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ABSTRACT

Working Time Accounts and Turnover[1]

Working time account is an organization tool that allows firms smoothing their demand for hours employed. Descriptive literature suggests that working time accounts reduce turnover and inhibit increase in unemployment during recessions. In a model of optimal choice of hours by a firm I show that working time account does not necessarily guarantee lower turnover. Turnover may be reduced or increased depending on whether a firm meets economic downturn with surplus or deficit of hours and on how productive this firm is. The model predicts that working time accounts contributed positively to reducing turnover in Germany during the Great Recession.

JEL Classification: J23, J63

Keywords: labour demand, working hours, working time accounts, turnover, Great Recession, Germany

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1 Introduction

European unemployment has increased dramatically during the recession that followed the global financial crises of 2007-2008 (the so-called *Great Recession*). Though while a number of major OECD countries have reported soaring unemployment rates, notably the US where unemployment rate has increased by unprecedented 5.5 percentage points reaching 9.9% at its peak (OECD, 2013), unemployment rate in Germany has shown nearly no changes.[2] Comparing further Germany and the US, although both countries have experienced a sharp decline in real GDP and a substantial reduction in person-hours worked, two important differences can be pointed out. First, while in the US a wave of firings went through, in Germany instead there was a large-scale decrease in hours worked per person with little job losses. In other words the post-crises adjustment at the German labour market took place on the intensive, rather than on the extensive margin. Second, composition of sectors affected by the crises and patterns of sector-specific post-crises recovery differ widely in the two countries. In Germany it is rather the exporting branch of the manufacturing sector that was hit strongly by the crisis (as measured by the drop in the value added). In the US, to the contrary, housing market, construction, retail services and financial services have suffered most. Germany has recovered faster than the US.[3]

In a landmark descriptive study Burda and Hunt (2011) look into multiplicity of factors that could help explain the surprisingly weak reaction of the German unemployment to the crises. Among others, they put forward a particular flexible working hours scheme called *working time accounts*. Potential of working time accounts is likewise emphasized by Möller (2010) and Rinne and Zimmermann (2013). Working time account is essentially a bookkeeping tool used by firms to track under- and overtime work. Firms that operate working time accounts for their personnel may, for instance, let employees work overtime but do not need to pay for this overtime work. Instead overtime work is written onto an account as a "debt" of the firm to its employee, such that at some point in the future the employee may work less, running down overtime hours accumulated on her account. Hourly wage rate as well as per period pay stay constant regardless of whether the employee currently has surplus or deficit on her working time account. There exist limits on the amount of accumulated surplus and deficit of hours. Finally by the end of the pre-specified time interval, called compensation period, the account must be balanced, i.e. both firms' debt to worker and workers' debt to firm, measured in hours, should be equal to zero.[4]

Shortly before the financial crises almost 45% of all German employees were in possession of a working time account (Zapf, 2012). Pre-crises years show a distinct pattern of changes in the balances of working time accounts at German establishments. While years 2005-2007 saw gradual increase in surplus of hours, year 2008 has been marked with their unusual extremely

[2]In fact German unemployment rate has continued to fall, loosing 0.5 percentage points in the first quarter of the recession. It did not change in the second quarter and started to go up only thereafter, picking 0.7 percentage points during the next two quarters. With the entire recession lasting one year, the economy entered recession with the unemployment rate of 7.7% and left recession with the unemployment rate of 7.9%. Once the recession was over unemployment rate started falling again (see OECD, 2013).

[3]For excellent descriptions of the US and German labour markets during the Great Recession see Eslby et al. (2010) and Burda and Hunt (2011), respectively.

[4]See Zapf and Herzog-Stein (2015) for an excellent review of the organization of working time accounts in Germany.

sharp fall (Zapf, 2012). Such dynamics has led the literature to suggest the mechanism through which working time accounts could have contributed to inhibiting the increase in German unemployment during the Great Recession. In particular, Burda and Hunt (2011) argue that by building up surpluses of hours worked in good times and running them down in bad times firms avoided firing workers immediately. A worker will not be fired unless she is compensated for the unpaid overtime hours worked previously. This compensation takes a form of working for a while at reduced hours with no change in workers salary, consistent with the stylized fact of falling hours worked per person in Germany during the Great Recession. Since the crises in Germany was rather a consequence of a drop in demand for German export goods at the world's market, the nature of the negative shock to the economy was temporary. By running down the surplus first, working time accounts postponed job destruction and gave many jobs sufficient time to survive until world's demand started showing signs of recovery. With increasing pace of recovery slashing jobs has become increasingly unnecessary. This lack of job destruction has found reflection in the absence of increase of the unemployment rate.

In the present paper I show that the theoretical relationship between working time accounts and turnover is more general than the one described by Burda and Hunt (2011). I demonstrate that working time accounts do not always restrain turnover at the firm level when a negative demand shock hits the goods market. Sometimes turnover can be amplified. The ultimate impact of working time accounts on turnover depends on two factors: (i) on the productivity of a firm relative to its wage cost, and (ii) on whether a firm has surplus or deficit on its working time accounts balance in face of a demand downturn. I find that at relatively high-productive firms working time accounts lead to *lower* turnover if a firm has surplus of hours and *higher* turnover if a firm has deficit of hours prior to the adverse demand shock. At relatively low-productive firms converse is true: working time accounts lead to *higher* turnover if there is surplus of hours and *lower* turnover if there is deficit of hours in face of the negative demand shock. In all the cases above turnover is always compared to that of an identical firm without working time accounts.

Intuition for the possible harmful impact of working time accounts on high-productive firms that face negative demand shock with deficit of hours can be illustrated by looking at the combined influence of returns on investment made in the past and direct profit obtained at present. With deficit of hours prior to the shock there is too little profit left to be invested and hence too little returns to be obtained comparatively to an identical firm without working time account. Once negative demand shock realizes, the loss may be too large to be compensated by low returns on the past investment even despite the increase in hours required to balance the account. As a result, jobs at a firm with working time account can be destroyed by a weaker shock than jobs at an identical firm without the account, where hours worked always remained constant. Intuition for the harmful effect of working time accounts on low-productive firms that face negative demand shock with surplus of hours is similar. Low productivity reduces profit and hence the size of investment made in the past as such. Despite returns on this investment will still be higher than at an identical firm without working time account due to surplus of hours, the necessity to reduce hours in order to balance the account once negative demand shock realizes may lead to a loss too large to be compensated by returns on the past investment. As a result jobs at a firm with working time accounts can again become more vulnerable.

Although I demonstrate that the general relationship between working time accounts and turnover is ambiguous at best, it is worth emphasizing that the hypothesis of Burda and Hunt (2011) is still an element in the set of my results. This makes the insight of the present paper particularly important. It suggests that the countries that wish to introduce working time accounts in the way these are organized in Germany may as well achieve some success in harnessing the rise of unemployment during recessions. However, depending on persistence of the shock that triggers the recession and the particular type of the firm hit by this shock, working time accounts may also lead to a completely opposite result. This insight is new to the literature.

To obtain all my results I construct a basic dynamic model of labour demand by a firm that operates a working time account. In this model the firm is a local monopolist that faces uncertainty about future demand at the goods market and chooses working hours subject to constraints imposed by the working time account regulations. There is no borrowing, but the firm may invest in a riskless asset. Intertemporal transfer of profits via investment is instrumental for the working time account to function. To the best of my knowledge this paper is the first to formalize the mechanics of the working time account.

The paper is organized as follows. Section 2 presents the basic model of a firm with a working time account and solves the problem of optimal hours choice. Section 3 discusses properties of the optimal solution and analyses the relationship between working time accounts and turnover. Section 4 concludes and sets directions for future research.

2 The model

2.1 Market structure and characteristics of a firm

- Output and demand at the goods market

A firm is equipped with production technology $Y_t = Ah_t$, where A is the productivity of the firm and h_t are *actual* hours worked per worker. For simplicity I assume that one firm employes just one worker. Therefore in what follows the terms "job destruction" and "bankruptcy of a firm" will have the meaning equivalent to a single worker loosing her job. Let m_t denote the demand for produced good. I specify the demand function as in Bentolila and Bertola (1990). I suggest that the firm is a local monopolist, such that the reduced-form demand function is

$$m_t = z_t p_t^{1/(\epsilon-1)}, \quad \epsilon \in (0,1)\,, \tag{1}$$

where p_t is the price of a good and ϵ is the inverted price mark-up that reflects the monopoly power of the firm. Similarly to Bentolila and Bertola (1990), scale parameter z_t in this demand function is subject to stochastic fluctuations at the goods market. I would generally suggest that z_t is a realization of a random variable Z_t, where $Z_t \sim F(z_t)$ and F is stationary. Stochastic fluctuations of z_t will constitute the only source of uncertainty influencing the optimal choice of hours employed by the firm in my model.

