Incomplete Self-Enforcing Labor Contracts

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Abstract

We consider a model economy populated by risk-neutral firms with multiple jobs and risk-averse workers. Following the implicit contract literature, we assume that workers have limited access to the intertemporal trade markets. Following the directed search literature, we assume that unemployed workers can choose which firms to visit after having observed what terms of trade are offered. Further, we assume that the participation to an employment relationship must be self-enforcing and that contractual prescriptions cannot be contingent upon the job title. Under these two restrictions on the contract space, firms face a trade-off between the efficient provision of insurance to its senior workers and efficient recruitment of junior hires. We characterize the ex-ante contractual solution to this trade-off. We find that the optimal contract prescribes that—in response to small and negative shocks to firm's productivity—the wage paid to senior employees is set equal to the wage offered to junior employees. This firm-wide wage is greater than the ex-post efficient hiring wage and lower than the full-insurance wage. In general equilibrium, we find that these distortions lead to a larger increase in the unemployment rate in response to a negative shock to aggregate productivity.

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1. INTRODUCTION

When the elasticity of labor supply is calibrated to match the estimates obtained from micro-data, the neoclassical growth model cannot account for the cyclical behavior of wages, employment and labor productivity. On the supply side, the model grossly overestimates the magnitude of wage fluctuations given the observed volatility of employment. On the demand side, the model overestimates the magnitude of employment fluctuations given the observed volatility in labor productivity (Kydland, 1995; Hall, 1997; Galí et alii, 2005).

The implicit contracts literature developed in response to this failure of the neoclassical growth model. Azariadis (1975) and Baily (1974) noticed that—if workers cannot insure themselves against labor productivity shocks in the formal markets—then risk-neutral firms find profitable to step-in and offer insurance to their employees. Wage and employment are the outcome of the combined trade of labor and insurance services. In the equilibrium labor-insurance trade, the wage efficiently insures workers' consumption. Therefore, if the marginal utility of consumption does not depend on leisure, the wage must be constant across states of the world. In the equilibrium labor-insurance trade, employment must maximize the value of production, i.e. it must be ex-post efficient in every state of the world. Therefore, abstracting from income effects, the allocation of labor is the same as in the neoclassical growth model. Overall, the implicit contract theory can account for the volatility of wages vis-à-vis the cyclical behavior of employment, but it does not help explain the volatility of employment in face of the cyclical behavior of aggregate productivity.

Building on the original insight of Azariadis and Baily, this paper develops a theory where labor-insurance contracts not only dampen wage fluctuations but also magnify the response of employment to aggregate productivity shocks.

Following the earlier literature on implicit contracts, we assume that the economy is populated by risk-neutral firms and risk-averse workers who have no access to the interdate/state consumption markets. Departing from the earlier literature, we assume that both the firm and the worker can renege on their contractual agreement in order to pursue more profitable trading opportunities. Moreover, we modify the original implicit contract environment by assuming that the labor market is characterized by search frictions—i.e. when a worker applies for a job she does not know whether she will be qualified for it or not. The first assumption reflects the view that there are significant costs and risks associated with seeking the judicial enforcement of a labor contract or the renegotiation of an existing agreement. The second assumption reflects the well established view that finding a job and filling a vacant position are time-consuming activities.

In our environment, firms face a trade-off between efficiently providing insurance to senior employees and efficiently recruiting workers to man vacant positions. On the one hand, a necessary condition for efficient insurance provision is to pay senior employees a constant wage across dates and states of the world. On the other hand, in order to attract the efficient number of applicants, the hiring wage has to depend upon the aggregate state of the economy and the realization of firm-specific shocks. Therefore, there might be circumstances where the terms-of-trade that efficiently insure senior employees are more generous than the terms-of-trade that attract the efficient number of applicants. In such circumstances, if one of the applicants turns out to be qualified for a filled position, the firm has the incentive to hire her and lay-off the senior worker. Because of this tension between employment insurance and recruitment, the optimal contract must specify not only what wage the employee is paid in each date and state, but also what terms-of-trade the firm offers to workers hired in the future.

Under the assumption that firms are active for two periods only, the optimal contract partitions the productivity space into four regions. If in the second period the productivity of the firm is sufficiently high, then the optimal contract implements the ex-post efficient allocation. Specifically, senior workers are paid the same wage as in the first period and junior hires are offered the wage that attracts the efficient number of applicants. Because the efficient hiring wage is greater than the efficient insurance wage, the firm has no incentive to replace senior workers with junior hires. If the productivity of the firm is such that the efficient hiring wage is smaller but sufficiently close to the insurance wage, then the optimal contract prescribes a common wage for senior and junior employees. The firm-wide wage is strictly greater than the efficient hiring wage and strictly smaller than the efficient insurance wage, but it guarantees that senior workers do not lose their job to junior hires. If the productivity of the firm is such that the efficient hiring wage is sufficiently smaller than the insurance wage, the contractual remedy to the replacement problem takes on a different form. In this region, the hiring wage is distorted downward to reduce the number of applicants attracted by the firm and, in turn, the probability that some of them is qualified to replace a senior employee. Conditional on remaining employed, a senior worker receives the efficient insurance wage. Finally, there is a productivity threshold below which the firm withdraws from the job market. The location of the four regions depends on the history of aggregate economic conditions.

Under limited commitment, the labor-insurance contract not only affects the distribution of output between firms and workers, it also affects the allocation of labor by distorting the hiring wage. In the last part of the paper, we characterize the effect of the hiring wage distortions on the cyclical dynamics of the unemployment rate. When the economy is hit by a small negative shock to aggregate productivity, established firms offer a firm-wide wage in order to protect senior employees from the risk of being replaced. On the one hand, because the firm-wide wage is greater than the efficient hiring wage, an excessive number of applicants are attracted to established firms. On the other hand, the upward distortion in the hiring wage offered by established firms crowds out the creation of new firms. Overall, if the probability of filling a vacancy is concave in the number of applicants, the net effect of the distortion of the hiring wage is to magnify the response of the unemployment rate to the negative productivity shock. When the economy is hit by a positive shock to aggregate productivity, established firms enter the region where there is no tension between the goals of insurance provision and efficient recruitment. Therefore, the number of workers attracted by established firms, the number of new firms entering the market and, ultimately, the response of the unemployment rate are unaffected by the labor-insurance contract.

1.1. Related Literature

Our paper's goal relates to an earlier strand of literature that modifies the basic implicit contract framework to obtain allocative effects on employment. In this literature, it is assumed that the firm privately observes the realization of the state of the world. Given asymmetric information between the trading partners, the contract space has to be restricted in order to guarantee that the firm reports the true realization of the state of the world. Chari (1983) and Kahn and Green (1983) prove that the optimal contract distorts the allocation of labor away from the ex-post efficient level in order to satisfy the truth-telling constraint. Unfortunately, as long as leisure is a normal good, the allocative distortions dampen employment fluctuations. Grossman and Hart (1981, 1983) show that asymmetric information can lead to amplification of employment fluctuations under the additional assumption that firms are risk-averse.

Our paper's findings might be relevant for a recent debate in the macro-search literature. Shimer (2005) noticed that the basic random search model à la Mortensen and Pissarides (1999) cannot account for the relative volatility of unemployment and labor productivity over the business cycle. This quantitative observation has brought about a wealth of theoretical studies that modify the basic search framework and create mechanisms of amplification of unemployment responses to productivity shocks. Hall (2005) and Gertler and Trigari (2006) show that –if hiring wages are sufficiently sticky— small fluctuations of aggregate productivity translate into large movements of the profit rate, of firm creation and, ultimately, unemployment. While the amplification mechanism highlighted in these papers is similar to ours, we provide an explicit theory of sticky hiring wages. Also Kennan (2004) and Menzio (2005) obtain endogenously rigid hiring wages, but they stress a rather different mechanism. They consider a version of the basic search framework where firms have private information about their labor productivity. Under some conditions on the distribution of firm-specific shocks, they show that informational rents accruing to the firms are procyclical and hiring wages are nearly acyclical. Hall (2005) and Nagypál (2005) uncover mechanisms of amplification that are based on the endogenous cyclical dynamics in the quality of the pool of job applicants.

1.2. Structure of the Paper

Section 2 lays out the environment of the economy. Section 3 studies the optimal labor–insurance contract under full-commitment. Section 4 characterizes the optimal contract under limited commitment. Section 5 compares the cyclical properties of the general equilibrium consequences of labor-insurance contracts under limited commtment. In Section 6 we discuss some of the key assumptions of the model and briefly conclude.

2. THE MODEL

2.1. Physical Environment

The economy is populated by a continuum of workers with measure 1. Each worker's preferences over streams of consumption $\tilde{c} = \{c_t\}_{t=0}^{\infty}$ can be represented by a von Neumann-Morgerstern utility function

$$U(\tilde{c}) = \sum_{t=0}^{\infty} \beta^t \cdot u(c_t); \tag{1}$$

the discount factor β belongs to the interval $(0,1)$ and $u : \mathbb{R}_+ \to \mathbb{R}$ is strictly increasing and strictly concave. In period t, there is a large number of idle firms that can enter the market and become active by investing $I > 0$ units of the consumption good. In addition, in period t, there is a number of established firms that have entered the market in the previous period and are still active. For the sake of simplicity, we assume that no firms established before date $t-1$ are still active in period t. Each active firm i is composed by a small exogenous measure of positions $n \in (0,1)$ and each manned position produces $p_{i,t} \geq 0$ units of the consumption good. The objective of a firm is to maximize the expected sum of profits with discount factor β.

Firms and workers are matched through a directed search process. At the beginning of the period, each newly established firm i posts a labor contract $\omega_{i,j}$ for every position $j \in [0,n]$. The labor contract $\omega_{i,j}$ specifies the terms of trade and employment offered to a worker who is hired at position j in every state/date. In particular, the contract $\omega_{i,j}$ includes the terms of trade and employment for a worker hired in period t. At the beginning of the period, a randomly selected fraction $\sigma \in (0,1)$ of the matches between workers and old firms is exogenously destroyed. In addition, some of the existing matches may be endogenously terminated and some filled positions may be offered to new applicants. After having observed the contracts posted by new firms and the realization of the match-specific shocks at old firms, each unemployed worker decides which firm to visit and which position to apply for. If a worker is hired by firm i for position j, the terms of trade and employment are determined by the contract $\omega_{i,j}$. If a worker is not hired, she receives $b > 0$ units of the consumption good as unemployment benefit and she continues searching in period $t+1$. Similarly, if a worker is displaced from her job during period t, she collects the unemployment benefit b and starts searching for work in period $t+1$.

The matching process is frictional. Denote with q the ratio of applicants to positions associated with a contract worth W to a newly hired worker. Following Acemoglu and Shimer (1999), we refer to q as the position's expected queue length, an endogenous measure of competition for positions worth W. If a worker applies to a position with expected queue length q, she is hired with probability $\lambda(q)$, where $\lambda : \mathbb{R}_+ \to (0,1)$ is twice continuously differentiable, strictly increasing. Conversely, if a firm attracts an expected queue length q for a certain position, it hires a worker with probability $\eta(q)$, where $\eta : \mathbb{R}_+ \to (0,1)$ is twice continuously differentiable and strictly increasing. In addition, we assume the boundary conditions $\lambda(\infty) = \eta(0) = 0$ and $\lambda(0) = \eta(\infty) = 1$.

The market for contingent claims is incomplete. In particular, we assume that workers can neither trade units of the consumption good in future dates/states nor their future labor services. While a discussion of this extreme form of market incompleteness is deferred to Section 6, at this point it should be noted that this assumption is used by most papers in the labor contracts literature (see for example Azariadis, 1975; Harris and Holmstrom, 1982; Holmstrom, 1983; Beaudry and DiNardo, 1991; Boldrin and Horvath, 1995; Rudanko, 2006).

The economic system is subject to firm-specific shocks, aggregate shocks and sunspots. The aggregate state of the economy x follows a L-state Markov chain, where $\pi_{m,l}$ is the probability that x_t equals x_l given that $x_{t-1} = x_m$. The realization of the aggregate state of the economy determines the distribution of the firm-specific productivity $p_{i,t}$. In particular, if x_t is equal to x_l, the probability that $p_{i,t}$ takes the value p_k is $\pi_{l,k}$, for $k = 1, ...K$. Finally, $\tilde{\theta}_t$ is a random variable without intrinsic economic content which is distributed as a uniform over the interval $[0, 1]$. The realization of firm-specific shocks, aggregate shocks and sunspots is publicly observable before idle firms make their entry decision.

2.2. Contractual Environment

In period t, the contract ω posted by a newly established firm specifies the contingent transfers from the firm to a worker hired in period t. In particular, it defines: the wage $w_1 \in \mathbb{R}_+$ paid in the first period of the employment relationship; the wage $w_{1,2}\left(\zeta_{t+1}\right) \in \mathbb{R}_+$ paid in the second period of the relationship, conditional on the realization $\zeta_{t+1} = \{x_{t+1}, p_{t+1}, \theta_{t+1}\}$ of the aggregate shock, the firm-specific shock and the sunspot; the severance benefit $s_{1,2}\left(\zeta_{t+1}\right) \in (-b, \infty)$ paid to the worker if she is not employed in the second period. Secondly, the contract ω specifies the terms of employment for a worker hired in period t. In particular, it defines: the probability δ_1 that the worker is sent to the unemployment pool at the end of the first production period; the probability $\delta_2\left(\zeta_{t+1}\right) \in [\sigma, 1]$ that the worker is laid-off in the second period; the probability $\rho\left(\zeta_{t+1}\right) \in [0, 1]$ that the worker's job is offered to a new applicant. Finally, the contract ω specifies the terms of trade between the firm and a worker hired in period $t + 1$. In particular, ω defines: the wage $w^u_{2,2}\left(\zeta_{t+1}\right) \in \mathbb{R}_+$ paid by the firm to a worker hired to man an unfilled position; the wage $w^e_{2,2}\left(\zeta_{t+1}\right) \in \mathbb{R}_+$ paid to a worker hired for a position held by a senior employee.

