such a shock, $1-p^{+}_{i,h}-p^{-}_{i,h}$, together with their age-varying variance, is crucial for us to match the age-varying share of individuals experiencing only small earnings changes (Figure IVb).

As workers accumulate different shocks over their life-cycles, our process implies that the variance of log residual earnings is increasing over the life-cycle. However, Figure V shows that residual earnings inequality is decreasing during the first years. We interpret this initial decline resulting from heterogeneity in the initial transitory components:

$$\iota^{j}_{i,0} \sim exp(N(\mu^{j}_{0}, \sigma^{j}_{0})), \text{ for } j = -, +. \tag{14}$$

The above equation completes our model specification. Our econometric process resembles several features from Guvenen et al. (2016). In particular, we adopt deterministic heterogeneity $(\alpha_i, \beta_i)$, mixture probabilities of two persistent components, and i.i.d shocks. However, our model places more emphasis on the life-cycle features of the data by incorporating age-varying shocks ($\mu^{j}_{h}, \sigma^{j}_{k,h}$ for $j = -, +$; $k = \xi, \iota$). Their only source of age dependence results from the age-varying probability of drawing a particular shock. Besides that, we have departed from their normality assumption of shocks and we allow shocks to the positive and negative components of log residual earnings to be a mixture of persistent and transitory shocks, instead of only a persistent component. In the following, we show that age-varying heterogeneity in positive and negative shocks explains fairly well the higher order moments that residual earnings growth displays in the data.

## 4.2 Empirical Results

This section presents the estimates of the labor income process described in Equations (1) to (14). We estimate the model by the method of simulated moments (MSM) and use the block bootstrapping procedure suggested by Horowitz (2003) to obtain standard errors that we report in Table A4.

Table 1: Parameter Estimates of the Labor Income Process

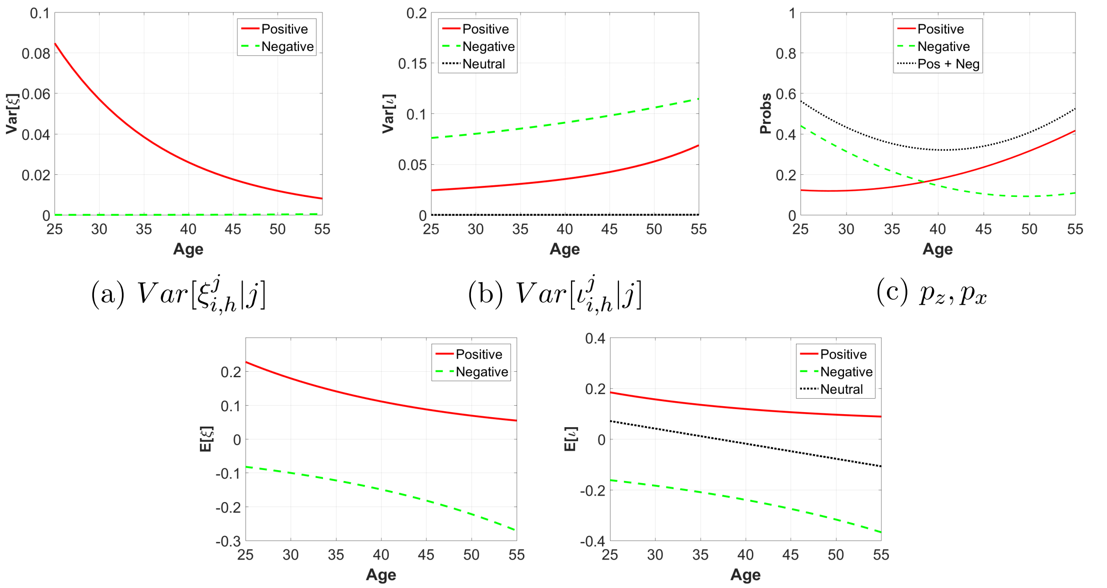
| *Model:* | (1) | (2) | (3) | (4) | (5) | (6) |
|---|---|---|---|---|---|---|
| | Full | No | No | No | *Macro* | *Micro* |
| **Parameters** | Model | $\iota$ | $h$ | $\{\sigma_\beta, \theta\}$ | | |
| $\rho^-$ | 0.9667 | 0.0360 | 0.9881 | 0.9531 | 0.9905 | 0.9465 |
| $\rho^+$ | 0.9857 | 0.9999 | 0.9820 | 0.9818 | | |
| $\sigma_\beta$ | 0.0000 | 0.0030 | 0.0003 | - | - | - |
| $\sigma_\alpha$ | 0.0053 | 0.1262 | 0.0944 | 0.0072 | 0.2149 | - |
| $\theta^+$ | 0.1783 | - | 0.1322 | - | - | - |
| $\theta^-$ | 0.0443 | - | 0.0567 | - | - | - |
| $\sigma_{\hat{\iota}}$ | - | - | - | - | 0.1860 | 0.1751 |
| $\sigma_{\hat{\xi}}$ | - | - | - | - | 0.0797 | 0.1670 |
| **Obj. Function** | **95.62** | **152.17** | **188.95** | **128.61** | **-** | **-** |

Notes: The table displays selected parameter estimates of the earnings process described by Equations (1)-(14). Additional parameter estimates are displayed in Table A3. Table A4 displays standard errors. The process is estimated by the method of simulated moments. We use the *SIAB R 7510* sample selection described in Section 2.2. Column (1) is the full model. Columns (2)-(4) shut down transitory shocks, age-dependence, and both profile growth rates and persistence of transitory shocks, respectively. The last two columns display parameter estimates of the model in equation (15).

We target three main sets of empirical moments over the life-cycle: (i) moments of unconditional residual earnings growth: the mean, skewness, kurtosis, fraction of shocks above 5%, and the autocovariance function; (ii) moments of conditional positive and negative residual earnings growth: the means, variances, share of positive changes, and the first two autocovariances; and (iii) the variance of unconditional residual log earnings[15]. In total, we estimate 32 parameters using 461 moments. Section A.1 in the Appendix describes further details about the estimation procedure and the set of moments.

Table 1 reports selected parameter estimates of the stochastic process of labor earnings. Table A3 in the Appendix reports the remaining parameter values. Column (1) is the full specification of our econometric model. We estimate that persistent shocks, both positive and negative, follow close to a unit-root processes.

[15] We also include as moment that mean residual log earnings are zero at age 24.




(a) $Var[\xi^j_{i,h}|j]$ (b) $Var[\iota^j_{i,h}|j]$ (c) $p_z, p_x$

(d) $\mathbb{E}[\xi^j_{i,h}|j]$ (e) $\mathbb{E}[\iota^j_{i,h}|j]$

Notes: The Figures display age specific estimates from the earnings process described by Equations (1)-(14). Figure VIa displays the estimates of the variances of persistent shocks to the positive and negative components. Figure VIb displays the estimates of the variances of the three transitory shocks. Figure VIc displays the probabilities of drawing a shock to the positive and negative components. Figure VId displays the estimates for the means of persistent shocks. Figure VIe displays the estimates for the means of transitory shocks.

Figure VI: Model Predictions

Transitory positive shocks are mildly persistent, $\theta^+ = 0.18$, and negative transitory shocks are close to i.i.d. We estimate that there is no unobserved heterogeneity in individual earnings growth rates, $\sigma_\beta = 0$.

Figure VI displays the age variation in the mean and variance of the different shocks. We find large heterogeneity between positive and negative persistent shocks. Positive persistent shocks are heavily dispersed early in life. Their variance decreases from 0.08 at age 24 to 0.01 at age 50. In contrast, the dispersion of negative persistent shocks is close to zero for most of the life-cycle, reaching 0.002 at age 50. The range of the mean for both positive and negative persistent shocks is somewhat similar, but their life-cycle behavior differs. While positive shocks decrease in size throughout the life-cycle, negative persistent shocks are relatively small early in life but become relatively large late in life. A close to zero variance in negative persistent shocks does not imply that there is no risk from the individual perspective with regard to those. Figure VIc shows that early in life, up to 44% of individuals receive such a shock and this probability decreases to 10% late in life. Therefore, given their size, there is substantial risk associated with negative persistent shocks throughout the life-cycle.