Assuming that the firm produces a non-storable good, output needs to equal demand at the goods market, implying

$$m_t = Ah_t. \tag{2}$$

- Working hours and working time accounts

Consider now hours employed. I make an important distinction between *actual* hours and *contracted* hours employed by the firm. Despite a worker has actually worked h_t for her firm, the firm does not pay the worker on the basis of h_t. Wage bill of the firm is calculated on the basis of a *contracted* amount of hours $\bar{h}$ instead, where $\bar{h}$ does not change over time. At any given t it need not be that $h_t = \bar{h}$. Consequently, there may exist either surplus or deficit of actual hours worked relative to contracted hours. Surplus will be viewed as a credit from worker to firm and deficit will be viewed as a credit from firm to worker. In addition at any given t there exist objective constraints on the actual hours worked, which tell that a person cannot work more than $h^{\max}$ and less than $h_{\min}$, i.e. $h_{\min} \leq h_t \leq h^{\max}$.[5]

At any t the surplus/deficit of hours worked is written onto a *working time account.* Let us denote the balance of the working time account by b_t. In addition let $b^{\max}$ stand for the upper limit of surplus accumulation, $b^{\max} > 0$, and let $b_{\min}$ stand for the lower limit of deficit accumulation, $b_{\min} < 0$. At the moment of opening the working time account, which I set to zero, the balance of the account is necessarily zero, $b_0 = 0$. For all dates to follow the balance of the working time account may take any value between $b_{\min}$ and $b^{\max}$. However, it must hold that at the end of each compensation period the account must be balanced, such that total amount of actual hours worked is equal to total amount of contracted hours within each compensation period. Equivalently, at the end of each compensation period all credit from worker to firm must be compensated by the firm as well as all credit from firm to worker must be compensated by the worker. Denoting the length of the compensation period by τ I therefore require that $b_{j\tau} = 0$, where $j = 1, 2, ...$[6]

Maintaining that time is discrete, the above argument leads us to the law of motion for the balance of the working time account

$$b_t = b_{t-1} + (h_t - \bar{h}), \tag{3}$$

where $b_{\min} \leq b_t \leq b^{\max}$, $b_{j\tau} = 0$ with $j = 0, 1, 2, ...$ and $t = 1, 2, ...$

- Profit function and borrowing constraints

Consider now the profit function of a firm. Using equations (1) and (2) in Appendix A.1 I show that profit of a firm reads

$$\pi_t (h_t) = z_t^{1-\epsilon} [A h_t]^{\epsilon} - w\bar{h}. \tag{4}$$

A firm operates as long as it is able to pay its wage costs. If in any t wage bill cannot be paid, the firm goes bankrupt and disappears from the market immediately. As a result, there arises demand for credit when in a given period t firms' revenues become insufficient to pay workers their contracted wage. This occurs, for instance, when a negative shock hits the

[5] At the extreme $h_{\min}$ cannot be less than zero hours per day and $h^{\max}$ cannot be more 24 hours per day. Furthermore, with $h_{\min} \leq h_t \leq h^{\max}$, clearly it also holds that $h_{\min} < \bar{h} < h^{\max}$.

[6] According to Zapf and Herzog-Stein (2015) in 2007 in Germany the average limit of surplus accumulation was equal to +103 hours, the average limit of deficit accumulation was equal to −63 hours and the average duration of compensation period was about 38 weeks.

goods market. Consistent with the credit crunch during the Great Recession, I do not allow firms to finance labour costs through borrowing at the financial market. Important however is that despite not being able to borrow, a firm can still invest its profit into a riskless asset with an interest rate r.

2.2 Optimal choice of hours

The task of a firm is to choose the sequence of hours that maximizes the sum of expected discounted profits subject to working time accounts regulations and conditions for survival of the firm at the market. In what follows I set up the optimization problem and derive the optimal solution for the hours employed.

- Time horizon and uncertainty

I assume that a firm lives only for two periods (i.e. $t = 1{,}2$) and the compensation period for a working time account is equal to two model periods (i.e. $\tau = 2$). This implies the following dynamics of the balance of a working time account: $b_0 = 0$, $b_1 \gtreqless 0$ and $b_2 = 0$. In principle a firm may live infinitely long. However, when drafting its optimal demand for hours the firm should respect the length of the compensation period in order to have its working time account balanced at due dates. Therefore it is only interesting what happens *within* a single compensation interval. For this reason a two-period model where the life of the firm is equal to the length of the compensation period is sufficient to study the effect of a working time account.

The demand level at the goods market reveals itself at the beginning of each period. A firm drafts its optimal demand for hours at the beginning of the first period. Thus the firm observes z_1 but still needs to form expectations about the value of z_2. These expectations are formed at $t = 1$ with respect to F.

- Objective function and constraints

Consider the first period. Under the assumption that the firm observes z_1 I can guarantee that the wage bill of the firm active at the market will always be paid in the first period, i.e. $\pi_1(h_1) \geq 0$ is always respected in the optimal choice of hours. By the end of the first period the firm possesses $(1 + r)\pi_1(h_1)$ accumulated by means of investing into a riskless asset with interest rate r.

Consider the second period. If the realized value of z_2 in the second period is small enough, such that $\pi_2(h_2)$ becomes negative, part of the wage bill in the second period will be paid using the profit from the first period together with returns on investing this profit in the riskless asset, $(1 + r)\pi_1(h_1)$. If the realized value of z_2 is too small, the necessity to pay the wage bill in the second period may consume the entire amount of $(1 + r)\pi_1(h_1)$. Should this amount be insufficient to cover the wage bill the firm goes bankrupt and disappears from the market. Thus the most the firm can loose is $(1 + r)\pi_1(h_1)$, which provides the lower bound on the size of loss in the second period and defines the limit of liability of the firm towards worker. I write the profit in the second period constrained by limited liability of a firm, $\hat{\pi}_2$, as

$$\hat{\pi}_2(h_2) = \max\{-(1 + r)\pi_1(h_1), \pi_2(h_2)\}. \tag{5}$$

Let $\beta \equiv 1/(1+r)$ denote the period discount factor. Then the value of a firm writes

$$V = \max_{\{h_1,h_2\}} \{\pi_1(h_1) + \beta E_1(\hat{\pi}_2(h_2))\}, \tag{6}$$

where E_1 is the expectation operator at $t = 1$. Note that (5) and (6) imply that $V \geq 0$.

Consider now the working time account regulations. The assumed two-period structure of the model provides an easy characterization of the balance of the working time account at the end of each period. Using (3) we can see that

$$t = 1: \quad b_1 = h_1 - \bar{h} \gtreqless 0, \tag{7}$$

$$t = 2: \quad b_2 = b_1 + (h_2 - \bar{h}) = 0, \tag{8}$$

where $b_2 = 0$ reflects the necessity to balance the account once the compensation period is over. From (7)-(8) follows that $h_2 = 2\bar{h} - h_1$. This means that once the choice of hours in the first period is made, it immediately pins down the choice of hours in the second period, so the problem of the firm reduces to choosing $h_1 : h_{\min} \leq h_1 \leq h^{\max}$. From (6) it is evident that this choice remains to be influenced by uncertainty about demand level at the goods market in the second period.

- Optimal solution

With all above, the problem of hours choice under working time accounts regulations and limited liability of the firm towards workers writes

$$V = \max_{\{h_1\}} \left\{\pi_1(h_1) + \beta E_1\left(\hat{\pi}_2\left(2\bar{h} - h_1\right)\right)\right\} \tag{9}$$

subject to:

$$h_{\min} \leq h_1 \leq h^{\max}, \tag{10}$$

$$\pi_1(h_1) \geq 0. \tag{11}$$

First order condition for the firms' problem in (9) follows immediately:

$$\pi_1'(h_1) - \beta E_1\left(\hat{\pi}_2'\left(2\bar{h} - h_1\right)\right) = 0. \tag{12}$$

Interpretation of this first order condition is quite standard. It tells that marginal benefit of a unit of labour today should be equal to the expected discounted marginal benefit of a unit of labour tomorrow.