Consider a worker hired in period t under the contract ω. In the first period of employment, the worker's period utility is $u(w_1)$. At the end of the first production period, the match is destroyed with probability δ_1 and the worker starts searching for a new job in period $t + 1$. If the trading partners enter period $t+1$ together, the worker is employed in the production process and she receives the period utility $u(w_{1,2})$ with probability $(1 - \delta_2) \cdot (1 - \rho\eta\,(q^e))$, where q^e is the expected queue length attracted by the wage $w^e_{2,2}$. With complementary probability, the worker is not employed in the production process and she receives the period utility $u(w_{1,2} + b)$. Whether employed or not, the worker resumes searching in period $t + 2$. Therefore, the expected utility of a worker hired in period t under the contract ω is

$$E\left(U\left(\tilde{c}\right)|\omega, x_t\right) = u(w_1) + \beta\left(1-\delta_1\right) E\left[W_{1,2}\left(\omega_2\left(\zeta_{t+1}\right)|\zeta_{t+1}\right)|x_t\right] + \beta\delta_1 E\left[Z_{t+1}|x_t\right], \qquad (2)$$

where the contract's continuation value $W_{1,2}\left(.\right)$ is given by

$$W_{1,2}\left(\omega_2\left(\zeta_{t+1}\right)|x_{t+1}\right) = u(b+s_{1,2}) + \beta E(Z_{t+2}|x_{t+1}) + \left[\left(1-\delta_2\right)\left(1-\rho\eta\left(q^e\right)\right)\right]\cdot\left[u\left(w_{1,2}\right) - u(b+s_{1,2})\right] \qquad (3)$$

and Z_τ is the value of searching for a job in period τ[1].

Consider a firm that has entered the market in period t, posted the contract ω and hired n_1 workers. In the first period of activity, the firm's profits are $n_1 \cdot (p_1 - w_1)$. In the second period of activity, a fraction $(1-\delta_1)\cdot(1-\delta_2)\cdot(1-\rho\eta\left(q^e\right))$ of the workers hired in period t are employed and paid $w_{1,2}$ units of the consumption good. A fraction $\delta_2 + \rho\eta\left(q^e\right)$ of the workers hired in period t and not laid-off at the end of the first production process is not employed and receives the severance benefit $s_{1,2}$. A measure $\eta\left(q^e\right)\cdot\left[n_1\left(1-\delta_1\right)\cdot\left(1-\delta_2\right)\cdot\rho\right]$ of workers is hired from the pool of unemployment at the wage $w^e_{2,2}$. A measure $\eta\left(q^u\right)\cdot\left[n - n_1\left(1-\delta_1\right)\cdot\left(1-\delta_2\right)\right]$ of workers is hired from the pool of unemployment at the wage $w^u_{2,2}$. Each position filled in the second period of activity returns p_2 units of output. Summarizing, the expected profits from posting the contract ω are

$$P_1\left(\omega, n_1|x_t, p_t\right) = n_1\left(p_1 - w_1\right) + \beta E\left[P_2\left(\omega_2\left(\zeta_{t+1}\right), n_1|\zeta_{t+1}\right)|x_t\right], \qquad (4)$$

where the continuation profits $P_2(.)$ are given by

$$\begin{aligned} &P_2\left(\omega_2\left(\zeta_{t+1}\right), n_1|\zeta_{t+1}\right) = \\ &\quad n_1\left(1-\delta_1\right)\left(1-\delta_2\right)\left(1-\rho\eta\left(q^e\right)\right)\cdot\left(p_2 - w_{1,2}\right) - n_1\left(1-\delta_1\right)\left[\delta_2 + \left(1-\delta_2\right)\rho\eta\left(q^e\right)\right]\cdot s_{1,2} + \\ &\quad \left[n_1\left(1-\delta_1\right)\left(1-\delta_2\right)\rho\right]\eta\left(q^e\right)\cdot\left(p_2 - w^e_{2,2}\right) + \\ &\quad \left[n - n_1\left(1-\delta_1\right)\left(1-\delta_2\right)\right]\eta\left(q^u\right)\cdot\left(p_2 - w^u_{2,2}\right). \end{aligned} \qquad (5)$$

3. FIRST BEST CONTRACT

3.1. Definition of the First Best Contract

In this section, we assume that both firms and workers can perfectly commit to their contractual agreements and that all the contractual contingencies specified in Section 2 are available. Under

[1] The notation in (2) and (3) implicitely conjectures that the equilibrium value of search Z_τ only depend on the contemporaneous realization of the aggregate shock x_τ. In section 5, this conjecture is vindicated.

these two assumptions, the contract offered by a newly established firm is the solution to the profit maximization problem

$$\max_{\{\omega, n_1, W_1\}} P_1 (\omega, n_1 | x_t, p_t), \text{ s.t.}$$

$$W_1 = E (U (\tilde{c}) | \omega, x_t), \tag{6}$$

$$n_1 = n \cdot \eta (q (W_1 | x_t)).$$

The firm's profit maximization problem is constrained by the labor supply curve $n \cdot \eta (q (W|x))$, which returns the measure of positions filled by the firm as a function of the value of the contract advertised. In turn, the labor supply curve is an increasing transformation of the function $q (W|x)$, which returns the equilibrium expected queue length at a position offering W.

The expected queue length function $q (W|x)$ is an equilibrium outcome. In period t, the distribution of labor contracts offered in the market and the expected queue lengths at every position determine the value Z_t of searching for a job. Formally, if $\overline{W}_t$ denotes the set of values associated with some contract offered in the market and $q (W|x_t)$ denotes the average queue length at a job worth $W \in \overline{W}_t$, then the value of searching is given by

$$Z_t = \max_{W \in \overline{W}_t} \{\lambda (q (W|x_t)) \cdot W + (1 - \lambda (q (W|x_t))) \cdot (u (b) + \beta E (Z_{t+1}))\}. \tag{7}$$

Taking the distribution of contracts and expected queue lengths as given, workers choose which firm to visit and which position to apply for. A worker might choose to apply for a certain position only if the expected utility of searching it is equal to Z_t. Formally, for all W such that $q (W|x_t)$ is strictly positive[2]

$$Z_t = \lambda (q (W|x_t)) \cdot W + (1 - \lambda (q (W|x_t))) \cdot (u (b) + \beta E (Z_{t+1})). \tag{8}$$

From equation (8), it follows that jobs worth less than Z_t do not attract any applicants. Moreover, (8) implies that the queue length $q (W|x)$ is strictly increasing in W for all $W > Z_t$ and that $q (\infty|x) = \infty$.

3.2. Characterization of the First Best Contract

If contracts are complete, in every state ζ_{t+1} the worker's consumption and employment can be chosen independently. Therefore, an optimal complete contract must prescribe a system of wage and severance payments that optimally insures the worker—i.e. $s_{1,2}$ and $w_{1,2}$ are such that the worker's marginal utility of consumption across dates and states is kept constant—and employment, replacement and recruitment policies that satisfy productive efficiency—i.e. the wages $w^u_{2,2}$ and $w^e_{2,2}$ and the probabilities δ_2 and ρ maximize the total output accruing to the two original contracting parties. First, notice that the worker's marginal utility of consumption is strictly decreasing and

[2] Following Moen (1997) and Acemoglu and Shimer (1999), we assume that equation (??) determines the queue length also for jobs that offer a contract worth W, where W does not belong to the equilibrium set $\overline{W}_t$.

independent from her employment status. Therefore, a complete contract provides optimal insurance when $w_{1,2}\left(\zeta_{t+1}\right)$ and $b+s_{1,2}\left(\zeta_{t+1}\right)$ are kept constant across states of the world and are equal to the initial wage w_1. Next, notice that the firm and the worker can appropriate $p_{t+1}+\rho\eta\left(q^e\right)\left(b-w_{2,2}^e\right)$ units of output by staying together and $b+\eta\left(q^u\right)\left(p_{t+1}-w_{2,2}^u\right)$ by separating. Because the probability $\eta\left(q\right)$ of filling a vacancy is equal to zero for any hiring wage $w_{2,2}$ smaller than the unemployment benefit b, it follows that the optimal recruitment policy is to set δ_2 equal to σ whenever $p_{t+1}>b$ and that the optimal replacement policy is to set ρ equal to 0. Finally, notice that, when the match is destroyed, the optimal recruitment policy is a wage $w_{2,2}^u$ that maximizes the value of an unfilled vacancy. The logic of these observations is made precise in the following proposition.

Proposition 3.1: (First Best Contract) *Let ω^* be a solution to (6), then: (i) in all states ζ_{t+1} such that $\delta_1^*\cdot\delta_2^*\left(\zeta_{t+1}\right)<1$, both $w_{1,2}^*(\zeta_{t+1})$ and $b+s_{1,2}^*(\zeta_{t+1})$ are equal to w_1^*; (ii) $\delta_2^*(\zeta_{t+1})$ is equal to 1 (σ) if p_{t+1} is smaller (greater) than b; (iii) $\rho^*(\zeta_{t+1})$ is equal to 0 for all ζ_{t+1}; (iv) in all states ζ_{t+1}, the wage $w_{2,2}^{u*}\left(\zeta_{t+1}\right)$ maximizes $\eta\left(q^u\right)\cdot\left(p_{t+1}-w\right)$ subject to $q^u=q\left(u(w)+\beta E(Z_{t+2})|x_{t+1}\right)$.*

Proof. See the Technical Appendix. ||

The proposition qualifies and extends the findings of the earlier implicit contract literature to a frictional labor market with multi-worker firms. Like in static models of implicit contracts, the wage is rigid across states of the world ζ_{t+1} while the employment policy satisfies productive efficiency. Moreover, we show that the rigidity of the wage paid to senior workers has no effects on the terms of trade and employment offered to junior hires. Therefore, in the second period of activity, the firm's employment is the same as if workers were risk-neutral. On the contrary, in the first period of activity, the employment policy δ_1 might violate productive efficiency because the firm cannot insure the worker's consumption in the second period if she has left the location. As a consequence, there exists an interval of realizations of the firm-specific productivity such that the worker would be employed even though the match should be destroyed under risk-neutrality (Holmstrom, 1983). Nevertheless, this productive inefficiency disappears whenever the product of existing jobs is close to the average product of jobs offered in the market. When this condition is satisfied, we can conclude that implicit contracts provide a rational for wage rigidity but have no allocative effects even in the context of a labor market with search frictions and multi-worker firms.

4. INCOMPLETE SELF-ENFORCING CONTRACT

4.1. Definition of an Incomplete Self-Enforcing Contract

In this section, we assume that both a firm and a worker are allowed to renege on the labor contract that governs their relationship. Further, it is assumed that once a trading partner has reneged, the match is permanently destroyed. Under this limited-commitment assumption, the contract ω posted by a firm has to be self-enforcing, i.e. at every stage of the employment relationship, the prescriptions of the contract have to be such that both the firm and the worker prefer to follow them than to renege and leave the match. Secondly, this section introduces an element of contractual incompleteness. While we allow the contract ω to specify terms of trade and employment that depend on the state of the economy and the history of the current match, we rule out contingencies based on the history of the position—i.e. whether someone held the position in the past or not.

Limited commitment and incompleteness restrict the space of feasible contracts. During the last period of activity of the firm, the terms of trade between the firm and a worker have to satisfy the individual rationality constraint of both parties—i.e. $w_{1,2}$ and $w_{2,2}$ have to belongs to the interval $[b, p_{t+1}]$. Conditional on not employing the worker in the second production process, the severance transfers have to satisfy both parties' individual rationality constraint—i.e. $s_{1,2}$ has to be equal to zero. In the second period of activity, the firm's employment choice between a senior and a junior employee has to be individually rational—i.e. if the continuation wage promised to a senior employee $w_{1,2}$ is greater *(smaller)* than the hiring wage $w^e_{2,2}$ then ρ is equal to 1 *(0)*. At the end of the first period of activity, the decision of continuing or terminating the employment relationship has to be individually rational. Finally, the assumption that contracts are incomplete implies that the wages $w^u_{2,2}$ and $w^e_{2,2}$ take on the same value $w_{2,2}$ in every state of the world.

Let z_{t+1} denote the flow value of search in period $t+1$—i.e. $u(z_{t+1})$ is equal to the difference between Z_{t+1} and $E\left(Z_{t+2}|x_{t+1}\right)$. Then, the set of feasible contracts ω is defined by the following constraints

$$E\left[W_{1,2}\left(\omega_2\left(\zeta_{t+1}\right)|x_{t+1}\right)|x_t\right] \geq E\left[Z_{t+1}|x_t\right], \text{ if } \delta_1 < 1;$$

$$\rho\left(\zeta_{t+1}\right) = \begin{cases} 1 \text{ if } w_{1,2}\left(\zeta_{t+1}\right) > w_{2,2}\left(\zeta_{t+1}\right), \\ 0 \text{ if } w_{1,2}\left(\zeta_{t+1}\right) \leq w_{2,2}\left(\zeta_{t+1}\right); \end{cases} \tag{9}$$

$$w_{1,2}\left(\zeta_{t+1}\right) \in \left[b, p_{t+1}\right] \text{ if } \delta_2\left(\zeta_{t+1}\right) < 1;\ s_{1,2}\left(\zeta_{t+1}\right) = 0;$$

$$w_{2,2}\left(\zeta_{t+1}\right) \in \left[b, \max\{p_{t+1}, z_{t+1}\}\right].$$

The formalization (9) of the contract space allows us to identify the tension between insurance and hiring created by the limited-commitment and incompleteness assumptions. First, because severance payments are not self-enforcing, there exists a link between the employment status and the consumption of a senior employee. Secondly, because of limited-commitment, there exists a link between the employment status of a senior employee and the hiring wage offered to a worker who's applying to a filled position. Finally, because the hiring wage cannot be made contingent on the position being filled or vacant, there exists a link between the firm's recruitment policy and the employment status of a senior employee. Overall, there exists a tension between the provision of insurance to senior workers and the recruitment of junior hires to fill vacant jobs. The remainder of this section identifies under what conditions this tension between insurance and hiring is relevant and what is the contractual solution to it.