Transitory shocks are quantitatively more important to understand negative rather than positive residual earnings growth. Negative transitory shocks are of larger size than their positive counterparts, particularly late in life. Moreover, their variance is larger. The mean and variance of purely transitory shocks, $\iota_{i,h}^{n}$, are close to zero for most of the life-cycle. Put differently, in years where individuals are unlikely to receive shocks to their positive or negative component, they face little earnings risk. Figure VIc shows that these probabilities are particularly low around the age of 40; 65% of workers only receive a small transitory shock during a year.

Figure A4 in the Appendix compares the targeted moments in the model with the data. Moreover, Table A2 shows the loss function with respect to different moments. While the overall fit is good, we briefly discuss one shortcoming. Positive shocks are highly dispersed early in life; yet, cross-sectional inequality does not increase fast; if any it decreases for the first 2-3 years. Our model can rationalize a non-increasing profile only when these positive shocks are mostly transitory. As a result, the first autocovariance of positive earnings growth is too negative in our model compared to the data.

## 4.3 Discussion of the Empirical Results

Consistent with Blundell et al. (2015), we find no unobserved heterogeneity in individual earnings growth rates. Similar to us, their identification comes from the autocovariance function of earnings growth with sufficient long lags. At the same time, ruling out $\sigma_\beta^2 > 0$ from this moment is difficult because the autocovariance function is measured with some noise. Yet, we can provide some plausible ranges. Table A4 shows that within two standard errors, $\sigma_\beta^2 < 10^{-4}$. This value is significantly smaller than the values found by the literature that estimates this parameter jointly with modestly persistent earnings shocks (see Guvenen (2009) for a review).

We find that a rich specification of transitory shocks is crucial for correct identification of unobserved heterogeneity in individual earnings growth rates. Hryshko (2012) shows in a simpler model that omitting transitory shocks downward biases the estimate for persistent shocks and upward biases $\sigma_\beta^2$. Following this idea, Column (2) of Table 1 displays estimates of a model where there are no transitory shocks. We find a much lower $AR(1)$ estimate for the negative persistent parameter: $\rho^- = 0.04$. This, in turn, leads to a larger estimate of profile heterogeneity,

$\sigma_\beta = 0.3\%$. The intuition is simple. When neglecting transitory shocks, the moments estimator implies $\rho << 1$ to match the negative autocovariance function at lags one and two. Yet, $\rho << 1$ alone implies that the autocovariance function is negative at intermediate lags. To obtain an autocovariance function which is closer to zero at those lags, $\sigma_\beta >> 0$ is required. Table A2 shows that relative to our full model, this specification does worse in matching the (conditional) autocovariance function.

One of the novelties in our econometric process are parameters that are age-varying. We find this to be key in fitting the moments of residual earnings growth over the life-cycle. In Column (3), we restrict the mean and variances of all shocks to a constant across ages. In this case, all life-cycle dynamics are driven by changes in the sampling probabilities of these age-invariant shocks, similar to Guvenen et al. (2016). Relative to our full model, the loss function almost doubles. Figure A5 in the Appendix shows that the model generates little age variation in the moments of residual earnings growth. In particular, the model fails to match the decrease in the variance of positive shocks, the age variation in the share of positive shocks, and the resulting decrease in skewness over the life-cycle.

Finally, we compare our model that features a positive and negative component to the earlier literature that models a single mean zero $AR(1)$ shock process:

$$\hat{y}_{i,h} = \alpha_i + \hat{Z}_{i,h} + \hat{\iota}_{i,h}, \qquad \mathbb{E}(\hat{\iota}_i) = 0, \ \ Var(\hat{\iota}_i) = \sigma^2_{\hat{\iota}} \tag{15}$$

$$\hat{Z}_{i,h} = \rho \hat{Z}_{i,h-1} + \hat{\xi}_{i,h}, \qquad \mathbb{E}(\hat{\xi}_i) = 0, \ \ Var(\hat{\xi}_i) = \sigma^2_{\hat{\xi}}. \tag{16}$$

This earlier literature identifies transitory and persistent shocks from either the autocovariance function of residual earnings growth, or the variance of log residual earnings over the life-cycle. Heathcote et al. (2010a) show that what they refer to as *Micro* estimation (targeting moments of the autocovariance function of earnings growth) leads to substantially larger persistent shocks than a *Macro* estimation (targeting cross-sectional inequality over the life-cycle). As a consequence, the *Micro* estimation leads to a too large increase in cross-sectional inequality over the life-cycle and the *Macro* estimation implies too much mean reversion of shocks[16]. Columns (5) and (6) present the parameter estimates resulting from $GMM$ estimators of the two identification strategies. Both strategies imply that shocks are highly persistent. As expected, the standard deviation of persistent shocks is almost twice as large in the *Micro* approach.

[16] Daly et al. (2016) show that eliminating beginning and end of earnings spell observations helps to reconcile the two approaches within the $AR(1)$ framework.

In the estimation of our full model, we target both sets of moments. Figure A4 in the Appendix shows that our full model is able to match the increase in residual earnings inequality over the life-cycle and the autocovariance function of residual earnings growth jointly. As in the data, the steepest increase in cross-sectional residual earnings inequality occurs before the age of 35 when the probability of large persistent positive shocks is high. Thereafter, earnings inequality grows at a rather slow rate. The reason for the relatively modest increase in earnings inequality over the life-cycle (compared to the *Micro* model) is not that the average persistent shocks is small in size, but that in a given year a substantial fraction of workers receive no persistent shock.

We compare the implications of our age-varying risk model to those of the two standard earnings processes using a structural model of consumption-savings decisions in the section below. Provided that our full model finds no heterogeneity in earnings profiles and low persistence in transitory shocks, we simplify our full model and exclude these parameters. Column (4) shows the resulting parameter values. Table A2 in the Appendix shows that this model does worse with respect to all moments. As expected, it matches poorly the autocovariance at lag 2. At the same time, the model matches most salient features which makes us confident to use it in a consumption-savings model.

# 5 Life-Cycle Consumption and Savings Model

We now turn to the implications of our earnings process for consumption and wealth inequality, and the degree to which workers can insure against idiosyncratic shocks. To this end, we introduce the estimated earnings uncertainty into a structural model with incomplete insurance markets[17].

## 5.1 Environment

For simplicity, we consider a partial equilibrium model with exogenous wages and interest rates. Individuals work for $H_W$ years in the labor market, and die with certainty at age $H > H_W$. They have CRRA preferences over consumption with risk aversion parameter $\gamma$ and discount the future with factor $\beta$. They have access to a one period risk free asset $a$ that pays certain returns $R = 1 + r$, and they face a zero borrowing constrained $a_{h+1} \geq 0$. Workers make consumption decisions to

[17] Similar to Aiyagari (1994), Hubbard et al. (1995), Imrohoroglu et al. (1995), Rios-Rull (1995), Huggett (1993), and Carroll (1997).

maximize expected life-time utility:

$$\max_{c^i_{h=1\ldots H}, a^i_{h=1\ldots H}} \left\{ \mathbb{E}_0 \sum_{h=1}^{H} \beta^{h-1} \frac{c_{i,h}^{1-\gamma}}{1-\gamma} \right\}$$

$$a^i_{h+1} = (1+r)a^i_h + Y^i_h - c^i_h$$

$$a^i_{h+1} \geq 0,$$

where $Y^i_h$ is post tax income of individual $i$ at age $h$. During working life, log gross earnings follow the sum of a common deterministic and an individual specific stochastic component:

$$E^i_h = exp(d_h + v_{i,h}) \text{ if } h \leq H_W. \tag{17}$$

The government reduces earnings inequality by applying a progressive income tax schedule. We apply the statutory income and social security tax schedule from Germany to map gross earnings into after tax income:

$$Y^i_h = G(E^i_h). \tag{18}$$

During retirement, workers face no further uncertainty and receive social security benefits. To avoid keeping track of individuals' average earnings, we assume social security benefits depend only on the fixed type $\alpha_i$[18]:

$$Y^i_h = F(\alpha_i) \text{ if } h > H_W. \tag{19}$$

***Calibration***

We calibrate the coefficient of relative risk aversion and the interest rate outside of our data. The former, $\gamma$, is set to 1.5, consistent with Attanasio and Weber (1995). Following Siegel (2002), we fix the value of $r$ to imply a yearly interest rate of 4%. To ensure that households have on average an adequate level of self-insurance, we match median wealth to earnings ratios reported by Bundesbank (2014). First, we calibrate $\beta$ to match a median wealth-to-earnings ratio of 2.5 at age 55, and second, we assign individuals initial assets equal to initial gross earnings implying a wealth-to-earnings ratio of one.