Given the profit function in (4), after some algebra (see Appendix A.2) we get

$$h_1 = \frac{2}{\frac{1}{z_1}\left[\beta E_1(z_2^{1-\epsilon})\right]^{1/(1-\epsilon)} + 1}\bar{h}, \tag{13a}$$

$$h_2 = \frac{2}{1 + z_1\left[\beta E_1(z_2^{1-\epsilon})\right]^{1/(\epsilon-1)}}\bar{h}, \tag{13b}$$

where (13b) follows from (13a) because of the necessity to balance the working time account at the end of the compensation period.

To complete the characterization of the optimal solution we need to make sure that inequality constraints (10)-(11) are always respected. First note that the optimal amount of hours employed in the first period may not be lower than $\tilde{h} = \frac{1}{A}\left(z_1^{\epsilon-1}[w\bar{h}]\right)^{1/\epsilon}$, where $\tilde{h}$ solves $\pi_1(\tilde{h}) = 0$. Second, the optimal amount of hours in the first period may not be lower than $h_{\min}$ and may not be higher than $h^{\max}$. Defining by h_1^* and h_2^* the optimal hours in periods one and two, respectively, these optimal hours become

$$h_1^* = \max\left\{\min\left\{h^{\max}, \frac{2}{\frac{1}{z_1}\left[\beta E_1(z_2^{1-\epsilon})\right]^{1/(1-\epsilon)}+1}\bar{h}\right\}, \max\left\{h_{\min}, \tilde{h}\right\}\right\}, \tag{14a}$$

$$h_2^* = 2\bar{h} - h_1^*, \tag{14b}$$

where, as before, (14b) follows from the necessity to balance the working time account. Figure 1 visualizes this solution. I discuss it in detail in the following section.

3 Hours, profits and impact of a working time account

3.1 Determinants of hours

The solution for optimal hours in (14) has several interesting analytical properties. First, we can see that no matter the period optimal hours always depend on two variables: the realized value of the demand level parameter at the goods market in the first period, z_1, and the expected value of the demand level parameter at the goods market in the second period, $E_1(z_2)$. Second, equation (14b) implies that whenever constraints do not bind a change in any of these two variables will make h_1^* and h_2^* move in opposite directions.

Figure 1 represents the optimal choice of hours in both periods as a function of the realized demand parameter z_1 for a fixed value of $E_1(z_2)$. Solid line in the left panel illustrates h_1^* and solid line in the right panel illustrates h_2^*. It is straightforward to show (see Appendix A.3) that optimal hours in the first period increase in z_1, and hence optimal hours in the second period fall in z_1, when constraints do not bind. This means that the better is the situation with demand at the goods market today, the more inclined is the firm to produce today, as compared to tomorrow. Binding constraints are reflected by flat lines at $h^{\max}$ and $\max\{h_{\min}, \tilde{h}\}$.

Dependence of optimal hours on the expected value of z_2 is just the opposite. As shown in Appendix A.3, for any given value of z_1 optimal hours in the first period decrease in $E_1(z_2)$ when constraints do not bind. From this follows that optimal hours in the second period increase in the expected value of the demand level in the second period. In Figure 1 this dependence is reflected by a vertical downward shift of the optimal hours curve in the first period (left panel) and a vertical upward shift of the optimal hours curve in the second period (right panel) for an increasing value of $E_1(z_2)$. It means that the better is the expected situation at the goods market tomorrow the less inclined will be the firm to produce today, and so the more production will be shifted to tomorrow, as compared to today. Again, binding constraints are reflected by flat lines at $h^{\max}$ and $\max\{h_{\min}, \tilde{h}\}$.

The above properties of optimal hours become particularly insightful if placed in the context of expansion/recession. If one associates the higher than average demand level at the goods market with an expansion and lower than average demand with a recession, then with values of z_1 sufficiently higher than $E_1(z_2)$ a firm will employ more hours in the expansion and with values of z_1 sufficiently lower than $E_1(z_2)$ a firm will employ less hours in the recession. Consequently, the optimal solution displays coherence with the observed fact that German firms have accumulated high surpluses on their working time accounts during the expansion and were running down these surpluses during the recession, as noted by Burda and Hunt (2011).

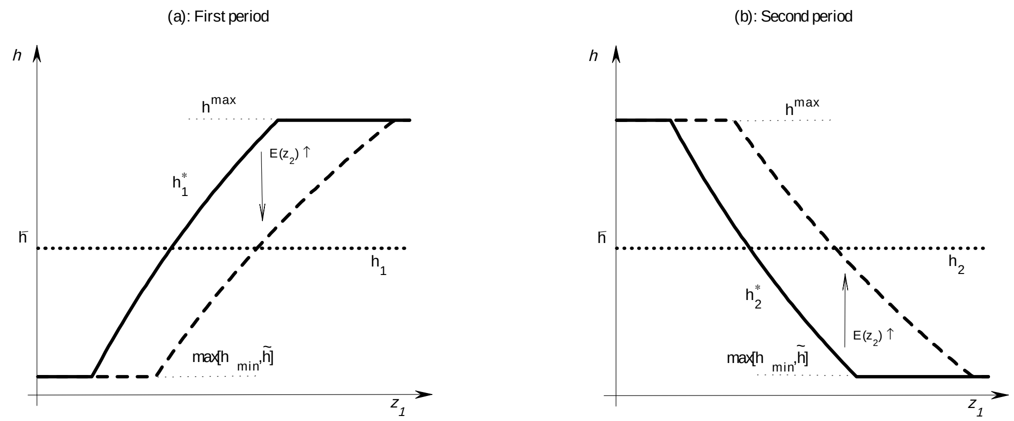

Figure 1 *Optimal hours*

Lastly, both panels of Figure 1 show a horizontal dotted line at $\bar{h}$. This line represents for the sake of comparison the demand for hours of an identical firm that, for some exogenous reason, does not operate a working time account. Since the actual hours at such a firm are always equal to contracted hours, it is evident that $h_1 = h_2 = \bar{h}$. Clearly, this hours schedule is independent of z_1 and $E_1(z_2)$, as the firm lacks the necessary instrument to react to demand fluctuations at the goods market. The differences $h_1^* - \bar{h}$ and $h_2^* - \bar{h}$ reflect the change to the balance of the working time account within each period.

3.2 Determinants of profits

Consider now profit levels implied by the optimal choice of hours in presence of a working time account. Being a function of optimal hours, profit of a firm in any period clearly depends on the parameters that determine optimal hours in this period, i.e. on z_1 and $E_1(z_2)$. Apart form these, profit in the first period depends directly on the realized value of a demand level parameter in the first period, z_1, and profit in the second period depends directly on the realized value of a demand level parameter in the second period, z_2. Let us introduce the notation $\pi_1^* \equiv \pi_1(h_1^*)$ and $\pi_2^* \equiv \pi_2(h_1^*)$. Using (4) and the optimal solution for hours it is straightforward to show that π_1^* increases in z_1 and π_2^* increases in z_2. This dependence is captured by Figure 2.

For a given value of $E_1(z_2)$ solid line in the left panel of Figure 2 plots π_1^* against the realized value of z_1 and solid line in the right panel of Figure 2 plots π_2^* against the realized value of z_2. These solid lines differ in shape because there is no indirect dependence of π_2^* on z_2 via hours. Still both profit functions are increasing, which tells that the higher is the demand at the goods market today the higher is the profit made today.