4.2. Necessary Conditions for Optimality

Under the limited-commitment and incompleteness assumptions, the contract offered by a newly established firm maximizes the expected profits $P_1(\omega, n_1|.)$ subject to the labor supply curve and the feasibility constraints (9). It is immediate to verify that the firm's profit function is not concave in ω and that the set of feasible contracts is not convex. Therefore, the optimal labor contract cannot be characterized through the standard Kuhn-Tucker conditions. In the following pages with implement a novel local/global solution technique.

Let ω^* be an optimal incomplete self-enforcing contract. Further, suppose that—for all realizations of the sunspot θ in some interval Θ in state $\{x_l, p_k\}$—the contract ω^* prescribes that senior employees should be paid the continuation wage $w_{1,2}^*$ and junior employees should be offered the wage $w_{2,2}^*$, where $w_{1,2}^*$ is smaller than $w_{2,2}^*$. Consider an alternative contract $\hat{\omega}$. In all states of the world ζ_{t+1} different from $\{x_l, p_k, \Theta\}$, the prescriptions of the contract $\hat{\omega}$ coincide with the prescriptions of ω^*. In the states $\{x_l, p_k, \Theta\}$, the contract $\hat{\omega}$ prescribes the alternative wages $\hat{w}_{1,2}$ and $\hat{w}_{2,2}$ and the replacement probability $\hat{\rho}$. Moreover, the initial wage $\hat{w}_1$ is such that the worker is indifferent between the contracts $\hat{\omega}$ and ω^*.

Suppose that the alternative contract $\hat{\omega}$ prescribes that the replacement probability $\hat{\rho}$ should be equal to zero. In this case, the contract $\hat{\omega}$ is feasible if the continuation wage $\hat{w}_{1,2}$ is smaller than the hiring wage $\hat{w}_{2,2}$, if $\hat{w}_{1,2}$ belongs to the interval $[b, p_k]$ and if $\hat{w}_{2,2}$ belongs to the interval $[b, \max\{z_l, p_k\}]$. If the firm was to offer the contract $\hat{\omega}$ rather than ω^*, there would be three effects on expected profits. First, in the states $\{x_l, p_k, \Theta\}$, the profits for each position manned by a senior employee would increase by $w_{1,2}^* - \hat{w}_{1,2}$. Secondly, in the states $\{x_l, p_k, \Theta\}$, the profits associated with each vacant position would increase by the difference between $\eta(\hat{q}) \cdot (p_k - \hat{w}_{2,2})$ and $\eta(q^*) \cdot (p_k - w_{2,2}^*)$. Finally, in the first period of activity, the profits produced by each employee would increase by $\hat{w}_1 - w_1^*$. If the probability measure μ of the states $\{x_l, p_k, \Theta\}$ is sufficiently small and for $\delta_1^* = 0$ and $\delta_2^* = \sigma$, the total differential in expected profits can be approximated by

$$\begin{aligned}\Delta(\hat{\omega}, \omega^*) = \quad & \beta\mu \cdot n_1^*(1-\sigma) \cdot \left[w_{1,2}^* - \hat{w}_{1,2}\right] + \\ & \beta\mu \cdot \left[n - n_1^*(1-\sigma)\right] \cdot \left[\eta(\hat{q}) \cdot (p_k - \hat{w}_{2,2}) - \eta(q^*) \cdot \left(p_k - w_{2,2}^*\right)\right] \\ & n_1^*(1-\sigma) \cdot \frac{1}{u'(w_1^*)} \left\{\beta\mu \cdot \left[u(\hat{w}_{1,2}) - u(w_{1,2}^*)\right]\right\}.\end{aligned} \tag{10}$$

The difference between the expected profits generated by the alternative contract $\hat{\omega}$ and the optimal contract ω^* has an interpretation in terms of ex-post efficiency. In fact, let $T^{nr}(w_1, w_{1,2}, w_{2,2})$ denote the weighted sum of the firm's period profits and worker's period utility given that the worker's continuation wage is $w_{1,2}$, the hiring wage is $w_{2,2}$ and the replacement probability is zero. Formally, let $T^{nr}(.)$ be defined as

$$T^{nr}(.) = n_1(1-\sigma) \cdot \left\{\tfrac{u(w_{1,2}) - u(b)}{u'(w_1)} + (p - w_{1,2})\right\} + \left[n - n_1(1-\sigma)\right] \cdot \eta(q)(p - w_{2,2}). \tag{11}$$

Then, it is immediate to verify that the alternative contract $\hat{\omega}$ is more profitable than ω^* if and only if $\hat{\omega}$ is more efficient than ω^* in the states $\{x_l, p_k, \Theta\}$—i.e. $\Delta(\hat{\omega}, \omega^*)$ is positive if and only if $T^{nr}(w_1^*, \hat{w}_{1,2}, \hat{w}_{2,2})$ is greater than $T^{nr}\left(w_1^*, w_{1,2}^*, w_{2,2}^*\right)$ when p is equal to p_k and x is equal to x_l.

Next, suppose that the alternative contract $\hat{\omega}$ prescribes that the replacement probability $\hat{\rho}$ should be equal to one. In this case, the contract $\hat{\omega}$ is feasible if the continuation wage $\hat{w}_{1,2}$ is strictly greater than the hiring wage $\hat{w}_{2,2}$, if $\hat{w}_{1,2}$ belongs to the interval $[b, p_k]$ and if $\hat{w}_{2,2}$ belongs to the interval $[b, \max\{z_l, p_k\}]$. If the firm was to offer the contract $\hat{\omega}$ rather than ω^*, there would be three effects on expected profits. First, in the states $\{x_l, p_k, \Theta\}$, the profits for each position manned by a senior employee would increase by the difference between $w_{1,2}^*$ and $\eta(\hat{q})\cdot\hat{w}_{2,2} + (1-\eta(\hat{q}))\cdot\hat{w}_{1,2}$. Secondly, in the states $\{x_l, p_k, \Theta\}$, the profits associated with each vacant position would increase by the difference between $\eta(\hat{q})\cdot(p_k - \hat{w}_{2,2})$ and $\eta(q^*)\cdot\left(p_k - w_{2,2}^*\right)$. Finally, in the first period of activity, the profits produced by each employee would increase by $\hat{w}_1 - w_1^*$. Overall, if the probability measure μ of the states $\{x_l, p_k, \Theta\}$ is sufficiently small, the total profit differential can be approximated by

$$\begin{aligned}\Delta(\hat{\omega}, \omega^*) = \quad & \beta\mu \cdot n_1^*(1-\sigma)\cdot\left[w_{1,2}^* - \eta(\hat{q})\,\hat{w}_{2,2} - (1-\eta(\hat{q}))\,\hat{w}_{1,2}\right] + \\ & \beta\mu\cdot[n - n_1^*(1-\sigma)]\cdot\left[\eta(\hat{q})\cdot(p_k - \hat{w}_{2,2}) - \eta(q^*)\cdot\left(p_k - w_{2,2}^*\right)\right] \\ & n_1^*(1-\sigma)\cdot\frac{1}{u'(w_1^*)}\left\{\beta\mu\cdot\left[\eta(\hat{q})\,u(b) + (1-\eta(\hat{q}))\,u(\hat{w}_{1,2}) - u(w_{1,2}^*)\right]\right\}.\end{aligned} \tag{12}$$

Denote with $T^r(w_1, w_{1,2}, w_{2,2})$ denote the weighted sum of the firm's period profits and worker's period utility given that the worker's continuation wage is $w_{1,2}$, the hiring wage is $w_{2,2}$ and the replacement probability is one. Formally, let $T^r(.)$ be defined as

$$T^r(.) = n_1(1-\sigma)(1-\eta(q))\cdot\left\{\tfrac{u(w_{1,2})-u(b)}{u'(w_1)} + (p - w_{1,2})\right\} + n\cdot\eta(q)(p - w_{2,2}). \tag{13}$$

Then, the alternative contract $\hat{\omega}$ is more profitable than ω^* if and only if $\hat{\omega}$ is more efficient than ω^* in the states $\{x_l, p_k, \Theta\}$—i.e. $\Delta(\hat{\omega}, \omega^*)$ is positive if and only if $T^r(w_1^*, \hat{w}_{1,2}, \hat{w}_{2,2})$ is greater than $T^{nr}\left(w_1^*, w_{1,2}^*, w_{2,2}^*\right)$.

A generalization of the type of argument developed in the previous paragraphs leads to the following set of necessary conditions.

Lemma 4.1: (Necessary Conditions for Optimality) *Let ω^* be an optimal incomplete self-enforcing contract. Suppose that ω^* is such that $\delta_1^* = 0$ and the constraint (9.a) is not binding. (i) If in state ζ_{t+1} the contract prescribes that $\delta_2^* = \sigma$ and $w_{1,2}^* \leq w_{2,2}^*$, then $T^{nr}(w_1^*, w_{1,2}^*, w_{2,2}^*|.)$ is equal to the maximum between*

$$\begin{aligned}&\max_{w_{1,2},w_{2,2}} T^{nr}(w_1^*, w_{1,2}, w_{2,2}|.),\, s.t.\, \{w_{1,2}, w_{2,2}\} \in [b, p_{t+1}] \times [b, \max\{z_{t+1}, p_{t+1}\}],\, w_{1,2} \leq w_{2,2}, \\ &\max_{w_{1,2},w_{2,2}} T^{r}(w_1^*, w_{1,2}, w_{2,2}|.),\, s.t.\, \{w_{1,2}, w_{2,2}\} \in [b, p_{t+1}] \times [b, \max\{z_{t+1}, p_{t+1}\}],\, w_{1,2} > w_{2,2}.\end{aligned} \tag{14}$$

(ii) If in state ζ_{t+1} the contract prescribes that $\delta_2^* = \sigma$ and $w_{1,2}^* \leq w_{2,2}^*$, then $T^r(w_1^*, , w_{1,2}^*, w_{2,2}^*|.)$ is equal to the maximum of (14). (iii) If in state ζ_{t+1} the productivity of labor p_{t+1} is greater (smaller) than b, then δ_2^* is equal to σ (1).

Proof. See the Technical Appendix. ||

When contracts are self-enforcing and incomplete, if the replacement probability ρ is equal to zero, then the continuation wage paid to senior employees is constrained to be smaller or equal than the hiring wage offered to junior hires. On the other hand, if the continuation wage is greater than the hiring wage, then the replacement probability ρ is constrained to be equal to one. Moreover, both the hiring and the continuation wages have to satisfy the individual rationality constraint for the firm and the worker. Lemma 4.1 states that—subject to these three constraints—the continuation and hiring wage prescribed by an optimal incomplete self-enforcing contract ω^* are ex-post efficient.

4.3. Optimal Contract without Replacement

This subsection characterizes the continuation wage $w_{1,2}^{nr*}(p_k|x_l)$ and the hiring wage $w_{2,2}^{nr*}(p_k|x_l)$ that maximize the weighted sum of worker's utility and firm's profits and that guarantee that senior employees are never replaced by junior employees. More formally, this subsection characterizes the solution to the maximization problem (14.a) as a function of the firm-specific productivity p_k.

As a preliminary to the analysis of $w_{1,2}^{nr*}(p_k|x_l)$ and $w_{2,2}^{nr*}(p_k|x_l)$, it is useful to consider two benchmarks. First, consider the continuation wage $w^I(p_k|x_l)$ that optimizes the provision of insurance to senior employees and that satisfies the individual rationality constraint for both trading partners—i.e. $w^I(p_k|x_l)$ is the maximizer of the difference between $u(w_1^*)^{-1} \cdot [u(w_{1,2}) - u(b)]$ and $(p_k - w_{1,2})$ over the interval $[b, p_k]$. If the firm-specific productivity of labor p_k is lower than the initial wage w_1^*, the optimal insurance wage $w^I(p_k|x_l)$ is equal to p_k. For all realizations of the firm-specific shock p_k greater than w_1^*, the optimal insurance wage is equal to the initial wage. Secondly, consider the hiring wage $w^H(p_k|x_l)$ that maximizes the value of a vacant position—i.e. $w^H(p_k|x_l)$ is the maximizer of $\eta(q) \cdot (p - w_{2,2})$ subject to q smaller or equal than $q(u(w_{2,2}) + \beta E(Z)|x_l)$. The optimal hiring wage is strictly greater than the flow value of search z_l and strictly smaller than the firm-specific productivity of labor. Moreover, under mild regularity conditions, the optimal hiring wage is a strictly increasing function of the firm's productivity. Denote with k_1 the level of firm-specific productivity such that the optimal insurance wage is equal to the optimal hiring wage—i.e. let k_1 be implicitly defined as

$$w^H(k_1|x_l) = w^I(k_1|x_l) = w_1 \text{ if } w_1 \geq z_l, \quad k_1 = z_l \text{ if } w_1 < z_l. \tag{15}$$

When the productivity of the firm is greater than k_1, the optimal insurance wage and the optimal hiring wage guarantee that the firm has no incentives to replace senior with junior employees. Therefore, for all p_k greater than k_1, the wages $w^I(p_k|x_l)$ and $w^H(p_k|x_l)$ are the solution to the maximization problem (14.a). On the contrary, when the productivity of the firm is smaller than k_1, the optimal insurance wage is strictly greater than the optimal hiring wage and would induce the firm to replace senior employees. In this region, the solution to (14.a) involves paying all the employees of the firm the same wage. For all p_k in the interval $[k_2, k_1]$, the firm-wide wage is such that there is under-provision of insurance to senior employees—in the sense that $w_{1,2}^{nr*}(p_k|x_l)$ is strictly smaller than $w^I(p_k|x_l)$—and over-recruitment of junior employees—in the sense that $w_{2,2}^{nr*}(p_k|x_l)$ is strictly greater than $w^H(p_k|x_l)$. More specifically, the firm-wide wage $w_2^{nr*}(p_k|x_l)$ is the unique solution to

$$\frac{u'(w_2^{nr*}(p_k|x_l)) - u'(w_1^*)}{u'(w_1^*)} = -\frac{n - n_1(1-\sigma)}{n_1(1-\sigma)} \frac{d\left[\eta\left(q(w_2^{nr*}(p_k|x_l)|x_l)\right) \cdot (p_k - w_2^{nr*}(p_k|x_l))\right]}{dw_{2,2}} \qquad (16)$$

For all p_k in the interval $[z_l, k_2]$, the firm-wide wage is equal to the marginal product of labor.