Workers work up to age 55 and spend ten years in retirement, thereafter. We

[18] Bundesministerium (2015) shows that the retirement replacement rate has decreased over the last decades and is projected to continue to do so. We assume households expect the replacement rate from 2010.

match average earnings during working life, $d_h$, by estimating cohort average age profiles as in Deaton and Paxson (1994). In what we call our *Baseline* model, the stochastic log earnings component, $v_{i,h}$, follows the process estimated in Column (4) of Table 1.

We compare the implications of this model to those from the standard approaches used in the literature, the *Macro* and *Micro* approach. In those models, $v_{i,h}$ follows the process estimated in Columns (5) and (6), and we assume shocks follow normal distributions[19]. To make the models comparable, we recalibrate $\beta$ in each model to match the median wealth-to-income ratio of 2.5 at age 55[20].

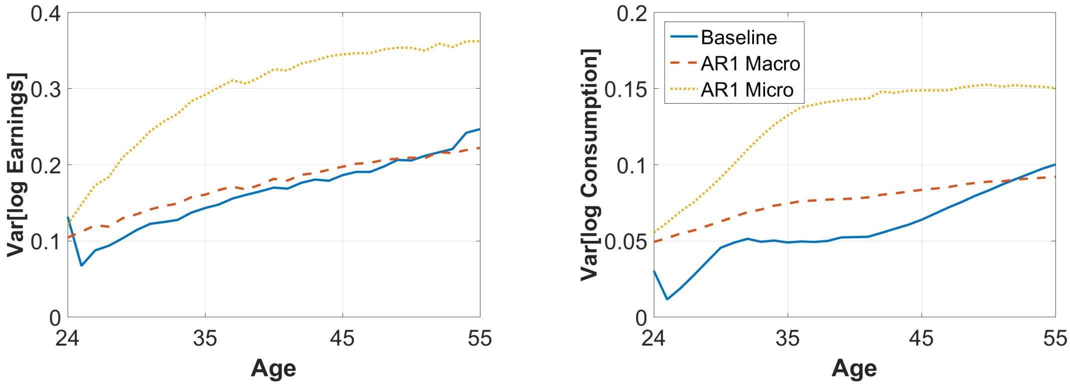
## 5.2 Counterfactual Experiment

Before discussing the quantitative implications of the models, let us briefly discuss which earnings process entails more risk for the individual. As can be seen in Figure VIIa, the *Micro* model implies substantially more earnings inequality, thereby risk, than our *Baseline* model and the *Macro* model. Our *Baseline* model features tail risk not present in the other two models. This feature alone makes earnings more risky to the individual. However, in our *Baseline* model, a substantial fraction of workers, particular those in prime-age, receive no persistent earnings shocks in a given year making earnings relatively less risky. Moreover, early in life when the buffer stock of savings is at its lowest, earnings risk is positively skewed, implying that large negative shocks are relatively unlikely. The opposite is true late in life.

How well do precautionary savings insure workers against earnings uncertainty? One way the literature addresses this question is by studying how fast consumption dispersion increases over the life-cycle. Figure VIIb displays the corresponding profiles for our three models. Unfortunately, cross-sectional consumption data is sparse in Germany. Fuchs-Schúndeln et al. (2010) find that consumption inequality increases between 5 and 10 log points from age 25 to 55, depending on the econometric specification. The *Micro* model implies an increase at the upper end, and our *Baseline* model implies an increase at the lower end of this range. The increase implied by the *Macro* model is slightly below this estimated range. Both the *Macro* and *Micro* model imply that consumption inequality increases in a concave fashion over the life-cycle. In contrast, our *Baseline* model shows

[19] We ensure that mean earnings by age are the same in all three models

[20] The discount factor, $\beta$, is smallest in our *Baseline* model and largest in the *Micro* model. Put different, for the same discount factor, workers accumulate on average the most savings in our *Baseline* model.




(a) $Var[log\ Earnings]$ (b) $Var[log\ Consumption]$

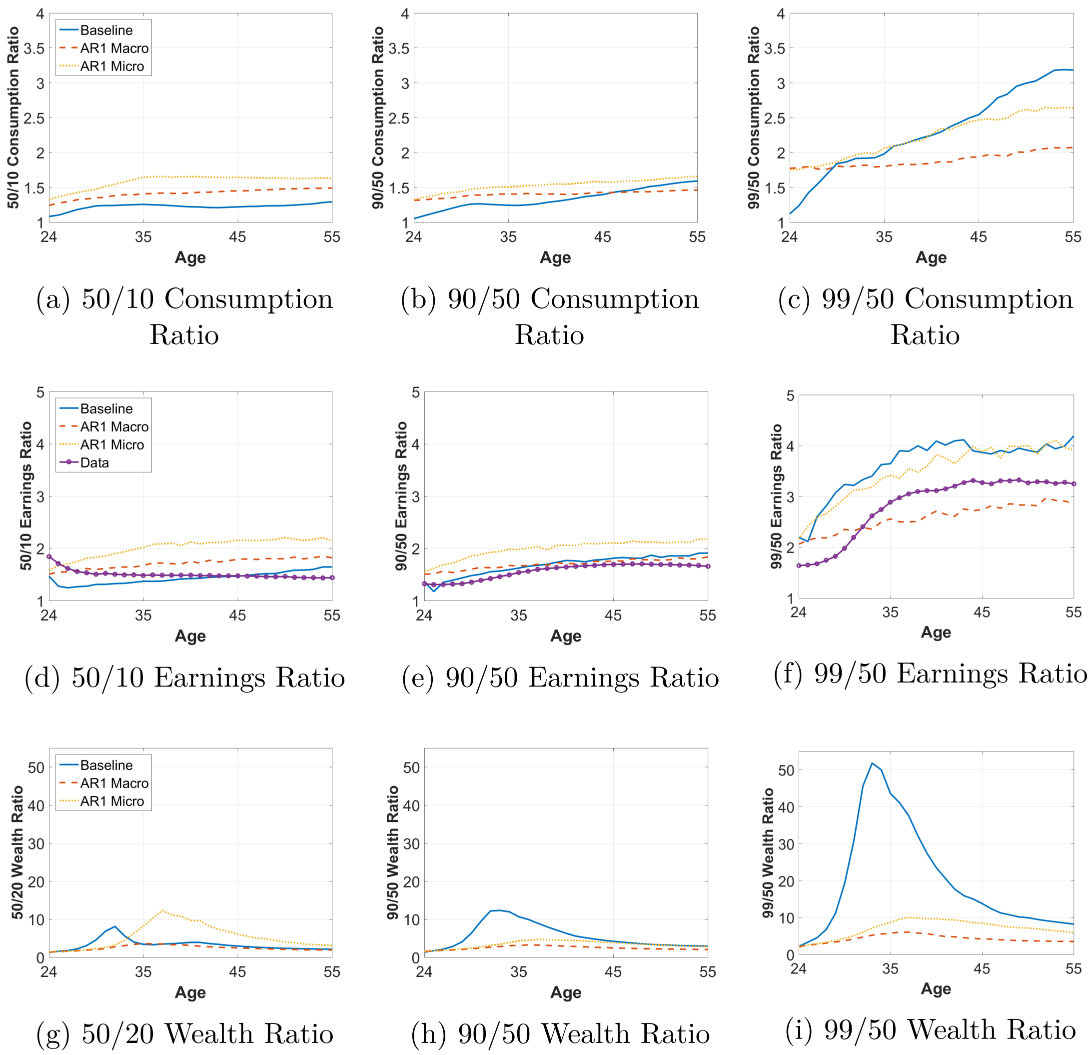
Notes: Figure VIIa displays the variance of log earnings by age from the structural models described in Section 5.1. Figure VIIb displays the corresponding variance of log consumption by age.