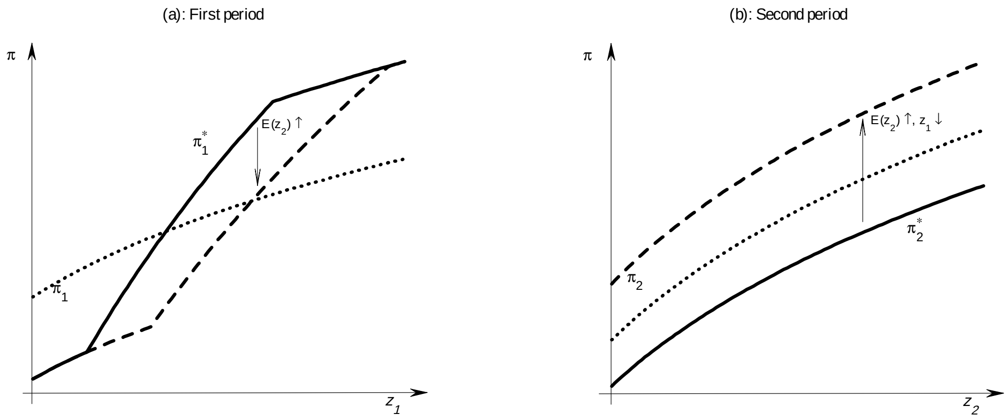

Figure 2 *Optimal profits*

Changes in the expected value of demand level parameter z_2 that induce changes in hours result into qualitatively similar changes in profits. Since profit function is monotone in hours, it follows that π_1^* falls in $E_1(z_2)$ and π_2^* increases in $E_1(z_2)$, ceteris paribus. In Figure 2 the dependence of profits on the expected value of the demand level parameter in the second period is reflected by a vertical downward shift of the solid line in the left panel and a vertical upward shift of the solid line in the right panel as $E_1(z_2)$ goes up. This tells that the better is the expected situation at the goods market tomorrow, the lower will be firm's profit today and the higher will be the firm's profit tomorrow. Finally, since an increase in z_1 lowers optimal hours in the second period, there is also a negative dependence between π_2^* and z_1.

What makes Figure 2 particularly interesting is the comparison of π_1^* and π_2^* with profits of an identical firm that for some exogenous reason does not operate a working time account. These profits are depicted by a dotted line in the left and in the right panel (denoted by π_1 and π_2, respectively). Since hours employed by such a firm are simply $h_1 = h_2 = \bar{h}$, the corresponding profit functions solely depend on realized demand level parameters, and the dependence is strictly positive. Figure 2 shows that it is not always the case, that profits of a firm with working time account are higher than profits of a firm without such an account. This leads us to question under which circumstances will a firm be ready to open working time account as such.

3.3 Adoption of a working time account

To see when a firm will choose to open a working time account we need to consider the values of a firm with and without the account. First of all, from (5) and (6), limited liability of a firm towards worker implies that value of a firm is nonnegative no matter if the firm operates a working time account or not. Let V^* denote the value of a firm with working time account, i.e. with hours policy $\{h_1^*, h_2^*\}$ as in (14). Let $\bar{V}$ denote the value of an identical firm without working time account, i.e. with hours policy $\{\bar{h}, \bar{h}\}$. Left panel of Figure 3 plots the ratio $V^*/\bar{V}$ as a function of z_1 and right panel of Figure 3 plots the same ratio as a function of $E_1(z_2^{1-\epsilon})$. Parameter values for this illustration are reported in Appendix A.4.

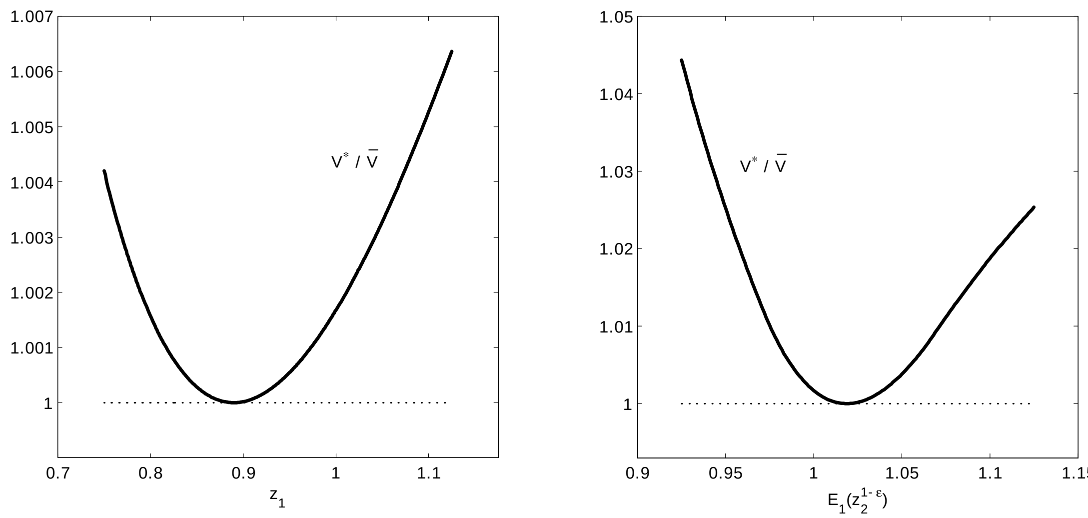

Figure 3 *Value of a firm*

We see that in both cases the value of a firm with working time account always exceeds the value of a firm without the account (except at $h_1^* = h_2^* = \bar{h}$). Indeed $V^* \geq \bar{V}$ should always hold because $\bar{V}$ is the value of a firm obtained under the same set of constraints as V^* plus an additional constraint that restricts hours as $h_1^* = h_2^* = \bar{h}$. This means that a firm will always choose to open a working time account for its employee.

3.4 Working time account and turnover

Given that a firm will always decide to open a working time account it would be tempting to suggest that a firm with working time account will always be able to withstand stronger demand downturns if compared to an identical firm without working time account. As a result, working time account will arguably always reduce turnover. Whether this is true or not shows the following analysis.

- Directions of influence

Consider a threshold level of the realized demand parameter in the second period that leads to destruction of a firm. Let z_2^* denote this threshold level for a firm with working time account and let $\bar{z}_2$ denote a threshold level for a firm without working time account. Then for any realization of z_2 such that $z_2 < z_2^*$ ($z_2 < \bar{z}_2$) demand downturn at the goods market leads a firm with (without) working time account to bankruptcy. In Appendix A.5 I show that the respective threshold values are given by

$$z_2^* = \left(\frac{w\bar{h} - (1+r)\,\pi_1\,(h_1^*)}{[Ah_2^*]^{\epsilon}} \right)^{1/(1-\epsilon)}, \tag{15}$$

$$\bar{z}_2 = \left(\frac{w\bar{h} - (1+r)\,\pi_1\,(\bar{h})}{[A\bar{h}]^{\epsilon}} \right)^{1/(1-\epsilon)}. \tag{16}$$

Both thresholds unambiguously increase in wage rate and decrease in productivity, i.e. the higher is the wage rate (the lower is the productivity) the weaker shock is needed to destroy the firm. We also see that in general z_2^* and $\bar{z}_2$ are not equal to each other. The intriguing question therefore is: Is it always true that $z_2^* < \bar{z}_2$? If this is the case, then for intermediate realizations of the demand parameter z_2 such that $z_2^* < z_2 < \bar{z}_2$ a firm with working time account will survive the demand downturn, whereas an identical firm without working time account will not. Consequently, working time account will contribute to reduction of turnover and hence to restraining the increase of unemployment.

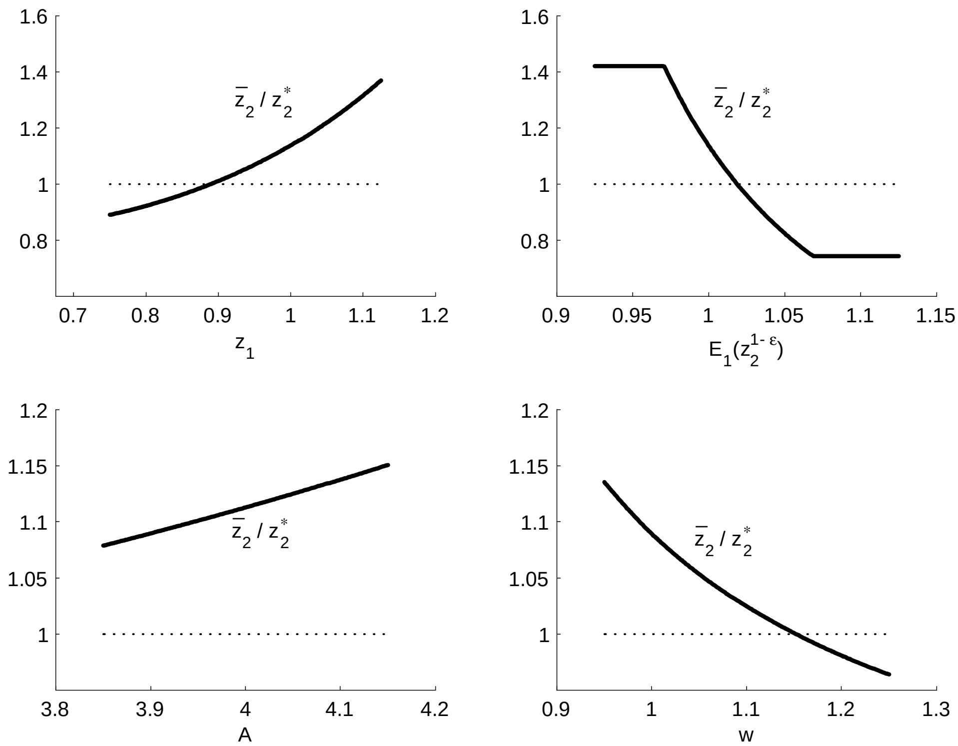

Figure 4 *Implications for turnover*

Surprisingly, we find that $z_2^* < \bar{z}_2$ may not always hold. Figure 4 illustrates the ratio of bankruptcy thresholds, $\bar{z}_2/z_2^*$, as a function of a set of model parameters, namely: z_1,