The characterization of the wages $w_{1,2}^{nr*}(p_k|x_l)$ and $w_{2,2}^{nr*}(p_k|x_l)$ is summarized by Lemma 4.2 and illustrated by Figure 1.

Lemma 4.2: (Optimal Contract without Replacement) *Assume that the value of an unfilled vacancy $\eta(q(u(w) + \beta E(Z')|x_l)) \cdot (p - w)$ is a strictly concave function of w, for all w in the interval $[z_l, p_k]$. Then there exist two cutoffs k_1 and a k_2 such that: (i) for $p_k > k_1$, $w_{1,2}^{nr*}(p_k|x_l)$ is equal to $w^I(p_k|x_l)$ and $w_{2,2}^{nr*}(p_k|x_l)$ is equal to $w^H(p_k|x_l)$; (ii) for $p_k \in [k_2, k_1]$ and $i = 1, 2$, the wage $w_{i,2}^{nr*}(p_k|x_l)$ is equal to $w_2^{nr*}(p_k|x_l)$, where $w_2^{nr*}(p_k|x_l)$ belongs to the interval $\left(w^H(p_k|x_l), w^I(p_k|x_l)\right)$; (iii) for $p_k \in [z_l, k_2]$ and $i = 1, 2$, the wage $w_{i,2}^{nr*}(p_k|x_l)$ is equal to p_k.*

Proof. See the Technical Appendix. ||

4.4. Optimal Contract with Replacement

Consider the continuation wage $w_{1,2}^{r*}(p_k|x_l)$ and the hiring wage $w_{2,2}^{r*}(p_k|x_l)$ that maximize the sum of worker's utility and firm's profits given that senior employees are always replaced with junior hires. More formally, consider the maximization problem (14.b) ignoring the constraint $w_{1,2} < w_{2,2}$.

Because it does not affect the job-destruction probability, the continuation wage $w_{1,2}^{r*}(p_k|x_l)$ that maximizes the welfare measure T^r is such that the marginal utility of consumption is kept constant whenever the individual rationality constraints are not binding—i.e. $w_{1,2}^{r*}(p_k|x_l)$ is equal to the optimal insurance wage $w^I(p_k|x_l)$. On the other hand, the greater is the wage $w_{2,2}^{r*}(p_k|x_l)$ offered to junior employees, the lower is the survival probability of any existing match. Therefore, the hiring wage $w_{2,2}^{r*}(p_k|x_l)$ that maximizes the welfare measure T^r is strictly smaller than the optimal hiring wage $w^H(p_k|x_l)$. More specifically, when the productivity of the firm is smaller than some cutoff k_3, the hiring wage $w_{2,2}^{r*}(p_k|x_l)$ is equal to the flow value of unemployment z_l. In this case, the firm does not attract any applicants and senior employees enjoy maximal employment security. When the productivity of the firm is greater than k_3, the hiring wage $w_{2,2}^{r*}(p_k|x_l)$ is the unique solution to

$$\frac{n}{n_1(1-\sigma)} \frac{d\left[\eta\left(q\left(w_{2,2}|x\right)\right)(p_k - w_{2,2})\right]}{dw_{2,2}} = \left[\frac{u(w_{1,2}^I(p|x)) - u(b)}{u'(w_1)} + p_k - w^I(p|x)\right] \frac{d\eta\left(q\left(w_{2,2}|x\right)\right)}{dw_{2,2}}. \qquad (17)$$

If the productivity of the firm and the aggregate state of the economy are such that $w_{2,2}^{r*}(p_k|x_l)$ is strictly smaller than $w_{1,2}^{r*}(p_k|x_l)$, then these two wages represent the solution to the maximization problem (14.b). On the contrary, when $w_{2,2}^{r*}(p_k|x_l)$ is greater than $w_{1,2}^{r*}(p_k|x_l)$, these two wages do not belong in the feasible set of (14.b). Nevertheless—because T^r is smaller than T^{nr} for all wage pairs $\{w_{1,2}, w_{2,2}\}$—in this case the optimal contract ω^* prescribes that senior employees are paid $w_{1,2}^{nr*}(p_k|x_l)$, junior employees are offered $w_{2,2}^{nr*}(p_k|x_l)$ and that the probability of replacement ρ^* is equal to zero. Therefore, for the purpose of characterizing the optimal contract ω^*, we can safely consider $w_{1,2}^{r*}(p_k|x_l)$ and $w_{2,2}^{r*}(p_k|x_l)$ instead of the solution to (14.b).

Lemma 4.3: (Optimal Contract with Replacement) *Assume that the function T^r is strictly concave in $w_{2,2}$, for all $w_{2,2}$ in the interval $[z_l, p_k]$. Then there exist a cutoff k_3 such that: (i) for $p_k \geq k_3$, $w_{1,2}^{r*}(p_k|x_l)$ is equal to $w^I(p_k|x_l)$ and $w_{2,2}^{nr*}(p_k|x_l)$ solves (17); (ii) for $p_k \in [z_l, k_3]$, the wage $w_{1,2}^{r*}(p_k|x_l)$ is equal to $w^I(p_k|x_l)$ and $w_{2,2}^{nr*}(p_k|x_l)$ is equal to z_l.*

Proof. See the Technical Appendix. ||

4.5. Characterization Results

If contracts were complete, it would be optimal to offer $w^H(p_k|x_l)$ to those junior workers hired to fill open positions, to offer $w^I(p_k|x_l)$ to senior employees and to minimize their risk of unemployment by offering an unattractive salary to junior employees hired as substitutes. In general, this ex-post efficient allocation is not feasible when contracts are incomplete. When the hiring wage is constrained to be independent from the nature of hiring, the optimal contract either minimizes the employment risk of senior employees by restricting $w_{2,2}$ to be greater than $w_{1,2}$ or introduces employment risk and allows for an unconstrained hiring wage. In the previous pages, we have derived the properties of the contract with and without employment risk. In this subsection, we finally identify which alternative is best as a function of firm's productivity.

When the realization of firm's productivity is sufficiently high, the wage required to maximize the value of the open positions is greater than the wage required to optimize the provision of insurance to senior employees. In this case, the ex-post efficient self-enforcing allocation can be implemented with incomplete contracts. Specifically, the incomplete self-enforcing contract can prescribe to offer $w^H(p_k|x_l)$ to junior employees and $w^I(p_k|x_l)$ to senior employees. Because $w^H(p_k|x_l)$ is greater than $w^I(p_k|x_l)$, none of the applicants tries to get a position currently held by a senior and the incomplete contract does not introduce any employment risk. As discussed in the previous sub-sections, the critical level of productivity at which the optimal hiring wage curve crosses the optimal insurance wage is k_1. These remarks lead to the following proposition.

Proposition 4.4: *Let ω^* be an optimal incomplete self-enforcing contract such that the constraint (9.a) is not binding. For all $\zeta_{t+1} = \{x_{t+1}, p_{t+1}, \theta_{t+1}\}$ such that p_{t+1} is greater than k_1, the contract ω^* prescribes that: (i) $\rho^*(\zeta_{t+1})$ is equal to 0, (ii) $w_{1,2}^*(\zeta_{t+1})$* is equal to $w^I(p_{t+1}|x_{t+1})$; *(iii) $w_{2,2}^*(\zeta_{t+1})$ is equal to $w^H(p_{t+1}|x_{t+1})$.*

Proof. See the Technical Appendix. ||

When the productivity of the firm falls short of k_1, the ex-post efficient allocation is not feasible under incomplete contracts. In fact, if the firm was to offer the optimal hiring wage $w^H(p_k|x_l)$ to junior employees and promise the optimal insurance wage $w^I(p_k|x_l)$ to senior employees, there would be some unemployed workers applying to positions currently held by seniors. And the firm would lay-off and replace as many tenured workers as possible because $w^I(p_k|x_l)$ is greater than $w^H(p_k|x_l)$. As discussed in Section 4.3, this moral hazard problem can be eliminated by distorting the hiring and the insurance wages away from the optimum and pay a tenure-independent firm-wide wage $w_2^{nr*}(p_k|x_l)$. On the one hand, the firm-wide wage is strictly greater than the optimal hiring wage $w^H(p_k|x_l)$ and therefore an inefficiently high number of applicants is attracted to the firm's vacancies. On the other hand, the firm-wide wage is strictly smaller than the optimal insurance wage $w^I(p_k|x_l)$ and therefore imposes some extra consumption risk on the workers. Alternatively, the firm can eliminate the consumption risk by offering $w^I(p_k|x_l)$ to senior employees and reduce the employment risk by distorting the hiring wage downwards. If the firm's productivity is below the cutoff k_1 but arbitrarily close to it, the cost of distorting the hiring and insurance wages by setting a firm-wide wage $w_2^{nr*}(p_k|x_l)$ becomes arbitrarily small. On the contrary, the cost of distorting the hiring wage downwards and imposing some employment risk on senior employees does not vanish as the firm's productivity approaches the cutoff k_1. If the wage $w_{2,2}^{r*}(p_k|x_l)$ converges towards z_l, the employment risk vanishes but nobody applies to the firm's open positions. If the wage $w_{2,2}^{r*}$ converges towards $w^H(p_k|x_l)$, the efficient number of applicants is attracted towards the firm's openings but senior employees are laid-off too often. These remarks lead to the following proposition.

Proposition 4.5: *Let ω^* be an optimal incomplete self-enforcing contract such that the constraint (9.a) is not binding. There exists an $\epsilon > 0$ such that for all $\zeta_{t+1} = \{x_{t+1}, p_{t+1}, \theta_{t+1}\}$ with p_{t+1} in the interval $(k_1 - \epsilon, k_1)$, the contract ω^* prescribes that: (i) $\rho^*\left(\zeta_{t+1}\right)$ is equal to 0, (ii) $w_{1,2}^*\left(\zeta_{t+1}\right)$ is equal to $w_2^{nr*}(p_{t+1}|x_{t+1})$; (iii) $w_{2,2}^*(\zeta_{t+1})$ is equal to $w_2^{nr*}(p_{t+1}|x_{t+1})$; (iv) $w_2^{nr*}(p_{t+1}|x_{t+1})$ is strictly greater than $w^H(p_{t+1}|x_{t+1})$ and strictly smaller than $w^I(p_{t+1}|x_{t+1})$.*

Proof. See the Technical Appendix. ||

Minimizing the risk of unemployment by setting a firm-wide wage is not always the optimal way to cope with the moral hazard created by contractual incompleteness. To illustrate this point consider a realization of firm's productivity in the interval between the cutoffs k_3 and k_2. On the one hand, if the firm offers a common wage for senior and junior employees, it is optimal to set $w_2^{nr*}(p_k|x_l)$ equal to the productivity of labor p_k. On the other hand, if the firm offers different wages for workers with different tenure, it is optimal to offer an attractive and profitable wage to junior employees, i.e. $w_{2,2}^{r*}(p_k|x_l) \in (z_l, p_k)$, and to provide perfect consumption insurance to senior employees, i.e. $w_{2,2}^{r*}(p_k|x_l) = w^I(p_k|x_l)$. Notice that the payoffs generated by setting a firm-wide wage can be replicated with an appropriate selection of tenure-specific wages, i.e. $w_{2,2}^{r*}(p_k|x_l) = z_l$ and $w_{1,2}^{r*}(p_k|x_l) = p_k$. By revealed preferences, the weighted sum of worker's and firm's payoffs must be greater under the second alternative.

Proposition 4.6: *Let ω^* be an optimal incomplete self-enforcing contract such that the constraint (9.a) is not binding. For all $\zeta_{t+1} = \{x_{t+1}, p_{t+1}, \theta_{t+1}\}$ such that p_{t+1} is greater than k_3 and smaller than k_2, the contract ω^* prescribes that: (i) $\rho^*\left(\zeta_{t+1}\right)$ is equal to 1, (ii) $w^*_{1,2}\left(\zeta_{t+1}\right)$ is equal to $w^I(p_{t+1}|x_{t+1})$; (iii) $w^*_{2,2}(\zeta_{t+1})$ is equal to $w^{r*}_{2,2}(p_{t+1}|x_{t+1})$, where $w^{r*}_{2,2}(p_{t+1}|x_{t+1})$ belongs to the interval (z_{t+1}, p_{t+1}).*

Proof. See the Technical Appendix. ||

Under what conditions on the fundamentals of the economy is the cutoff k_3 smaller than k_2? Suppose that the probability of filling an open position can be represented as the product between an efficiency parameter A and a concave function $h(w|x_l)$. The optimal hiring wage $w^H(p_k|x_l)$ does not depend on the efficiency of the matching process because A affects equally the marginal cost and the marginal benefit of increasing $w_{2,2}$. Similarly, the cutoffs k_1 and k_3 do not depend on the efficiency parameter A. On the other hand, the cutoff k_2 is decreasing in the efficiency of the matching process. Indeed, for A sufficiently small k_2 is equal to the first period wage w^*_1, while k_3 remains strictly smaller than w^*_1. When search frictions are sufficiently large, the optimal incomplete contract prescribes inefficient separations in some states of the world.