Figure VII: Earnings and Consumption Inequality

no flattening of the increase in inequality late in life which is consistent with the evidence shown by Fuchs-Schúndeln et al. (2010). Guvenen (2007) discusses that standard earnings risk models generate concave consumption profiles because precautionary savings are growing with age, but risk is not age-varying. He shows that learning about deterministic differences in individual earnings growth profiles can reconcile the model with the data. We find that age-varying earnings risk achieves the same objective. Particularly, age-varying sampling probabilities are key. Late in life, the probability to receive a persistent shock, particularly a positive shock, increases. This extra risk, relative to prime-aged workers, offsets the extra amount of precautionary savings that individuals have.

Not only the shape of consumption inequality over the life-cycle, but also the overall level of inequality is different in the three models. Consumption inequality is largest in the *Micro* model and smallest in the *Baseline* model. The large consumption inequality in the *Micro* model is little surprising given its large earnings inequality. More surprising is that consumption inequality is larger for most of the life-cycle in the *Macro* model compared to our *Baseline* model, even though the life-cycle profiles of earnings inequality are similar.

To better understand these differences in consumption and earnings inequality, we now turn to inequality at the top and at the bottom of the distribution. The first two panels of Figure VIII show bottom inequality (50/10 ratio), upper inequality (90/50 ratio), and top inequality (99/50 ratio) for consumption and earnings. For most of the life-cycle, both lower and upper consumption inequality are larger in the *Macro* model than in our *Baseline* model. These patterns closely




(a) 50/10 Consumption Ratio

(b) 90/50 Consumption Ratio

(c) 99/50 Consumption Ratio

(d) 50/10 Earnings Ratio

(e) 90/50 Earnings Ratio

(f) 99/50 Earnings Ratio

(g) 50/20 Wealth Ratio

(h) 90/50 Wealth Ratio

(i) 99/50 Wealth Ratio

Notes: Figure VIII displays selected percentile ratios of consumption, earnings and wealth by age from the structural models described in Section 5.1.

Figure VIII: Inequality over the Life-Cycle

follow upper and lower earnings inequality. The figures also show the same ratios for earnings inequality in the data. Our *Baseline* model nicely fits the fact that bottom inequality does not increase at all over the life-cycle and upper inequality grows at a moderate pace. At the same time, our *Baseline* model implies relatively large and rapidly increasing top-consumption inequality. Those in the top 1% consume more than 3 times more than the median at age 55, but only 2.1 times more in the *Macro* model (2.6 in the *Micro* model). Again, top earnings inequality shows a very similar pattern. The increase in top earnings inequality over the life-cycle is similar to the data in our *Baseline* model; yet, the overall level is somewhat too large.

De Nardi et al. (2016) show that existing life-cycle savings models fail to rationalize sufficient cross-sectional wealth inequality given the observed earnings inequality in US data. Particularly, the models imply too little wealth holdings of the richest workers. Bundesbank (2014) shows that wealth is also top-concentrated in Germany: The 95th percentile owns 12 times more wealth than the median. The bottom panel of Figure VIII shows that our *Baseline* model implies much more top wealth inequality than the *Micro* and *Macro* models[21]. We find that the age variation in positive shocks is key to understand this phenomenon. Large (Figures VIa, VId) but rare (Figure VIc) persistent positive shocks early in life lead to high earnings for a lucky few. These workers have the incentives to accumulate high life-cycle savings. Looking at the cross-section of all workers, we find that the 99/50 wealth percentile is 3.4 in the *Macro* model, 5.8 in the *Micro* model, and 10.0 in the *Baseline* model. This number still falls short of the inequality reported by Bundesbank (2014), but this data includes business owners. Cagetti and Nardi (2006) show that they have incentives to accumulate wealth not present in our model. Large top inequality also implies larger overall cross-sectional wealth inequality. The Gini coefficient of wealth is 0.50 in our *Baseline* model, 0.46 in the *Micro* model, and 0.33 in the *Macro* model. Again, inequality is yet larger in the data (0.76). Part of this difference is likely to result from our sample selection; we have discarded workers with low labor market attachment which decreases bottom inequality.

Temporary and persistent shocks should affect consumption in different ways. Kaplan and Violante (2010) propose an alternative way to measure the insurability of shocks that allows us to differentiate between the two types of shocks and that partially takes into account the variance of shocks:

$$\phi_\xi = 1 - \frac{Cov(\Delta ln(c_{i,h}), \xi_{i,h})}{Var(\xi)} \tag{20}$$

$$\phi_\iota = 1 - \frac{Cov(\Delta ln(c_{i,h}), \iota_{i,h})}{Var(\iota)} \tag{21}$$

$\phi_\xi$ and $\phi_\iota$ measure how much persistent and transitory shocks affect consumption, respectively. A $\phi$ close to one implies that a worker is well insured against the respective shocks.

The left panel of Table 2 shows how consumption fluctuates in response to permanent earnings shocks over a worker's life-cycle. One needs to be careful in

[21] For wealth, due to the financially constrained households at the beginning of their working life, we use the 50/20 ratio for bottom inequality.

Table 2: Insurance Coefficients and Welfare

| Model | $\phi_\xi$ $\overline{\phi_\xi}$ | Age: 30 | 35 | 45 | 55 | $\phi_\iota$ $\overline{\phi_\iota}$ | Age: 30 | 35 | 45 | 55 | Welfare Cost |
|---|---|---|---|---|---|---|---|---|---|---|---|
| *Macro* | 56 | 41 | 45 | 59 | 94 | 91 | 87 | 87 | 94 | 94 | 5.75 |
| *Micro* | 59 | 50 | 49 | 60 | 92 | 88 | 88 | 82 | 90 | 92 | 9.11 |
| *Baseline* | 38 | 4 | 14 | 46 | 94 | 56 | 25 | 33 | 66 | 87 | 5.77 |

Notes: Values are in percentage points. The insurance coefficients, $\phi_\xi$ and $\phi_\iota$, are calculated following Equations 20-21, respectively. $\overline{\phi_\xi}$ and $\overline{\phi_\iota}$ are their respective population averages. The welfare cost is specified in Equation 22. The models are described in Section 5.1

comparing the coefficients across models because the risk structures are different. In the *Baseline* model, receiving a negative persistent shock implies that the individual also receives a negative transitory shock. This is not true in the other two specifications where the signs of persistent and transitory shocks are independent.

The three models imply that consumption responses to persistent earnings hocks become smaller as workers age. This has two reasons. First, workers accumulate assets over the life-cycle that serve as self-insurance. Second, shocks become effectively more transitory as workers move closer to retirement because a smaller share of future earnings consists of labor market earnings. Across models, consumption responses are largest in our *Baseline* model for most of the life-cycle. The difference between the models is largest around age 30 when precautionary savings are low. Recall that in our *Baseline* model persistent shocks are more extreme than in the other two models. As a result, when hit by a negative persistent shock, workers are more likely to come close to their borrowing constraints, triggering larger consumption responses.

A similar picture emerges with regard to transitory shocks. Both the *Macro* and *Micro* model imply that workers are well insured against transitory shocks, particularly late in life. On average, about 10 percent of transitory shocks translate into consumption within the same year. Insurance in our *Baseline* model is much lower. On average, 44 percent of transitory shocks translate into consumption[22]. Again, shocks in our *Baseline* model have fatter tails, as a consequence, workers

[22]Empirical studies for Germany are missing. Commault (2017) reviews US literature on tax rebates that finds between 9 to 38 percent of a transitory income change translate into contemporaneous consumption. Our model is at the upper end of this range. One has to keep in mind that we abstract from all insurance mechanism besides progressive taxation and self-insurance.

are more likely to come close to their borrowing constrained after an adverse transitory shock.