$E_1(z_2^{1-\epsilon})$, A and w. Parameter values for this illustration are reported in Appendix A.4. Figure 4 clearly shows that for a range of values of z_1, $E_1(z_2^{1-\epsilon})$ and w bankruptcy threshold of a firm with working time account *exceeds* that of a firm without working time account. Consequently, in this range of values for intermediate realization of the demand level parameter z_2 in the second period, namely for $\bar{z}_2 < z_2 < z_2^*$, a firm with working time account gets destroyed whereas a firm without working time account survives the downturn. This tells that in fact working time account sometimes *increases* turnover and thereby contributes to increasing unemployment.

Looking at the first row of Figure 4 we can see that detrimental effect of the working time account obtains either when the current state of demand z_1 is too low, while expected value of the future state of demand stays unchanged, or when expected value of the future state of demand $E_1(z_2^{1-\epsilon})$ is too high, while the current state of demand stays unchanged. Rearranging (14a) we can show that optimal choice of hours in the first period is always less than the contracted amount hours if $z_1^{1-\epsilon} < \beta E_1(z_2^{1-\epsilon})$, i.e. if the current state of demand is sufficiently low relative to the expected state of demand in the next period. Thus, the first row of Figure 4 suggests that working time account is likely to enhance turnover when demand downturn at the goods market is met with a deficit at the working time accounts' balance. Looking at the second row of Figure 4 we can see that the ratio of bankruptcy thresholds positively depends on productivity A and negatively depends on hourly wage rate w. For high enough values of hourly wage, keeping productivity unchanged, bankruptcy thresholds flip and working time account again contributes to higher turnover and hence higher unemployment.

Observations made with the help of Figure 4 are not coincidental. In fact these are manifestations of the general result on how working time account impacts turnover. This result is summarized in Proposition 1.

Proposition 1 *When productivity of a firm is high enough relative to its wage cost, working time account: (i) reduces turnover if a firm meets demand downturn with surplus of actual hours employed; (ii) increases turnover if a firm meets demand downturn with deficit of actual hours employed.* □

Proof. See Appendix A.6. ■

Necessary condition for Proposition 1 to hold is given in the Proof, though it is not particularly intuitive. However, if the firm is productive enough to insure that revenues from the first period cover the present value of wage costs of both periods in absence of working time account, i.e. whenever $z_1^{1-\epsilon}\left[A\bar{h}\right]^{\epsilon} \geq w\bar{h} + \frac{w\bar{h}}{1+r}$, it is sufficient for Proposition 1 to apply. This sufficient condition is likewise given in the Proof and discussed in Appendix A.6.

Proposition 1 highlights the key message of the paper. It tells that the general dependence between working time accounts and turnover is ambiguous. While the literature existing to this date has emphasized only the positive side of this dependence, namely turnover-reducing effect of a working time account, I show that turnover-enhancing effect is also present.

This turnover-enhancing effect deserves somewhat more attention. First, in this model the firm commits to its working hours policy only within each compensation period. Therefore, to meet the demand downturn with deficit it must be that: $^{a)}$ when the firm was choosing its working hours policy demand conditions at the goods market were already poor, which motivated the choice of initially running into deficit, and *b)* contrary to expectations, aggregate demand conditions did not improve thereafter, i.e. production continued to be too low to repay wage credit given by workers to the firm. Thus the detrimental effect of the working time account for high-productive firms materializes in *protracted recessions*, where recovery of demand at the goods market takes longer than initially expected.

Second, the very relationship between productivity and wage rate also plays a role. While for high-productive firms surplus of hours on working time account insures against higher turnover, it turns out that for low-productive firm the result is completely opposite. The result follows from the Proof of Proposition 1 and is summarized by the corollary below.

Corollary 1 *When productivity of a firm is low enough relative to its wage cost, working time account: (i) increases turnover if a firm meets demand downturn with surplus of actual hours employed; (ii) reduces turnover if a firm meets demand downturn with deficit of actual hours employed.* □

Necessary condition is given in the Proof of Proposition 1. Though, for Corollary 1 to apply, it is sufficient that the firm operates at the break even point or at least epsilon-above it in absence of a working time account. This is also shown in Appendix A.6. Thus, under normal circumstances, it takes a weaker negative demand shock at the goods market to destroy a low-productive firm with working time account vis-a-vis the firm of the same productivity without an account (at least at the lowest end of the productivity distribution).

- Mechanism of influence

For the above description of the impact of the working time account it is important to see the mechanism according to which the account affects turnover. The impact goes through the two channels: the intertemporal shifting of hours and the intertemporal shifting of profits.

Consider first the situation in which a firm with working time account meets downturn with surplus of hours on the account and compare this firm to an identical firm without working time account. If a firm with working time account meets downturn with surplus of hours, its profit in the first period is higher and its profit in the second period is lower than respective profits of a firm without working time account, due to intertemporal shifting of hours. As there is more profit to invest in the first period, there are more returns to get for the second period than at a firm without working time account, due to intertemporal shifting of profits. Thus, facing downturn in the second period, a firm with working time account has lower direct profit in the second period but higher returns on investment form the first period than an identical firm without working time account. It will be able to withstand a stronger demand downturn only if higher returns on investment in the first period outweigh the higher loss due to reduced hours in the second period. The higher is the productivity of a firm relative to wage cost, the higher is the importance of returns on investment. Since a firm with working time account invested more in the past, the result of Proposition 1 applies and the firm with working time account withstands stronger shock than the identical firm without the account. Once productivity of a firm relative to wage cost gets lower, the lower becomes the importance of returns on investment and the higher becomes the importance of hours in the second period, so according to Corollary 1 the firm with working time account gets destroyed by a weaker shock than an identical firm without the account.

Consider now the situation in which a firm with working time account meets downturn with deficit of hours on the account. The mechanics is just the opposite. Once meeting downturn with deficit of hours, the profit in the first period is lower and the profit in the second period is higher than respective profits of an identical firm without working time account, as implied by intertemporal shifting of hours. Lower profit in the first period means lower investment in the first period and hence lower returns in the second period than at a firm without working time account, as implied by intertemporal shifting of profits. Thus, facing downturn in the second period, a firm with working time account has higher direct profit in the second period but lower returns on investment from the past than an identical firm without working time account. To be able to withstand a stronger demand downturn the loss incurred in the second period, despite being lower than at an identical firm without working time account, should still not be too large, such that it could be covered by the relatively lower investment made in the past. The higher is the productivity of a firm relative to wage cost, the higher is the weight of returns on investment. Since a firm with working time account invested less in the past, according to Proposition 1 this firm needs a weaker shock to be destroyed than an identical firm without working time account. Once productivity gets sufficiently low relative to wage cost, direct effect of higher hours acquires more importance than returns on investment from the past, so Corollary 1 applies and a firm with working time account withstands a stronger demand downturn.