When the productivity of the firm falls between the flow value of search z and $\min\{k_2, k_3\}$, the optimal incomplete contract is indeterminate. On the one hand, if the firm offers a common wage to all its employees, it is optimal to set w^{nr*}_2 equal to the productivity of labor p. On the other hand, if the firm offers different wages for workers with different tenure, it is optimal to offer an unattractive wage to junior employees, i.e. $w^{r*}_{2,2}(p_k|x_l) \leq z_l$, and to provide perfect consumption insurance to senior employees, i.e. $w^{r*}_{2,2}(p_k|x_l) = w^I(p_k|x_l)$ which in turn is equal to the productivity of labor p_k. From the perspective of the firm and its senior employees, the two alternatives are identical.

Proposition 4.7: *Let ω^* be an optimal incomplete self-enforcing contract such that the constraint (9.a) is not binding. For all $\zeta_{t+1} = \{x_{t+1}, p_{t+1}, \theta_{t+1}\}$ such that p_{t+1} is greater than z_{t+1} and smaller than $\min\{k_2, k_3\}$, the contract ω^* prescribes one of the following: (i) $\rho^*\left(\zeta_{t+1}\right)$ is equal to 0 and $w^*_{1,2}\left(\zeta_{t+1}\right)$, $w^*_{2,2}(\zeta_{t+1})$ are both equal to p_{t+1}; (ii) $\rho^*\left(\zeta_{t+1}\right)$ is equal to 1, $w^*_{1,2}\left(\zeta_{t+1}\right)$ is equal to $w^I(p_{t+1}|x_{t+1})$ and $w^*_{2,2}(\zeta_{t+1})$ is equal to z_{t+1}.*

Proof. See the Technical Appendix. ||

When the productivity of the firm falls between the flow value of unemployment b and the flow value of search z, the moral hazard problems created by contractual incompleteness disappear. In fact, the ex-post self-enforcing allocation requires to employ all of the senior employees at the optimal insurance wage and to leave unfilled all of the open positions. The optimal incomplete contract can attain the ex-post efficient allocation by setting $w_{1,2}$ equal to the optimal insurance wage $w^I(p_k|x_l)$ and by offering an unattractive wage to new applicants. Figure 3 uses the results derived in this subsection and illustrates the optimal wages $w^*_{1,2}$ and $w^*_{2,2}$ as a function of firm's productivity.

5. GENERAL EQUILIBRIUM

Under the assumptions of limited commitment and contractual incompleteness, the optimal agreement between a firm and a worker imposes restrictions upon the firm's future hiring policy. In particular, we have shown that the firm's hiring wage might be distorted away from the ex-post efficient level in order to mitigate the firm's incentives to replace senior with junior employees. In this section, we characterize the general equilibrium of the model economy and highlight the effect of this hiring distortion on unemployment dynamics.

In order to obtain an intuitive characterization of the general equilibrium, it is convenient to rescale the agent's payoffs in order to make the firm's problem time-independent—as if the firm was infinitely lived. Therefore, we assume that at date t and in state $\{x_l, p_k, \theta\}$, the value of the contract ω for a worker hired at date $t-1$ and a worker hired at date t are respectively given by

$$W_{1,2}(\omega_2|.) = u(w_{1,2}) + \beta\{u(b) + \beta E[Z'|x_l] + (1-\sigma)(1-\rho\eta(q^e)) \cdot [W_{1,2}(\omega_2|.) - u(b) - \beta E[Z'|x_l]]\},$$

$$W_{2,2}(\omega_2|.) = u(w_{2,2}) + \beta\{u(b) + \beta E[Z'|x_l] + (1-\sigma) \cdot [W_{2,2}(\omega_2|.) - u(b) - \beta E[Z'|x_l]]\}. \tag{18}$$

Similarly, the firm's value of an open position and the value of employing a senior and a junior employee are respectively given by

$$J_{0,2}(\omega_2|.) = \beta\{J_{0,2}(\omega_2|.) + \eta(q_2)[J_{2,2}(\omega_2|.) - J_{0,2}(\omega_2|.)]\},$$

$$J_{1,2}(\omega_2|.) = (p_k - w_{1,2}) + \beta \left\{ \begin{array}{l} (1-\sigma)(1-\rho\eta(q^e)) \cdot J_{1,2}(\omega_2|.) + \\ \\ (1-\sigma)\rho\eta(q^e) \cdot J_{2,2}(\omega_2|.) + \sigma(1-\eta(q^u)) \cdot J_{0,2}(\omega_2|.) \end{array} \right\}, \tag{19}$$

$$J_{2,2}(\omega_2|.) = (p_k - w_{2,2}) + \beta\{J_{2,2}(\omega_2|.) + \sigma(1-\eta(q^u)) \cdot [J_{0,2}(\omega_2|.) - J_{2,2}(\omega_2|.)]\}.$$

In order to afford a closed-form solution for the general equilibrium, we specialize the stochastic process of aggregate and firm-specific productivity shocks. In particular, we assume that the economy is either in an expansionary or recessionary state, i.e. the aggregate-wide shock x_t is a two-state random variable $\{\underline{x},\overline{x}\}$. In the expansionary state, each active firm produces $\overline{p}$ units of output per worker. In the recessionary state, each active firm produces $\underline{p} > b$ units of output per worker, where $\underline{p}$.is strictly smaller than $\overline{p}$. The economy switches from the expansionary to the recessionary state (and viceversa) with probability $1-\pi$, where $\pi > 1/2$.

5.1. Recursive Competitive Search Equilibrium

In a symmetric equilibrium, the state of the economy can be described by the tuple $\hat{\zeta} = \{x, \theta, F_2, n_2, \omega_2, a\}$. The first element $x \in \{\underline{x},\overline{x}\}$ denotes the period's realization of the economy-wide shock. The second element $\theta \in [0,1]$ denotes the period's realization of the sunspot. The number of active firms that have been established in the past is $F_2 \geq 0$. The measure of workers matched to each of the F_2 established firms is $n_2 \in [0, n]$. The contractual agreement between workers and established firms is ω_2. Finally, $a \in [0,1]$ denotes the measure of workers that are unemployed at the beginning of the period.

In a symmetric equilibrium, each firm entering the market in the current period offers the contract ω_1, which solves the maximization problem (6) subject to the self-enforcement and incompleteness constraints (9). Denoting with W_1 the worker's expected value of the contract ω_1, the applicants' queue length q_1 attracted by each vacancy at newly established firms is given by $\eta\left(q\left(W_1|x\right)\right)$. In a symmetric equilibrium, each established firm offers the continuation contract ω_2. Denote with $w_{1,2}$ and $w_{2,2}$ the continuation and hiring wages prescribed by the contract ω_2 and with δ_2 and ρ the destruction and replacement probabilities. Then, each established firm employs $n_2 \cdot (1-\delta_2) \cdot (1-\rho) \cdot (1-\eta\left(q_2\right))$ senior workers at the wage $w_{1,2}$ and $\left[n - n_2\left(1-\delta_2\right)\left(1-\rho\right)\right] \cdot \eta\left(q_2\right)$ junior workers at the wage $w_{2,2}$, where q_2 is equal to $q\left(W_{2,2}\left(\omega_2|x\right)|x\right)$.

In equilibrium, the measure of applicants received by the firms has to be equal to the measure a of workers searching for jobs. Each of the F_2 established firms receives $\left[n - n_2\left(1-\delta_2\right)\left(1-\rho\right)\right] \cdot q_2$ applicants. Each of the F_1 newly established firms receives $n \cdot q_1$ applicants, where F_1 satisfies the free entry condition

$$\begin{gathered}\left(P_1\left(\omega_1, n \cdot q_1|x,p\right) - I\right) \cdot F_1 = 0, \\ P_1\left(\omega_1, n \cdot q_1|x,p\right) - I \leq 0.\end{gathered} \tag{20}$$

By equating the aggregate demand and the aggregate supply of applicants, we obtain the following market clearing condition

$$a = F_2 \cdot \left[n - n_2\left(1-\delta_2\right)\left(1-\rho\right)\right] \cdot q_2 + F_1 \cdot n \cdot q_1. \tag{21}$$

A symmetric *recursive competitive search equilibrium* is a tuple $\{F_1, n_1, \omega_1\}$, a function $Z(x)$ and a law of motion for the state of the economy $Q\left(\hat{\zeta}'|\hat{\zeta}\right)$ such that: (i) the contract ω_1 and the employment level n_1 solves the firm's problem (6) subject to (9); (ii) the free-entry condition (20) and the market clearing condition (21) are satisfied; (iii) the function $Z(x)$ satisfies the Bellman equation (7); (iv) the cumulative distribution function over next period's state of the economy $\hat{\zeta}'$ is consistent with the stochastic process for the exogenous random variables and with the optimizing behavior for the endogenous variables. Specifically, the future aggregate productivity shock x' is equal to x with probability π and different from x with probability $(1-\pi)$. The future sunspot θ' is distributed as a uniform over the interval $[0,1]$. The future number of established firms F_2' is given by F_1, the number of employees n_2' is given by $n \cdot q_1$ and the contract ω_2' is given by the continuation of ω_1. The future supply of applicants a' is given by

$$a' = a + F_2 \cdot (1-\delta_1) \cdot n_2 - F_1 \cdot n_1 \cdot \eta\left(q_1\right). \tag{22}$$

Before turning to the characterization of the general equilibrium, we have to vindicate the conjecture that the value of searching Z is determined uniquely by the realization of the economy-wide shock x. If Z' is a function of x' only, then the optimal contract posted by new firms and the average number of applicants q_1 depend on x and Z only. Therefore, the firm's profits are a function of x and Z only. Moreover, if F_1 is strictly positive, then the free-entry condition (20) implies that the firm's profits have to equal the cost of entry I. Therefore, (20) represents an implicit equation for the value of search Z as a function of x only. In conclusion, if the parameters of the model are such that there is entry in every state $\hat{\zeta}$, then the conjecture adopted in Sections 3 and 4 is vindicated.

5.2. Unemployment and Vacancy Dynamics

Consider the case where aggregate productivity shocks are rare and small, i.e. when $\pi \to 1$ and $\left|\overline{p} - \underline{p}\right| < \overline{\Delta}$. Suppose that the economy is in the recessionary state $\underline{x}$. A newly established firm advertises the labor contract that maximizes expected profits (6) subject to the contractual restrictions (9). Because the firm's problem is affected by the aggregate state of the economy $\hat{\zeta}$ only through the realization of the aggregate shock x and of the value of searching for a job $Z(x)$, it follows that every newly established firm offers the same labor contract $\underline{\omega}$. According to $\underline{\omega}$, all workers employed in the first production process are paid a relatively low wage $\underline{w}_1$. In the second production process, senior employees are still paid $\underline{w}_1$ independently from the realization of the aggregate state of the economy, i.e. $\underline{w}_{1,2}(\overline{x}) = \underline{w}_{1,2}(\underline{x}) = \underline{w}_1$. In the second production process, junior employees are offered a relatively high wage $\underline{w}_{2,2}(\overline{x}) = \overline{w}_1$ if the economy enters the expansionary state and the low wage $\underline{w}_{2,2}(\underline{x}) = \underline{w}_1$ if the economy remains in a recession. Indeed, the labor contract $\underline{\omega}$ achieves productive efficiency—i.e. senior workers are employed are never replaced by junor employees and the hiring wages $\underline{w}_{2,2}(\underline{x})$ and $\underline{w}_{2,2}(\overline{x})$ maximize the value of open positions.

When the economy is in the expansionary state $\overline{x}$, every newly established firm offers the labor contract $\overline{\omega}$. According to $\overline{\omega}$, all workers employed in the first production process are paid the relatively high wage $\overline{w}_1$. If the economy remains in the expansionary state, all workers employed in the second production process are paid $\overline{w}_1$, which is both the optimal insurance and the optimal hiring wage. If the economy enters a recession, senior and junior employees are paid a common wage $\overline{w}_2(\underline{x})$ that is strictly greater than the ex-post efficient hiring wage $\underline{w}_1$ and strictly smaller than the optimal insurance wage $\overline{w}_1$. In order to guarantee that senior employees are not replaced by junior hires when labor becomes relatively cheaper, the optimal contract $\overline{\omega}$ moves away from productive efficiency by distorting the hiring wage upward.

What are the general equilibrium consequences of such distortion? First, notice that the upward distortion on the hiring wage prescribed by $\overline{\omega}_2(\underline{x})$ does not affect the value of searching for a job during a recession, because $Z(\underline{x})$ only depends on the state of the economy ony through the realization of x. Therefore, equation (8) implies that the upward distortion on the hiring wage leads to longer queue lenghts at each of the $F_2 \cdot [n - n_2 \cdot (1 - \sigma)]$ positions open at the established firms. Secondly, notice that the value $\underline{W}_1$ of the contract posted by new firms depends on the aggregate state of the economy only through the value of search $Z(\underline{x})$. Therefore, equation (8) implies that the distortion on the hiring wage posted by established firms does not affect the measure of applicants attracted by each new firm. From the two previous remarks and the market clearing condition (21), it follows that the entry of new firms is crowded out. Overall, there are two countervailing effects on the rate of unemployment

$$u = 1 - F_2 \cdot \{n_2 (1-\sigma) + [n - n_2 (1-\sigma)] \cdot \eta (q_2)\} - F_1 \cdot n \cdot \eta (q_1) . \tag{23}$$

On the one hand, the distortion on the hiring wage decreases the rate of unemployment by increasing the employment level at established firms. On the other hand, it increases the unemployment rate by crowding out the entry of new firms. The the net effect on unemployment is given by $F_2 \cdot [n - n_2 (1-\sigma)] \cdot \eta (q_2)$ times $\eta'(q_2) - q_1^{-1} \cdot \eta(q_1)$, where the average queue length at established firms q_2 is strictly greater than the queue length at new firms q_1. From the previous expression it follows that, if the matching function $\eta(q)$ is strictly concave, the distortion on the hiring wage magnifies the response of the unemployment rate to a negative shock to productivity.