A yet further measure of uninsurable idiosyncratic risk is the amount of consumption an unborn worker is willing to give up to gain access to full insurance markets. To this end, define by $c_s(a, w^-, w^+, \iota, \alpha)$ the optimal consumption function of an agent without complete markets at age $s$. Under complete markets, consumption does not depend on the idiosyncratic states, but only on the aggregate distribution of idiosyncratic states, i.e., the amount of aggregate resources. Call this aggregate distribution $\hat{\lambda}_s$ and let $\hat{c}_s$ be the corresponding consumption function. Hence, we are interested in the $\omega^U$ that solves

$$\int E_0 \sum_{s=0}^{H} \beta^s U([1+\omega_s^U]c_s(a, w^-, w^+, \iota, \alpha))d\lambda_s(a, w^-, w^+, \iota, \alpha) = \int E_0 \sum_{s=0}^{H} \beta^s U(\hat{c}_s)d\hat{\lambda}_s, \quad (22)$$

where $\lambda_s$is the distribution functions of workers over states in out *Baseline* model. The last column in Table 2 shows that the *Micro* model has the largest welfare losses; an unborn worker is willing to pay around 9 percent of life-time consumption to gain access to complete markets. This is not surprising provided the *Micro* approach generates an exaggerated increase of cross-sectional inequality in earnings and hence in consumption. The welfare costs implied by our *Baseline* model and the *Macro* model are around 5.8 percent of life-time consumption. Put differently, in our *Baseline* model, workers react stronger to shocks which increases their willingness to pay to insure against these shocks. Yet, this effect almost cancels with the welfare improving effect stemming from a low probability to experience in a given year a shock to the positive or negative component of log earnings.

# 6 Conclusion

In this paper, we estimate age variation in earnings risk. Early in working-life, workers experience rare but large positive shocks, both transitory and persistent in nature. As workers move into prime-age, the probability of receiving persistent shocks becomes small and most earnings changes result from small transitory shocks. For elderly workers, rare, but large (persistent and transitory) negative earnings shocks become the main source of uncertainty.

Though more complex than an age-invariant $AR(1)$ process with Gaussian shocks, our parametric process is still simple enough to introduce it into a model of consumption decisions with incomplete financial markets. The age-varying risk structure help us to reconcile two stylized facts from the data. First, large persistent positive shocks early in life imply high life-time income for a small group of workers. These workers have incentives to accumulate large life-cycle savings. In the cross-section, the 99/50 wealth ratio increases by a factor of 2.9 relative to a model with an $AR(1)$ process with Gaussian shocks. Second, relative to prime-age, workers face more risk late in their working life. As a result, even though precautionary savings are highest, cross-sectional consumption inequality keeps growing in a linear fashion at the end of working-life.

In our framework, residual earnings growth results from exogenous shocks to individuals. However, many of the features in our risk process resemble outcomes from individuals choices under labor market risk, as in Low et al. (2010). Large positive shocks (job-to-job transitions, finding stable employment) occur early in life. Large negative shocks (losing a high tenured job, periods of non-employment) occur late in life. However, the degree to which models of endogenous labor market choices can match the full dynamics of earnings growth over the life-cycle documented here is still an open question.

Our age-varying risk process also raises several questions regarding social insurance. On average, earnings risk is negatively skewed, implying that insurance against catastrophic events is highly valuable to society. Yet, early in life, when self-insurance is lowest, earnings risk is positively skewed; thus, decreasing the need of insurance. The optimal size and design of the welfare state is, therefore, an even more complex question than that of age independent Gaussian shocks. Finally, the risk structure also has implications on the level of attainable private (and public) insurance. Krueger and Perri (2006) analyze privately efficient risk sharing contracts. We show that prime-aged workers face close to no risk; thus, have little incentives to enter into any private, or support large public, risk sharing.

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# A Appendix

## A.1 Estimation

### A.1.1 Constructing the Moments

We model log earnings as the sum of deterministic and stochastic components that may depend on cohort and time effects[23]. Let $Y^c_{i,h,t}$ be the log earnings of individual $i$, at age $h$, belonging to the birth cohort $c$, in year $t$:

$$Y^c_{i,h,t} = f(\pi_t, X_{i,h,t}) + y^c_{i,h,t}, \tag{A.23}$$

where $f(\pi_t, X_{i,t,h})$ are predictable differences in earnings such as education, region, age and industrial sector, and $y^c_{i,h,t}$ represents the process of earnings. Rewriting the above process in first differences yields

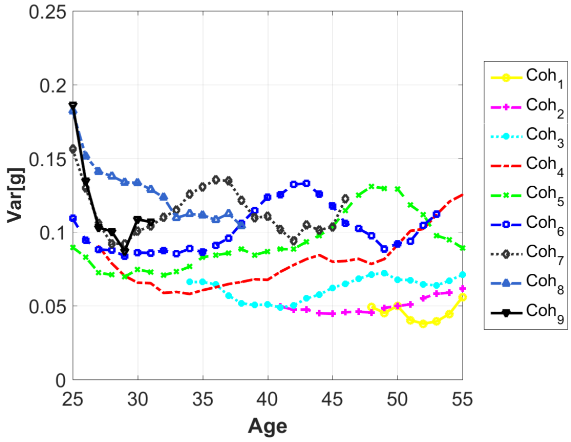
$$\Delta Y^c_{i,h,t} = \Delta f(\pi_t, X_{i,t,h}) + \Delta y^c_{i,h,t}. \tag{A.24}$$

First, we remove predictable changes in log earnings, such as calendar and age effects, by running for each year cross-sectional regressions on a dummy of workers' education, age, interaction of the two, region of residence, and 14 major industries. Denote the corresponding residual by $g^c_{i,h,t}$:

$$g^c_{i,h,t} \equiv \Delta y^c_{i,h,t} \tag{A.25}$$

So far, our specification allows the moments of residual earnings growth to be calendar year and birth cohort specific. As an illustration of such effects, Figure A1 presents the variance of residual earnings growth for each of our 9 cohorts. There are two salient features. First, there is a calendar year effect with large variances for all cohorts about 5 years after the German reunification. For example, for the 5th birth cohort, born between 1951-1957 (green line) the German reunification occurs at ages 34-40, and the time effect increases the variances after age 45. Second, there is also a visible cohort effect, with later cohorts facing substantial higher variances than earlier cohorts. As mentioned in the main text, we follow Blundell et al. (2015) and eliminate these effects by averaging all moments (variance, skewness, kurtosis, etc.) across cohorts, assigning equal weight to each. Therefore, our results can be interpreted as the risk a typical cohort faces.

[23]Both cohort and time effects are present in Germany. The Hartz reforms after 2003 and the reunification of West and East Germany are a few examples.




Notes: Figure A1 displays the variance of residual earnings growth by age and birth cohorts. Birth cohorts 1-9 belong to years of birth 1923-1929,1930-1936,...,1980-1986, respectively.

Figure A1: By Cohort Variance of Residual Earnings Growth

Regarding the variance of log residual earnings over the life-cycle we follow Deaton and Paxson (1994) and obtain the age component from regressing the variance of log earnings on a cohort and age dummies.

### A.1.2 Moments Selection and Estimation

We simulate life-cycle employment histories for 20,000 workers who enter the labor market at the age of 24 and work until the age of 55. Denote by $f_n(\theta)$ the $n^{th}$ model moment and by $m_n$ the $n^{th}$ data moment. Similar to Guvenen et al. (2016), define

$$F(\theta)_n = \frac{f_n(\theta) - m_n}{\omega_n}, \tag{A.26}$$

where $\omega_n$ is an adjustment factor. We use a moment specific adjustment factor to jointly deal with two issues presented by the data. First, the moments are measured on different scales. For example, kurtosis is in absolute value about 500 times larger than the second autocovariance. Had we minimized the sum of absolute squared deviations ($\omega_n = 0$), the optimization would not have put emphasis on moments with low absolute sizes. At the same time, we have several moments which are close to zero, such as the autocovariance function, but fluctuate substantially in relative terms from one age to the next. Thus, had we minimized the sum of relative squared deviations ($\omega_n = abs(m_n)$), the optimization would have concentrated almost exclusively on these large relative deviations close to zero that are likely generated by small samples.