- Germany during the Great Recession: Policy experience and implications

Despite its relative simplicity the model developed above provides an able framework to mirror the pattern of turnover in Germany during the Great Recession as well as consider counterfactuals. Four observations in the literature appear relevant. First, Burda and Hunt (2011) argue that German firms have met the Great Recession with high surpluses on their working time accounts. Second, Möller (2010) states that the recession has primarily hit German exporting firms in manufacturing, which are regarded by Möller (2010) as "strong firms in economically strong regions". Third, Dustmann et al. (2014) emphasize low per unit labour costs throughout the entire period of interest. Fourth, the recession itself was lasting just one year, in contrast to much longer recovery in many OECD countries. These four observations fit well into the prediction of Proposition 1, which states that positive surpluses, high productivity relative to wage costs and no protraction of the recession make working time accounts reduce turnover. Nevertheless my results show that although working time accounts contribute to explaining German success, their influence does not need to be positive universally. Ambiguity of their effect depends on the nature of the shock. Two important situations in which presence of working time accounts can be more destructive than their absence are when recessions take much longer than expected and when recessions primarily hit low-productive firms.[7]

[7] An example of such setting would be Spain where shock to construction industry during the Great Recession was permanent rather than temporary.

Once policy maker weighs introduction of working time accounts in view of this ambiguous impact, another tool from the German institutional palette can help if combined with introduction of the accounts. This tool is the so-called short-time work. Short-time work supports the firm by temporarily paying salaries to employees using public funds. Thereby it postpones layoffs and hedges from protraction of the recession, weakening possible detrimental effect of working time accounts. Indeed Möller (2010) documents a pattern in which firms that were eventually applying for access to short-time work would first run down the surpluses on their working time accounts, if operating ones. Balleer et al. (2016) demonstrate, though, that it is important to distinguish the automatic stabilization effect of short-time work form a discretionary intervention. They find that while discretionary intervention did not help during the Great Recession, mere presence of short-time work in capacity of automatic stabilizer has significantly contributed to missing increase in unemployment. Combining my results with those of Balleer et al. (2016), policy advise would go for the introduction both working time accounts and short-time work simultaneously and permanently.

4 Conclusion

In this paper I construct a simple yet powerful model of demand for working hours by a local monopolist who operates a working time account. Optimal hours are chosen in face of uncertain demand conditions at the goods market and under consideration of constraints imposed by working time accounts regulations. Firms do not have access to credit, but can save at a risk-free rate. Motivated by the hypothesis of Burda and Hunt (2011) on the performance of working time accounts in Germany during the Great Recession, I use this model to investigate the connection between working time accounts and turnover.

Contrary to initial expectations, I find that firms with working time accounts need not necessarily have lower turnover than firms without such accounts. Working time account may increase turnover when a high-productive firm runs deficit of actual hours worked and expects improvement of demand at the goods market in future. Working time account may also increase turnover when a low-productive firm runs surplus of actual hours worked and expects deterioration of demand at the goods market in future. In both situations a firm without working time account will be able to withstand stronger demand downturns than an identical firm with the account. At the same time the model also encompasses the pattern described by Burda and Hunt (2011). I show that when a high-productive firm has surplus on its working time account and expects demand downturn at the goods market in future it will be able to withstand a stronger negative demand shock than a firm without working time account, provided that the recession that follows is not protracted.

The main message of this paper is that working time accounts may not be perceived by policy makers with too much optimism. While they are indeed a useful tool for enhancing flexibility of labour demand, their effect on turnover is ambiguous and depends on productivity of a firm, its wage costs and the nature of the shock that hits goods market. If we look at the German example that motivated this study, the model predicts that working time accounts should have indeed contributed to reducing turnover and hence restraining the rise of unemployment in Germany during the Great Recession. However the favorable combination of factors that has lead to this prediction may not necessarily be repeated in recessions to follow. This applies not only to Germany but also to any other country that considers introduction of similar working hours regulation in future.

One intriguing question for the future research is the quantification of an increase in unemployment that did not materialize due to working time accounts preventing it. To answer this question a researcher can, for instance, build a model of equilibrium unemployment in which firms explicitly operate working time accounts. Formalization of a working time account used in this paper can be taken as a first step towards constructing such an analytical equilibrium framework.

Appendix

A.1 Profit function

Consider the demand function given in (1). Solving (1) for price we get $p_t = z_t^{1-\epsilon} m_t^{\epsilon-1}$. Inserting (2) for output, revenue $p_t m_t$ becomes

$$\begin{aligned} p_t m_t &= z_t^{1-\epsilon} m_t^{\epsilon-1} m_t \\ &= z_t^{1-\epsilon} m_t^{\epsilon} = z_t^{1-\epsilon} \left[A h_t\right]^{\epsilon}. \end{aligned}$$

Since wage costs are given by $w\bar{h}$ profit function writes

$$\pi_t \left(h_t\right) = z_t^{1-\epsilon} \left[A h_t\right]^{\epsilon} - w\bar{h}.$$

Profit function is an explicit function of actual hours worked, h_t.

A.2 Optimal solution for hours

Optimal solution for h_1 follows form the first order condition (12). We get

$$\begin{aligned} \pi_1' \left(h_1\right) &= \beta E_1 \left(\pi_2' \left(2\bar{h} - h_1\right)\right) \\ \epsilon z_1^{1-\epsilon} A^{\epsilon} \left[h_1\right]^{\epsilon-1} &= \beta E_1 \left(\epsilon z_2^{1-\epsilon} A^{\epsilon} \left[2\bar{h} - h_1\right]^{\epsilon-1}\right) \\ z_1^{1-\epsilon} h_1^{\epsilon-1} &= \left[2\bar{h} - h_1\right]^{\epsilon-1} \beta E_1(z_2^{1-\epsilon}) \\ z_1^{(1-\epsilon)/(\epsilon-1)} h_1 &= \left[2\bar{h} - h_1\right] \left[\beta E_1(z_2^{1-\epsilon})\right]^{1/(\epsilon-1)} \\ z_1^{(1-\epsilon)/(\epsilon-1)} h_1 &= 2\bar{h} \left[\beta E_1(z_2^{1-\epsilon})\right]^{1/(\epsilon-1)} - h_1 \left[\beta E_1(z_2^{1-\epsilon})\right]^{1/(\epsilon-1)}, \end{aligned}$$

such that

$$\begin{aligned} h_1 &= \frac{2\bar{h} \left[\beta E_1(z_2^{1-\epsilon})\right]^{1/(\epsilon-1)}}{z_1^{(1-\epsilon)/(\epsilon-1)} + \left[\beta E_1(z_2^{1-\epsilon})\right]^{1/(\epsilon-1)}} = \frac{2\bar{h}}{\left[\beta E_1(z_2^{1-\epsilon})\right]^{-1/(\epsilon-1)} z_1^{(1-\epsilon)/(\epsilon-1)} + 1} \\ &= \frac{2\bar{h}}{\left[\beta E_1(z_2^{1-\epsilon})\right]^{1/(1-\epsilon)} \left[\frac{1}{z_1}\right]^{(1-\epsilon)/(1-\epsilon)} + 1} = \frac{2\bar{h}}{\left[\beta \frac{1}{z_1^{1-\epsilon}} E_1(z_2^{1-\epsilon})\right]^{1/(1-\epsilon)} + 1}, \end{aligned}$$

and finally

$$h_1 = \frac{2}{\frac{1}{z_1} \left[\beta E_1(z_2^{1-\epsilon})\right]^{1/(1-\epsilon)} + 1} \bar{h}.$$

A.3 Properties of optimal hours

- Dependence of h_1^* on z_1

Considering (14a) when constraints do not bind,

$$\frac{\partial h_1^*}{\partial z_1} = \frac{\partial}{\partial z_1}\left(\frac{2}{\frac{1}{z_1}\left[\beta E_1(z_2^{1-\epsilon})\right]^{1/(1-\epsilon)}+1}\bar{h}\right)$$

$$= \frac{2\bar{h}}{\left(\frac{1}{z_1}\left[\beta E_1(z_2^{1-\epsilon})\right]^{1/(1-\epsilon)}+1\right)^2}\frac{1}{z_1^2}\left[\beta E_1(z_2^{1-\epsilon})\right]^{1/(1-\epsilon)} > 0$$

- Dependence of h_1^* on $E_1(z_2)$

Define $E \equiv E_1({z_2}^{1-\epsilon})$. Considering (14a) when constraints do not bind,

$$\frac{\partial h_1^*}{\partial E} = \frac{\partial}{\partial E}\left(\frac{2}{\frac{1}{z_1}\left[\beta E\right]^{1/(1-\epsilon)}+1}\bar{h}\right)$$

$$= -\frac{2\bar{h}}{(1-\epsilon)\,z_1\left(\frac{1}{z_1}\left[\beta E\right]^{1/(1-\epsilon)}+1\right)^2}\beta^{1/(1-\epsilon)}E^{\epsilon/(1-\epsilon)} < 0.$$

Since h_1^* decreases in $E_1({z_2}^{1-\epsilon})$ and ${z_2}^{1-\epsilon}$ is a monotone increasing transformation of z_2, h_1^* decreases in $E_1(z_2)$.