Proposition 5.1: (Unemployment and Vacancy Fluctuations) *Suppose that $\eta(q)$ and $J_{0,2}(w)$ are strictly concave functions. If $\overline{p} - \underline{p}$ and σ are positive and sufficiently small, then: when the economy transits from the expansionary to the recessionary state: (i) the increase in unemployment is larger than if labor contracts satisfied productive efficiency; (ii) the decrease in vacancy creation is larger than if labor contracts satisfied productive efficiency.*

Proof. See the Technical Appendix. ||

6. SOME REMARKS ON ASSUMPTIONS

Limited Commitment and Incompleteness

The assumption that a worker may leave her current employer without financial obligation is common in the labor contract literature as it reflects prohibitions against involuntary servitude (Harris and Holmstrom, 1982; Holmstrom, 1983; Beaudry and DiNardo, 1991). The assumption that a firm may leave its employee without financial obligation is often entertained on the grounds of the high costs of enforcing labor contracts (Thomas and Worrall, 1988). Finally, the assumption that the hiring contract offered by a firm cannot be made contingent upon the history of the position that is being filled is novel to our paper. We take this assumption as an exogenous feature of the environment, reflecting the difficulty for a benevolent court to verify the history of a specific position within a firm.

Limited-commitment on the side of the firm and contractual incompleteness are the two assumptions necessary to create a tension between insurance provision and efficient hiring. If the firm could commit to a long-term contract, then it would never accept applications for positions held by senior employee. If the contracts were complete, the firm could keep applicants away from the positions held by senior employee by offering an unattractive hiring wage. The assumption that workers cannot commit is natural but entirely orthogonal to our argument.

Borrowing and Saving

In our model, workers could neither borrow nor save. Relaxing one or both of these constraints would lead to a different design for the optimal contract. If workers were allowed to save their labor earnings, the tension between insurance provision and efficient recruitment would be partially resolved by front-loading the contract's value. Specifically, the firm would promise a relatively higher entry wage and a relatively lower continuation wage to workers hired in the first production period. Because of higher entry wages, senior workers would partially insure their consumption level through their own savings. Because of lower continuation wages, the firm would less frequently have the incentive to replace senior employees with junior hires. Nevertheless, in reality, front-loading a contract's value would also give senior workers a strong incentive to search on the job and lad to excessive turnover. In the spirit of the implicit contract literature, our model does not allow workers to borrow. The economic justification for such assumption is that borrowing is infeasible because the main source of a worker's collateral is her human capital and moral hazard problem would arise if a firm offered to guarantee its workers' loans.

Richer contract space

In the contractual environment described in Section 2, actions are contingent only on match-specific and economy-wide histories. Implicitly, our framework does not allow actions to be contingent on the realization of firm-wide histories such as the turnover rate or the average firm wage. If such contingencies were allowed, they would be used to resolve the tension between insurance provision and efficient hiring. For example, the wage promised to senior employees could include a large bonus if the firm's turnover rate happens to exceed its natural level. To avoid paying the bonus, the firm would refrain from hiring junior employees as replacements of senior workers. Nevertheless, this type of contractual clause is likely to be subject to renegotiation as both the firm and non-replaceable workers would gain from waiving it.

7. CONCLUDING REMARKS

This paper studies the relationship between insurance contracts and unemployment volatility in a labor market with search frictions. Following the insight of the early implicit contract literature, we assume that workers demand insurance from their employers because they are excluded form the market for intertemporal trade. Following the insight of the search literature, we assume that involuntary unemployment arises from matching frictions. When contracting parties can commit to the employment relationship and all contingencies are available, the optimal labor contract prescribes that the wage of a worker is kept constant across dates/states and that she is efficiently employed. In this contractual environment, the rigidity of the wage affects the distribution of rents between workers and firms across states of the world, but has no allocative role. When contracting parties can unilaterally break the employment relationship and firms cannot direct applicants towards vacant positions, there exists a tension between the optimal provision of insurance to senior employee and efficient recruitment of junior hires. Specifically, when the economic conditions are such that the optimal hiring wage falls below the optimal insurance wage, the firm has an incentive to replace senior with junior employees. In order to reduce the replacement risk, the optimal contract prescribes that in response to small negative productivity shocks, the firm pays senior and junior employees a common wage. The firm-wide wage is smaller than the insurance wage and higher than the optimal recruiting wage. In this contractual environment, hiring wages are downwardly sticky and the allocation of labor is distorted. In general equilibrium, the stickiness of the hiring wage offered by established firms crowds out the entry of new firms and amplifies the response of the unemployment rate to negative shocks to productivity.

A. TECHNICAL APPENDIX

Proof of Proposition 3.1: The profit maximization problem (6) can be decomposed into a two-stage problem. In the first stage, a newly established firm chooses the value of the contract W_1 and how many workers to hire $n_1 \in [0, n]$, subject to the constraint imposed by the labor supply curve. In the second stage, the firm selects the optimal contract ω among those that deliver the value W_1, taking as given the initial level of employment n_1. In order to characterize the necessary properties of an optimal contract, it is sufficient to solve analyze the second-stage problem

$$\max_{\omega} P_1 \left(\omega, n_1 | x_t, p_t\right), \text{ s.t. } E\left(U\left(\tilde{c}\right)|\omega, x_t\right) \geq W_1. \tag{24}$$

(i) Suppose that there is a set of states $\{x_l, p_k, \Theta_1\}$ with positive probability measure μ such that the contract ω^* prescribes $w^*_{1,2} \neq b + s^*_{1,2}$. Consider an alternative contract $\hat{\omega}$ such that $\hat{\omega}_2\left(\zeta_{t+1}\right)$ is equal to $\omega^*_2\left(\zeta_{t+1}\right)$ if $\zeta_{t+1} \notin \{x_l, p_k, \Theta\}$ and $\hat{\omega}_2\left(\zeta_{t+1}\right)$ is equal to $\left\{\hat{w}_{1,2}, \hat{s}_2, \delta^*_2, \rho^*, w^e_{2,2}, w^u_{2,2}\right\}$ if $\zeta_{t+1} \in \{x_l, p_k, \Theta_1\}$. Specifically, let $\hat{w}_{1,2}$ be the solution to the equation

$$\mu \cdot u\left(\hat{w}_{1,2}\right) = \int_{\Theta_1} \left\{\left[\left(1-\delta^*_2\right)\left(1-\eta\left(q^e\right)\rho^*\right)\right] \cdot \left[u\left(w^*_{1,2}\right) - u\left(b+s^*_{1,2}\right)\right] + u\left(b+s^*_{1,2}\right)\right\} d\theta \tag{25}$$

and $\hat{s}_{1,2}$ is equal to $\hat{w}_{1,2} - b$. Because $\hat{\omega}$ gives the same expected utility to the worker as ω^*, the contract is feasible. Moreover, because of the concavity of the utility function $u(c)$, $\hat{w}_{1,2}$ is strictly smaller than the average of $w_{1,2}$ and $b + s_{1,2}$. Therefore, under the contract $\hat{\omega}$ the profits of the firm are strictly greater than under ω^*. The contract ω^* is not optimal: a contradiction. In a similar way, we can prove that $w^*_{1,2}$ is equal to w^*_1.

(ii) Suppose that there is a set of states $\{x_l, p_k, \Theta_1\}$ with positive measure μ such that the firm-specific shock p_k is greater b and the contract ω^* prescribes $\delta^*_2 > \sigma$. Consider an alternative contract $\hat{\omega}$ that makes the same prescriptions as ω^* with the exception that $\hat{\delta}$ is equal to σ and $\hat{\rho}$ is equal to 0 whenever ζ_{t+1} belongs to $\{x_l, p_k, \Theta_1\}$. In light of part (i), $w^*_{1,2}$ is equal to $b + s^*_{1,2}$ and the worker is indifferent between being employed or not employed in the second production process. Because $\hat{\omega}$ modifies the terms of employment but prescribes the same transfers as ω^*, we can conclude that the expected utility of the worker is unchanged and $\hat{\omega}$ is a feasible contract. By switching from ω^* to $\hat{\omega}$, the firm's expected profits change by

$$P_1(\hat{\omega}) - P_1(\omega^*) = n_1(1-\delta_1)\begin{cases} \int_{\Theta_1}\{(p_k - b)\cdot[\delta_2^* + (1-\delta_2^*)\rho^*\eta(q^e) - \sigma]\}\,d\theta - \\ \\ \int_{\Theta_1}\left\{\left(p_k - w_{2,2}^{u*}\right)(\delta_2^* - \sigma)\,\eta(q^u)\right\}d\theta - \\ \\ \int_{\Theta_1}\left\{\left(p_k - w_{2,2}^{e*}\right)[(1-\delta_2^*)\rho^*\eta(q^e)]\right\}d\theta \end{cases} \tag{26}$$

Because the probability of filling a vacancy $\eta(q^u)$ is equal to 0 for all $w_{2,2}^{u*} \leq b$, then $p_k - b$ is strictly greater than $\eta(q^u)\cdot\left(w_{2,2}^{u*} - b\right)$. Similarly, $(p_k - b)\cdot\rho^*\eta(q^e)$ is greater or equal than $\left(p_k - w_{2,2}^{e*}\right)\cdot\rho^*\eta(q^e)$. These two remarks imply that $P_1(\hat{\omega})$ is strictly greater than $P_1(\omega^*)$: a contradiction. In a similar way, we can prove part (iii).
(iv) The result follows immediately from the firm's objective function. ||

Proof of Lemma 4.1:. (i)–(ii) Suppose that there is a set of states $\{x_l, p_k, \Theta_1\}$ where the optimal contract ω^* violates inequality (13.a). Denote with $\left\{w_{1,2}^{nr}, w_{2,2}^{nr}\right\}$ the couple of wages that maximizes $T^{nr}(w_1^*, w_{1,2}, w_{2,2}|.)$, subject to $w_{1,2} \leq w_{2,2}$, $w_{1,2} \in [b, p_k]$, $w_{1,2} \in [b, \max\{p_k, z_l\}]$. Consider a sequence $\{\Theta_n\}_{n=1}^{\infty}$ of subsets of Θ_1 such that the probability mass of the state $\{x_l, p_k, \Theta_n\}$ converges to zero and the Δ_n converges to a strictly positive value, where Δ_n is defined as

$$\begin{aligned} &\Delta_n = \mu(\Theta_n)^{-1}\beta\int_{\Theta_n} T^{nr}\left(w_1^*, w_{1,2}^{nr}, w_{2,2}^{nr}|.\right) - \\ &\mu(\Theta_n)^{-1}\beta\int_{\Theta_n}\left\{\mathbf{1}(w_{1,2}^*(\Theta_n) \leq w_{2,2}^*(\Theta_n))\cdot T^{nr}\left(w_1^*, w_{1,2}^*(\Theta_n), w_{2,2}^*(\Theta_n)|.\right)\right\}d\theta - \\ &\mu(\Theta_n)^{-1}\beta\int_{\Theta_n}\left\{\mathbf{1}(w_{1,2}^*(\Theta_n) > w_{2,2}^*(\Theta_n))\cdot T^{r}\left(w_1^*, w_{1,2}^*(\Theta_n), w_{2,2}^*(\Theta_n)|.\right)\right\}d\theta. \end{aligned} \tag{27}$$

For every n, consider the contract $\hat{\omega}_n$ that prescribes $\hat{\delta}_2 = \delta_2^* = \sigma$, $w_{1,2}^{nr} \leq w_{2,2}^{nr}$ and $\hat{\rho} = 0$ at date $t+1$ and in state $\{x_l, p_k, \Theta_n\}$, that replicates the contract ω^* at date $t+1$ and in any state different from $\{x_l, p_k, \Theta_n\}$ and that specifies a wage $\hat{w}_1$ such that the worker is indifferent between $\hat{\omega}$ and ω^*. By assumption, the contract ω^* is such that the constraint (9.a) is not binding. Therefore, for $\mu(\Theta_1)$ sufficiently small, the contract $\hat{\omega}$ satisfies the constraint (9.a) as well. Moreover, the contract $\hat{\omega}$ satisfies (9.b)–(9.d) by construction. The contract $\hat{\omega}$ is feasible. By switching from ω^* to $\hat{\omega}_n$, the firm's profits change by $\mu(\Theta_n)\cdot\Delta_n$ plus a higher order function of $(w_1^* - \hat{w}_n)$. Because $|w_1^* - \hat{w}_n|$ is bounded above by $\mu(\Theta_n)\cdot[u(p_k) - u(b)]$, there exists an N such that for all $n \geq N$, the firm strictly prefers $\hat{\omega}_n$ to ω^*. A contradiction. Part (ii) of the lemma is proved in the same fashion. Part (iii)
(iii) The proof replicates the argument developed in Proposition 3.1 and is therefore omitted. ||

Proof of Lemma 4.2:. Consider the maximization problem (14.a) for a triple $\{w_1, x, p\}$ such that p is greater than z. Because (14.a) specifies a continuous objective function and a non-empty continuous and compact-valued feasible set, the set of solutions to (14.a) is non-empty, upper hemi-continuous with respect to $\{w_1, x, p\}$ and compact-valued. Because the value of a vacant position is equal to zero for all $w_{2,2} \leq z$ and strictly greater than zero for $w_{2,2} \in (z,p)$, there are no solutions to (14.a) such that $w_{2,2}$ is smaller than z. Therefore, the strict concavity of the objective function over the rectangle $[b,p] \times [z,p]$ is sufficient to guarantee the uniqueness of the solution to (14.a). Finally, because the concave programme (14.a) satisfies Slater's condition, the Kuhn-Tucker conditions are necessary and sufficient for optimality.