Using moment specific adjustment factors allows us to use absolute deviations but reduce the emphasis on moments with large absolute numbers. Unfortunately, it gives us a degree of discretion. We choose the adjustment factors in an iterative fashion such that the implied loss function displayed in Table A2 is consistent with the model fit we observe in Figure A4. We opt to give the variance of log earnings over the life-cycle and the mean earnings growth by age (which is zero by construction in the data) somewhat larger weights as we want to ensure a good fit with these moments. The resulting simulated minimum distance estimator is given by:

$$\hat{\theta} = \underset{\theta}{argmin} \boldsymbol{F}(\theta)' I \boldsymbol{F}(\theta) \tag{A.27}$$

Most sets of moments contain 31 year moments. This is the case for the skewness, kurtosis, fraction of positive shocks, fraction of shocks above 5%, unconditional mean, variance of log earnings, unconditional autocovariance, conditional mean and conditional variance. This amounts to $31 \times 11 = 341$ moments. The conditional first and second autocovariances are observed for 30 and 29 years, respectively. These amount to $30 \times 2 + 29 \times 2 = 118$ moments. Lastly, the mean and the variance of log residual earnings at age 24 amounts to 2 moments. The total number of moments that we target is then $N = 341 + 118 + 2 = 461$.

Given our large parameter set, the issue of finding a global minimum arises. We first obtain reasonable starting values by experimenting with different combinations of parameters. We tested different global minimum algorithms and a pattern search algorithm performed best in finding a minimum. Provided the optimal parameters, we compare the minimum to (possibly) other minima where we start the algorithm from different starting points. We find that the pattern search algorithm, in general, is able to converge to the same minimum from different starting points.

We obtain standard errors by 100 block bootstraps. Re-optimizing in each iteration using a global search algorithm is infeasible numerically. Therefore, we use a local optimizer, more specific, a sequential quadratic programming algorithm. Implicitly, we assume that a change in the data sample does not lead to a too large change in our estimates; therefore, possibly downward biasing the standard errors.

## A.2 Additional Figures and Tables

Table A1: Summary Statistics, *SIAB R 7510*.

| Year | Average € Daily Wage | Average € Annual Earnings | Average Age | Observations |
|---|---|---|---|---|
| 1975 | 89 | 31,093 | 38 | 84,520 |
| 1976 | 91 | 32,159 | 38 | 93,163 |
| 1977 | 94 | 33,317 | 39 | 100,266 |
| 1978 | 96 | 34,157 | 39 | 106,533 |
| 1979 | 100 | 35,200 | 40 | 113,825 |
| 1980 | 101 | 35,565 | 40 | 120,347 |
| 1981 | 100 | 35,287 | 41 | 126,584 |
| 1982 | 98 | 34,486 | 41 | 126,436 |
| 1983 | 98 | 34,419 | 41 | 125,288 |
| 1984 | 103 | 36,186 | 41 | 125,263 |
| 1985 | 106 | 36,931 | 40 | 124,294 |
| 1986 | 108 | 37,702 | 40 | 124,976 |
| 1987 | 112 | 39,068 | 40 | 124,888 |
| 1988 | 104 | 35,988 | 39 | 112,372 |
| 1989 | 109 | 38,289 | 39 | 119,823 |
| 1990 | 113 | 39,321 | 39 | 121,520 |
| 1991 | 116 | 40,564 | 39 | 122,187 |
| 1992 | 117 | 40,813 | 39 | 122,513 |
| 1993 | 115 | 40,268 | 39 | 121,614 |
| 1994 | 114 | 39,941 | 39 | 118,973 |
| 1995 | 117 | 41,023 | 39 | 116,538 |
| 1996 | 118 | 41,025 | 39 | 112,438 |
| 1997 | 117 | 40,539 | 39 | 110,949 |
| 1998 | 119 | 41,764 | 40 | 107,283 |
| 1999 | 123 | 41,937 | 40 | 100,374 |
| 2000 | 122 | 42,873 | 40 | 96,059 |
| 2001 | 122 | 41,410 | 41 | 91,234 |
| 2002 | 122 | 42,032 | 41 | 85,807 |
| 2003 | 123 | 41,768 | 41 | 80,718 |
| 2004 | 104 | 34,948 | 40 | 59,828 |
| 2005 | 105 | 36,056 | 41 | 54,846 |
| 2006 | 106 | 36,765 | 41 | 50,814 |
| 2007 | 107 | 36,835 | 42 | 47,290 |
| 2008 | 108 | 37,610 | 42 | 43,542 |
| 2009 | 107 | 37,035 | 43 | 38,947 |
| 2010 | 109 | 37,116 | 44 | 34,160 |

Notes: Summary Statistics of the *SIAB R 7510*. See Section 2.2 for the description of the sample selection. All Euro values are deflated using the CPI to 2010 values.

Table A2: Objective Function Decomposition

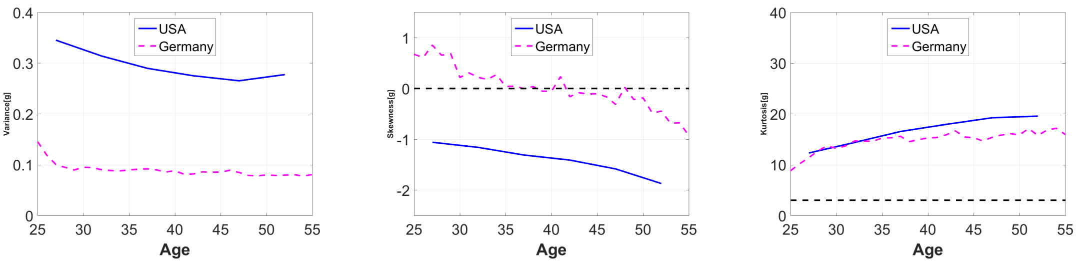
| *Model:* | (1) | (2) | (3) | (4) |
|---|---|---|---|---|
| | Full | No | No | No |
| **Moments** | Model | $\iota$ | $h$ | $\{\sigma_\beta, \theta\}$ |
| $\mathbb{E}[g^+]$ | 6.17 | 11.40 | 12.11 | 7.70 |
| $\mathbb{E}[g^-]$ | 4.62 | 5.83 | 20.84 | 3.88 |
| $Var[g^+]$ | 8.37 | 7.21 | 14.28 | 9.07 |
| $Var[g^-]$ | 7.21 | 7.86 | 14.48 | 7.79 |
| $Skew[g]$ | 4.12 | 5.92 | 19.15 | 3.67 |
| $Kurt[g]$ | 9.24 | 10.38 | 10.78 | 8.51 |
| % of Positive Innovations | 4.93 | 10.57 | 20.73 | 6.43 |
| $E[g_h^- g_{h+1}]$ | 0.89 | 8.51 | 2.87 | 1.20 |
| $E[g_h^+ g_{h+1}]$ | 29.13 | 49.67 | 22.83 | 34.37 |
| $E[g_h^- g_{h+2}]$ | 5.80 | 4.71 | 5.88 | 21.10 |
| $E[g_h^+ g_{h+2}]$ | 3.08 | 12.86 | 1.81 | 9.58 |
| $E[g]$ | 2.31 | 1.15 | 4.86 | 3.27 |
| % of Innovations $> 5\%$ | 3.86 | 8.56 | 33.57 | 4.73 |
| Uncond. Autocovariance | 2.23 | 5.11 | 2.53 | 3.96 |
| Initial $\mathbb{E}$[log earnings] | 0.01 | 0.01 | 0.00 | 0.00 |
| Initial $Var$[log earnings] | 0.00 | 0.00 | 0.00 | 0.00 |
| $Var$[log earnings] | 3.65 | 2.39 | 2.22 | 3.35 |
| **Total** | **95.62** | **152.17** | **188.95** | **128.61** |

Notes: The table displays a decomposition of the loss function. The process is estimated by the method of simulated moments. We use the *SIAB R 7510* sample selection described in Section 2.2. Column (1) estimates our *Baseline* specification outlined in 4.1. Columns (2)-(4) shut down transitory shocks, age-dependence, and both profile growth rates and persistence of transitory shocks, respectively.