A.4 Parameters

Figures 3 and 4 are plotted using the following choice of parameters. I assume that the distribution of demand shocks has a unit mean, implying that $E_1(z_2^{1-\epsilon}) = 1$. I further normalize wage rate to unity, which also implies a scaled productivity measure (in or application $A = 4.1$). Lastly, z_1 is set to one as well. Table A.1 shows the ranges of variation of parameters on the horizontal axis in Figures 3 and 4. Its first block refers to Figure 3 and its second block to Figure 4. In this table, leading parameter of each row is the parameter on the x-axis which I let varying.

The rest of the parameters remain invariant all the time. These parameters are chosen to mimic German economy shortly before the Great Recession. I let one period in the model last six months. First, this corresponds to the time window within which the economy may technically enter recession (two consecutive quarters). Second, the length of the compensation period in manufacturing frequently lasts up to one year (Zapf and Herzog-Stein, 2015). Period interest rate is set to $r = 0.0188$, which corresponds to the average annual long-term interest rate of 3.8% in 2006-2009 (OECD, 2013). Period amount of hours worked at a firm without working time account, $\bar{h}$, is set to $\bar{h} = 670$ based on average annual hours actually worked per worker in dependent employment in 2006-2007 (OECD, 2013). I further set $h^{\max} = 1.15 \times \bar{h}$ and, symmetrically, $h_{\min} = 0.85 \times \bar{h}$. Lastly, inverted mark-up, ϵ, is set to $\epsilon = 1/1.19$ which is implied by the estimated price mark-up of 19% in German manufacturing (Christopoulou and Vermeulen, 2012).

	Adoption of a working time account ($V^*/\bar{V}$)			
	z_1	$E_1(z_2^{1-\epsilon})$	A	w
z_1	[0.750,1.125]	1	4.1	1
$E_1(z_2^{1-\epsilon})$	1	[0.925,1.125]	4.1	1

	Turnover ($z_2^*/\bar{z}_2$)			
	z_1	$E_1(z_2^{1-\epsilon})$	A	w
z_1	[0.725,1.115]	1	4.1	1
$E_1(z_2^{1-\epsilon})$	1	[0.925,1.125]	4.1	1
A	1	1	[3.85,4.15]	1
w	1	1	4.1	[0.95,1.25]

Table A.1 *Parameter values*

A.5 Bankruptcy thresholds

Consider a firm with a working time account. The firm is on the bankruptcy threshold if invested profit from the first period, together with return on this investment, is just sufficient to cover for the loss in the second period. I therefore look for z_2^* which solves $(1+r)\,\pi_1\left(h_1^*\right)+\pi_2\left(h_2^*\right)=0$. This gives

$$\begin{aligned}(1+r)\,\pi_1\left(h_1^*\right)+\left[z_2^*\right]^{1-\epsilon}\left[Ah_2^*\right]^{\epsilon}-w\bar{h}&=0\\ \left[z_2^*\right]^{1-\epsilon}\left[Ah_2^*\right]^{\epsilon}&=w\bar{h}-(1+r)\,\pi_1\left(h_1^*\right)\\ z_2^*&=\left(\frac{w\bar{h}-(1+r)\,\pi_1\left(h_1^*\right)}{\left[Ah_2^*\right]^{\epsilon}}\right)^{1/(1-\epsilon)}.\end{aligned}$$

Similar argument applies to a firm without a working time account. With $\bar{h}$ replacing h_1^* and h_2^* I get

$$\bar{z}_2=\left(\frac{w\bar{h}-(1+r)\,\pi_1\left(\bar{h}\right)}{\left[A\bar{h}\right]^{\epsilon}}\right)^{1/(1-\epsilon)}.$$

A.6 Proposition 1 and Corollary 1

Proof of Proposition 1. Assume that $z_2^*<\bar{z}_2$ holds. Inserting (15) and (16) we get

$$\begin{aligned}\left(\frac{w\bar{h}-(1+r)\,\pi_1\left(h_1^*\right)}{\left[Ah_2^*\right]^{\epsilon}}\right)^{1/(1-\epsilon)}&<\left(\frac{w\bar{h}-(1+r)\,\pi_1\left(\bar{h}\right)}{\left[A\bar{h}\right]^{\epsilon}}\right)^{1/(1-\epsilon)}\\ \left[w\bar{h}-(1+r)\,\pi_1\left(h_1^*\right)\right]\left[\frac{\bar{h}}{h_2^*}\right]^{\epsilon}&<w\bar{h}-(1+r)\,\pi_1\left(\bar{h}\right)\end{aligned}$$

$$\left[w\bar{h} - (1+r)\left\{z_1^{1-\epsilon}\left[Ah_1^*\right]^\epsilon - w\bar{h}\right\}\right]\left[\frac{\bar{h}}{h_2^*}\right]^\epsilon < w\bar{h} - (1+r)\left\{z_1^{1-\epsilon}\left[A\bar{h}\right]^\epsilon - w\bar{h}\right\}$$

$$(2+r)\,w\bar{h}\left[\frac{\bar{h}}{h_2^*}\right]^\epsilon - (1+r)\,z_1^{1-\epsilon}\left[Ah_1^*\right]^\epsilon\left[\frac{\bar{h}}{h_2^*}\right]^\epsilon < (2+r)\,w\bar{h} - (1+r)\,z_1^{1-\epsilon}\left[A\bar{h}\right]^\epsilon$$

$$(2+r)\,w\bar{h}\left\{\left[\frac{\bar{h}}{h_2^*}\right]^\epsilon - 1\right\} < (1+r)\,z_1^{1-\epsilon}\left[A\bar{h}\right]^\epsilon\left\{\left[\frac{h_1^*}{h_2^*}\right]^\epsilon - 1\right\}$$

$$\left[\frac{\bar{h}}{h_2^*}\right]^\epsilon - 1 < \frac{1+r}{2+r}\frac{z_1^{1-\epsilon}\left[A\bar{h}\right]^\epsilon}{w\bar{h}}\left\{\left[\frac{h_1^*}{h_2^*}\right]^\epsilon - 1\right\}$$

$$\left[\bar{h}\right]^\epsilon - \left[h_2^*\right]^\epsilon < \frac{1+r}{2+r}\frac{z_1^{1-\epsilon}\left[A\bar{h}\right]^\epsilon}{w\bar{h}}\left\{\left[h_1^*\right]^\epsilon - \left[h_2^*\right]^\epsilon\right\}$$

and finally

$$\left\{\left[\bar{h}\right]^\epsilon - \left[h_2^*\right]^\epsilon\right\} - \frac{1+r}{2+r}\frac{z_1^{1-\epsilon}\left[A\bar{h}\right]^\epsilon}{w\bar{h}}\left\{\left[h_1^*\right]^\epsilon - \left[h_2^*\right]^\epsilon\right\} < 0. \tag{A.6.1}$$

Consider statement (i) of the proposition. Surplus at the working time account in the first period means that $h_1^* > \bar{h} > h_2^*$, implying that $\left[h_1^*\right]^\epsilon > \left[\bar{h}\right]^\epsilon > \left[h_2^*\right]^\epsilon$ and $\left[h_1^*\right]^\epsilon - \left[h_2^*\right]^\epsilon > \left[\bar{h}\right]^\epsilon - \left[h_2^*\right]^\epsilon > 0$. Consequently, (A.6.1) implies that any $\frac{1+r}{2+r}\frac{z_1^{1-\epsilon}\left[A\bar{h}\right]^\epsilon}{w\bar{h}} \geq 1$ is sufficient for $z_2^* < \bar{z}_2$ to hold. Rearranging (A.6.1), $z_2^* < \bar{z}_2$ holds as long as

$$\frac{2+r}{1+r}\frac{\bar{h}^{1-\epsilon}}{z_1^{1-\epsilon}}\frac{\left[\bar{h}\right]^\epsilon - \left[h_2^*\right]^\epsilon}{\left[h_1^*\right]^\epsilon - \left[h_2^*\right]^\epsilon} < \frac{A^\epsilon}{w}. \tag{A.6.2}$$