Denote with μ the multiplier associated with the constraint $w_{1,2} - w_{2,2} \leq 0$. If $\mu^* = 0$, then $\left\{w_{1,2}^{nr*}(p|x), w_{2,2}^{nr*}(p|x)\right\}$ satisfies the Kuhn-Tucker conditions if and only if

$$\begin{aligned} w_{1,2}^{nr*}(p|x) &= w^I(p|x) \equiv \arg\max_{w\in[b,p]}\left\{\tfrac{u(w)-u(b)}{u'(w_1)} + (p-w)\right\}, \\ w_{2,2}^{nr*}(p|x) &= w^H(p|x) \equiv \arg\max_{w\in[z,p]} \eta\left(q\left(w|x\right)\right)\cdot(p-w). \end{aligned} \tag{28}$$

On the other hand, if $\mu^* > 0$, then $\left\{w_{1,2}^{nr*}(p|x), w_{2,2}^{nr*}(p|x)\right\}$ satisfies the Kuhn-Tucker conditions if and only if $w_{i,2}^{nr*}(p|x)$ is equal to $w_2^{nr*}(p|x)$, where

$$\begin{aligned} w_2^{nr*}(p|x) &= \{w_2 \text{ if } \Phi^{nr}(w_2) = 0,\ z \text{ if } \Phi^{nr}(z) < 0,\ p \text{ if } \Phi^{nr}(p) > 0\}, \\ \Phi^{nr}(w_2|x,p) &= \left(\tfrac{u'(w_2)}{u'(w_1)} - 1\right) + \tfrac{n-n_1(1-\sigma)}{n_1(1-\sigma)} \cdot \tfrac{d[\eta(q(w_{2,2}|x))(p-w_{2,2})]}{dw_{2,2}}. \end{aligned} \tag{29}$$

Finally, because the solution to (14.a) is unique, the multiplier μ^* is zero if and only if the optimal hiring wage $w^H(p|x)$ is greater or equal than the optimal insurance wage $w^I(p|x)$.

(i) The optimal hiring wage $w^H(p|x)$ belongs to the open interval (z,p) for all $p > z$. Moreover, because the value of an unfilled vacancy is a strictly concave function of w and the cross-derivative is strictly positive, the optimal hiring wage $w^H(p|x)$ is a continuous strictly increasing function of the firm's productivity p. Together, these two properties imply that the hiring wage $w^H(p|x)$ is strictly greater than the optimal insurance wage $w^I(p|x)$ if and only if the realization of firm's productivity p is greater than the cutoff k_1.

(ii) From the proof of part (i), it follows that (29) is the solution to the maximization problem (14.a) for all realizations of firm's productivity $p \in (z, k_1)$. Notice that the derivative of $\eta\left(q\left(w_2|x\right)\right)\cdot(p-w_2)$ with respect to w_2 is negative if and only if w_2 is greater than $w^H(p|x)$. Moreover, notice that $u'(w_2) - u'(w_1)$ is positive if and only if w_2 smaller than w_1. Because $w^H(p|x)$ is strictly smaller than w_1 for all $p \in (z, k_1)$, it follows that the firm-wide wage $w_2^{*nr}(p|x)$ belongs to the open interval $\left(w^H(p|x), w_1\right)$.

(iii) It can be directly verified that the wage $w_2^{*nr}(p|x)$ is equal to the marginal product of labor if and only if p is smaller or equal than the cutoff k_2, where k_2 is implicitly defined by the equation $\Phi^{nr}(k_2|x, k_2) = 0$. ||

Proof of Lemma 4.3:. Given a vector $\{w_1, x, p\}$ in $\mathbb{R}^3_+$ with $p > z(x)$, consider the constrained optimization problem

$$\max_{w_{1,2},w_{2,2}} T^r(w_1, w_{1,2}, w_{2,2}|x,p), s.t. (w_{1,2}, w_{2,2}) \in [b,p]\times[z,p] \;=$$

$$\max_{w_{2,2}\in[z,p]} \left\{ n\cdot\eta(q)\cdot(p-w_{2,2}) + n_1(1-\sigma)\cdot(1-\eta(q))\cdot \max_{w_{1,2}\in[b,p]}\left[\frac{u(w_{1,2})-u(b)}{u'(w_1)} + (p-w_{1,2})\right]\right\}. \tag{30}$$

First, consider the second-stage optimization problem in (30). For every tuple $\{w_1, x, p, w_{2,2}\}$, the objective function is differentiable and strictly concave with respect to $w_{1,2}$ and the feasible set is non-empty and convex. Therefore, the solution $w^{r*}_{1,2}(p|x)$ to the second-stage problem is unique. For every $w_{2,2}$ in the interval $[z,p]$, $1-\eta(q)$ is strictly positive. Therefore, the solution $w^{r*}_{2,2}(p|x)$ to the second-stage problem is independent from $w_{2,2}$. Next, consider the first-stage optimization problem. For every triple $\{w_1, x, p\}$, the objective function is differentiable and strictly concave with respect to $w_{2,2}$ and the feasible set is non-empty and convex. Therefore, the solution $w^{r*}_{2,2}(p|x)$ to the first-stage problem is unique. The necessary and sufficient conditions for the optimality of $\left\{w^{r*}_{1,2}(p|x), w^{r*}_{2,2}(p|x)\right\}$ are

$$w^{r*}_{1,2}(p|x) = w^I(p|x),$$

$$w^{r*}_{2,2}(p|x) = \{w_{2,2} \text{ if } \Phi^r(w_{2,2}) = 0,\ z \text{ if } \Phi^r(z) < 0,\ p \text{ if } \Phi^r(p) > 0\},$$

$$\Phi^r(w_{2,2}|x,p) = \frac{n}{n_1(1-\sigma)} \frac{d\left[\eta\left(q\left(u(w_{2,2})+\beta E(Z')|x\right)\right)(p-w_{2,2})\right]}{dw_{2,2}} - \left[\frac{u(w^I_{1,2}(p|x))-u(b)}{u'(w_1)} + p - w^I(p|x)\right]\frac{d\eta(q)}{dw_{2,2}}. \tag{31}$$

(i) Because $u(w^I_{1,2}(p|x)) - u(b)$ is strictly greater than $u'(w_1)\cdot\left(p - w^I(p|x)\right)$, $w^{r*}_{2,2}(p|x)$ is strictly smaller than the optimal hiring wage $w^H(p|x)$. Because the function T^r is strictly concave in $w_{2,2}$ and the cross-derivative $T^r_{w_{2,2},p}$ is strictly positive, the wage $w^{r*}_{2,2}(p|x)$ is a non-decreasing function of firm's productivity p.
(ii) It can be directly verified that $w^{r*}_{2,2}(p|x)$ is equal to z if and only if p is smaller or equal than k_3, where the cutoff k_3 is the unique solution to $\Phi^r(z|x,k_3) = 0$. For all p greater than k_3, the wage $w^{r*}_{2,2}(p|.)$ is strictly increasing. ||

Proof of Proposition 4.4: Take any triple $\{w_1, x_l, p_k\}$ such that p_k is greater than $k_1(w_1, x_l)$. First, notice that the function T^r is smaller than T^{nr} for every couple $\{w_{1,2}, w_{2,2}\}$ such that $w_{1,2} \in [b, p_k]$ and $w_{2,2} \in [b, p_k]$, i.e.

$$T^{nr}(.) - T^r(.) = n_1(1-\sigma)\eta(q)\left\{\frac{u(w_{1,2})-u(b)}{u'(w_1)} + w_{2,2} - w_{1,2}\right\} \geq 0. \tag{32}$$

Moreover, T^r is strictly smaller than T^{nr} for every couple $\{w_{1,2}, w_{2,2}\}$ such that $w_{1,2} \in [b, p_k]$ and $w_{2,2} \in [z_l, p_k]$. Secondly, notice that the solution of the maximization problem (14.a) is such that the no-replacement constraint is not binding, i.e. $w^{nr*}_{1,2}(p_k|x_l)$ is smaller than $w^{nr*}_{2,2}(p_k|x_l)$. Because of the concavity of T^{nr}, the wages $w^{nr*}_{1,2}(p_k|x_l)$ and $w^{nr*}_{2,2}(p_k|x_l)$ are also a maximizer of T^{nr} over the rectangle $[b, p_k] \times [z_l, p_k]$. Combining the first and the second remarks, we conclude that the maximum of (14.a) is strictly greater than the maximum of (14.b). $||$

Proof of Proposition 4.5: Take any triple $\{w_1, x_l, p_k\}$ such that p_k is greater than z_l and smaller than $k_1(w_1, x_l)$. For any realization of firm's productivity p in the interval (z_l, k_1), the solution of the maximization problem (14.a) is a firm-wide wage $w_2^{nr*}(p|x_l)$ that is strictly greater than $w^H(p|x_l)$ and strictly smaller than $w^I(p|x_l)$. For any realization of firm's productivity p in the interval (z_l, k_1), the solution of the maximization problem (14.b) is $\{w^I(p|x_l), w_{2,2}^{r*}(p|x_l)\}$, where $w_{2,2}^{r*}(p|x_l)$ is strictly smaller than the optimal hiring wage $w^H(p|x_l)$. For all p in the interval (z_l, k_1), the difference between the maximum of T^{nr} and the maximum of T^r is

$$\begin{aligned} & T^{nr}(w_1, w_2^{nr*}, w_2^{nr*}|.) - T^r(w_1, w^I, w_{2,2}^{r*}|.) = \\ & n_1(1-\sigma) \cdot \left\{ \tfrac{u(w_2^{nr*}) - u(w^I)}{u'(w_1)} + w^I - w_2^{nr*} \right\} + \\ & n_1(1-\sigma)\eta\left(q\left(w_{2,2}^{r*}\right)\right) \cdot \left\{ \tfrac{u(w^I) - u(b)}{u'(w_1)} + w_2^{nr*} - w^I \right\} + \\ & n\left[\eta\left(q\left(w_2^{nr*}\right)\right)\left(p - w_2^{nr*}\right) - \eta\left(q\left(w_{2,2}^{r*}\right)\right)\left(p - w_{2,2}^{r*}\right)\right]. \end{aligned} \tag{33}$$

When p converges to k_1, the wage $w_2^{nr*}(p|x_l)$ converges to $w^I(k_1|x_l)$ and $w^H(k_1|x_l)$. If $w_{2,2}^{r*}(p|x_l)$ converges to $w^H(k_1|x_l)$ for $p \to k_1$, then the limit of (33) is strictly positive because the first and third term vanish and the second term converges to a strictly positive value. If $w_{2,2}^{r*}(p|x_l)$ converges to z_{t+1} for $p \to k_1$, then the limit of (33) is strictly positive because the first and second terms vanish and the third term converges to a strictly positive value. If $w_{2,2}^{r*}(p|x_l)$ converges to any value in the interval $\left(z_{t+1}, w^H(k_1|.)\right)$, then the limit of (33) is strictly positive because both the limits of the second and the third term are strictly positive. By continuity, there exists a left neighborhood of k_1 where (33) is strictly positive. $||$

Proof of Proposition 4.6: Take any triple $\{w_1, x_l, p_k\}$ such that p_k is greater than k_3 and smaller than k_2. For any realization of firm's productivity p in the interval (z_l, k_2), the solution of the maximization problem (14.a) is the firm-wide wage $w_2^{nr*}(p|x_l) = p$. For any realization of p in the interval (k_3, k_1), the solution of the maximization problem (14.b) is the couple $\{w^I(p|x_l), w_{2,2}^{r*}(p|x_l)\}$, where the hiring wage $w_{2,2}^{r*}(p|x_l)$ is strictly smaller than $w^H(p|x_l)$ and strictly greater than z_l. Next, notice that the maximum of (14.b) is equal to

$$T^{nr}(w_1, w_2^{nr*}(p|x_l), w_2^{nr*}(p|x_l)|.) = n_1(1-\sigma) \cdot \left\{ \frac{u(p) - u(b)}{u'(w_1)} \right\}. \tag{34}$$

The right hand side in (34) is equal to $T^r(w_1, p, z_l)$. By strict concavity, the maximum of (14.b) is strictly greater than $T^r(w_1, p, z_l)$. $||$

Proof of Proposition 4.7: The result follows from the argument used in the proof of Proposition 4.6. $||$

Proof of Proposition 5.1: The proof is divided into six claims.

Claim 1: For $\pi = 1$, $x = \underline{x}$ and for all $\underline{\hat{Z}}$ such that $(1-\beta(1-\sigma)) \cdot \underline{\hat{Z}}$ is smaller than $u(\underline{p}) + \beta\sigma\left(u(b) + \underline{\hat{Z}}\right)$, the optimal incomplete self-enforcing contract $\underline{\omega}$ prescribes that: (i) the terms of employment are $\delta_1 = 0$, $\delta_2 = \sigma$; (ii) the wages $\underline{w}_1$, $\underline{w}_{1,2}(\underline{x})$ and $\underline{w}_{2,2}(\underline{x})$ are equal to $\arg\max J_{0,2}(w|\underline{x})$.

Proof. We prove the claim by showing that $\underline{\omega}$ coincides with the maximizer $\underline{\omega}^+$ of (6) subject to $s_2 = 0$. (i) Let $\underline{W}_1^M$ denote the maximum utility that a firm can promise to an employee subject to the zero profits condition, i.e. $\underline{W}_1^M$ is equal to $(1-\beta(1-\sigma))^{-1} \cdot \left[u(\underline{p}) + \beta\sigma\left(u(b) + \underline{\hat{Z}}\right)\right]$. Following the same argument of Proposition 3.1, we can prove that if $\underline{W}_1^M$ is greater than $u(b) + \beta\underline{\hat{Z}}$, then $\rho = 0$ and $\delta_2 = \sigma$. Moreover, if $\underline{W}_1^M$ is greater than $\underline{Z}$, then $\delta_1 = 0$. (ii) The result follows from the necessary optimality conditions for $\underline{w}_1$, $\underline{w}_{1,2}(\underline{x})$ and $\underline{w}_{2,2}(\underline{x})$ in (6).