Table A3: Additional Parameter Estimates from Table 1

| *Model:* | (1) | (2) | (3) | (4) |
|---|---|---|---|---|
| | Full | No | No | No |
| **Parameters** | Model | $\iota$ | $h$ | $\{\sigma_\beta, \theta\}$ |
| $\delta_I^-$ | 0.4692 | 0.4856 | 0.5220 | 0.4691 |
| $\delta_{II}^-$ | -0.0295 | -0.0300 | -0.0273 | -0.0295 |
| $\delta_{III}^-$ | 0.0006 | 0.0006 | 0.0006 | 0.0006 |
| $\delta_I^+$ | 0.1252 | 0.0574 | 0.2920 | 0.1252 |
| $\delta_{II}^+$ | -0.0034 | -0.0022 | -0.0072 | -0.0034 |
| $\delta_{III}^+$ | 0.0004 | 0.0004 | 0.0002 | 0.0004 |
| $\gamma_{a,\iota}^+$ | 0.7086 | - | 1.4064 | 0.5519 |
| $\gamma_{b,\iota}^+$ | 0.0258 | - | - | 0.0269 |
| $\gamma_{a,\iota}^-$ | 1.1785 | - | 1.0987 | 1.1654 |
| $\gamma_{b,\iota}^-$ | -0.0127 | - | - | -0.0111 |
| $\gamma_{a,\xi}^-$ | 0.0059 | 0.8546 | 0.0456 | 0.0046 |
| $\gamma_{b,\xi}^-$ | 0.0022 | -0.0100 | - | 0.0052 |
| $\gamma_{a,\xi}^+$ | 0.9780 | 1.0829 | 1.4180 | 0.9729 |
| $\gamma_{b,\xi}^+$ | 0.0054 | 0.0003 | - | 0.0075 |
| $\lambda_a^+$ | -1.9084 | -1.7075 | -3.5051 | -1.8867 |
| $\lambda_b^+$ | -0.0534 | -0.0667 | - | -0.0496 |
| $\lambda_a^-$ | -2.5392 | -1.6768 | -2.2860 | -2.5621 |
| $\lambda_b^-$ | 0.0397 | 0.0351 | - | 0.0389 |
| $\gamma_{a,\iota^n}$ | 0.0000 | 0.0039 | 0.0069 | 0.0022 |
| $\gamma_{b,\iota^n}$ | 0.0004 | 0.0014 | - | 0.0004 |
| $\lambda_{a,\iota^n}$ | 0.0769 | 0.1426 | 0.1164 | 0.0735 |
| $\lambda_{b,\iota^n}$ | -0.0059 | -0.0164 | - | -0.0098 |
| $\gamma_{0,\iota}^+$ | 0.4201 | 0.0000 | 0.0387 | 0.0015 |
| $\gamma_{0,\iota}^-$ | 1.0023 | 0.8487 | 0.9769 | 0.0004 |
| $\lambda_{0,\iota}^+$ | -0.8590 | -0.8442 | -1.8794 | 0.0716 |
| $\lambda_{0,\iota}^-$ | -1.6969 | -1.3769 | -1.6114 | -0.0061 |
| **Objective Function** | **95.62** | **152.17** | **188.95** | **128.61** |

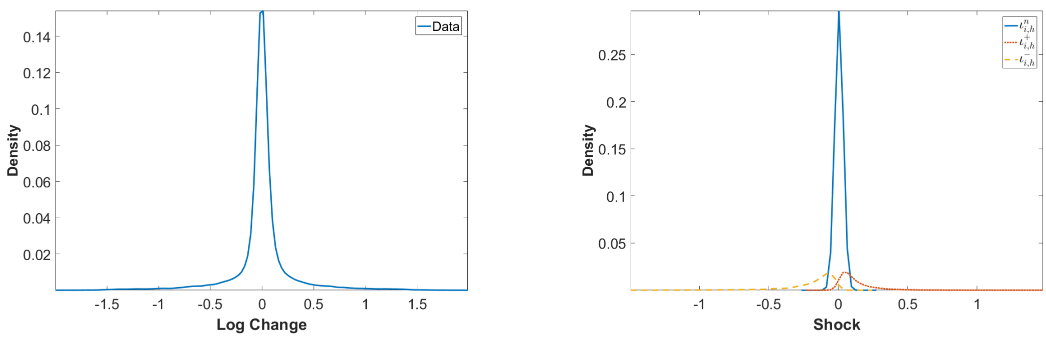
Notes: The table displays complementary estimates to Table 1.




(a) Variance (b) Skewness (c) Kurtosis

Figures A2a, A2b and A2c display, respectively, the variance, skewness and kurtosis of residual earnings growth by age for the US and Germany. The German data is described in Section 2.2. For the US, we take for each age groups (25-29,...,50-54) the average over the percentiles reported in Guvenen et al. (2016).

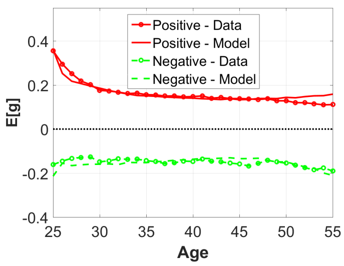
Figure A2: US and German Higher Order Moments



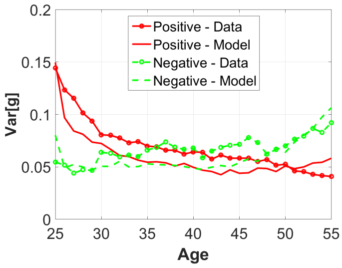

(a) Data (b) Model

Figure A3a displays the kernel distribution of residual earnings growth at the age of 36 in our data described in Section 2.2. Figure A3b displays the densities of transitory shocks from the model described in Section 4.1 at age 36.

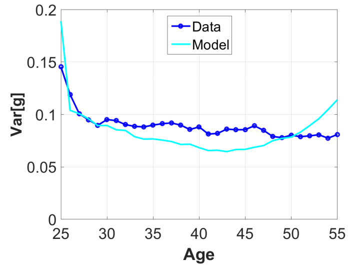
Figure A3: Density of Residual Earnings Growth