Consider statement (ii) of the proposition. Deficit at the working time account in the first period means that $h_1^* < \bar{h} < h_2^*$, implying that $\left[h_1^*\right]^\epsilon < \left[\bar{h}\right]^\epsilon < \left[h_2^*\right]^\epsilon$ and $\left[h_1^*\right]^\epsilon - \left[h_2^*\right]^\epsilon < \left[\bar{h}\right]^\epsilon - \left[h_2^*\right]^\epsilon < 0$. Consequently, (A.6.1) implies that any $\frac{1+r}{2+r}\frac{z_1^{1-\epsilon}\left[A\bar{h}\right]^\epsilon}{w\bar{h}} \geq 1$ is sufficient for $z_2^* < \bar{z}_2$ *not* to hold. Rearranging (A.6.1) again, $z_2^* < \bar{z}_2$ will be violated as long as (A.6.2) holds. ■

Inequality (A.6.2) provides the necessary condition for Proposition 1 to hold. From the above proof follows that $\frac{1+r}{2+r}\frac{z_1^{1-\epsilon}\left[A\bar{h}\right]^\epsilon}{w\bar{h}} \geq 1$ is the sufficient condition. Rearranging the latter inequality it is easy to see that

$$(1+r)\,z_1^{1-\epsilon}\left[A\bar{h}\right]^\epsilon \geq (2+r)\,w\bar{h}$$

$$(1+r)\,z_1^{1-\epsilon}\left[A\bar{h}\right]^\epsilon \geq (1+r)\,w\bar{h} + w\bar{h}$$

$$z_1^{1-\epsilon}\left[A\bar{h}\right]^\epsilon \geq w\bar{h} + \frac{w\bar{h}}{1+r}.$$

This defines the relationship between the revenue of the first period and discounted wage costs of both periods for a firm without the account.

Corollary 1 follows immediately from (A.6.1). Consider statement (i) of the corollary. Since surplus at the working time account in the first period implies $\left[h_1^*\right]^\epsilon - \left[h_2^*\right]^\epsilon > \left[\bar{h}\right]^\epsilon - \left[h_2^*\right]^\epsilon > 0$, we must require that $\frac{1+r}{2+r}\frac{z_1^{1-\epsilon}\left[A\bar{h}\right]^\epsilon}{w\bar{h}}$ is sufficiently small for $z_2^* < \bar{z}_2$ *not* to hold.

For any given r, the lowest value is the break-even point $z_1^{1-\epsilon}\left[A\bar{h}\right]^{\epsilon} = w\bar{h}$. Thus for a firm without working time account at the break-even point it has to be that

$$\frac{\left[\bar{h}\right]^{\epsilon} - \left[h_2^*\right]^{\epsilon}}{\left[h_1^*\right]^{\epsilon} - \left[h_2^*\right]^{\epsilon}} < \frac{1+r}{2+r}. \tag{A.6.3}$$

Consider the left hand side of (A.6.3). First let us demonstrate that it is a increasing function of h_1^*, i.e. that $\frac{\partial}{\partial h_1^*}\left(\frac{\left[\bar{h}\right]^{\epsilon}-\left[h_2^*\right]^{\epsilon}}{\left[h_1^*\right]^{\epsilon}-\left[h_2^*\right]^{\epsilon}}\right) > 0$, where $\bar{h} < h_1^* < h^{\max}$. Invoking that $h_2^* = 2\bar{h} - h_1^*$, one can show (see Technical Appendix) that

$$\frac{\partial}{\partial h_1^*}\left(\frac{\left[\bar{h}\right]^{\epsilon} - \left[h_2^*\right]^{\epsilon}}{\left[h_1^*\right]^{\epsilon} - \left[h_2^*\right]^{\epsilon}}\right) > 0 \quad \Leftrightarrow \quad 2\left(\frac{\bar{h}}{2\bar{h} - h_1^*}\right)^{1-\epsilon} > \left(\frac{h_1^*}{2\bar{h} - h_1^*}\right)^{1-\epsilon} + 1.$$

Since $2\bar{h} > h_1^*$ implies $\left(\frac{2\bar{h}}{2\bar{h}-h_1^*}\right)^{1-\epsilon} + 1 > \left(\frac{h_1^*}{2\bar{h}-h_1^*}\right)^{1-\epsilon} + 1$, it is sufficient to show that $2\left(\frac{\bar{h}}{2\bar{h}-h_1^*}\right)^{1-\epsilon} > \left(\frac{2\bar{h}}{2\bar{h}-h_1^*}\right)^{1-\epsilon} + 1$. Rearranging the latter,

$$2\left(\frac{\bar{h}}{2\bar{h} - h_1^*}\right)^{1-\epsilon} - \left(\frac{2\bar{h}}{2\bar{h} - h_1^*}\right)^{1-\epsilon} > 1 \quad \Leftrightarrow \quad \left(\frac{\bar{h}}{2\bar{h} - h_1^*}\right)^{1-\epsilon}\left[2 - 2^{1-\epsilon}\right] > 1 \quad \Leftrightarrow$$

$$\frac{\bar{h}}{2\bar{h} - h_1^*}\frac{1}{\left[2 - 2^{1-\epsilon}\right]^{1-\epsilon}} > 1. \tag{A.6.4}$$

Since $h_1^* > \bar{h}$, we know that $\frac{\bar{h}}{2\bar{h}-h_1^*} > 1$. Furthermore, for any $\epsilon \in (0,1)$ we have $0 < 2-2^{1-\epsilon} < 1$, which means that $\frac{1}{\left[2-2^{1-\epsilon}\right]^{1-\epsilon}} > 1$. As a result (A.6.4) holds, and so the left hand side of (A.6.3) is an increasing function of h_1^*.

Next one can show that the l.h.s. converges to 1/2 once $h_1^* \to \bar{h}$. Applying L'Hôpital's rule,

$$\lim_{h_1^*\to\bar{h}}\frac{\left[\bar{h}\right]^{\epsilon} - \left[h_2^*\right]^{\epsilon}}{\left[h_1^*\right]^{\epsilon} - \left[h_2^*\right]^{\epsilon}} = \frac{\lim\limits_{h_1^*\to\bar{h}}\frac{\partial}{\partial h_1^*}\left(\left[\bar{h}\right]^{\epsilon} - \left[2\bar{h} - h_1^*\right]^{\epsilon}\right)}{\lim\limits_{h_1^*\to\bar{h}}\frac{\partial}{\partial h_1^*}\left(\left[h_1^*\right]^{\epsilon} - \left[2\bar{h} - h_1^*\right]^{\epsilon}\right)}$$

$$= \frac{\lim\limits_{h_1^*\to\bar{h}}\left(\epsilon\left[2\bar{h} - h_1^*\right]^{\epsilon-1}\right)}{\lim\limits_{h_1^*\to\bar{h}}\left(\epsilon\left[h_1^*\right]^{\epsilon-1} + \epsilon\left[2\bar{h} - h_1^*\right]^{\epsilon-1}\right)} = \frac{\epsilon\left[\bar{h}\right]^{\epsilon-1}}{\epsilon\left[\bar{h}\right]^{\epsilon-1} + \epsilon\left[\bar{h}\right]^{\epsilon-1}} = \frac{1}{2}. \tag{A.6.5}$$

Since $\frac{1+r}{2+r}$ is a constant arbitrary close to 1/2, both $\frac{\partial}{\partial h_1^*}\left(\frac{\left[\bar{h}\right]^{\epsilon}-\left[h_2^*\right]^{\epsilon}}{\left[h_1^*\right]^{\epsilon}-\left[h_2^*\right]^{\epsilon}}\right) > 0$ and (A.6.5) imply that there exists $\tilde{h}_1^*$ such that $\frac{\left[\bar{h}\right]^{\epsilon}-\left[h_2^*\right]^{\epsilon}}{\left[h_1^*\right]^{\epsilon}-\left[h_2^*\right]^{\epsilon}} < \frac{1+r}{2+r}$ is violated. Increasing h_1^* from $\tilde{h}_1^*$ towards its upper limit of $\frac{\left[\bar{h}\right]^{\epsilon}-\left[h_{\min}\right]^{\epsilon}}{\left[h^{\max}\right]^{\epsilon}-\left[h_{\min}\right]^{\epsilon}}$ allows increasing the productivity-wage ratio and moving away from the break-even point. Statement (ii) of Corollary 1 is verified in the same way.

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