Claim 2: For any sequence $\underline{\hat{Z}}_\pi$ converging to $\underline{\hat{Z}} < \underline{W}_1^M$, the sequence of optimal incomplete self-enforcing contracts $\underline{\omega}_\pi$ converges to $\underline{\omega}$ for $\pi \to 1$.

Proof. For any $\pi < 1$, the contract $\underline{\omega}_\pi$ is preferred by the firm to the feasible contract $\underline{\omega}$, i.e. $\underline{P}_\pi(\underline{\omega}_\pi, n_{1,\pi}(\underline{\omega}_\pi))$ is greater or equal than $\underline{P}_\pi(\underline{\omega}, n_{1,\pi}(\underline{\omega}))$. For any $\pi < 1$, the contract $\underline{\omega}_\pi^+$ is preferred by the firm to $\underline{\omega}_\pi$, i.e. $\underline{P}_\pi(\underline{\omega}_\pi^+, n_{1,\pi}(\underline{\omega}_\pi^+))$ is greater or equal than $\underline{P}_\pi(\underline{\omega}_\pi, n_{1,\pi}(\underline{\omega}_\pi))$. By the theorem of the maximum, the value $\underline{P}_\pi(\underline{\omega}_\pi^+, n_{1,\pi}(\underline{\omega}_\pi^+))$ is continuous in $\underline{Z}$ and π. By direct inspection, $\underline{P}_\pi(\underline{\omega}, n_{1,\pi}(\underline{\omega}))$ is continuous in $\underline{Z}$ and π. Therefore, the sequences $\underline{P}_\pi(\underline{\omega}_\pi^+, n_{1,\pi}(\underline{\omega}_\pi^+))$ and $\underline{P}_\pi(\underline{\omega}, n_{1,\pi}(\underline{\omega}))$ both converge to $\underline{P}(\underline{\omega}, n_1(\underline{\omega}))$. In turn, $\underline{P}_\pi(\underline{\omega}_\pi, n_{1,\pi}(\underline{\omega}_\pi))$ converges to $\underline{P}(\underline{\omega}, n_1(\underline{\omega}))$. Since the optimal contract $\underline{\omega}$ is unique, then $\underline{\omega}_\pi$ converges to $\underline{\omega}$.

Claim 3: The equilibrium value of search $\underline{Z}_\pi$ converges to $\underline{Z}$ for $\pi \to 1$.

Proof. For all $\pi \in (0,1]$, the equilibrium profits of the firm $\underline{P}_\pi(\underline{\omega}_\pi, n_{1,\pi}(\underline{\omega}_\pi))$ are strictly decreasing in $\underline{\hat{Z}}$ over the interval $(-\infty, \underline{W}_1^M]$. The equilibrium value of search $\underline{Z}_\pi$ is the unique solution to the free-entry condition (20). Because $\underline{P}_\pi(\underline{\omega}_\pi, n_{1,\pi}(\underline{\omega}_\pi))$ uniformly converges to $\underline{P}(\underline{\omega}, n_1(\underline{\omega}))$ for all $\underline{\hat{Z}}$, then $\underline{Z}_\pi$ converges to $\underline{Z}$.

Claim 4: The following inequalities hold: (i) $\overline{Z}$ is strictly greater than $\underline{Z}$; (ii) $\overline{w}_1$ is strictly greater than $\underline{w}_1$; (iii) $\overline{Z}-\underline{Z}$ and $\overline{w}_1-\underline{w}_1$ converge to zero when $\overline{p}-\underline{p} \to 0$.

Proof. (i) Because $\overline{p}$ is strictly greater than $\underline{p}$, it follows that $\overline{P}(\overline{\omega}, n_1(\overline{\omega}))$ is strictly greater than $\underline{P}(\underline{\omega}, n_1(\underline{\omega}))$ for all Z. From the free-entry condition (20), it follows that $\overline{Z}$ is strictly greater than $\underline{Z}$. (ii) The result follows immediately from the strict concavity of the functions $J_{0,2}(w)$ and $\eta(q)$.

Claim 5*:* For $\pi \to 1$ and $\overline{p} - \underline{p}$ sufficiently small, the optimal incomplete self-enforcing contract $\underline{\omega}$ prescribes that $\underline{w}_{1,2}(\overline{x})$ is equal to $\underline{w}_1$ and $\underline{w}_{2,2}(\overline{x})$ is equal to $\overline{w}_1$.

Proof. The contination wage $\underline{w}_{1,2}(\overline{x}) = \underline{w}_1$ maximizes the insurance value of the contract $\underline{\omega}$. The hiring wage $\underline{w}_{2,2}(\overline{x}) = \overline{w}_1$ maximizes the value of unfilled vacancies for $x = \overline{x}$. Since $\overline{w}_1$ is greater than $\underline{w}_1$, these wages do not lead to replacement of senior employee with junior hires, i.e. $\underline{\rho}(\overline{x}) = 0$. If $\overline{p} - \underline{p}$ is sufficiently small, then $\underline{w}_1$ is self-enforcing, i.e. $\underline{w}_1 \in [\overline{z}, \overline{p}]$.

Claim 6: For $\pi \to 1$ and $\overline{p} - \underline{p}$ sufficiently small, the optimal incomplete self-enforcing contract $\overline{\omega}$ prescribes that $\overline{w}_{1,2}(\underline{x}) = \overline{w}_{2,2}(\underline{x})$ and the common wage belongs to the open interval $(\underline{w}_1, \overline{w}_1)$.

Proof. The proof follows the same argument as in Proposition 4.5.

The remainder of the proof is provided in the main text. ||

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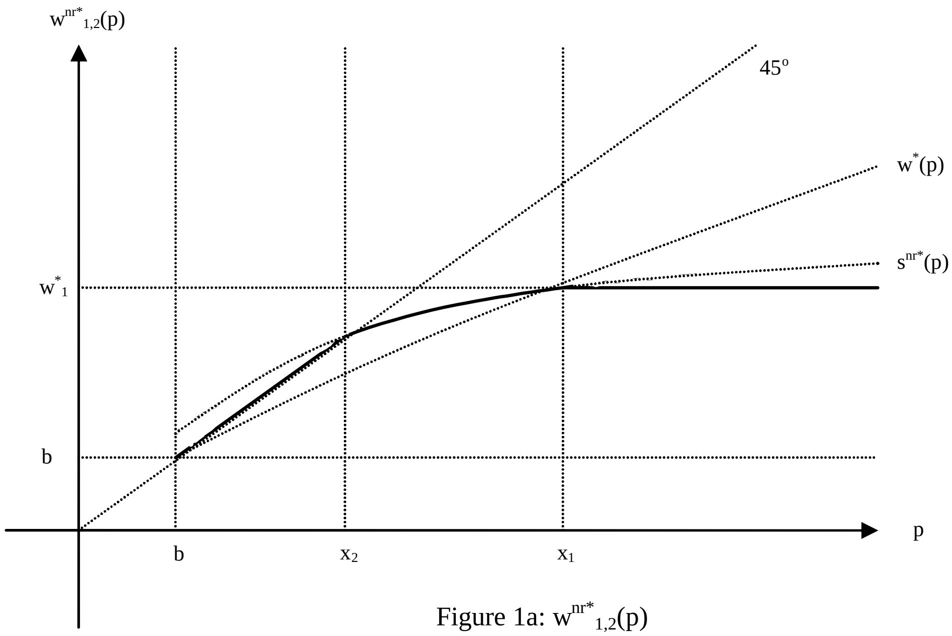

Figure 1a: $w^{nr*}_{1,2}(p)$

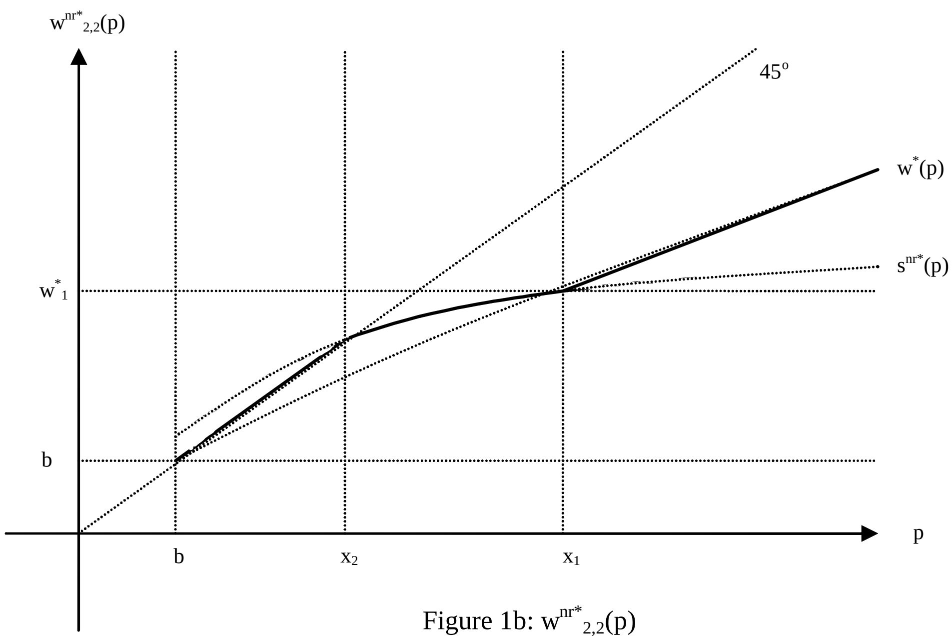

Figure 1b: $w^{nr*}_{2,2}(p)$

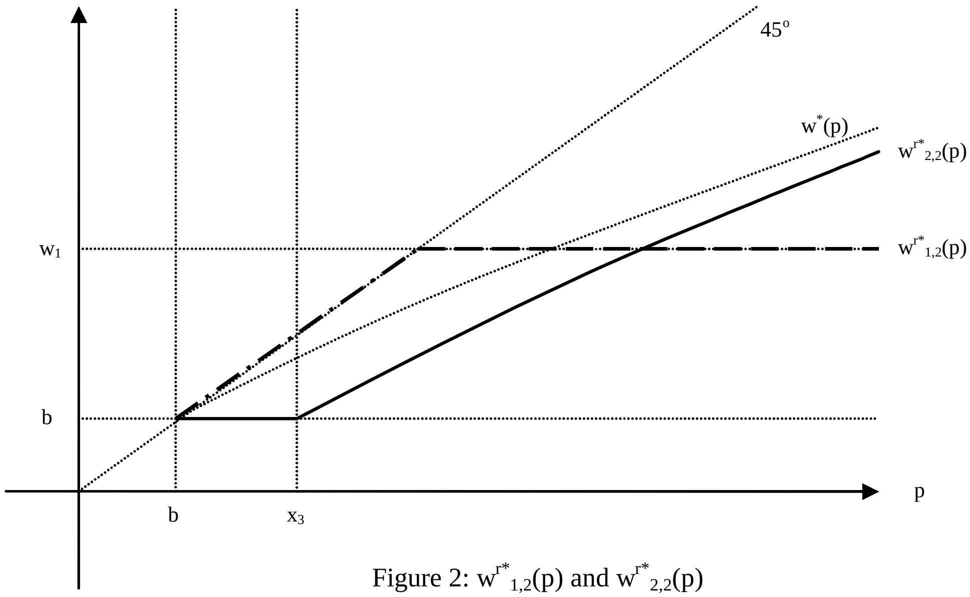

Figure 2: $w^{r*}_{1,2}(p)$ and $w^{r*}_{2,2}(p)$

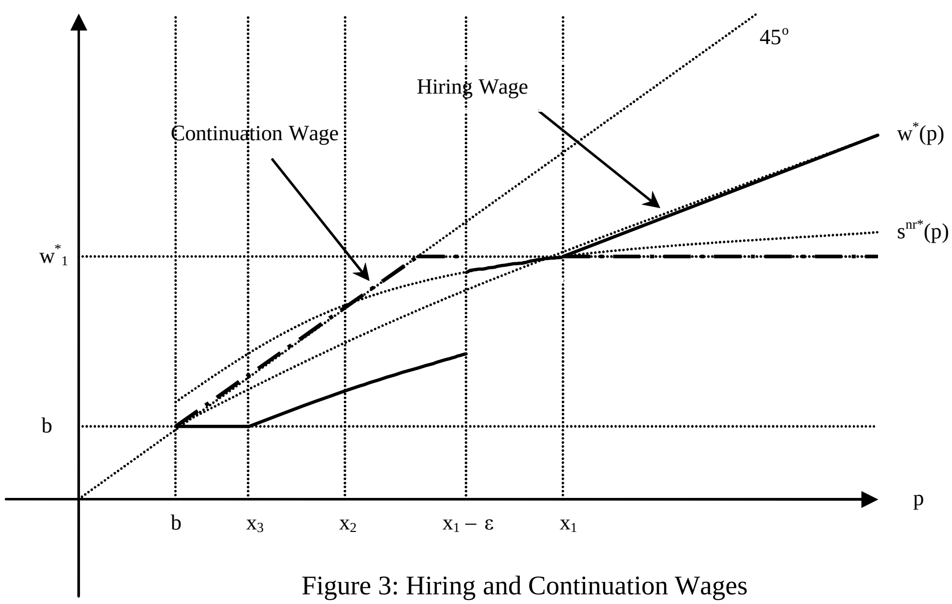

Figure 3: Hiring and Continuation Wages