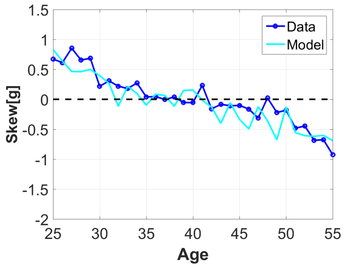

(a) $\mathbb{E}[g_{i,h}|g_{i,h} \gtrless 0]$



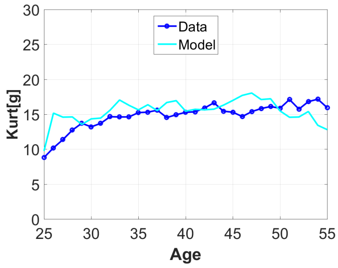

(b) $Var[g_{i,h}|g_{i,h} \gtrless 0]$



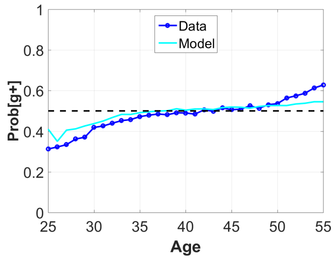

(c) $Var[g_{i,h}]$



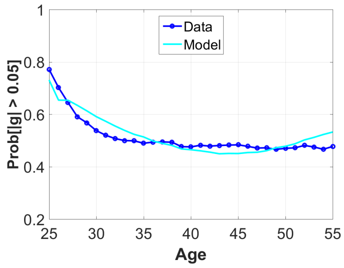

(d) $Skew[g_{i,h}]$



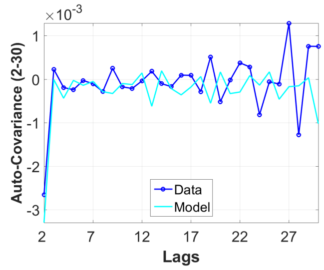

(e) $Kurt[g_{i,h}]$



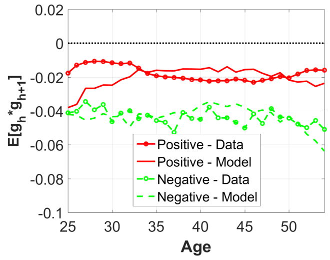

(f) $Prob(g_{i,h} > 0)$



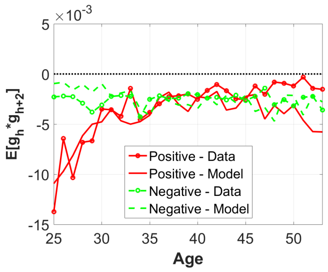

(g) $Prob(|g_{i,h}| > 5\%)$



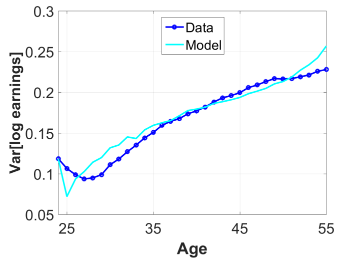

(h) $\mathbb{E}[g_{i,h+k}g_{i,h}]$



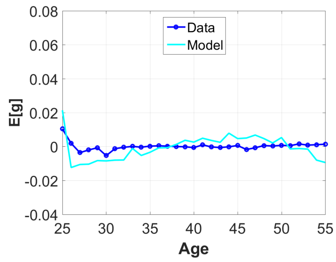

(i) $\mathbb{E}[g_{i,h+1}g_{i,h}|g_{i,h} \gtrless 0]$




(j) $\mathbb{E}[g_{i,h+2}g_{i,h}|g_{i,h} \gtrless 0]$

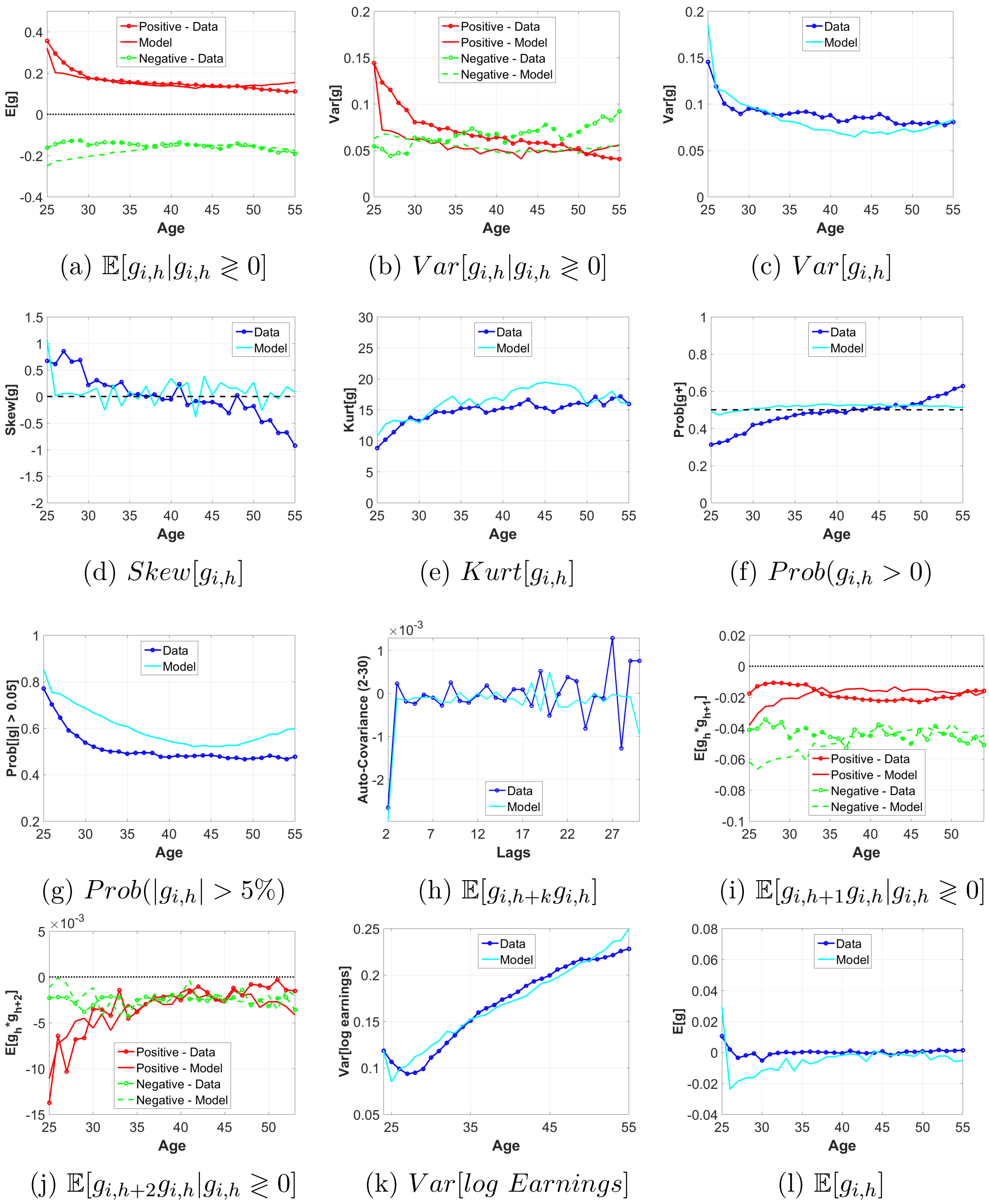



(k) $Var[log\ Earnings]$




(l) $\mathbb{E}[g_{i,h}]$

Figure A4: Model Fit - Column (1), Table 1




(a) $\mathbb{E}[g_{i,h}|g_{i,h} \gtrless 0]$ (b) $Var[g_{i,h}|g_{i,h} \gtrless 0]$ (c) $Var[g_{i,h}]$

(d) $Skew[g_{i,h}]$ (e) $Kurt[g_{i,h}]$ (f) $Prob(g_{i,h} > 0)$

(g) $Prob(|g_{i,h}| > 5\%)$ (h) $\mathbb{E}[g_{i,h+k}g_{i,h}]$ (i) $\mathbb{E}[g_{i,h+1}g_{i,h}|g_{i,h} \gtrless 0]$

(j) $\mathbb{E}[g_{i,h+2}g_{i,h}|g_{i,h} \gtrless 0]$ (k) $Var[log\ Earnings]$ (l) $\mathbb{E}[g_{i,h}]$

Figure A5: Model Fit - Column (3), Table 1

Table A4: Standard Errors, Column (1) Table 1.

| **Parameters** | SE | **Parameters** | SE |
|---|---|---|---|
| $\theta^{+}$ | 0.0355*** | $\delta_{I}^{-}$ | 0.0005*** |
| $\theta^{-}$ | 0.0177*** | $\delta_{II}^{-}$ | 0.0003*** |
| $\rho^{-}$ | 0.0116*** | $\delta_{III}^{-}$ | 0.0001*** |
| $\rho^{+}$ | 0.0036*** | $\delta_{I}^{+}$ | 0.0016*** |
| $\sigma_{\beta}$ | 0.0046 | $\delta_{II}^{+}$ | 0.0007*** |
| $\sigma_{\alpha}$ | 0.0314 | $\delta_{III}^{+}$ | 0.0002** |
| $\gamma_{a,\iota}^{+}$ | 0.0820*** | $\gamma_{a,\xi}^{-}$ | 0.1979 |
| $\gamma_{b,\iota}^{+}$ | 0.0281 | $\gamma_{b,\xi}^{-}$ | 0.0323 |
| $\gamma_{a,\iota}^{-}$ | 0.0823*** | $\gamma_{a,\xi}^{+}$ | 0.0343*** |
| $\gamma_{b,\iota}^{-}$ | 0.0177 | $\gamma_{b,\xi}^{+}$ | 0.0271 |
| $\lambda_{a}^{+}$ | 0.0911*** | $\lambda_{a}^{-}$ | 0.2571*** |
| $\lambda_{b}^{+}$ | 0.0606 | $\lambda_{b}^{-}$ | 0.0379 |
| $\gamma_{a,\iota^{n}}$ | 0.0210 | $\lambda_{a,\iota^{n}}$ | 0.0248*** |
| $\gamma_{b,\iota^{n}}$ | 0.0041 | $\lambda_{b,\iota^{n}}$ | 0.0176 |
| $\gamma_{0,\iota}^{+}$ | 0.3297 | $\lambda_{0,\iota}^{+}$ | 0.9778 |
| $\gamma_{0,\iota}^{-}$ | 0.0752*** | $\lambda_{0,\iota}^{-}$ | 0.3294*** |

Notes: The table displays standard errors for the estimated model of Column (1) in Tables 1 and A3. Standard errors are obtained by 100 block bootstraps. Estimates with superscripts {*,**,***} imply the parameter is different from zero at the 10, 5, and 1 percent significance level, respectively.